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# EFFECTIVE FIELD THEORIES OF QCD

264. WE-Heraeus-Seminar  
Physikzentrum Bad Honnef, Bad Honnef, Germany  
November 26 — 30, 2001

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## ABSTRACT

These are the proceedings of the workshop on “Effective Field Theories of QCD” held at the Physikzentrum Bad Honnef of the Deutsche Physikalische Gesellschaft, Bad Honnef, Germany from November 26 to 30, 2001. The workshop concentrated on Chiral Perturbation Theory in its various settings, its relations with lattice QCD and dispersion theory and the use of effective field theories in systems with one, two or more nucleons as well as at high baryon densities.

Included are a short contribution per talk and a listing of some review papers on the subject.

# 1 Introduction

The use of effective field theory techniques is an ever growing approach in various fields of theoretical physics. Along with the continuing application of Chiral Perturbation Theory, Nuclear Effective Field Theory and, more recently, QCD at high baryon density have been the focus of many investigations. We therefore decided to organize the next topical workshop. This meeting followed the series of workshops in Ringberg (Germany), 1988, Dobogókő (Hungary), 1991, Karrebæksmunde (Denmark), 1993, Trento (Italy), 1996 and Bad Honnef (Germany), 1998. All these workshops shared the same features, about 50 participants, a fairly large amount of time devoted to discussions rather than presentations and an intimate environment with lots of discussion opportunities.

This meeting took place in late fall 2001 in the Physikzentrum Bad Honnef in Bad Honnef, Germany and the funding provided by the WE-Heraeus-Stiftung allowed us to provide for the local expenses for all participants and to support the travel of a fair amount of participants. The WE-Heraeus foundation also provided the administrative support for the workshop in the person of the able secretary Heike Uebel. We extend our sincere gratitude to the WE-Heraeus Stiftung for this support. We would also like to thank the staff of the Physikzentrum for the excellent service given to us during the workshop and last but not least the participants for making this an exciting and lively meeting.

The meeting had 57 participants whose names, institutes and email addresses are listed below. 48 of them presented results in presentations of various lengths. A short description of their contents and a list of the most relevant references can be found below. As in the previous three of these workshops we felt that this was more appropriate a framework than full-fledged proceedings. Most results are or will soon be published and available on the archives, so this way we can achieve speedy publication and avoid duplication of results in the archives.

Below follows first the program, then the list of participants and a subjective list of review papers, lectures and other proceedings relevant to the subject of this workshop.

Johan Bijnens, Ulf-G. Meißner and Andreas Wirzba

## 2 The Program

### Monday, November 26th, 2001

#### *Early Afternoon Session*

Chairperson: Johan Bijnens

#### **Chiral Perturbation Theory**

14:00 Ulf-G. Meißner /  
Ernst Dreisigacker

Introductory Remarks

14:20 Barry R. Holstein  
(Amherst)

Effective Field Theory and Gravity

14:55 Jürg Gasser (Bern)

The Quark Condensate from  $K_{e4}$  Decays

15:35 Bastian Kubis (Jülich)

Isospin Violation in Pion-Kaon Scattering

16:00 Coffee

#### *Late Afternoon Session*

Chairperson: Andreas Wirzba

#### **Chiral Perturbation Theory**

16:30 Joaquim Prades (Granada)

Matching the Elektroweak Penguins  $Q_7$  and  $Q_8$   
at NLO

17:05 Paul Büttiker (Jülich)

Chiral and Dispersive Description of Pion Kaon  
Scattering

17:30 Elisabetta Pallante (Trieste)

Kaon Decays on and off the Lattice

17:55 Pere Talavera (Marseille)

On the Light Meson Formfactors

18:20 End of Session

18:30 Dinner

### Tuesday, November 27th, 2001

#### *Early Morning Session*

Chairperson: Ulf-G. Meißner

#### **Chiral Perturbation Theory**

09:00 Gerhard Ecker (Wien)

Hadronic Vacuum Polarization

09:40 Akaki Rusetsky (Bern)

Hadronic Atoms in Effective Field Theories

10:20 Santiago Peris (Barcelona)

Large- $N_c$  QCD and Weak Matrix Elements

10:45 Coffee

#### *Late Morning Session*

Chairperson: Ulf-G Meißner

#### **Chiral Perturbation Theory**

11:15 Eugene Golowich (Amherst)

Determination of  $\langle(\pi\pi)_{I=2}|Q_{7,8}|K^0\rangle$  in the  
Chiral Limit

11:50 Luca Girlanda (Padova)

Analysis and Interpretation of New Low-Energy  
Pion Pion Scattering Data

12:15 Jan Stern (Orsay)

New Approach to Three-Flavour Chiral Dynamics

12:55 End of Session

13:00 Lunch

#### *Early Afternoon Session*

Chairperson: Johan Bijnens

#### **Chiral Perturbation Theory**

14:20 Marc Knecht (Marseille)

Low-Energy Physics of the Standard Model and

- Large- $N_c$  QCD
- 14:50 Andreas Nyffeler (Marseille) Hadronic Light-by-Light Scattering Corrections to the Muon  $g-2$ : the Pion-Pole Contribution
- 15:10 Sebastian Descotes (Southampton)  $N_f$ -Sensitivity of Chiral Dynamics
- 15:35 Hartmut Wittig (Liverpool) Effective Chiral Lagrangian from Simulations of Partially Quenched QCD
- 16:00 Coffee
- Late Afternoon Session*
- Chairperson: Johan Bijnens **Chiral Perturbation Theory**
- 16:30 Vincenzo Cirigliano (Wien) Radiative Corrections to  $K_{l3}$  Decays
- 16:55 Roland Kaiser (San Diego) Light Quark Mass Ratios from Large  $N_c$  Chiral Perturbation Theory
- 17:20 Frederik Persson (Lund) Recalculation and Reanalysis of  $K \rightarrow 3\pi$
- 17:55 Bugra Borasoy (München) Decays of the  $\eta'$  in Chiral Perturbation Theory
- 18:20 End of Session
- 18:30 Dinner

**Wednesday, November 28th, 2001**

*Early Morning Session*

- Chairperson: Andreas Wirzba **Chiral Nucleon Dynamics**
- 09:00 Dieter Drechsel (Mainz) The Polarizability of the Nucleon: The View of Dispersion Relations
- 09:35 Pavel Pobylitsa (Bochum) Chiral Dynamics and Large- $N_c$  Baryons: Model-Independent Results for Parton Distributions
- 10:05 Thomas R. Hemmert (München) Chiral EFT with Explicit Spin 3/2 Fields: A Status Report
- 10:40 Coffee

