Parton distribution functions and Monte Carlo simulations of photon-photon collisions
ATP seminar

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Introduction

- In high-energy proton-proton collisions interactions happens between the quarks and gluons (partons), described by Quantum Chromodynamics (QCD)

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Collinear Factorization

Factorize long and short distance physics:

\[
\, d\sigma^{p+p\rightarrow k+X} = \sum_{i,j,X'} f_i(x_1, Q^2) \otimes f_j(x_2, Q^2) \otimes d\hat{\sigma}^{ij\rightarrow k+X'} + \mathcal{O}(1/Q^2)
\]

- \(d\hat{\sigma}^{ij\rightarrow k+X'}\) calculated using perturbative QCD
- \(f_i(x, Q^2)\) non-perturbative but universal functions
- \(x\) fraction of proton momentum carried by the parton (\(\hat{p} = xp\))
Parton distribution functions (PDFs)

- PDFs cannot be calculated from first principles of QCD
- However, the $Q^2$ dependence is given by DGLAP evolution equations:

$$\frac{\partial f_i(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \sum_j \int \frac{dz}{z} P_{ij}(z) f_j(x/z, Q^2)$$

where $j$ runs over the parton flavours

The leading order (LO) splitting functions

<table>
<thead>
<tr>
<th>$P_{qq}(z)$</th>
<th>$P_{qg}(z)$</th>
<th>$P_{gq}(z)$</th>
<th>$P_{gg}(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{4}{3} \left[ \frac{1+2z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right]$</td>
<td>$\frac{4}{3} \left[ \frac{1+(1-z)^2}{z} \right]$</td>
<td>$\frac{1}{2} \left[ z^2 + (1-z)^2 \right]$</td>
<td>$6 \left[ \frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) + \frac{11-2n_f}{12} \delta(1-z) \right]$</td>
</tr>
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</table>
### Parton distribution functions (PDFs)

- **Problem:** The $x$-dependence not given by QCD
- **Solution:** Use experimental data to fix the non-perturbative input

### Global DGLAP analysis

1. Parametrize $f_i(x, Q^2)$ at chosen initial scale $Q_0 \sim 1$ GeV

   $$ f_i(x, Q_0^2) = N_i x^{a_i} (1 - x)^{b_i} F(x, c_i, \ldots) $$

2. Use DGLAP equations to calculate $f_i(x, Q^2)$ at $Q > Q_0$

   $$ \frac{\partial f_i(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z) f_j(x/z, Q^2) $$

3. Calculate cross section with the evolved PDFs

4. Fit to data to obtain the values for parameters $\{a_i\}$
Deep inelastic scattering

- Probe hadron with a lepton beam

![Kinematics diagram](image)

**Invariant variables**

\[
Q^2 = -q^2 \\
x = \frac{Q^2}{2 p \cdot q} \\
y = \frac{p \cdot q}{p \cdot k}
\]

- Provided by the scattered lepton

**Cross section**

\[
\frac{d\sigma^{\text{DIS}}}{dx dQ^2} = \frac{4\pi\alpha^2_{\text{EM}}}{Q^4} \frac{1}{x} \left[ xy^2 F_1(x, Q^2) + (1 - y) F_2(x, Q^2) \right]
\]

- Structure functions \(F_i(x, Q^2)\) can be directly related to PDFs
- At LO: \(2x F_1(x, Q^2) = F_2(x, Q^2) = \sum_q e_q^2 f_q(x, Q^2)\)
DGLAP evolution

The scale evolution of $F_2(x, Q^2)$ (HERA data, CTEQ6NLO fit)

- DGLAP evolution describes the data very well
  ⇒ With higher scales more partonic substructure observed

[JHEP 0207 (2002) 012]
Also other data available for the fits (from LHC and Tevatron)
For proton PDFs $\sim 4000$ data points available ($\sim 3000$ from DIS)
Precise data constraints at $x \gtrsim 10^{-4}$
The present status of proton PDF fits

PDF comparison

\[ xu(x, Q), \text{comparison} \]

Relative uncertainties

\[ xu(x, Q), \text{comparison} \]

- Good agreement between different analyses
- Uncertainties < 5% where there are data
Comparison to recent LHC data

Inclusive jet production in p+p

\[
\frac{d\sigma_{\text{jet}}}{dp_Tdy} = \sum_{i,j} \int dx_1 dx_2 x_1 f_i(x_1, Q^2) x_2 f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij\rightarrow \text{jet}}}{dp_Tdy}
\]

