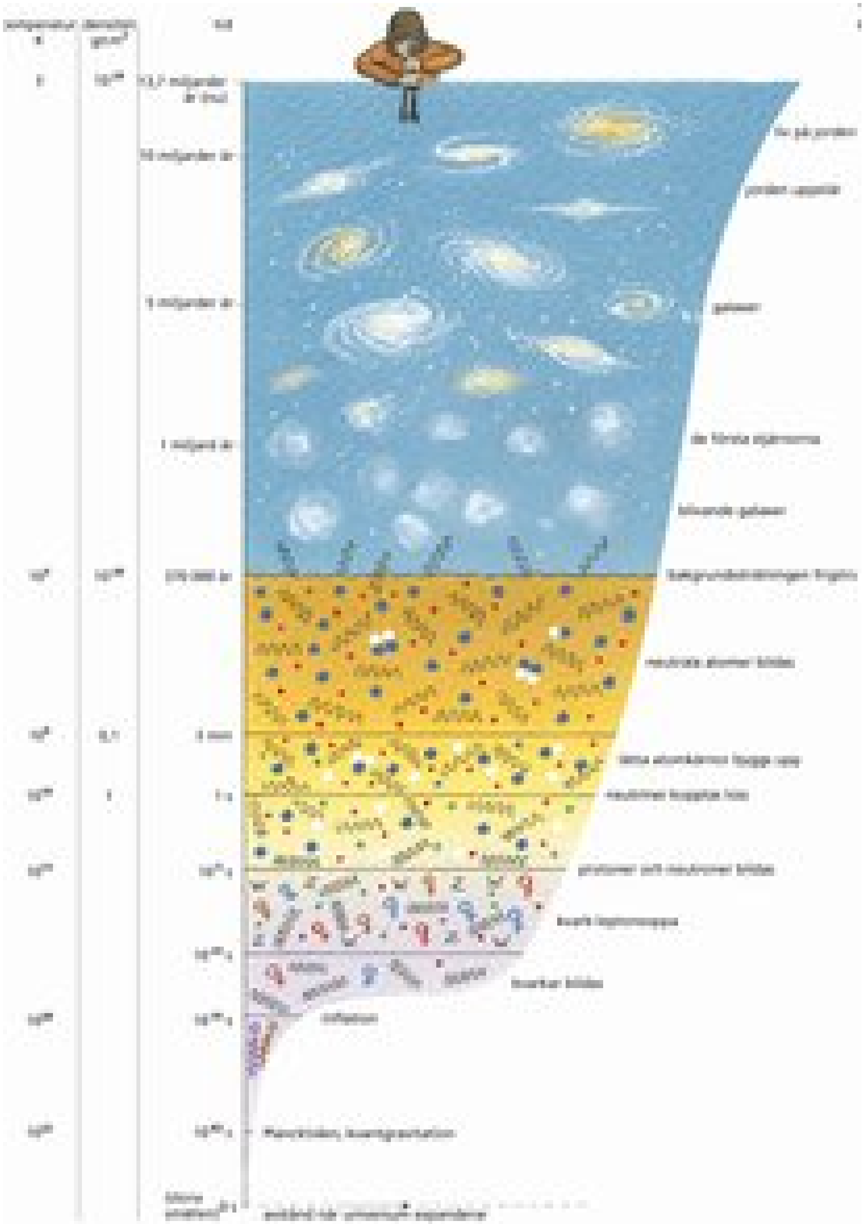
An Accelerating Universe



Bengt E Y Svensson

3 March, 2010

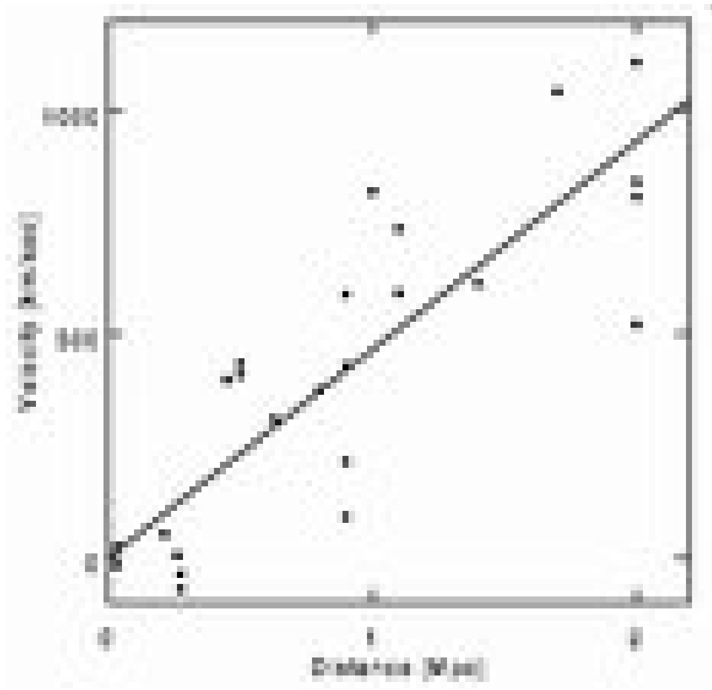
Cosmology

Universal expansion: $v = H_o\ d$

(Hubble, ~1929)

$v = c\ z$

z = redshift = $\dfrac{\lambda_r - \lambda_e}{\lambda_e}$



H_o ñHubble constantò

Todayôs value: H_o ~ 72 km/s Mpc

(t_o ~ 13,7 Gy)

Cosmological principle

$ds^2 = c^2 \; dt^2 - a(t)^2 \, d\mathbf{x}^2$

$d\mathbf{x}^2 = dr^2 \, (1-Kr^2)^{-1} + r^2 \, d\omega^2$

$K = +1, -1, 0$

$d\omega^2 = d^2\theta + \sin^2 d\phi^2$

$1 + z = a(t_o) / a(t)$

Hubble parameter: $H = \dot{a} / a = - \dot{z} / (1 + z)$,

$$dt = \frac{1}{H}\frac{da}{a} = -\frac{1}{H}\frac{dz}{1+z}$$

Age of universe: $t_o = \int_0^\infty \frac{1}{H}\frac{dz}{1+z}$

deceleration parameter: $q = - a \, \ddot{a} \, / \, \dot{a}^2$

Observations:

$q_o \approx -0{,}6 \Rightarrow$ accelerating universe!

light : $ds = 0 \;\Rightarrow\; dr = \pm\, c\, dt / a(t)$

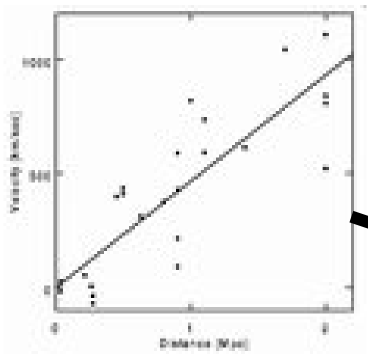
(Luminosity) distance: $l = L / (4\pi\, d^2)$ ->

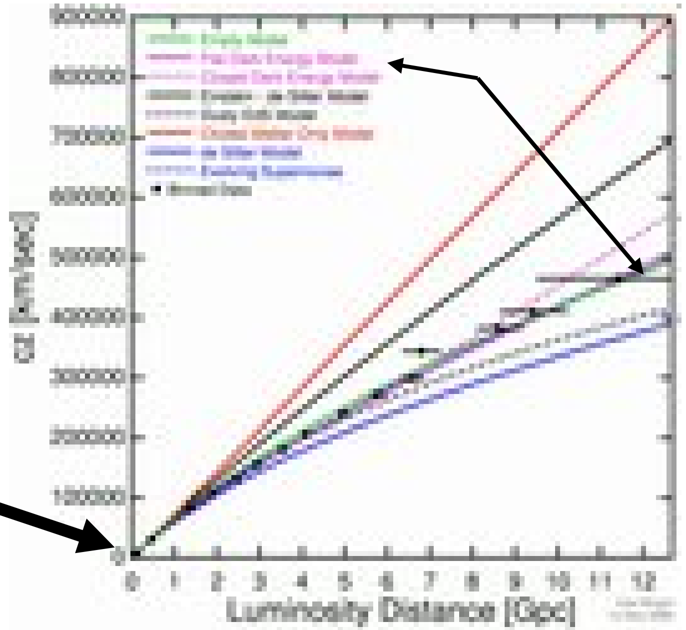
-> $[L / 4\pi\, a(t_o)^2\, r_e^2] \times [1/(1 + z)] \times [1/(1 + z)]$

$$r_e = c \int_0^z \frac{dz'}{H(z')}$$

$$d_L = a(t_o)\,(1 + z)\; c \int_{t_e}^{t_o} \frac{dt'}{a(t')} =$$

$= c\; H_o^{-1}$ [z + ½ (1 ï q_o) z^2 + é .]