*Late Morning Session*

- Chairperson: Andreas Wirzba **Chiral Nucleon Dynamics**
- 11:15 Matthias Frink (Jülich) Analysis of the Pion-Kaon Sigma Term
- 11:40 Maxim V. Polyakov (Bochum) New Soft Pion Theorem for Hard Near Threshold Pion Production
- 12:05 Eulogio Oset (Valencia) Chiral Dynamics in Meson-Baryon, N-N and Weak LambdaN-NN Interactions
- 12:45 End of Session
- 13:00 Lunch

*Early Afternoon Session*

- Chairperson: Ulf-G. Meißner **Chiral Few-Nucleon Dynamics**
- 14:30 Silas R. Beane (Seattle) Perturbative Theories of Nuclear Forces
- 15:10 Joan Soto (Barcelona) Renormalizing the Lippmann-Schwinger Equation for the OPE Potential
- 15:35 Harald W. Griesshammer Very Low Energy Deuteron Compton Scattering and

(München) Dispersive Effects in Nucleon Polarisabilities  
 16:00 Coffee  
*Late Afternoon Session*  
 Chairperson: Ulf-G. Meißner  
 16:30 Hermann Krebs (Jülich) **Chiral Few-Nucleon Dynamics**  
 Pion Electroproduction on the  
 Deuteron Near the Threshold  
 16:55 Bira van Kolck (Tucson) One Two Three ... Infinity: Nucleons in EFT  
 17:35 Hans-Werner Hammer Few-Body Physics in Effective Field  
 (Ohio) Theory  
 18:15 End of Session  
 18:30 Dinner

**Thursday, November 29th, 2001**

*Early Morning Session*

Chairperson: Johan Bijnens **Chiral Few- and Many-Nucleon Dynamics**  
 09:00 Walter Glöckle Applications of Chiral Nuclear Forces  
 (Bochum) to Few-Nucleon Systems  
 09:40 Evgeny Epelbaum Nuclear Forces from Chiral EFT: Recent  
 (Jülich) Developments  
 10:20 Norbert Kaiser (München) Chiral Dynamics of Nuclear Matter  
 10:55 Coffee

*Late Morning Session*

Chairperson: Johan Bijnens **Chiral Many-Body Dynamics and High Density**  
 11:25 José A. Oller (Jülich) In-Medium ChPT beyond the Mean-Field  
 Approach  
 12:05 Mark Alford (Glasgow) Color Superconductivity in High-Density QCD  
 12:45 End of Session  
 13:00 Lunch

*Early Afternoon Session*

Chairperson: Andreas Wirzba **High-Density QCD**  
 14:30 Krishna Rajagopal Crystalline Color Superconductivity  
 (Cambridge)  
 15:10 Dirk Rischke (Frankfurt) The Prefactor of the Color-Superconducting Gap  
 15:50 Coffee

*Late Afternoon Session*

Chairperson: Andreas Wirzba **Effective Theories of High-Density QCD  
 and Lattice QCD**  
 16:20 Roberto Casalbuoni Effective Theories for QCD at High Density  
 (Firenze)  
 17:00 Ismail Zahed (Stony Brook) Dense QCD  
 17:40 Harald Markum (Wien) Color Superfluidity in Two Color QCD  
 on the Lattice  
 18:05 End of Session

18:30 Dinner

### Friday, November 30th, 2001

#### *Early Morning Session*

Chairperson: Ulf-G. Meißner

09:00 Martin Savage (Seattle)

09:40 Tilo Wettig (New Haven)

10:15 Christof Gatttringer  
(Regensburg)

10:40 Coffee

#### *Late Morning Session*

Chairperson: Ulf-G. Meißner

11:10 Klaus Schilling  
(Wuppertal)

11:45 C.-J. David Lin  
(Southampton)

12:10 Johan Bijnens

12:20 Lunch and End of Workshop

#### **Lattice QCD**

Baryons in Quenched Chiral Perturbation Theory

Chiral Symmetry and the Spectrum of the QCD  
Dirac Operator

Calorons Near the QCD Phase Transition

#### **Lattice QCD**

$\eta'$  Mass from QCD on the Lattice

Non-leptonic Kaon Decays from Lattice QCD

Farewell

## 3 Participants and their email

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## 4 A short guide to review literature

Chiral Perturbation Theory grew out of current algebra, and it soon was realized that certain terms beyond the lowest order were also uniquely defined. This early work and references to earlier review papers can be found in [1]. Weinberg then proposed a systematic method in [2], later systematized and extended to use the external field method in the classic papers by Gasser and Leutwyler [3],[4], which, according to Howard Georgi, everybody should put under his/her pillow before he/she goes to sleep. The field has since then extended a lot and relatively recent review papers are: Ref.[5] with an emphasis on the anomalous sector, Ref.[6] giving a general overview over the vast field of applications in various areas of physics, Ref.[7] on mesons and baryons, and Ref.[8] on baryons and multibaryon processes. This is updated in Ref.[9]. In addition there are books by Georgi[10], which, however, does not cover the standard approach, including the terms in the lagrangian at higher order and a more recent one by Donoghue, Golowich and Holstein[11].

There are also the lectures available on the archives by H. Leutwyler [12] E. de Rafael [13], A. Pich [14], G. Ecker [15] as well as numerous others (the single nucleon sector is covered in most detail in [16]). The references to the previous meetings are [17],[18],[19],[20],[21]. There are also the proceedings of the Chiral Dynamics meetings at MIT (1994) [22], in Mainz (1997) [23] and at Jefferson Lab [24]. The DAΦNE handbook [25] also contains useful overviews. The few-nucleon sector is covered in the proceedings of the Caltech and Seattle workshops [26], [27], respectively, in the reviews by van Kolck [28] and by Beane, Bedaque, Haxton, Philips and Savage [29]. There also exist reviews on the novel developments in dense QCD, see Refs.[30]. A recent condensed overview can be found in [31].