▶ Very good description of the data for jet production
⇒ Factorization works well at the LHC
▶ Small PDF uncertainties
In addition to p+p, also p+Pb and Pb+Pb collisions at the LHC
Structure functions modified in nuclear DIS:

\[ \frac{1}{A} \sigma_{\text{DIS}}(A) = \frac{1}{2} \sigma_{\text{DIS}}(D) \]

Modifications absorbed into process independent nuclear PDFs:

\[ f_i^A(x, Q^2) = R_i^A(x, Q^2) f_i(x, Q^2) \]

Global DGLAP analyses
- Provide the nuclear modifications \( R_i^A(x, Q^2) \)
- Test factorization of nuclear effects
Nuclear PDF analyses

Kinematic coverage of nuclear data:

Gluon nuclear modification:

- Much less data than for proton
- Sizeable uncertainties in the fits
- Discrepancies between analyses
- Fits not at the same level of accuracy as for protons
- Recent data from p+Pb collisions consistent with the predictions
  ⇒ Factorization of nuclear effects holds also at LHC energies
Monte Carlo event generators

- Hadronic collisions are complex events

Need to simulate:
1. Hard process
2. Parton shower
3. Multiple interactions
4. Beam remnants
5. Hadronization
6. Decays

... 

MC generators

- Herwig
- Sherpa
- Pythia

[JHEP 02 (2009) 007]
Typical simulation pattern

1. Generate hard process using
   - PDFs
   - Perturbative QCD
2. Generate parton shower using DGLAP equations
   - Initial state radiation (ISR)
   - Final state radiation (FSR)
3. Construct beam remnants
4. Hadronize event
   - String model

Very good description of charged particle yield
Photon-photon collisions
Photon-photon collisions

- Next large collider project most likely an electron-positron collider
  - Linear collider
  - Circular collider (FCC-ee)
- Clean environment to make precision measurements
- However, high-energy $e^\pm$ radiate photons
  \[ \Rightarrow \] background from photon-photon interactions
- High-energy photons can fluctuate to hadronic state

**Direct processes**

- Can be directly calculated

**Resolved processes**

- Need photon PDFs!
Photon PDFs

- Determined via global DGLAP analysis

### Photon scale evolution

- Inhomogeneous DGLAP equations for photons

\[
\frac{\partial f_i^\gamma(x, Q^2)}{\partial \log(Q^2)} = \frac{\alpha_{\text{EM}}}{2\pi} e_i^2 k_i(x) + \frac{\alpha_s(Q^2)}{2\pi} \sum_j \int_1^x \frac{dz}{z} P_{ij}(z) f_j(x/z, Q^2)
\]

where \( k_i(x) = 3(x^2 + (1 - x)^2) \) arise from the \( \gamma \to q\bar{q} \) splitting

- Solution has two components:

\[
f_i^\gamma(x, Q^2) = f_i^{\gamma,\text{pl}}(x, Q^2) + f_i^{\gamma,\text{had}}(x, Q^2)
\]

  - Point-like part, special solution of full equation
  - Hadron-like part, general solution of homogeneous part

⇒ Need non-perturbative input which is fixed by the data

\[
f_i^{\gamma,\text{had}}(x, Q_0^2) = N_i x^{a_i} (1 - x)^{b_i}
\]
Data for photon PDFs

- Photon structure functions can be measured in $e^- + e^+$ collisions

**“Photon DIS”**

- Other electron emits a virtual photon ($\gamma^*$)
  - $\Rightarrow$ This electron is measured
- Other electron is not detected as the scattering angle is small
  - $\Rightarrow$ Photon from this electron has small virtuality
- Also $W_{\gamma\gamma}$ need to be measured to construct kinematics

- Data available mainly from different LEP experiments ($\mathcal{O}(200)$ points)
- Precision and kinematic coverage more limited than for proton PDFs
Several groups have performed photon PDF analyses

Reasonable agreement between the data and the fits

Some differences between different analyses

Due to the point-like component $F_2^\gamma(x, Q^2)$ rises with $Q^2$ for all values of $x$ ($\neq F_2(x, Q^2)$ of protons)
Comparison to p+p collisions