(Angular) distance: angle = (object size) / (distance to object)

$$d_A(t) = a(t_e)\ r_e = d_L / (1 + z)^2$$

Cosmodynamics

Einstein Field Equations (EFE) $$G_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu}$$

Fluid:

$$T_{\mu\nu} = (\rho + p/c^2) u_\mu u_\nu - p/c^2 g_{\mu\nu}$$

$$Too = \rho c^2$$

$$T_{11} = (p/c^2) a^2$$

Field: Action $$S = \int d^4 x \sqrt{-g} \mathcal{L}$$

$$\mathcal{L} = ½ \partial_\mu\varphi \partial^\mu\varphi - V(\varphi) g_{\mu\nu}$$

Spatially homogeneous: $\partial_k = 0$

$$T_{oo} = \frac{1}{2c^2}\dot{\varphi}^2 + V(\varphi) = \rho_{field} c^2$$

$$T_{11} = a^2 [\dot{\varphi}^2/2 - c^2 V(\varphi)] = (p_{field}/c^2) a^2$$

EFE $\Rightarrow$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K c^2}{a^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3 c^2} \rho c^2$$

ñFriedmann equationò

$\Rightarrow$

$$H^2 = \frac{8\pi G}{3} [\rho + \rho_\Lambda + \rho_K]$$

$$\rho_K = - \frac{K}{a^2} / (\frac{8\pi G}{3 c^2}) \qquad \rho_\Lambda = \Lambda / (\frac{8\pi G}{c^2})$$

$$\rho = \rho_\gamma + \rho_\nu + \rho_m \text{é é} .$$

$$\rho_m = \rho_D + \rho_B$$

Energy conservation $$\frac{d}{dt}(\rho\ a^3) = -\ (p/c^2)\ \frac{d}{dt}\ a^3$$

In combination with $\frac{d}{dt}$ (Friedmann):

$$\frac{\ddot{a}}{a} = -\ \frac{8\pi\ G}{3}\ (\rho\ +\ 3\ p/c^2)\ +\ \Lambda\ c^2\ /\ 3$$

ñacceleration equationò

Equation of state: $p/c^2 = w\ \rho$

$w = 0$ for non-relativistic matter (ρ_m , ρ_D , ρ_B)

$= 1/3$ for relativistic matter (ρ_γ , ρ_ν , é .)

$= -1$ for ρ_Λ

$= -1/3$ for ρ_K

$$\frac{d}{dt}(\rho\, a^3) = -(p/c^2)\,\frac{d}{dt}a^3$$

$$p/c^2 = w\,\rho\ ;\ \ w \text{ constant}$$

$$\Longleftrightarrow\quad \rho = \rho_o \left(\frac{a}{a_o}\right)^{-3(1+w)}$$

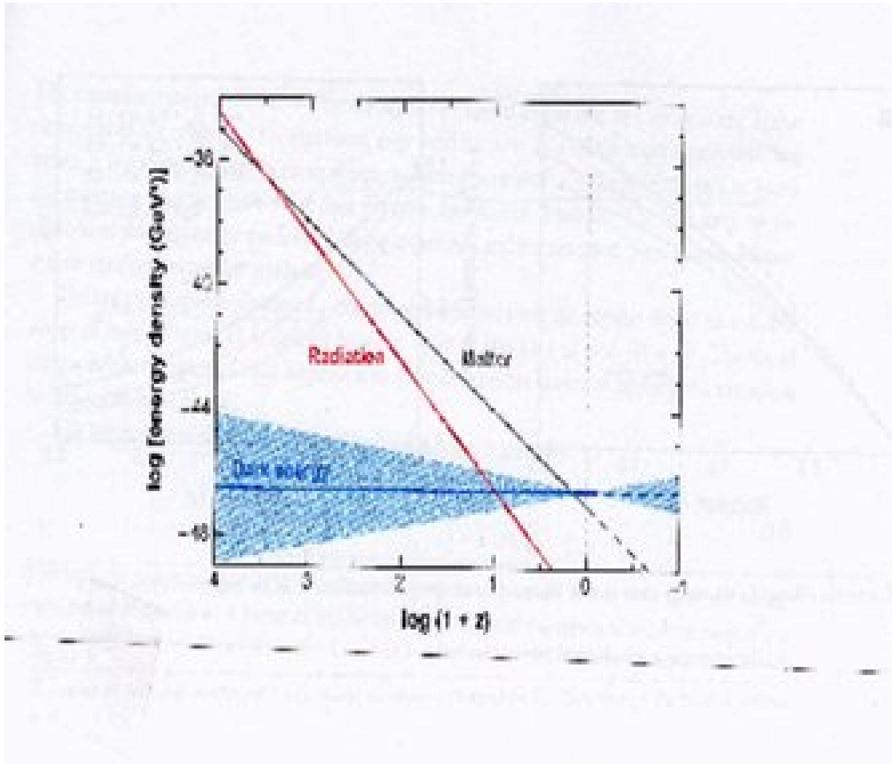
$$\rho_m = \rho_{m,o}\left(\frac{a_o}{a}\right)^3 = \rho_{m,o}\,(1+z)^3$$

Same for ρ_D , ρ_B

$$\rho_\gamma = \rho_{\gamma,o}\left(\frac{a_o}{a}\right)^4 = \rho_{\gamma,o}\,(1+z)^4$$

$$\rho_\Lambda = \rho_{\Lambda,o} \quad \text{constant!}$$

$$\rho_K = \rho_{K,o}\left(\frac{a_o}{a}\right)^2 = \rho_{K,o}\,(1+z)^2$$



Introduce $\rho_{crit} = H^2 / \left(\frac{8\pi G}{3}\right) \sim 0{,}97 \times 10^{-26}$ kg/m^3

$\Omega = \rho / \rho_{crit}$

Friedmann: $1 = \Omega_\gamma + \Omega_\nu + \Omega_m + \Omega_\Lambda + \Omega_K$

Present values: $\Omega_\gamma \sim \Omega_\nu \sim 10^{-5} \sim 0$

$\Omega_m \sim 0{,}27$

$\Omega_D \sim 0{,}24$

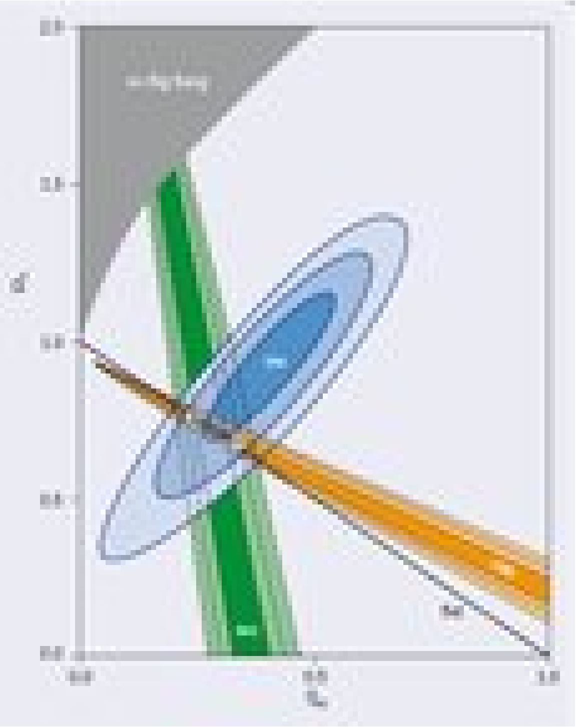
$\Omega_B \sim 0{,}04$

$\Omega_\Lambda \sim 0{,}73$ [$\Lambda \sim 1{,}5 \times 10^{-54}$ m^{-2},

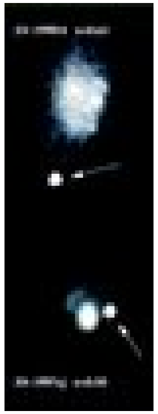
$\rho_\Lambda \sim (2 \times 10^{-3}\ \mathrm{eV})^4$ ($c = \hbar = 1$)]

