FYTA12/FYTB03

Exercise sheet 4

Friday, March 4th, 2016

Warm-up

Exercise 1: Four-vectors

Consider the four-vector $A = (1, 2, 3, 4)$

(a) Calculate the square of A.

(b) Is the four-vector A timelike, lightlike or spacelike?

(c) When one speaks about a four-vector, one normally means the contravariant four-vector. If $A = (1, 2, 3, 4)$ is the contravariant four-vector, what is then the covariant four-vector?

Exercises

Exercise 2: More four-vectors

Consider a particle with four-momentum $P = (3, 2, 1, 0)$ GeV/c.

(a) Use $P^2 = m^2c^2$ to calculate the mass of the particle.

(b) Use $E = \gamma mc^2$ to calculate the γ-factor for the particle.

(c) Use the γ-factor from (b) to calculate the speed of the particle.

(d) Use $P = \gamma m(c, \overline{v})$ to derive the same speed.

Exercise 3: Index gymnastics

Calculate:

(a) $g_{\mu\nu}g^{\mu\nu}$

(b) $g_{\mu\nu}g^{\nu\alpha}$

(c) $g_{\mu\nu}g^{\nu\alpha}p_\alpha$

Exercise 4: Lorentz boosts in different directions don't commute

Prove that Lorentz boosts in different directions don't commute. (Consider for example a boost in the x-direction and a boost in the y-direction.)

Exercise 5: A decaying Σ hyperon

A Σ-particle at rest decays to a neutron and a π-meson

$$\Sigma^+ \to n + \pi^+.$$

The masses for n and π^+ are assumed to be known, and the kinetic energy (i.e. the total energy minus the rest energy) for the pion is measured to be 92 MeV.

(a) Determine the magnitude of the spatial momentum of the pion.

(b) Determine the total energy of the neutron.

(c) Determine the mass of the Σ-particle.

Exercise 6: $\gamma + \gamma \to e^+ + e^-$

Two colliding photons can produce an e^+e^--pair,

$$\gamma + \gamma \to e^+ + e^-.$$

(a) Determine the threshold energy for this reaction (i.e. the lowest possible total energy that allows this reaction to take place).

(b) In the cosmic microwave background (CMB) the photons have an energy corresponding to the temperature 2.7 K. What is the (approximate) energy of such photons, Boltzmann's constant can be expressed as $k_B = 8.6173324 \cdot 10^{-5} eV/K$?

(c) Consider the creation of an e^+e^--pair in the collision between a (highly energetic) photon A and a background CMB photon B. Assume that the photons move in opposite directions. What is the minimal energy of photon A in order for this reaction to take place?

Exercise 7: $\pi^- + p \to n + \gamma$

A π^--meson and a proton, both at rest, interact and create a neutron and a photon,

$$\pi^- + p \to n + \gamma.$$

What is the energy of the photon?

Exercise 8: Proton collision at the LHC

At the Large Hadron Collider (LHC) at CERN, close to Geneva, protons will be accelerated in opposite directions, and collide head on with a total center of momentum energy of 13 TeV (i.e. 6.5 TeV per proton). Determine the energy of one proton in the other protons rest system.

Some particle masses:

$$
\begin{aligned}
m_\gamma &= 0 \\
m_{e^-} = m_{e^+} &= 0.511 \text{ MeV}/c^2 \\
m_{\mu^-} = m_{\mu^+} &= 105.7 \text{ MeV}/c^2 \\
m_{\pi^+} = m_{\pi^-} &= 139.6 \text{ MeV}/c^2 \\
m_{\pi^0} &= 135.0 \text{ MeV}/c^2 \\
m_{K^+} = m_{K^-} &= 494 \text{ MeV}/c^2 \\
m_{K^0} &= 498 \text{ MeV}/c^2 \\
m_p &= 938.3 \text{ MeV}/c^2 \\
m_n &= 939.5 \text{ MeV}/c^2
\end{aligned}
$$

Answers:

1 (a) $1^2 - 2^2 - 3^2 - 4^2 = -28$ (b) spacelike (c) $(1, -2, -3, -4)$

2 (a) 2 GeV/c^2 (b) 3/2 (c,d) $c\sqrt{5}/3$.

3 (a) 4 (b) ${g_\mu}^\alpha = \delta_\mu^\alpha$ (c) p_μ

5 (a) 185 MeV/c (b) 960 (958) MeV (c) 1190 MeV/c^2

6 (a) 1.02 MeV (b) 0.2 meV (c) $\approx 10^{15}$ eV

7 130 MeV