Written Exam, FYTB03 and FYTA12, Classical Physics I
March 21st, 2014, 10:15–15:15.

Allowed aids: (a) one A4 sheet with notes; (b) pens/pencils, erasers, rulers and other basic drawing utensils; (c) drinks, coffee/tea, fruit, candy; pillow, towel and similar necessities.

Total of 30 points; 15 required to pass.

NB! The tasks are not necessarily ordered in difficulty.

Read the text in each task carefully before attempting to solve it.

Carefully define your notation, and never use a formula without motivating why it applies.

Please try to avoid disturbing noise with your fruit, drinks etc.

The **results** will be displayed in the Theoretical Physics corridor, as soon as they exist.

1. [6p]

Several kilometers up in the atmosphere a muon is created. The muon thinks that it lives for 2×10^{-6}s. Nevertheless the muon reaches the Earth.

a) [2p] Explain in words why this is possible.

b) [4p] Consider a muon which is created 3 km from the Earth's surface. Assuming that it lives for 2 μs, at what speed does it have to travel to reach the Earth, if it is moving vertically? Express the answer as $v = c(1 - x)$ where x is a small number (in which you may expand).

2. [6p]

A sledge slides down an exceptionally icy hill (assume frictionless) with a constant slope α. Introduce a coordinate x (counted positively downwards) for describing the position of the sledge along the slope.

a) [1p] Write down the Lagrangian.

b) [2p] Write down Lagrange's equations.

c) [3p] Now, on top of the sledge, a (spherically symmetric) ball, which can roll along the sledge, is placed. Introduce a coordinate y for describing the ball's position along the sledge. The ball has a moment of inertia I w.r.t. its center. Write down the Lagrange equations for x and y and solve the equations of motion for y.

3. [8p]

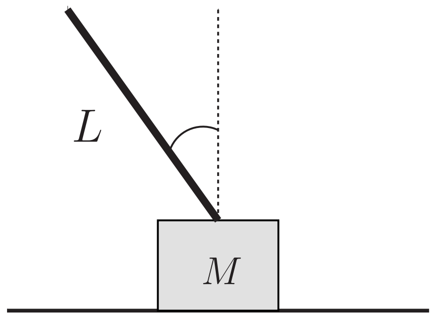
A relativistic train is moving in the positive x-direction with speed u. A relativistic football with mass m and speed v is approaching the train from behind at an angle θ w.r.t. the rail. The football bounces against the back of the train. The train has an effectively infinite mass M, and you can safely neglect gravity.

a) [2p] Choose your y-coordinate such that the ball moves in the xy-plane and write down the four-momenta of the train and the ball.

b) [2p] Write down the Lorentz matrix for boosting to the rest frame of the train.

c) [4p] After the bounce, what angle will the football have w.r.t. the rail (in the rail's rest frame)?

4. [6p]



A wagon with mass M can slide without friction along a rail. On top of the wagon a homogeneous rod is attached. The rod can rotate freely (in the plane spanned by a vertical line and the rail) around its connection point.

a) [2p] Introduce suitable generalized coordinates for describing the system and write down the Lagrangian. *Hint*: The moment of inertia of a homogeneous rod around center is $mL^2/12$.

b) [2p] Write down Lagrange's equations.

c) [2p] Assume that the system starts at rest, with the rod forming an angle $\phi(t = 0) = \pi/4$ with a vertical line. Determine the acceleration of the wagon at $t = 0$.

5. [4p]

We have derived the Euler-Lagrange equation,

$$0 = \frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'}, \tag{1}$$

for the function f which is extremizing the integral

$$\int_A^B f(y(x), y'(x))dx, \tag{2}$$

by using a small perturbing function $\alpha\eta(x)$, where α is small and $\eta(x)$ is an arbitrary function which vanishes in A and B, and which relates to y as $y(x, \alpha) = y(x) + \alpha\eta(x)$.

Sometimes in mechanics, the potential energy depends on the second derivative as well. Let us therefore consider the case that we have an integral

$$\int_A^B f(y(x), y'(x), y''(x))dx. \tag{3}$$

Prove that this integral is extremized by a function f fulfilling

$$0 = \frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} + \frac{d^2}{dx^2}\frac{\partial f}{\partial y''} \tag{4}$$

if $\eta(A) = \eta(B) = 0$ and $\eta'(A) = \eta'(B) = 0$.

GOOD LUCK!