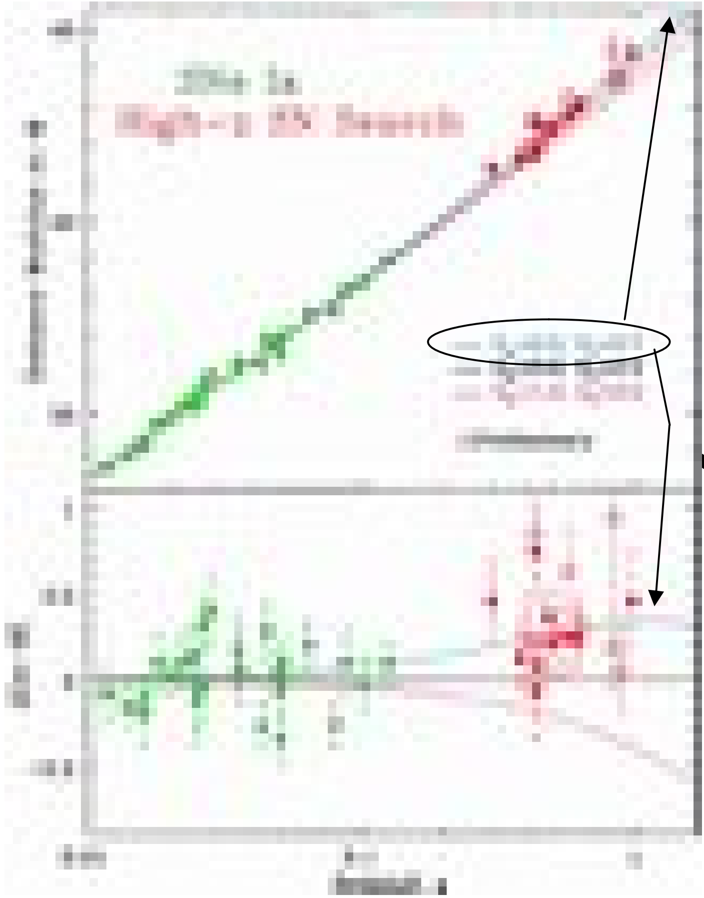
$$q = -\frac{\ddot{a}\,a}{\dot{a}^2} = -\tfrac{1}{2} \sum_i \Omega_i\,(1 + 3 w_i) =$$

$$= -0{,}5 \times [0{,}27 - 2 \times 0{,}73] = +0{,}6$$



Hubble Diagram of Type Ia Supernovae

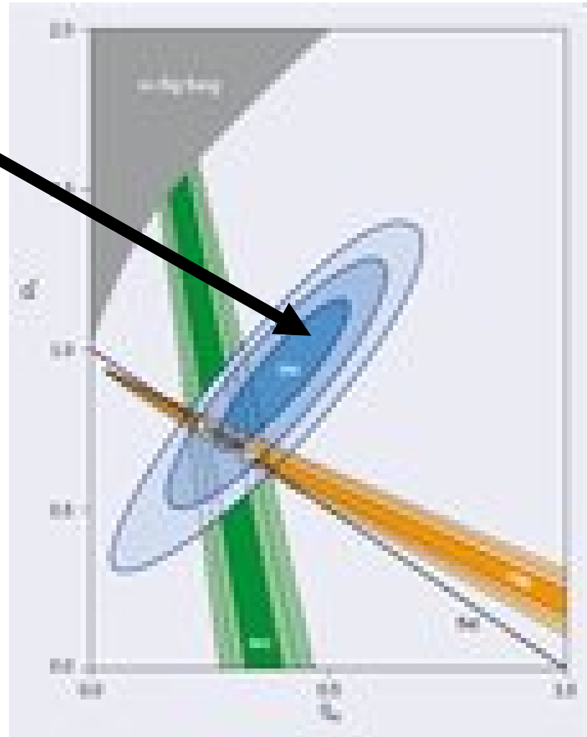




$$d_L(t_e) = a(t_o)\,(1+z)\,c\int_{t_e}^{t_o}\frac{dt'}{a(t')} =$$

$$= (1+z)\,c\int_0^z\frac{dz'}{H(z')}$$

$$H^2(z) = H_o^2\,[\Omega_m(1+z)^3 + \Omega_\Lambda]$$

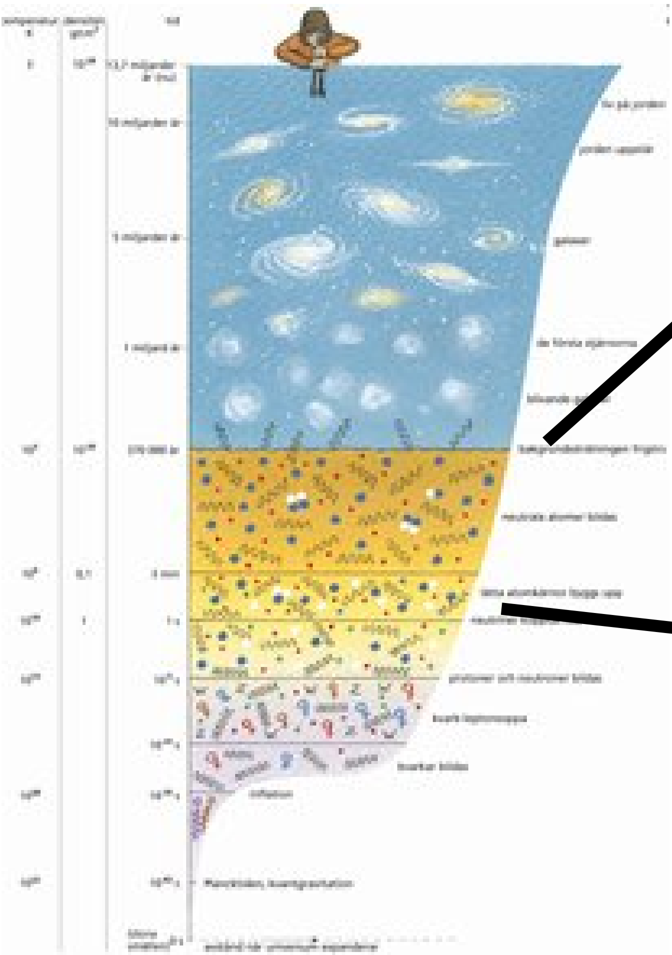


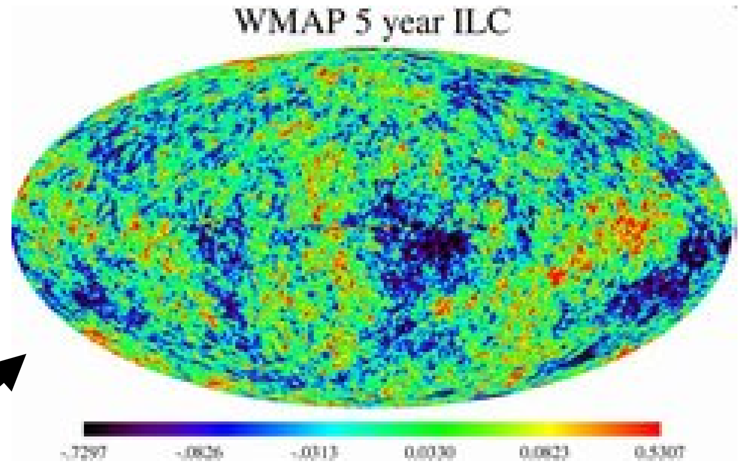
$\Omega_\Lambda > 0$ ñdark energyò

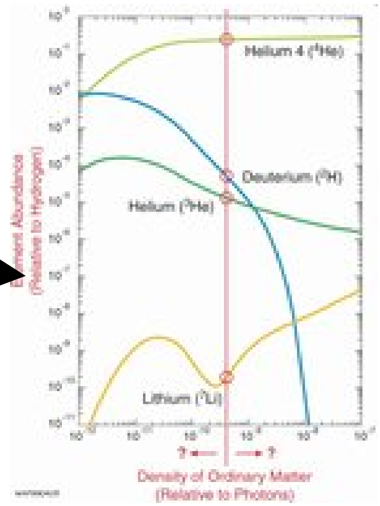
$\Omega_\Lambda \sim \Omega_m$ ñcoincidence problemò