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## Gravity and Effective Field Theory

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We use effective field theory techniques to examine the quantum corrections to the gravitational metrics of charged particles, with and without spin. Gravity couples to the energy momentum tensor  $T_{\mu\nu}$  of a particle[1]. We calculate the energy momentum tensor in a power series in  $\alpha$ , using usual Feynman diagram techniques. The results are expressed in terms of form factors  $F(q^2)$  of the various allowed Lorentz structures in the matrix element of  $T_{\mu\nu}$ . Normally, form factors can be expanded in a power series in  $q^2$  around  $q^2 = 0$ , with the coefficients being related to the structure of the particle. However, in momentum space the masslessness of the photon implies the presence of nonanalytic pieces  $\sim \sqrt{-q^2}$ ,  $q^2 \log -q^2$ , etc. in the form factors of the energy-momentum tensor. By transforming to coordinate space we show how the former reproduces the classical non-linear terms of the Reissner-Nordström[2] and Kerr-Newman[3] metrics or order  $G\alpha/r^2$  while the latter can be interpreted as quantum corrections to these metrics, of order  $G\alpha\hbar/mr^3$ .

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# The quark condensate from $K_{e4}$ decays

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Roy equations [1] combined with chiral symmetry allow one to predict [2] the low-energy behavior of the  $\pi\pi$  scattering amplitude with high precision. The prediction is based on the hypothesis that the quark condensate is the leading order parameter of spontaneously broken chiral symmetry. This has been questioned in Refs. [3,4], where it is emphasized that experimental evidence for this scenario is not available. The recent result from the high statistics  $K_{e4}$  experiment E865 at Brookhaven [5] closes this gap, because those data allow one to determine the size of the leading term in the quark mass expansion of the pion mass. E865 confirms [5] the hypothesis that underlies our prediction of the  $\pi\pi$   $S$ -wave scattering lengths: more than 94% of the pion mass stems from the quark condensate. The generalized framework of  $SU(2) \times SU(2)$  chiral perturbation theory developed in [3], that allows for a small or vanishing condensate, can therefore be dismissed.

*Note added:* In Ref. [6], the E865 data [5] have been analyzed without the use of chiral symmetry constraints.

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# Isospin violation in low-energy pion-kaon scattering

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In order to extract the two pion-kaon S-wave scattering lengths from  $\pi K$  bound state experiments [1], the measurable observables have to be related to combinations of these scattering lengths by modified Deser formulae that include next-to-leading order effects in isospin breaking. In particular, these involve the isospin violating corrections in the regular parts of the scattering amplitudes  $\pi^- K^+ \rightarrow \pi^0 K^0$  ( $\pi^- K^+$ ) at threshold. We have evaluated these corrections up to one-loop order, including leading effects both in the light quark mass difference  $m_u - m_d$  and the electromagnetic fine structure constant  $\alpha = e^2/4\pi$  [2] (see also [3]). The numerical results are

$$a_0(\pi^- K^+ \rightarrow \pi^0 K^0) = \tag{1}$$

$$-\sqrt{2} a_0^- \left\{ \underbrace{(1 \pm 0.8\%)}_{\mathcal{O}(p^4)} + \underbrace{0.8\%}_{\mathcal{O}(e^2)} + \underbrace{0.5\%}_{\mathcal{O}(m_u - m_d)} - \underbrace{(0.8 \pm 0.7)\%}_{\mathcal{O}(p^2 e^2)} + \underbrace{(0.7 \pm 0.2)\%}_{\mathcal{O}(p^2(m_u - m_d))} \right\},$$

$$a_0(\pi^- K^+ \rightarrow \pi^- K^+) = \tag{2}$$

$$(a_0^- + a_0^+) \left\{ \underbrace{(1 \pm 16.1\%)}_{\mathcal{O}(p^4)} + \underbrace{1.2\%}_{\mathcal{O}(e^2)} - \underbrace{(0.3 \pm 3.2)\%}_{\mathcal{O}(p^2 e^2)} + \underbrace{0.2\%}_{\mathcal{O}(p^2(m_u - m_d))} \right\}.$$

The isospin symmetric one-loop representation allows for a very precise prediction of the isovector scattering length  $a_0^-$  that enters the pion-kaon atom lifetime formula. Isospin breaking corrections for the charged-to-neutral channel give rise to a small shift of about one percent, and only a slight increase of the error range. The uncertainty in a chiral prediction for the  $2S-2P$  energy level shift is however dominated by the large uncertainty in the isoscalar scattering length  $a_0^+$ . Isospin breaking corrections in the relevant charged-to-charged channel are again small, though with a larger error range, which still allow for a sufficiently accurate extraction of the combination  $a_0^- + a_0^+$  from experiment.

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# Matching the Electromagnetic Penguins $Q_7$ and $Q_8$

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This talk describes the work done together with Hans Bijmens and Elvira Gámiz in [1]. Exact analytical expressions for the  $\Delta S = 1$  coupling  $\text{Im } G_E$  in terms of observable spectral functions are given. This coupling determines the size of the  $\Delta I = 3/2$  contribution to  $\varepsilon'$ . We show analytically how the scheme and the scale dependences vanish to all orders in  $1/N_c$  and NLO in  $\alpha_S$  explicitly both for  $Q_7$  and  $Q_8$ . Numerical results are derived for both  $Q_7$  and  $Q_8$  from the  $\tau$ -data and known results on the scalar spectral functions. In particular, we study the effect of all higher dimension operators. The coefficients of the leading operators in the OPE of the needed correlators are derived to NLO in  $\alpha_S$ . Related work can be found in the recent papers [2,3,4].

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# Chiral and Dispersive Description of $\pi K$ Scattering

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ChPT provides the low energy effective theory of the standard model that describes the interactions involving hadronic degrees of freedom. It requires the introduction of additional low energy constants (LEC) at each order of the loop-expansion. In  $SU(3)$  at next-to-leading order, ten LECs  $L_i^r(\mu)$  enter the calculations. Recently, there has been a revival in interest in the constant  $L_4^r$  as it reflects the dependence of the decay constant  $F_\pi$  in the chiral limit on the number of chiral quarks [1]. However, these quantities cannot be fixed by chiral symmetry only; experimental information is needed to pin them down. Using the  $K_{l4}$  form factors and  $\pi\pi$  sum rules provide accurate estimates for some of the LECs (e.g.  $L_1^r, L_2^r, L_3^r$ ) but leave the large  $N_c$ -suppressed LEC  $L_4^r$  essentially undetermined. This is due to the fact that in such processes, contributions from  $L_4^r$  accidentally are suppressed. Such a suppression does not occur in  $\pi K$  scattering [2]. Therefore this process is suitable for a more reliable determination of  $L_4^r$  and also provides estimates for  $L_1^r, L_2^r$  and  $L_3^r$ . To fix these quantities we use dispersion relations [3], relying on analyticity, unitarity and crossing symmetry only, which are the suitable tools for a comparison of experimental data with ChPT. By combining fixed- $t$  and hyperbolic dispersion relations for  $T^+(s, t, u)$  and  $T^-(s, t, u)$  and rewriting the chiral amplitudes, we find an appropriate matching of the chiral and the dispersive representations of these amplitudes. Saturating the dispersive integrals with the available data, we find the following estimates (renormalization-scale  $\mu = m_\rho$ ):

$$L_1^r = 0.84 \pm 0.15, L_2^r = 1.36 \pm 0.13, L_3^r = -3.65 \pm 0.45, L_4^r = 0.22 \pm 0.30$$

yielding an estimate for  $L_4^r$  with improved reliability.

