

**FYTA12/FYTB03****Exercise sheet 2**

Friday, February 5th, 2016

**Warm-up****Exercise 1: Free fall**

A particle with mass  $m$  falls down towards the ground. Use horizontal coordinates  $x$  and  $y$ , and vertical coordinate  $z$ , counted positively upwards, to describe the system.

- (a) Write down the Lagrangian.
- (b) Identify any cyclic coordinates.
- (c) Identify the corresponding constants of motion.

**Exercises****Exercise 2: Angular momentum**

The earth moves in the sun's gravitational field. (You may consider the earth to be much lighter than the sun.)

- (a) Write down the Lagrangian using the distance  $r$  and an angle  $\phi$  as generalized coordinates.
- (b) Identify any cyclic coordinates.
- (c) Prove that angular momentum is conserved.

**Exercise 3: An elastic pendulum**

Write down Lagrange's equations for the motion of an elastic pendulum: A particle of mass  $m$  hangs in a spring with spring constant  $k$ , natural length  $\ell$ , and negligible mass. The spring can only be deformed longitudinally, but it can swing like a pendulum.

- (a) Write down the Lagrange equations.
- (b) Find the equilibrium point.
- (c) Approximate for small deviations, and find the Lagrange equations for horizontal and vertical oscillations.

**Exercise 4: An accelerating pendulum in a gravitational field**

The suspension point of a mathematical pendulum is lifted vertically with a constant acceleration  $a$ .

- (a) Determine the frequency for small oscillations of the pendulum. Contemplate the result.
- (b) The Lagrangian has an explicit time dependence. Is the energy for the system (the pendulum) conserved?

### Exercise 5: Three identical springs

Three identical springs are connected in a row and then attached between a pair of walls. Two identical masses are attached at the connection points between the central spring and the two others. Determine the frequencies for horizontal longitudinal vibrations.

(It can *not* be assumed that the distance between the walls is such that the springs have their natural length at equilibrium. If you use deviations from the equilibrium points as generalized variables, you must prove that what you think are the equilibrium points, are indeed the equilibrium points.)

### Exercise 6: Carbon dioxide

The carbon dioxide molecule can be considered a linear molecule with a central carbon atom, bound to two oxygen atoms with a pair of identical springs in opposing directions. Study the longitudinal motion of the molecule. If three coordinates are used, one of the normal mode frequencies vanishes. What does that represent physically? Calculate a numerical value for the ratio between the two other (non-zero) normal mode frequencies of the molecule. (The exercise should be solved using Lagrangian mechanics.)

*Answers:*

1 (a)

$$L = E_k - E_p = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz \quad (1)$$

(b)  $x$  and  $y$  are cyclic

(c)  $p_x = m\dot{x}$  and  $p_y = m\dot{y}$  are constants of motion

2 (a)

$$L = E_k - E_p = \frac{1}{2}m_e(r^2\dot{\phi}^2 + \dot{r}^2) + G\frac{M_s m_e}{r} \quad (2)$$

(b)  $\phi$  is cyclic

(c)

$$\frac{\partial L}{\partial \phi} = 0 \quad \Rightarrow \quad \text{const} = \frac{\partial L}{\partial \dot{\phi}} = m_e r^2 \dot{\phi}, \quad (3)$$

Here  $m_e r$  is multiplying the component of  $\vec{v}$  which is orthogonal to  $\vec{r}$ ,  $r\dot{\phi}$ , so this is the angular momentum.

3 Vertical oscillations:  $\omega_v^2 = k/m$ , horizontal:  $\omega_h^2 = k/(m + kl/g) = g/(l + gm/k)$

4 (a)  $(\frac{g+a}{l})^{1/2}$ , (b) No

5  $(k/m)^{1/2}$ , and  $(3k/m)^{1/2}$

6  $\sqrt{\frac{2m_o+m_c}{m_c}}$ , numerically 1.915 (or the inverse of this)