Thermal evolution

Temperatur $T \sim a(t)^{-1} \sim (1 + z)$



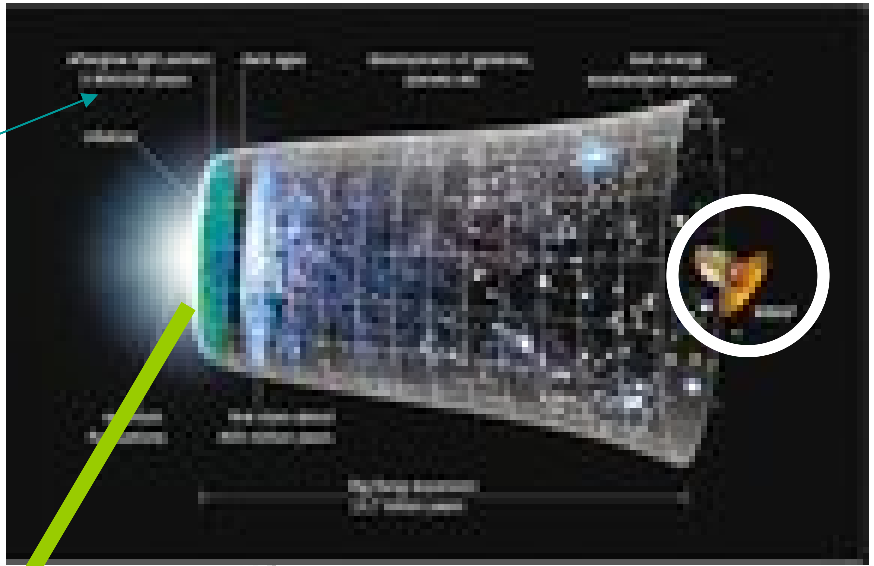


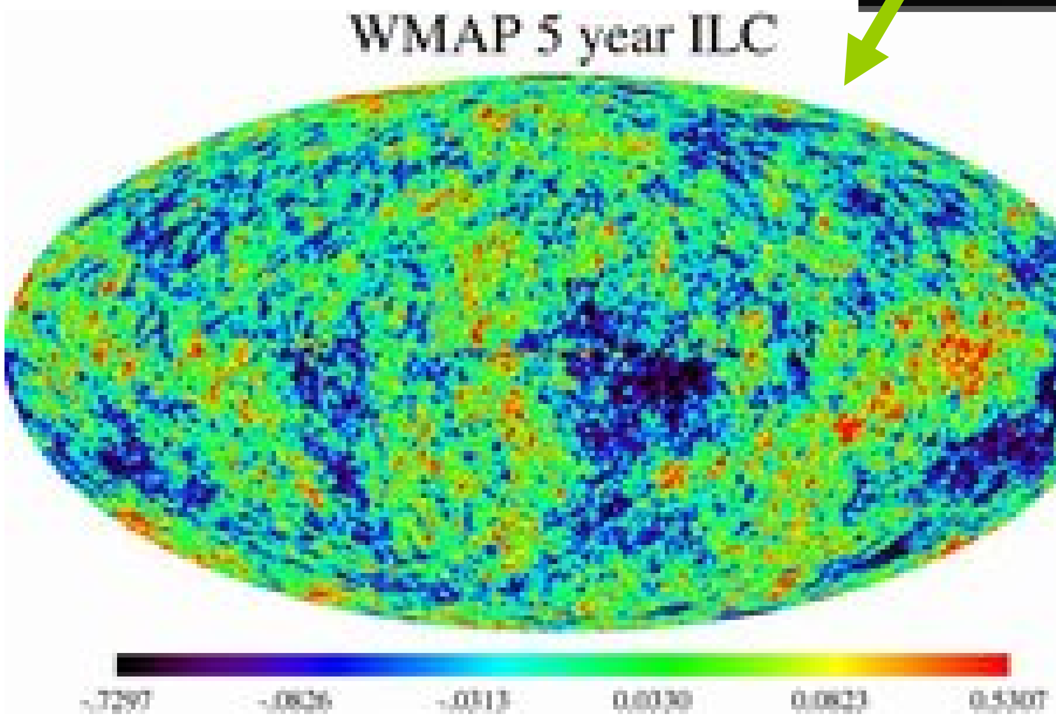


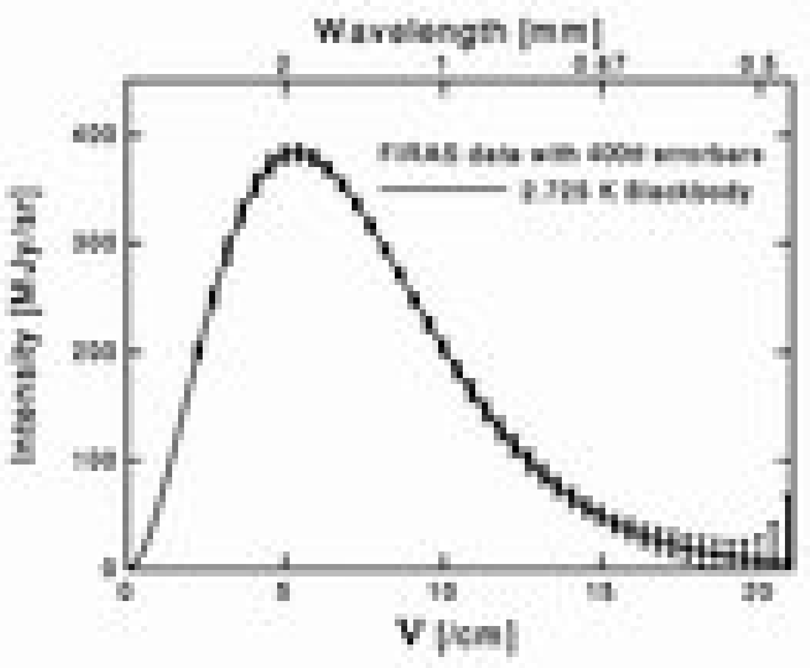
CMB =
Cosmic Microwave background

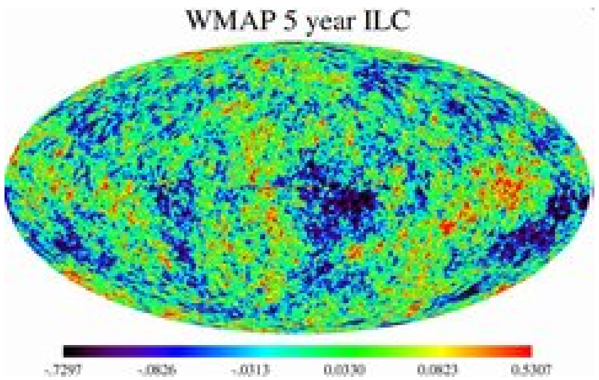
380 000 years !!

z ~ 1090









$$\Delta T(\vec{n}) = [T(\vec{n}) - < T >] / < T >$$

$$< \Delta T(\vec{n})\ \Delta T(\vec{n}') > = \sum_l \left(\frac{2l+1}{4\pi}\right) C_l\ P_l(\vec{n}\ \vec{n}')$$

$$C_l = (....)\int_0^\infty q^2\, dq\ P(q)\ j_l^2(q\, r_L)$$

P (q) is the *spectral function* and represents the intensity of the temperatur differences, (Fourier transform in comoving coordinates of correlation function), which in turn depend on the photon density irregularities, which in its turn are coupled to the baryon (but not dark matter) irregularities.

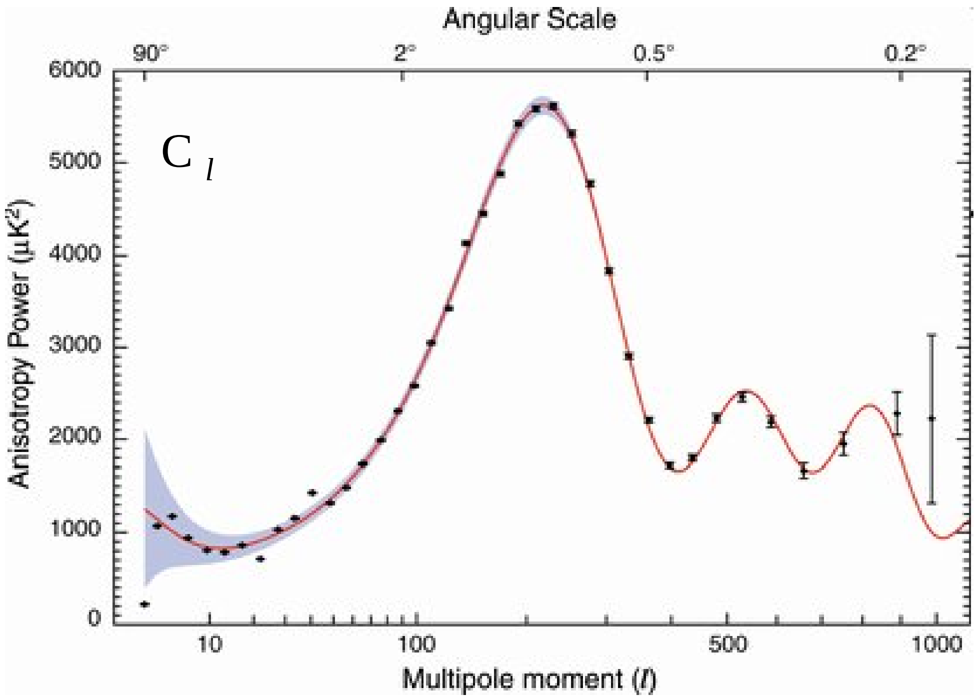
So we expect *P* (q) to peak at

baryon mass concentrations

C_l peáks at $q\, r_L \sim l$,

i e Ăprojects outñ P (q) at q $\sim l / r_L$

(and $l \sim \pi$ / angle)