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# Kaon decays on and off the lattice

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The theoretical prediction of kaon-decay amplitudes, along with the determination of the  $\Delta I = 1/2$  ratio and the CP-violating parameter  $\varepsilon'/\varepsilon$ , still remains a challenging problem because of the crucial interplay between long- and short-distance contributions. Relevant issues are the calculation of final state interactions (FSI), especially in the  $I = 0$  channel, and the consequences of implementing (partial) quenching in a lattice determination of  $K \rightarrow \pi\pi$  or  $K \rightarrow \pi$  amplitudes.

A Standard Model (SM) prediction of  $\varepsilon'/\varepsilon$  has been derived in [2], where FSI effects [1] in the  $I = 0, 2$  channels have been included by means of ChPT,  $1/N_c$  expansion and the Omnès resummation. The SM prediction  $\text{Re}(\varepsilon'/\varepsilon) = (17 \pm 9) \cdot 10^{-4}$  is in good agreement with the present experimental value  $\text{Re}(\varepsilon'/\varepsilon) = (17.2 \pm 1.8) \cdot 10^{-4}$ , although still affected by a large uncertainty which is dominated by the large- $N_c$  approximation and the uncertainty in the determination of the strange quark mass.

The calculation of weak matrix elements of strong penguin operators is implemented on the lattice within the (partially) quenched approximation, with  $K$  valence and ghost quarks and  $N$  sea quarks ( $N = 0$  in the fully quenched limit). It has been shown in [3] how partial quenching modifies the transformation properties of QCD penguin operators under chiral symmetry. In the (P)QChPT realization of  $(8_L, 1_R)$  penguin operators, new  $(8_L, 8_R)$  operators appear at leading order in the chiral expansion as pure quenching artifacts. It is crucial to uncover the size of the contamination due to the new operators [4], or alternatively develop a strategy which removes those effects *ab initio*. This issue is currently under study.

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# On the Light Meson Formfactors

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We show the next-to-next-to-leading order (NNLO) results [1] on the pseudoscalar vector formfactors in the three flavour case. Our results are confronted with earlier similar works in the NLO case [2] and the NNLO in the case of two flavours [3,4].

We discuss all the assumptions besides chiral symmetry that come in the calculation, special attention is devoted to the large- $N_c$  arguments. We update the value for the three flavour  $\mathcal{O}(p^4)$  parameters,  $L_i^r$ ,  $i = 1, \dots, 8$ , [5] and in addition we determine the low-energy constant  $L_9^r$ . Together with this we show also the ratio  $m_u/m_d$  for the quark-masses [6] as well as the dependence on  $m_s/\hat{m}$ .

Finally we discuss some ad hoc changes in the values of  $L_4^r$  and  $L_6^r$  to have a “well behaved chiral series” for the pseudoscalar masses. This suggests that values for  $L_4^r$  and  $L_6^r$  at the edge of the range given by large- $N_c$  can satisfy the requirement  $\mathcal{O}(p^2) > \mathcal{O}(p^4) > \mathcal{O}(p^6)$  for all existing calculated processes.

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# Hadronic Vacuum Polarization

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The hadronic vacuum polarization is an important ingredient for the calculation of the anomalous magnetic moment of leptons and of the running fine structure constant. For the muon anomalous magnetic moment, the two-pion intermediate state provides the dominant contribution. To incorporate the precise experimental results for  $\tau^- \rightarrow \pi^0 \pi^- \nu_\tau$  in the determination of hadronic vacuum polarization, the leading isospin-violating and electromagnetic corrections for this decay at low energies were calculated [1]. The corrections are small but relevant at the level of precision necessary for a comparison between the standard model prediction for the muon anomalous magnetic moment and experiment. More generally, the corrections account for the main part of the systematic differences between the measured form factors in  $\tau^- \rightarrow \pi^0 \pi^- \nu_\tau$  and  $e^+ e^- \rightarrow \pi^+ \pi^-$  at low to intermediate energies.

In the calculation of the running fine structure constant entering the analysis of electroweak precision measurements, a major theoretical uncertainty is due to the four-pion intermediate state. The first complete calculation of  $e^+ e^- \rightarrow 4\pi$  to  $O(p^4)$  in chiral perturbation theory is presented [2]. Although the chiral amplitude cannot be used directly in the region of main interest ( $E_{\text{cms}} > 1$  GeV) it exhibits the correct low-energy structure to be continued into the resonance region. Important additional contributions occurring at  $O(p^6)$  due to  $\omega$  and  $a_1$  exchange have been included. Cross sections for the final states  $2\pi^0 \pi^+ \pi^-$  and  $2\pi^+ 2\pi^-$  are displayed for  $E_{\text{cms}} \leq 1$  GeV.

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# Splitting Electromagnetic and Strong Interactions

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In order to fully exploit the high-precision experimental data on hadronic atoms which is available at present and which will be provided in the future, it is imperative to design a theoretical framework that describes this type of bound systems to an accuracy that matches the experimental precision [1]. The Deser-type relations which are then used to extract the “purely strong” scattering lengths from the measured values of the strong energy shift in the ground state, and the partial decay width into hadronic channels, contain isospin-breaking corrections which depend on the fine structure constant  $\alpha$  and on the quark mass difference  $m_d - m_u$ . The following question arises: The scattering lengths are calculated in QCD at  $m_u = m_d$ . How is this theory related to a framework that includes isospin violating effects,  $SU(3)_C \times U(1)_{em}$ ? [Note that, in view of the announced accuracy 0.3% in the future measurement of the energy shift of the  $\pi^-p$  atom by the Pionic Hydrogen collaboration at PSI, the question of the precise definition of the pure QCD limit is not an academic one.]

