

FYTA12/FYTB03**Exercise sheet 3**

Friday, February 19th, 2016

Warm-up**Exercise 1: Lorentz transformation**

An event takes place at $t = 0$ and $x = 0$ in a system S . What coordinates does it have in a system S' which is moving in the x -direction with speed u compared to S ? Compare this to another event at $t = 0$ but $x = 1$.

Exercises**Exercise 2: Differences in time and space**

(a) If two events are simultaneous in a system S , but appear at different positions, prove that there is no limit on the time difference which can be measured in other systems. Also, show that the spatial separation can vary between a smallest value and infinity.

(b) If two events appear in the same point in space, but at different times in S , show that the time order between the two events is the same in all systems.

Exercise 3: A trip to Andromeda

Assume that you live for another 50 years. At what speed, expressed as a fraction of the speed of light, would you have to travel to reach the Andromeda galaxy 2.5 million light-years away within that time span. (We assume that you survive an instant acceleration to that speed!)

Solve the exercise by

- (a) using that the distance to the Andromeda galaxy is shortened in your frame.
 - (b) using that, from the point of view of the earth (and Andromeda), you will appear to age slowly.
- If you could "ride a photon", then,
- (c) how far away would you think that Andromeda was?
 - (d) how long would it take you to reach Andromeda?

Exercise 4: Velocity addition

A particle is moving with velocity \vec{v}' in a system S' which in turn is moving with velocity u in the x -direction compared to a system S . Prove, *starting* from the Lorentz transformation, that the particle's velocity, in the S -system, is

$$v_x = \frac{v'_x + u}{1 + v'_x u/c^2}, \quad v_y = \frac{v'_y}{\gamma(1 + v'_x u/c^2)}, \quad v_z = \frac{v'_z}{\gamma(1 + v'_x u/c^2)}. \quad (1)$$

Exercise 5: Rapidity

Introduce the velocity measure *rapidity* η , defined by

$$\eta(v) = \operatorname{artanh}\left(\frac{v}{c}\right). \quad (2)$$

Consider the parallel velocity addition from exercise 4, and prove that the particle's rapidity in S is simply given by its rapidity in S' plus the rapidity of the S' -system compared to the S system,

$$\eta(v) = \eta(v') + \eta(u). \quad (3)$$

Thus rapidities in the same direction are additive. In this sense they play the same role as velocities for Galilean transformations.

Extra: Prove this using eq. 4 by considering two Lorentz boosts after each other.

Exercise 6: Rapidity

Prove that the Lorentz transformations can be written in terms of the rapidity $\eta = \operatorname{artanh}(u/c)$ as

$$\begin{aligned} ct' &= ct \cosh(\eta) - x \sinh(\eta) \\ x' &= -ct \sinh(\eta) + x \cosh(\eta). \end{aligned} \quad (4)$$

Extra: Use the above equation to prove eq. 3 by considering two Lorentz boosts after each other.

Exercise 7: A shrinking rod

A rod with length L' is at rest in the system S' which is moving with speed u in the x -direction compared to S . In S' the rod has an angle θ' compared to the x' -axis. What is the length of the rod in the S -system, and what is the angle w.r.t. the x -axis?

Exercise 8: Higgs Decay

A Higgs particle of mass M is moving in the z -direction with speed $\beta_H c$ compared to the lab system. It decays to a b -quark and a \bar{b} -quark, each of mass m . What is the speed and the direction of the b -quark compared to the lab system if, in the Higgs system,

- (a) it is emitted in the forward direction.
- (b) it is emitted in the backward direction.
- (c) it is emitted in the y -direction.

Solve the exercise by

- (1) Using that the b and \bar{b} get out back to back with equal speed in the Higgs system. Then use velocity addition for finding the velocity in the lab-system.

Check your result by verifying that

- (i) All terms have the right dimension.
- (ii) You recover the correct limit when $m_b/m_H \rightarrow 0$
- (iii) Your result makes sense when $\beta_H \rightarrow 1$

Some potentially useful relations:

$$\begin{aligned}
 \sinh x &= \frac{e^x - e^{-x}}{2} \\
 \cosh x &= \frac{e^x + e^{-x}}{2} \\
 \tanh x &= \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} \\
 \cosh^2 x - \sinh^2 x &= 1 \\
 \cosh(x + y) &= \cosh x \cdot \cosh y + \sinh x \cdot \sinh y, \\
 \sinh(x + y) &= \cosh x \cdot \sinh y + \sinh x \cdot \cosh y, \\
 \operatorname{artanh}(x) + \operatorname{artanh}(y) &= \operatorname{artanh}\left(\frac{x + y}{1 + xy}\right) \\
 \sinh(\operatorname{artanh}(x)) &= \frac{x}{\sqrt{1 - x^2}} \quad \text{for } -1 < x < 1 \\
 \cosh(\operatorname{artanh}(x)) &= \frac{1}{\sqrt{1 - x^2}} \quad \text{for } -1 < x < 1
 \end{aligned} \tag{5}$$

Answers:

1 $x' = 0$, $t' = 0$ and $x' = \gamma$, $t' = -\gamma u/c^2$, in particular t' is not 0!

3 (a, b) $(1 - 2 \times 10^{-10})c$, (c) 0 m (d) 0 s.

7 $L = L' \sqrt{(1 - u^2/c^2) \cos^2 \theta' + \sin^2 \theta'}$, $\tan \theta = \frac{\tan \theta'}{\sqrt{1 - u^2/c^2}}$

8 No answer this time.