Peaks at distances given by ñacoustic oscillationsò (= gravity <-> baryonic matter density oscillations) in baryon-photon plasma, given by

d_{sound} = ñsound horizonò , which depends on

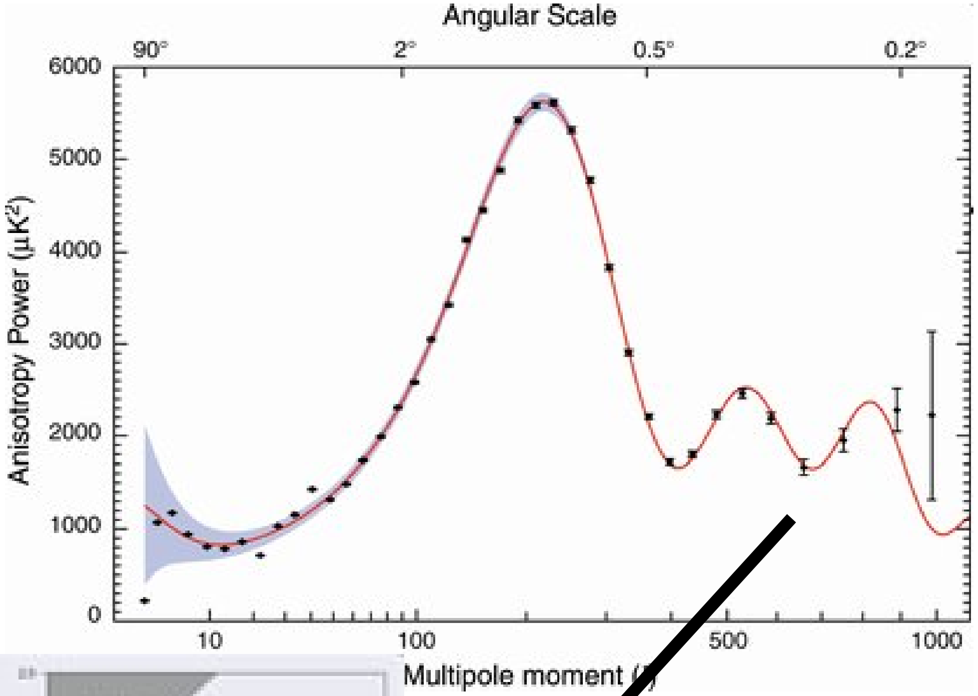
$$v_{sound}^2 = \frac{1}{3\left(1+3\rho_B/4\rho_g\right)}$$

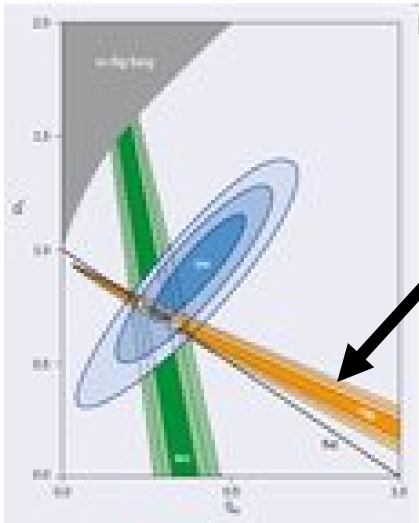
$$d_{sound} = a(t_L) \int_0^{t_l} \frac{dt\, v_{sound}}{a(t)}$$

So we expect peaks at

$l \sim l_{sound} \sim d_{sound} / d_A \sim 1^o$

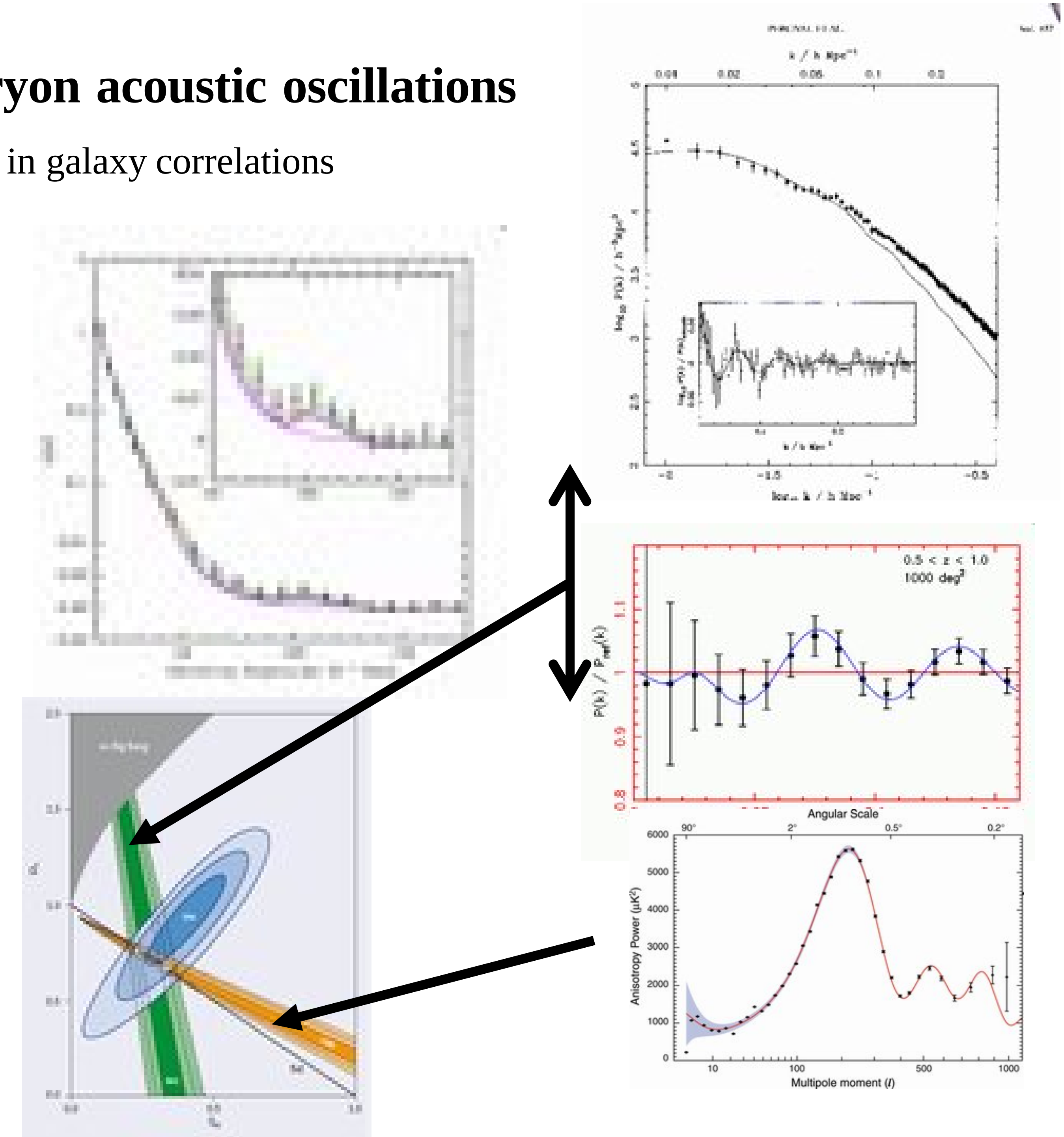
Observations: $l_{sound} \sim 200 \Rightarrow 1^o$





Baryon acoustic oscillations

Seen in galaxy correlations



Explaining cosmic acceleration

- Cosmological constant ~ vacuum energy
- Exotic matter
  - Quintessence
  - Phantom matter
  - k-essence
  - Braneworld models
  - ....
- Revise basic assumptions
  - Non-homogenous matter/energy distribution
  - Modified GR

Cosmological constant ~ vacuum energ y:

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu} \quad \Rightarrow \quad G_{\mu\nu} = \frac{8\pi G}{c^2} [T_{\mu\nu} + g_{\mu\nu} \Lambda / (\frac{8\pi G}{c^2})]$$

GR: Absolute value of energy important

QFT (or even Heisenberg): zero-point energy E_o

Each mode has $E_o = ½\, \hbar \omega = ½\, \hbar \sqrt{\vec{p}^2 + m^2}$

$$\Rightarrow \rho_{vac} = E/V = \frac{\hbar}{2V} \sum_{all\ fields} \left[g_i \sum_{all\ \vec{p}} \sqrt{\vec{p}^2 + m_i^2} \right] \propto p_{max}^4$$

Remember observations: $\rho_{vac} \sim (2 \times 10^{-3}\ eV)^4$

Also: Coincidence:

$$\rho_{vac} \sim \rho_m \text{ today}$$

Anthropic reasoning ??

