Written Exam, FYTB03 and FYTA12, Classical Physics I
March 20th, 2015, 13:15–18:15.

Allowed aids: (a) one A4 sheet with notes; (b) pens/pencils, erasers, rulers and other basic drawing utensils; (c) drinks, coffee/tea, fruit, candy; pillow, towel and similar necessities.

Total of 30 points; 15 required to pass.

NB! The tasks are not necessarily ordered in difficulty.

Read the text in each task carefully before attempting to solve it.

Carefully define your notation, and never use a formula without motivating why it applies.

All powers of c must be correct in the final answer.

Please try to avoid disturbing noise with your fruits, drinks etc.

1. [6p]

Consider a pendulum consisting of a massless rod of of length L and a mass m attached at the end of the rod.

a) [2p] Write down the kinetic and potential energy of the pendulum when the rod forms an angle ϕ w.r.t. the vertical axis.

b) [2p] Write down the Hamiltonian expressed in terms of ϕ and the corresponding generalized momentum p_ϕ.

c) [2p] Write down and solve Hamilton's equations in the small angle limit, when the pendulum is hanging almost straight down.

2. [8p]

At the Large Hadron Collider (LHC) at CERN, protons will be accelerated in opposite directions, and collide head on with a total center of momentum energy of 13 TeV (i.e. $E = 6.5$ TeV per proton). The mass m of the proton is approximately $1\text{GeV}/c^2$.

a) [2p] Choose suitable coordinates and write down the four-momenta of the protons in the LHC frame in terms of E and m.

b) [1p] Calculate the invariant mass using the four-momenta.

c) [3p] Make sensible approximations and determine the energy of one proton in the other proton's rest system.

d) [2p] A bumblebee has a mass of about 0.2g. How fast should the bumblebee fly to have the same kinetic energy as the moving proton in (c)? Use $1\text{GeV}/c^2 \approx 1.8 \times 10^{-27}$ kg. (For full points your answer may deviate with up to 30% from the correct value.)

3. [6p]

An intergalactic voyager wants to visit the Andromeda galaxy, 2.5 million light years away. She is prepared to spend 50 years of her life on the trip.

a) [2p] Neglecting the time for acceleration, how fast does she have to travel?

b) [1p] Let us temporarily ignore relativistic effects, how much time does it take to reach this speed if she is accelerating with $g \approx 10\,\text{m/s}^2$?

c) [3p] How long time does it take to reach this speed if we consider special relativistic effects, and she is accelerating with g in her own frame? *Hint:* $\text{arctanh}(1 - 2 \times 10^{-10}) \approx 11.5$

4. [6p] A so-called Wilberforce pendulum consists of a mass m hanging in a spiral spring that is free to oscillate in vertical and in torsional mode. The Lagrangian can be written

$$L = \frac{1}{2}m\dot{z}^2 + \frac{1}{2}I\dot{\theta}^2 - \frac{1}{2}kz^2 - \frac{1}{2}\delta\theta^2 - \frac{1}{2}\epsilon z\theta \tag{1}$$

where z is the deviation from the vertical equilibrium, θ describes the angle of rotation from equilibrium, I is the moment of inertia for rotation around the center of the spring, while k, δ and ϵ are constants

a) [3p] Write down Lagrange's equations.

b) [3p] Find the frequencies of the normal modes.

5. [4p] In so-called φ^4 field theory the Lagrangian density can be written

$$\mathcal{L}(\varphi, \partial^\mu\varphi) = \frac{1}{2}[\partial^\mu\varphi\partial_\mu\varphi - m^2\varphi^2] - \frac{1}{4}\lambda\varphi^4, \tag{2}$$

where φ is a scalar field, m is the mass of the field and λ is a constant. The action is given by

$$\int \mathcal{L}(\varphi, \partial^\mu\varphi)dx^0dx^1dx^2dx^3. \tag{3}$$

Consider $\varphi(x^\mu)$ as a generalized coordinate and derive the Lagrange equation.

GOOD LUCK!