Written Exam, FYTB03 and FYTA12, Classical Physics I
March 24th, 2016, 10:15–15:15.

Allowed aids: (a) one A4 sheet with notes; (b) pens/pencils, erasers, rulers and other basic drawing utensils; (c) drinks, coffee/tea, fruit, candy; pillow, towel and similar necessities.
Total of 30 points; 15 required to pass.
NB! The tasks are not necessarily ordered in difficulty.
Read the text in each task carefully before attempting to solve it.
Carefully define your notation, and never use a formula without motivating why it applies.
State clearly if you use natural units.
All powers of c must be correct in the final answer.
Please try to avoid disturbing noise with your fruits, drinks etc.

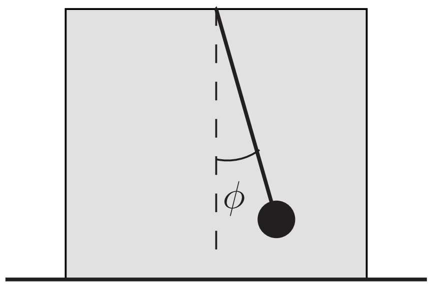
1. [5p]
Two relativistic trains meet on a double track stretch. Relative to the tracks, both trains move with speed $v = c/2$. In their respective rest systems, each train has length l and width b. You may assume that the trains move straight towards each other.

a) [2p] What speed does the train driver in one of the trains observe the other train to approach with?

b) [2p] How long does he perceive the other train to be?

c) [1p] How wide does he perceive the other train to be?

2. [11p]
A box of mass M is sliding without friction over ice. Inside the box an (approximately pointlike) pendulum of mass m is hanging from the top (see figure). The motion takes place in one plane.



a) [2p] Write down the Lagrangian using suitable generalized coordinates.

b) [2p] Write down Lagrange's equations, identify the cyclic coordinate and calculate the corresponding conserved momentum.

c) [2p] Solve Lagrange's equations in the small angle approximation.

d) [5p] Now assume that there are two identical pendulums hanging from the ceiling in the box. Find the frequencies of the normal modes of the motion in the limit where both angles stay small.

3. [6p]
We have derived the change in wavelength of a photon being emitted from a source which moves away from an observer at relativistic speed v by using time dilation and the fact that consecutive wave peaks have a longer distance to travel.

a) [3p] Derive the change in wavelength for a source that is moving straight away or towards the observer by using that the photon wave vector $k^\mu = \frac{\omega}{c}(1, \hat{e})$ transforms as a four-vector.

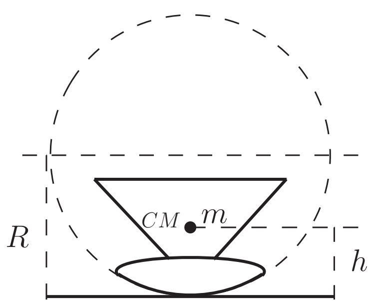
Use the result for the case of a source moving *towards* the observer to derive what a clock following the source would *look like* for the observer.

b) [1p] Would the color of the clock be redshifted, be the same, or be blueshifted?

c) [2p] Would the hands of the clock *look like* if they were moving faster or slower? How much faster/slower would they look? Motivate carefully.

4. [3p]
Prove that the relativistic Kronecker delta, $\delta^\mu{}_\nu$, is a Kronecker delta in all reference systems by using the rules for tensor transformation.

5. [5p]
A mother wants to build a cradle (vagga in Swedish) for her baby. She wants the frequency of the cradle to be ω, and makes the approximations that all mass m is located in the center of mass at height h above the floor, and that the amplitude of the cradle stays small. If she is using a round shape for the part of the cradle touching the floor, what radius R should she use to achieve the design frequency? (The problem should be solved using analytical mechanics.)



GOOD LUCK!