The answer is the following [2]: In two-flavour QCD, the quark mass  $m_u = m_d$  and strong coupling constant may be fixed from the requirement that the pion and nucleon masses are equal - by convention - to the experimental values of the charged pion and the proton masses, respectively. In the effective theory of  $SU(3)_C \times U(1)_{em}$ , the values of the strong LECs stay put. With this definition, the isospin-breaking corrections to the bound-state characteristics can be calculated in a general theory of hadronic atoms [1], which has been successfully applied to the calculation of the  $\pi^+\pi^-$  atom decay width [1], and to the energy shift of the  $\pi^-p$  atom [3].

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# Large- $N_c$ QCD and weak matrix elements

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The use of large  $N_c$  for the calculation of matrix elements of electroweak operators was pioneered in the work of [1]. With time, those ideas have evolved into a new approach, the Minimal Hadronic Approximation (MHA) to large- $N_c$  QCD. In this approximation resonances are introduced by requiring appropriate matching conditions with the Operator Product and the Chiral Expansions. In this way a scheme independent result is obtained.

In the talk I reviewed the use of the MHA in the calculation of several electroweak matrix elements in the Standard Model. These included the  $\pi^+ - \pi^0$  mass difference[2] which is a very interesting theoretical laboratory; matrix elements of EW penguins[3] which are of relevance in the determination of  $\epsilon'/\epsilon$ ; the decays  $\pi^0 \rightarrow e^+e^-$  and  $\eta \rightarrow \mu^+\mu^-$ [4] for which experimental results are available; and, finally,  $B_K$  in the chiral limit[5] which has also been computed on the lattice[6].

I thank V. Cirigliano, E. Golowich and D. Lin for very interesting discussions during the workshop and M. Golterman, T. Hambye, M. Knecht, M. Perrottet and E. de Rafael for a very pleasant collaboration.

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# Determination of $\langle(\pi\pi)_{I=2}|\mathcal{Q}_{7,8}|K^0\rangle$ in the Chiral Limit

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We discuss evaluation of the weak matrix elements  $\langle(\pi\pi)_{I=2}|\mathcal{Q}_{7,8}|K^0\rangle$  in the chiral limit [1]. The perturbative matching is accomplished fully within the scheme dependence used in the two loop weak OPE calculations. The effects of dimension eight (and higher dimension) operators are fully accounted for. We thus obtain expressions for the weak matrix elements in terms of two dispersive sum rules which involve weights of the V-A spectral function. Our numerical analysis of the sum rules is fortified by data from  $\tau$  decay, constraints from the classical chiral sum rules and use of the weak OPE, and a careful assessment of the attendant uncertainties is given. Our result implies that the electroweak penguin contribution to  $\epsilon'/\epsilon$  is

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{EWP}} = (-2.2 \pm 0.7) \times 10^{-3} \quad . \quad (1)$$

Comparison with other approaches is given [2]. According to the analysis carried out in Ref. [3],  $\mathcal{O}(p^2)$  corrections to our leading chiral component will be of order 30% and negative.

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# Analysis of new low-energy $\pi\pi$ scattering data

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We present an analysis[1] of the recently published E865 data[2] on charged  $K_{e4}$  decays and  $\pi\pi$  phases, to extract values of the two s-wave scattering lengths, of the subthreshold parameters  $\alpha$  and  $\beta$ , of the low-energy constants  $\bar{l}_3$  and  $\bar{l}_4$  as well as of the main two-flavour order parameters:  $\langle\bar{u}u\rangle$  and  $F_\pi$  in the limit  $m_u = m_d = 0$  taken at the physical value of the strange quark mass. Our analysis is exclusively based on the new solutions of the Roy Equations by Ananthanarayan, Colangelo, Gasser and Leutwyler [3] and on direct experimental information. In particular no use is made of the correlation between  $a_0^0$  and  $a_0^2$  as inferred from the scalar radius of the pion[4]. Instead, the phaseshifts extracted from  $K_{e4}$  data are supplemented with data on  $\delta_0^2$  below 800 MeV. The result

$$a_0^0 = 0.228 \pm 0.012, \quad a_0^2 = -0.0382 \pm 0.0038, \quad (1)$$

is compared with the theoretical relation between  $2a_0^0 - 5a_0^2$  and the scalar radius of the pion obtained in two-loop standard Chiral Perturbation Theory[5]. If the dispersive determination of the latter,  $\langle r^2 \rangle_s = (0.61 \pm 0.04) \text{ fm}^2$  is used, one finds a disagreement at the 1- $\sigma$  level. We argue that this discrepancy can be explained by an unexpectedly large value of the  $O(p^6)$  counterterms contributing to the above-mentioned relation. This in turn could be a manifestation of the exceptional status of the scalar channel, characterized by a strong  $\pi\pi$  continuum and OZI rule violation.

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# New Approach to the Three Flavor Chiral Dynamics

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As the number  $N_f$  of light quark flavors increases, vacuum fluctuations of  $\bar{q}q$  - pairs become more important reducing the chiral condensate and affecting the convergence of the standard  $ChPT$  expansion [1]. In practice, already two- and three- flavor dynamics may be rather different: In the limit  $m_u = m_d = m_s = 0$  the remaining massive quarks are heavy compared to  $\Lambda_{QCD}$  and they barely affect the vacuum structure. The corresponding three-flavor condensate  $\Sigma(3) = - \langle \bar{q}q \rangle |_{m_u=m_d=m_s=0}$  is close to the genuine condensate of the purely massless theory. On the other hand, the ground state in the  $N_f = 2$  chiral limit  $m_u = m_d = 0$  with  $m_s$  fixed at its physical value ( $m_s \sim 150 MeV \leq \Lambda_{QCD}$ ) is polluted by **massive**  $\bar{s}s$  pairs. The latter can modify the  $SU(2) \times SU(2)$  chiral structure of the vacuum through OZI - rule violating transition  $\bar{s}s \leftrightarrow \bar{u}u + \bar{d}d$ . The two-flavor condensate then consists of two distinct pieces:  $\Sigma(2) = \Sigma(3) + m_s Z_s + \dots > \Sigma(3)$ , where **the induced condensate**  $m_s Z_s$  reflects the vacuum correlation between  $0^{++}$  strange and non-strange pairs. The latter is related to the LEC  $L_6(\mu)$ . Sum rule estimates indicate that the induced condensate can be actually rather large [1], despite its suppression in the large  $N_c$  limit. We propose a reordering of the  $N_f = 3$  chiral expansion based on a resummation of large fluctuations such as  $m_s Z_s$ . As a first application we can study the behavior of various order parameters (c.f  $X(N_f) = 2m\Sigma(N_f)/M_\pi^2 F_\pi^2$ ) in the two limiting cases: i) For large  $N_c$ , i.e. no fluctuation, we recover the expected result  $X(3) = X(2) \sim 1$ . ii) In the large fluctuation limit (formally realized as a large  $N_f$  limit) we find  $X(3) \rightarrow 0$ , whereas  $X(2)$  stays close to 1 (at least for  $r = m_s/m > 15$ ). Numerically, the case  $N_f = 3$  appears closer to the large fluctuation limit than to the large  $N_c$  limit. The non-suppression of  $X(2)$  is observed in recent low-energy  $\pi\pi$  scattering experiments. We discuss how a (partial) suppression of  $X(3)$  could be seen in a combined  $N_f = 3$  analysis of masses, decay constants, and of more precise  $\pi\pi$  and  $\pi K$  scattering data[2].