Quintessence (~ à la inflation but without reheating)

Suppose $\Lambda = 0$

Scalar field Q $\mathcal{L} = \frac{1}{2}\, \partial_\mu Q\, \partial^\mu Q - V(Q)\, g_{\mu\nu}$

$$\frac{1}{2c^2}\dot{Q}^2 + V(Q) = \rho_Q c^2$$

$$\dot{Q}^2/2 - c^2 V(Q) = p_Q/c^2$$

(put c = 1)

$$w_Q = p_Q / \rho_Q = \frac{\frac{1}{2}\dot{Q}^2 - V(Q)}{\frac{1}{2}\dot{Q}^2 + V(Q}$$

so $1 \geq w_Q \geq -1$

$w_Q \sim -1$ requires $\dot{Q} \sim 0$

Field equation:

$$\ddot{Q} + 3H\dot{Q} + V_{,Q} = 0$$

òHubble frictionò

Quintessence (cont)

$$\ddot{Q} + 3\,H\,\dot{Q} + V_{,Q} = 0$$

For large H (early time, i e near big bang) , the Hubble friction terms implies ñslow rollò so that

$\ddot{Q} \sim 0$ and $\dot{Q} \sim - V_{,Q} / 3\,H$, which should be $<< (V(Q))^{½}$

This requires a shallow V(Q) , i e a very low (effective) mass

$$m_Q \sim V_{,QQ} < H_o \sim 10^{-32}\ \mathrm{eV}$$

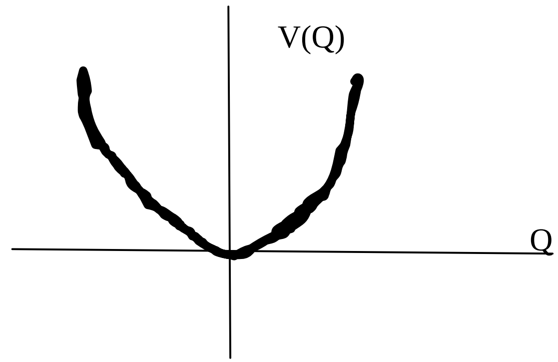
and for ρ_Q to give a contribution to ρ requires $Q \sim M_{Planck} = \sqrt{\dfrac{\hbar\, c}{8\,\pi\, G}}$

Example 1:

Axion $V(Q) = \mu^4 [\, 1 + \cos (Q/f)]$

$\mu \sim 0{,}002$ eV, $f \sim 10^{18}$ GeV

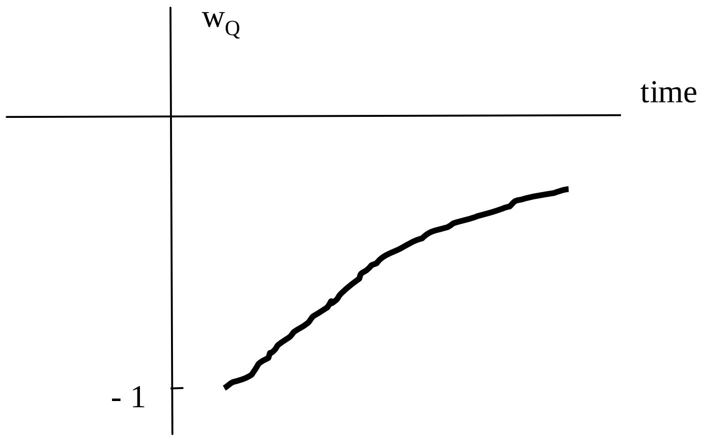
More generally a potential with a (local) minimum



Scenario:

Hubble friction freezes the field for most of cosmic history.

As Hubble friction relaxes, the field oscillates at the bottom of the potential.

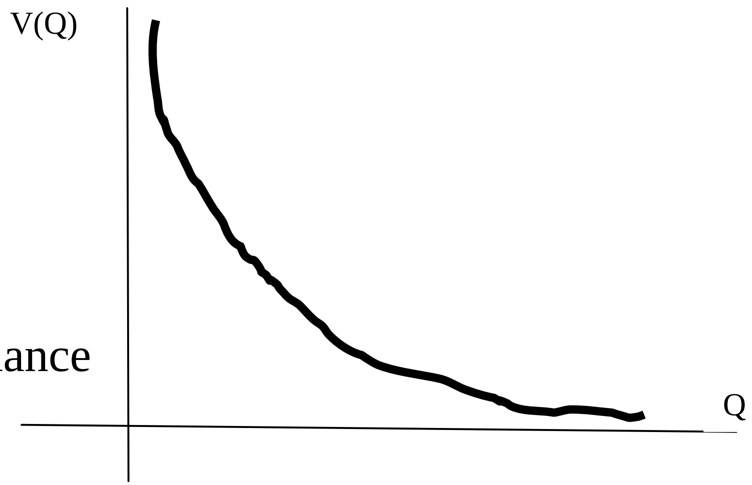


ñThawingò scenario

Example 2:

$V(Q) = \mu^4 (Q/M_{Planck})^{-n}$ (Peebles & Ratra 1988 !)

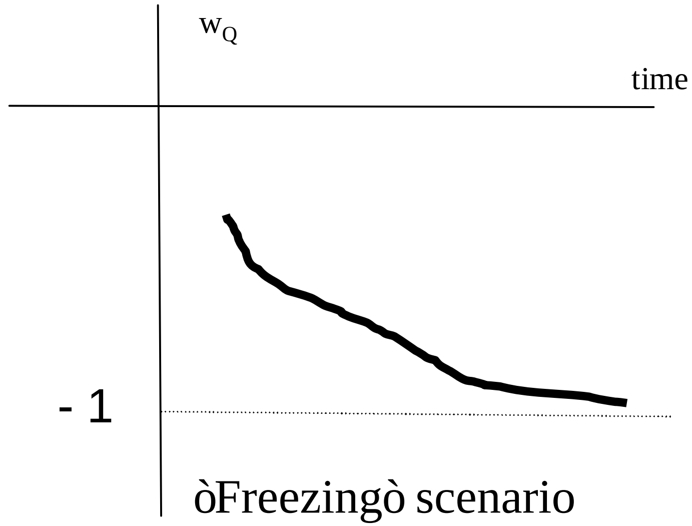
$V(Q) = \mu^4 \exp(-\lambda Q/M_{Planck})$



Scenario:

Freezing at early times, when ρ_Q is subdominant, followed by gradual overtaking, resulting in dominance and (essentially) an exponential growth of a(t).

ñTracker solutionò: The ñcoincidence problemò can be solved with clever adjustment of constants



òFreezingò scenario

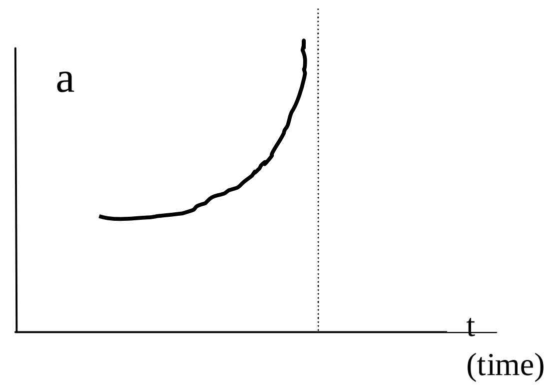
ñPhantom modelsò:

For quintessence: w_Q > - 1 (or possibly = - 1)

What if observation should imply w < - 1?