Comparison to proton PDFs

- Photon DGLAP evolution
  \[
  \frac{\partial f_i(Q^2)}{\partial \log(Q^2)} = \frac{\alpha_{EM}}{2\pi} k_i + \frac{\alpha_s(Q^2)}{2\pi} P_{ij} \otimes f_j(Q^2)
  \]
  has the $\gamma \rightarrow q\bar{q}$ splitting
- Partons take larger fraction of momenta

Partonic cross section

- Cross section lower due to small coupling constant ($\alpha_{EM}^2 \sim 10^{-4}$)
- The slope of the cross section less steep
  \[\Rightarrow\] More high-$p_T$ partons
Monte Carlo simulations

- **Goal:** Simulate photon-photon collisions with **PYTHIA8**

**Requires**

- Implement photon PDFs into code
- Modify the beam remnant handling
- Modify the initial state radiation
  - Add possibility to the end up to a photon
- Include multiple partonic interactions
  - Add parametrization for total cross section

**Further developments**

- Consider also virtual photons (currently only real photons)
- Modelling of soft interactions (so far only hard interactions)
Parton distribution functions

- Proton PDFs can be accurately determined with global DGLAP-based analysis
- Also fits for nuclear PDFs but precision still quite limited
- Energetic photons can fluctuate into hadronic state
  ⇒ Also photons have partonic structure
- Photon PDFs can be determined using $e^+ + e^-$ data

Monte Carlo simulations

- Provide very good description of proton-proton collisions
- Simulations of photon-photon collisions important for future $e^+ + e^-$ colliders
- Work ongoing to implement photon-photon collisions into PYTHIA8
Extra Slides

Backup
Inclusive hadron production

Charged hadron spectra in p+Pb:

$\sqrt{s_{NN}} = 5020$ GeV

$|\eta| < 0.3$

$p+Pb$

$1/N_c d^2 N^{\text{ch}}/d^2 p_T$ [GeV$^{-2}$c$^{-2}$]

Data/NLO

NLO calculations overshoot the data at $p_T > 10$ GeV/c


Nuclear modification ratio

$R_{pPb}^{h}(p_T, \eta) = \frac{1}{208} \frac{d^2\sigma_{pPb}^{h}}{dp_T d\eta} / \frac{d^2\sigma_{pp}^{h}}{dp_T d\eta}$

$\sqrt{s} = 5.0$ TeV

$|\eta| < 0.3$


$\langle T_{pPb} \rangle = 0.0983 \pm 0.0035$ mb$^{-1}$

$\epsilon = 5.0$ TeV

|$\eta$| < 0.3

EPS09 uncert.

▸ FF differences cancel in ratio

$\Rightarrow R_{pPb}$ not sensitive to FFs

▸ Enhancement in the data at $p_T \sim 3$ GeV/c

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I. Helenius (Lund U.)
Dijets in p+Pb

**Dijet pseudorapidity**

\[
\eta_{\text{dijet}} = \frac{\eta_1 + \eta_2}{2}
\]

- at \(\eta_{\text{dijet}} < 0\) data sensitive to antishadowing region
- at \(\eta_{\text{dijet}} > 0\) data sensitive to EMC effect
- Good description with EPS09

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CMS pPb 35 nb\(^{-1}\)
\[\sqrt{s_{\text{NN}}} = 5.02\text{ TeV}\]
\(p_{T,1} > 120\text{ GeV/c}\)
\(p_{T,2} > 30\text{ GeV/c}\)
\(\Delta\phi_{1,2} > 2\pi/3\)
All \(E_T^{\text{jet}} > 4\)

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[ JHEP 1310 (2013) 213 ]
$W^\pm$ production

Forward-backward asymmetry

\[ \frac{N(+\eta_{\text{lab}})}{N(-\eta_{\text{lab}})} \]

- Sum over $W^+$ and $W^-$
- $\eta_{\text{lab}} = \eta + 0.465$
  where $\eta$ pseudorapidity in nucleon-nucleon CMS frame
- Dominating processes:
  $u\bar{d} \rightarrow W^+$ and $d\bar{u} \rightarrow W^-$
- Sensitive to
  - $\eta_{\text{lab}} > 0$: $0.002 < x < 0.02$
  - $\eta_{\text{lab}} < 0$: $0.02 < x < 0.2$
- Good agreement with EPS09

The gluon nPDFs at small-$x$ remain badly constrained!

[arXiv:1503.05825]
[JHEP 03 (2011) 071]