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# Low-energy physics of the standard model and large- $N_C$ QCD

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The standard model involves several different mass scales. At the very lower end of the spectrum, the effective degrees of freedom reduce to the light pseudoscalar mesons, the light leptons and the photon. Their interactions are described by an effective lagrangian, whose general structure is governed by the symmetry properties of the standard model, but which involves scale dependent low-energy constants. Their values are not fixed by symmetry requirements, but refer to the non-perturbative QCD dynamics at the scales around 1 GeV and above.

Starting from the study of the low-energy and high-energy behaviours of the QCD three-point functions  $\langle VAP \rangle$ ,  $\langle VVP \rangle$  and  $\langle AAP \rangle$ , several  $\mathcal{O}(p^6)$  low-energy constants of the chiral Lagrangian have been evaluated within the framework of the minimal hadronic approximation to the large- $N_C$  limit of QCD in [1]. In certain cases, values that differ substantially from estimates based on a resonance Lagrangian are obtained. These differences arise through the fact that QCD short-distance constraints are in general not correctly taken into account in the approaches using resonance Lagrangians. Implications of our results for the  $\mathcal{O}(p^6)$  counterterm contributions to the vector form factor of the pion and to the decay  $\pi \rightarrow e\nu_e\gamma$ , and for the pion-photon-photon transition form factor are discussed. The representation obtained for the latter has subsequently been used for a re-evaluation of the pion pole contribution to the anomalous magnetic moment of the muon  $a_\mu$  (see [2] and A. Nyffeler's talk).

The hadronic light-by-light contribution to  $a_\mu$  has also been discussed from the effective field theory point of view in [3]. In particular, using a renormalization group argument, the coefficient of the leading logarithm arising from the two-loop graphs involving two anomalous WZW vertices was computed. The sign of this coefficient was found to be positive, in agreement with the evaluation of the hadronic light-by-light contribution to  $a_\mu$  obtained in [2].

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# Hadronic light-by-light scattering corrections to the muon $g - 2$ : the pion-pole contribution

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The main uncertainties in the standard model prediction for the muon  $g - 2$  originate from the hadronic contributions. The hadronic light-by-light scattering correction is particularly problematic, since one relies purely on model calculations. According to earlier estimates [1], the dominant term involves the exchange of a neutral pion between two anomalous vertices. Our motivation to reevaluate [2] this contribution was two-fold: recently [3] we derived a new representation for the pion-photon-photon transition form factor based on large- $N_C$  and short-distance properties of QCD. Furthermore, we wanted to push the analytical evaluation further than in [1]. For a rather general class of form factors, which also includes the VMD case and the case of a constant form factor, derived from a point-like Wess-Zumino-Witten vertex, we succeeded in performing all angular integrations in the corresponding two-loop integrals analytically, obtaining a two-dimensional integral representation for the pion-pole contribution to  $a_\mu$ :

$$a_\mu^{\text{LbyL};\pi^0} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \sum_i w_i(Q_1, Q_2) f_i(Q_1, Q_2).$$

It involves several model-independent weight-functions  $w_i$ , whereas the dependence on the form factors resides in the  $f_i$ . The weight functions in the dominant term are positive definite and peaked around  $Q_1 \sim Q_2 \sim 0.5$  GeV. This explains the fact that  $a_\mu^{\text{LbyL};\pi^0}$  is not very sensitive to the high-energy behavior of the form factor, but it is important to reproduce the data at small energies. Our result  $a_\mu^{\text{LbyL};\text{PS}} = +8.3 (1.2) \times 10^{-10}$  for the three pseudoscalar states differs essentially only by its *sign* from the results given in [1] and has been *confirmed* in the meantime [4]. The new value reduces the difference between  $a_\mu^{\text{exp}}$  and  $a_\mu^{\text{SM}}$  to  $1.6 \sigma$ .

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# $N_f$ -sensitivity of chiral dynamics

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Recent investigations in the  $0^{++}$  channel [1] suggest rather different mechanisms of chiral symmetry breaking in the limits of  $N_f = 2$  massless flavours ( $m_u, m_d \rightarrow 0$ , but physical  $m_s$ ) and  $N_f = 3$  ( $m_u, m_d, m_s \rightarrow 0$ ) [2]. Meson-meson scattering is a favoured place to test this scenario: it is necessary to combine  $\pi\pi$  results (2-flavour sector) including the analysis of the new  $K_{e4}$  data of the E865 collaboration [3], with  $\pi K$  scattering observables (3-flavour sector) for which dispersive estimates are now available [4].

We follow the procedure outlined in [1] for the masses and decay constants of  $(\pi, K, \eta)$ . We take their chiral expansion up to next-to-leading order in  $\hat{m}$  and  $m_s$ , in the isospin limit  $m_u = m_d = \hat{m}$ . We can then rewrite the low-energy constants  $L_{4,5,6,7,8}$  as functions of only 3 fundamental parameters:  $r = m_s/\hat{m}$ ,  $X(3) = 2m\Sigma(3)/(F_\pi^2 M_\pi^2)$  and  $F(3) = \lim_{\hat{m}, m_s \rightarrow 0} F_\pi$ . We then consider the chiral expansion of  $\pi\pi$  and  $\pi K$  scattering observables (namely  $\alpha_{\pi\pi}$ ,  $\beta_{\pi\pi}$  and  $\beta_{\pi K}$ ) up to next-to-leading order, and reexpress  $L_{4,5,6,8}$ . At each stage, we keep explicitly track of the NNLO remainders coming from each chiral expansion.