Friedmann: $H^2 = H_o^2 \, \Omega \, (1 + z)^{3(1+w)}$

$$\Rightarrow \quad a(t) = a(t_p) \, [-w + (1 + w)(t/t_p)]^{2/3(1+w)}$$



ñBig ripò

With w = - 1,1 , the big rip occurs after ~ 100 Gy

Phantom model:

Field theory with negative kinetic energy , - $\frac{1}{2}\dot{Q}^2$!

$$w_{ph} = p_{ph} / \rho_{ph} = \frac{-\frac{1}{2}\dot{Q}^2 - V(Q)}{-\frac{1}{2}\dot{Q}^2 + V(Q)} = -1 + \frac{\dot{Q}^2}{\frac{1}{2}\dot{Q}^2 - V(Q)}$$

But unstable as a quantum field theory!

Remark: This is roughly the same idea as F red Hoyle had in his ñc-fieldò to explain the steady-state theory.

K-essence

Field theory Lagrangian density

$$\mathcal{L} \;=\; \mathrm{K}\,(\,½\;\partial_{\mu} Q\,\partial^{\mu} Q\,) - \mathrm{V}(Q)$$

K is ñany functionò of $\mathrm{X} = ½\;\partial_{\mu} Q\,\partial^{\mu} Q$

$$w_{K} \;=\; \frac{K(X) \;\; - V(Q)}{2\,X\,K_{,X}(X) - K(X) \;\;+\;\; V(Q)} \;=\; -1 + \frac{2\,X\,K_{,X}(X)}{2\,X\,K_{,X}(X) - K(X) \;\;+\;\; V(Q)}$$

Chaplygin gas

(Michael Blomqvistôs master thesis 2004)

Postulate: Fluid with $p = - A / \rho$, A = constant

Friedmann => $\rho = \left(\sqrt{\left(A + B / a^{6}\right)} \right)$, B = integration constant

So

For early times, the Chaplygin gas behaves as a pressureless dust,

while

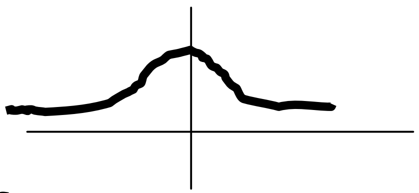
For later times, $\rho \sim$ constant => acceleration

Remarks

1. The Chaplygin gas is related to the so called ghost field approach where

Action $S_{ghost} = \int d^4 x \; V(\phi) \sqrt{- \det \left(g_{\mu\nu} + \partial_\mu \varphi \, \partial_\nu \varphi \right)}$

with a potential $V(\phi) = V_o / \cosh(\phi / \phi_o)$



2. There are grave instability problems attached to these approaches

Varying neutrino mass

Is $\Lambda^{1/4} \sim 2 \times 10^{-3}$ eV $\sim m_\nu$ a clue?

Try usual field but with potential $V(Q) = n_\nu m_\nu(Q) + V_o(Q)$

Neglecting $\dot{Q}$, one obtains

$$w = p / \rho =$$

$$= (- V_o) / [n_\nu m_\nu(Q) + V_o(Q)] =$$

$$= -1 + n_\nu m_\nu(Q) / V_o(Q)$$

Modified gravit y

Constraints:

* GR very good, at least up to solar system scale, ~100 μm to ~ 10^{12} m

* GR must be recovered also for early times

One approach:

ñf($\mathcal{R}$) gravityò

Action $\quad S = ½\,(\hbar / l_{\mathrm{Planck}}^2) \int d^4x \, \sqrt{-g} \;\; \mathcal{R} + S_{\mathrm{non\ grav}}$

where $\quad \mathcal{R}$ = the Ricci scalar, and $\; l_{\mathrm{Planck}} = \sqrt{\dfrac{8\pi\, G\, \hbar}{c^3}}$,

is generalized to

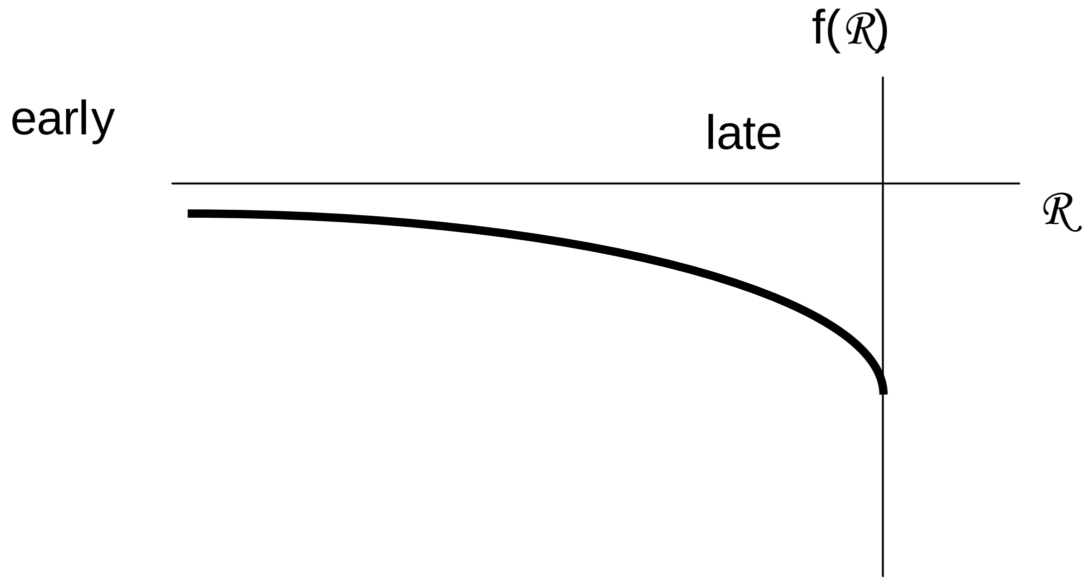
$$S_f = -\,(\hbar / l_{\mathrm{Planck}}^2) \int d^4x \, \sqrt{-g} \;\; \mathcal{R}\,[\,1 + f(\mathcal{R})\,] + S_{\mathrm{non\ grav}}$$

where f($\mathcal{R}$) is ñanyò non-constant function.

ñf($\mathcal{R}$) gravityò (cont)

$$S_f = -(\hbar / l_{Planck}^2) \int d^4x \sqrt{-g}\ \mathcal{R}\,[\,1 + f(\mathcal{R})\,] + S_{non\ grav}$$

$$\mathcal{R} = 6\,H^2\,[\,-q + 1\,] + 6\,K / a^2 = -\ 24\,\pi\,G\,(\,\rho + p\,)$$



Remark: For theoretical reasons, $\mathcal{R}$ is the only higher order (in g_{mn}) scalar that can appear (otherwise higher derivatives than 2nd in field equations)

ñf($\mathcal{R}$) gravityò (cont)

Two further remarks:

(i) $w_f = -1 + A / B$

[A,B explicit (non-positive definite) functions of H, f and derivatives of f.]

So essentially any value of w_f is possible

(ii) One can show that the new action is equivalent to introducing a new, scalar field degree of freedom. In fact, f($\mathcal{R}$) is a ñscalaronò field. One may then use the freedom to transform the metric so that one recovers essentially the quintessence model

As usual, a not-negligible amount of fine-tuning is needed to get a viable model

Another approach to modified gravity:

Scalar-tensor theories (c f Brans-Dicke)

$$S_{s\text{-}t} = \tfrac{1}{2}\,(\hbar / l_{Planck}{}^2) \int d^4 x \sqrt{-g} \,\Big[\, b(\phi)\, \mathcal{R} + \tfrac{1}{2}\, h(\phi)\, \partial_\mu \phi\, \partial^\mu \phi - U(\phi) \Big] + S_{non\ grav}$$

involving what amounts to a ϕ -dependent gravitational constant.

(Recall: Possible observation of time-varying fundamental constants.)