$\pi\pi$  and  $\pi K$  scattering observables constrain therefore  $[r, X(3), F(3)]$ . Lines of constant  $\alpha_{\pi\pi}$  and  $\beta_{\pi\pi}$  in the  $[r, X(3)]$  plane are almost perpendicular, so that an accurate knowledge of  $\pi\pi$  scattering (combined with the  $SU(3)$  chiral expansion of  $F_\pi, F_K, M_\pi, M_K$ ) yields information about the  $N_f = 3$  case. Moreover, the dispersive estimate of  $\beta_{\pi K}$  favours low values of  $F(3)$ . When we consider the 3 scattering observables altogether, nonvanishing NNLO remainders are needed for a consistent picture. Preliminary studies favour low values of  $F(3)$  (around 75 MeV) and  $X(3)$  (of order 0.5 or less), and  $r$  larger than 20.

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# Effective Chiral Lagrangian from Simulations of Partially Quenched QCD

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A method to determine some of the low-energy constants in the  $O(p^4)$  effective chiral Lagrangian, using lattice simulations of QCD is reviewed. It is shown how the constants  $L_5$  and  $L_8$  can be determined independently by studying the quark mass dependence of suitably defined ratios of pseudoscalar meson masses and decay constants [1]. It is pointed out that the physically relevant coefficients can be computed even in unphysical situations, as realised in the so-called *partially quenched* approximation [2,3], as long as the physical number of active quark flavours,  $N_f$ , is employed. Furthermore, an independent determination of  $L_8$  based on the (lattice) solution of the underlying field theory of QCD, serves to decide the question whether or not the up-quark could be massless [4].

As a first step in an ultimately realistic treatment of the problem, numerical results from simulations using  $N_f = 2$  flavours are discussed [5]. Results for  $L_5$  and  $L_8$  are obtained with a statistical error of 5 – 10%. Systematic effects due to neglecting higher orders are estimated to be  $\pm 0.25$ . Future simulations at smaller quark masses are required to corroborate this estimate. Lattice results for  $L_5$  and  $L_8$  for  $N_f = 2$  are broadly consistent with the usual phenomenological determinations in the physical three-flavour theory. These findings strongly disfavour the possibility of massless up-quark, provided that the quark mass dependence in the physical three-flavour case is not fundamentally different. The method may be extended to determine  $L_7$ , which involves the evaluation of disconnected contributions to mesonic two-point functions [3].

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# Radiative Corrections to $K_{\ell 3}$ Decays

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In this contribution [1], I have presented general formulae for the  $K_{\ell 3}$  form factors at one loop in the framework of CHPT with virtual photons and leptons [2]. Two features induced by the electromagnetic interactions are worth noticing. First, the form factors now depend not only on the momentum transferred between the kaon and pion, but also on a second kinematical variable. Second, there are contributions from four of the local counterterms  $X_i$ , specific to semileptonic processes [2].

Loops with virtual photons generate also infrared divergences. In order to deal with them, we have analysed the associated real photon emission processes. We have given a general description of the changes induced in the Dalitz plot density, and have proposed a model-independent procedure for including radiative corrections in the data analysis. This consists in incorporating the known long-distance electromagnetic effects into generalized kinematical densities, while including all the structure dependent (UV sensitive) terms as corrections to the form factors. Details of the new kinematical densities will depend eventually on the way the specific experimental set-up deals with real photon emission.

Within this framework, we have studied the effect of radiative corrections in the extraction of the CKM matrix element  $|V_{us}|$  from the  $K_{e3}^+$  mode. As compared to the pure  $\mathcal{O}(p^4)$  form factors [3], the inclusion of  $\mathcal{O}(e^2 p^2)$  electromagnetic contributions shifts  $f_+(0)$  by about  $(0.36 \pm 0.16)\%$ . Moreover, the radiative corrections produce an effective reduction of  $-1.27\%$  for the phase space integral. We note here that the uncertainty on the form factor up to  $\mathcal{O}(p^4, e^2 p^2)$  is well under control, and it affects the extraction of  $|V_{us}|$  only marginally compared to present experimental errors. This result opens the road to a precision determination of  $|V_{us}|$ , for which the next two important ingredients are: the inclusion of two-loop chiral corrections and a new high statistics measurement of branching ratios, properly accounting for radiative corrections.

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# Light quark mass ratios from large $N_c$ chiral perturbation theory<sup>1</sup>

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If the number of colours in QCD ( $N_c$ ) is treated as large, the spectrum involves an additional light degree of freedom because the mass of the  $\eta'$  disappears in the large  $N_c$  limit:  $M_{\eta'}^2 = O(1/N_c)$ . Also in this case, the low energy properties of QCD may be studied by means of an effective theory, see [1] and the references therein. To organize the expansion, we introduce a counting parameter  $\delta$  and assign the following weights to powers of momenta, quark masses and  $1/N_c$ :

$$p = O(\sqrt{\delta}) \quad , \quad m = O(\delta) \quad , \quad 1/N_c = O(\delta) \quad . \quad (1)$$

Within the given scope, we performed a calculation of the mass spectrum, the decay constants and the electromagnetic decay rates of the Goldstone particles. The calculation is valid to next-to-leading order in the expansion in  $\delta$  and also includes the logarithmically enhanced contributions of the subsequent order [5].

The framework has the property that it does not allow the Kaplan–Manohar transformation [1,2,3]. This implies that the effective representations for the various observables do lead to constraints on the quark mass ratios when compared with the experimental measurements. In this way, one may in particular express the ratio  $S \equiv 2m_s/(m_u + m_d)$  in terms of measured quantities and a single constant which determines the SU(3) breaking in the decay amplitudes  $\eta, \eta' \rightarrow \gamma\gamma$ . The low energy constant may be estimated theoretically by means of resonance saturation of a three point function [4]. The numerical result  $S = 26 \pm 2$  [5] lies remarkably close to the current algebra prediction  $S = 25.9$ .

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# Recalculation and Reanalysis of $K \rightarrow 3\pi$ .

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The theoretical study of kaon decay is hindered by the nonperturbative behaviour of QCD in the low-energy domain. One of the approaches successfully applied in the study of low-energy processes is Chiral Perturbation Theory. Using this, we have recalculated the CP conserving  $K \rightarrow 3\pi$  amplitude to one loop in the isospin limit. Using various symmetries we have then figured out a way to reduce these amplitudes into publishable form, see [1]. From the amplitudes we have calculated various decay rates and cross-sections which are compared with recent experimental data from the CPLEAR, HYPERON and NA48 collaborations. For this comparison we have used the data as they are presented in [2]. The result is a new fit of the  $p^4$  nonleptonic weak constants in the Chiral Lagrangian. The  $K_i$  presented in the table below are defined as linear combinations of these weak constants, see [3] and [4]. The three columns show the preliminary results we get for three different sets of input data, the strong constants  $L_1 - L_8$ . In the first column we use the values presented in the DAΦNE book. In the second and third column the input data are taken from a one- and two-loop fit respectively, see [5]. In the present results there seem to be some disagreement compared to the numbers presented in [3].