A third approach to modified gravity :

ñDegravitationò

Modify E F E

$$G_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu}$$

to $$[1 + F(L^2 \partial_\mu \partial^\mu)]\, G_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu}$$

where L is some (large) length and

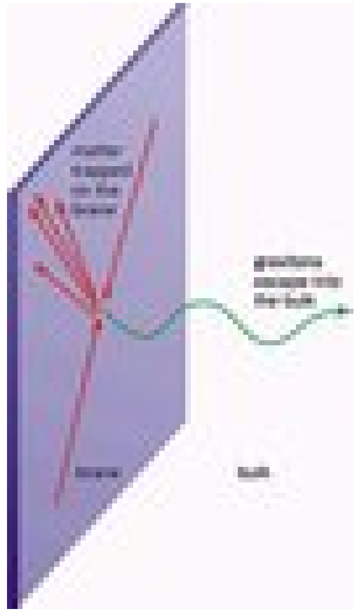
where F is a ñfiltering functionò obeying

F(x) -> 0 for x -> oo (small scales agree with GR)

F(x) >> 1 for x -> 0 (reducing the strength of gravity ï ñdegravitationingò - at large distances)

Brane-world models:

Dvali-Gabadadze-Porratiôs ñDGP-modelò



A 3 + 1 dimensional brane (= our world) imbedded in a 4 + 1 dimensional bulk

$$\text{Action}\ \ S_{DGP} = \tfrac{1}{2}\,(\hbar / l_5^{3} \int d^5 x \ \sqrt{-g_5}\ \mathcal{R}_5 +$$

$$+\ \tfrac{1}{2}\,(\hbar / l_{Planck}^{2}) \int d^4 x \ \sqrt{-g}\ \mathcal{R} + S_{\text{non grav}}$$

For small distances, gravity stays on the brane, so

gravitational potential $\sim r^{-1}$, $r \ll r_c = l_5^{3} / l_{Planck}^{2}$

For large distances, gravity can ñescapeò into the 5th dimension, so

gravitational potential $\sim r^{-2}$, $r \gg r_c = l_5^{3} / l_{Planck}^{2}$

so again, gravity is weakened on larger scales.

The DGP-model in more details

Action $S_{DGP} = ½\,(\hbar / l_5^{\,3}) \int d^5x \sqrt{-g_5}\; \mathcal{R}_5 + ½\,(\hbar / l_{Planck}^{\,2}) \int d^4x \sqrt{-g}\; \mathcal{R} + S_{non\ grav}$

=> DGP Friedmann equation:

$$H^2 \pm H/r_c = \frac{8\pi G}{3}\rho$$

Early times means large H means ~ ordinary Friedmann

Later times :

minus sign => $H \sim 1/r_c$ so constant H,

meaning $a \sim \exp(\alpha\ t) \Leftrightarrow$ acc expansion.

Also: coincidence problem ñsolvedò with $r_c \sim H_o$

plus-sign (& $H \ll 1/r_c$) => $H/r_c \sim \dfrac{8\pi G}{3}\rho$,

meaning no acceleration

INTERIM SUMMARY

1. With some fine-tuning, essentially all models are able

   to reproduce current observations

2. The cosmological constant gives a good over-all fit:

   ΛCDM ñconcordanceò model

3. Since the ΛCDM model gives so good an understanding of the early universe ï inflation, primordial nucleosyntheses, CMB, BAO ï any other model must be required not to interfere too much in the description of these phenomena. In other words, they should make themselves felt only *after* last scattering at z ~ 1090.

How might one differenti ate between models?

It is obviously important to study the period after z ~ 1090, in particular after z ~ 10 by

- improving on present observation : CMB, SNe, BAO, etc

- polarization in CMB

- more detailed studies of galaxy correlations, galaxy-CMB-correlations

- growth of structure after last scattering

Growth of structure after last scattering

* Initial conditions are known from CMB

* Study evolution of small structures from then until now:

Linear perturbation GR-theory

$ds^2 = c^2 \ dt^2 - a(t)^2 d\mathbf{x}^2 \ \ -> \ ds^2 = c^2 [1 + 2 \Psi(\mathbf{x},t)] dt^2 - a(t)^2 [1$ ï $2 \ \Phi(\mathbf{x},t)] d\mathbf{x}^2$

$\rho -> \rho (1 + \delta)$

$p \ -> \ p + \delta p$ etc

and then cancel all terms higher than linear in δ, Ψ, Φ, etc, in the EFE and energy-conservation equation, followed by Fourier transforming the linear quantities.

[But: Beware of gauge ambiguities!]

E g $\ddot{\delta}_q + 2 H \dot{\delta}_q + [v_{sound}^2 \ q^2 - 4 \pi G \ \rho] \ \delta_q = 0$

describes the growth of density perturbations.

Ways to study such phenomena observationally are, besides those already mentioned,, e g,

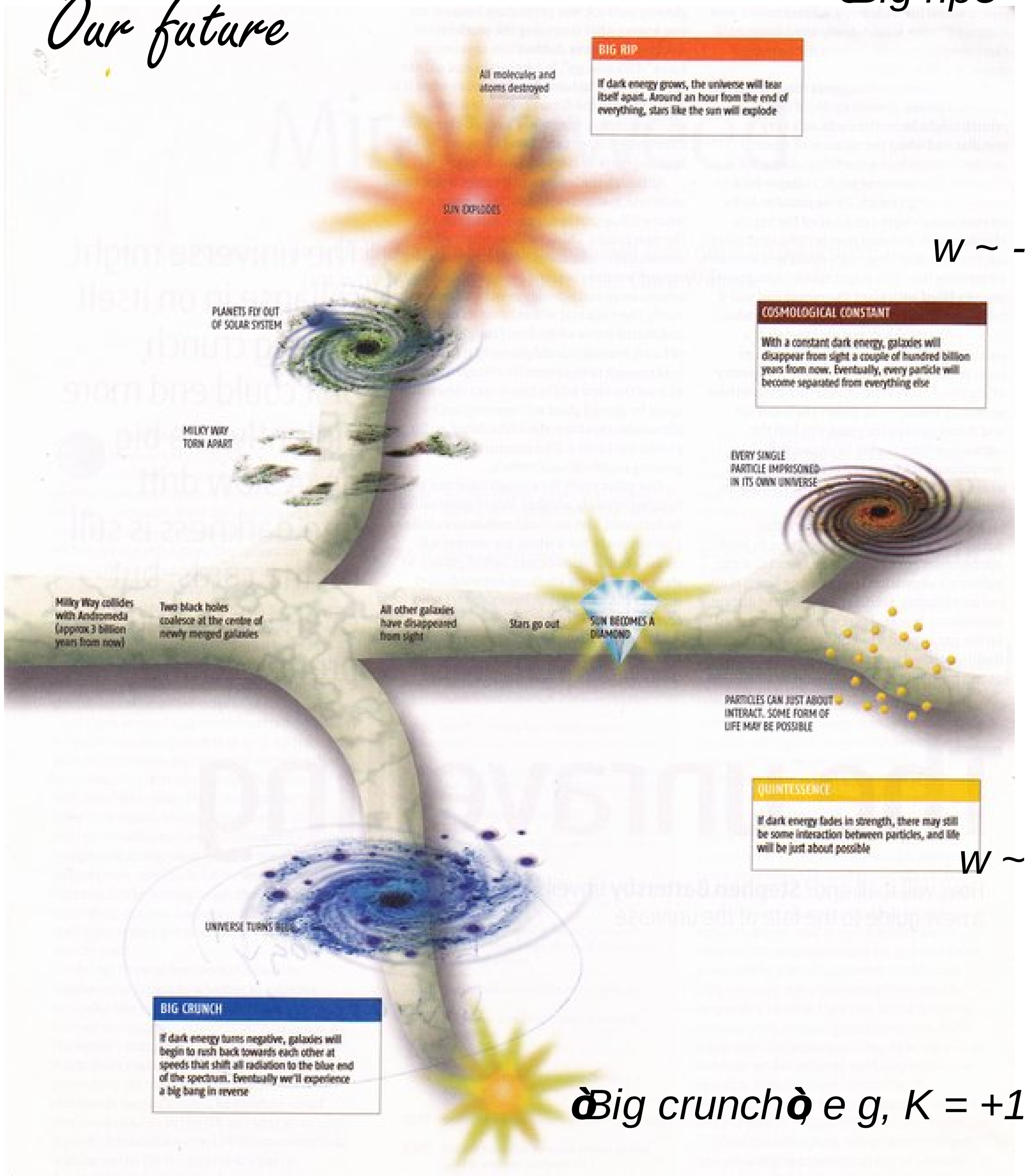
Weak gravitational lensing

(Integrated) Sachs- Wolfe effect

Using gamma ray burst as standard candles

??

Our future



ðBig ripò

$w \sim -1$, e g , Λ or ðfreezingò
quintessence

Not (only) geometry that decides fate, but (also, and more) the equation of state.

$w \sim 0$, e g, ðthawingò
quintessence

ðBig crunchò, e g, $K = +1$,or $\Lambda < 0$