Constant	$p^2$	$p^4$ (DAΦNE)	$p^4$ (1-l. ABT)	$p^4$ (2-l. ABT)
$C G_8 F_0^4 / f_\pi^2$	$8.50 \times 10^{-10}$	$4.23 \times 10^{-10}$	$4.23 \times 10^{-10}$	$4.23 \times 10^{-11}$
$C G_{27} F_0^4 / f_\pi^2$	$4.72 \times 10^{-11}$	$3.01 \times 10^{-11}$	$3.01 \times 10^{-11}$	$3.01 \times 10^{-11}$
$K_2$	-	$5.62 \times 10^{-9}$	$6.56 \times 10^{-9}$	$5.55 \times 10^{-9}$
$K_3$	-	$6.97 \times 10^{-9}$	$7.39 \times 10^{-9}$	$9.89 \times 10^{-9}$
$K_5$	-	$4.34 \times 10^{-12}$	$6.30 \times 10^{-13}$	$4.63 \times 10^{-12}$
$K_6$	-	$1.34 \times 10^{-10}$	$1.37 \times 10^{-10}$	$1.57 \times 10^{-11}$
$K_7$	-	$-4.54 \times 10^{-10}$	$-4.55 \times 10^{-10}$	$-4.55 \times 10^{-10}$

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# The $\eta' \rightarrow \eta\pi\pi$ decay in $U(3)$ chiral perturbation theory

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The dominant hadronic decay of the  $\eta'$ ,  $\eta' \rightarrow \eta\pi\pi$ , is investigated up to one-loop order in  $U(3)$  chiral perturbation theory [1]. Within this framework, the  $\eta'$  is included without employing  $1/N_c$  counting rules and loop integrals are evaluated using infrared regularization which preserves Lorentz and chiral symmetry. Important features of the  $\eta$ - $\eta'$  system, such as  $\eta$ - $\eta'$  mixing [2], are incorporated, and loop integrals of an  $\eta'$  are shown to be suppressed. Reasonable agreement with data is obtained without finetuning any parameters. The investigation of the  $\eta'$  meson may eventually lead to a better understanding of the role of gluons in chiral dynamics due to the axial  $U(1)$  anomaly.

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# The Polarizability of the Nucleon: The View of Dispersion Relations

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Real and virtual Compton scattering has recently attracted much attention on the sides of both theory and experiment. Considerable experimental progress has resulted in improved data on the polarizabilities from real Compton scattering (RCS), and first data have been obtained for generalized polarizabilities from virtual Compton scattering (VCS). In principle, polarizabilities can be read off a low-energy expansion of the respective scattering amplitudes in terms of small (real) photon momenta. In order to extract precision values, however, it is useful and mostly even necessary to analyze the data by means of dispersion relations (DR). Therefore, we have set up both unsubtracted and subtracted DR for the RCS [1] and VCS [2] amplitudes, thus relating these experiments with information on the reactions  $\gamma + N \rightarrow \gamma + N + X$  (s-channel contributions) and  $\gamma + \gamma \rightarrow Y$  (t-channel contributions). The necessary input for this scheme is taken from phase shift analyses, dispersion theory and phenomenological models describing these reactions.

There are two immediate advantages in using DR for such analyses. First, the kinematical range of the experiments can be extended to momenta beyond the applicability of low-energy expansions, which may increase the experimental sensitivity to the polarizabilities by large factors. Second, DR allow one to evaluate higher order polarizabilities [3], which can be predicted and thus compared to the results from ChPT but cannot be extracted from RCS and VCS directly. Furthermore, DR can simulate the practically impossible experiment of doubly virtual Compton scattering, related to quark structure functions and generalized Gerasimov-Drell-Hearn and Burkhardt-Cottingham integrals (Ref. [4]).

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# Positivity Bounds for Parton Distributions in Multicolored QCD

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The twist-2 parton distributions — unpolarized  $q_f(x)$ , polarized  $\Delta_L q_f(x)$  and transversity  $\Delta_T q_f(x)$  — obey the well known positivity bounds

$$|\Delta_L q_f| \leq q_f, \quad 2|\Delta_T q_f| \leq q_f + \Delta_L q_f. \quad (1)$$

The second inequality is known as Soffer inequality [1]. In the large  $N_c$  limit the above inequalities can be enhanced as follows [2]

$$|\Delta_L q_u| \leq \frac{1}{3} q_u, \quad 2|\Delta_T q_u| \leq \frac{1}{3} q_u + \Delta_L q_u \quad (N_c \rightarrow \infty). \quad (2)$$

The factor of 1/3 has nothing to do with  $1/N_c$ . The derivation of inequalities (2) uses no model assumptions and is based on the spin-flavor symmetry [3,4] due to which in the leading order of the  $1/N_c$  expansion the baryons with spin equal to isospin are degenerate.

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# Chiral Effective Field Theory with Explicit Spin 3/2 Degrees of Freedom—A Status Report

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Non-relativistic Baryon Chpt has been very successful in describing near-threshold properties in the single Baryon sector during the 1990s. However, when it comes to magnetic or spin-observables, the corresponding perturbative chiral expansion sometimes behaves rather poorly (e.g.  $\gamma_i$ , GDH\* sum-rules,...) or shows rather delicate cancellations of unnaturally large higher order contributions (e.g.  $\beta_{M1}$ ,  $P_i^{\pi^0 p}$ ,...). Typically, in these channels one expects rather large contributions stemming from the first nucleon resonance,  $\Delta(1232)$ . Therefore, in the mid 1990s a chiral effective field theory with explicit spin 3/2 degrees of freedom *and systematic powercounting* was developed [1]. An overview of recent calculations can be found in [2]. The appearance of explicit Deltas in the calculation leads to a resummation of the chiral expansion, pushing these important physical contributions into lower orders of perturbation theory. These can result in the desired effects (e.g. in the case of  $\gamma_i$  [3]), but it can also upset the delicate cancellation pattern of Baryon Chpt (e.g. as in  $\beta_{M1}$  [4]). In the latter case we argue that additional large higher order effects should also be promoted into lower orders of perturbation theory via an “anomalous counting” of corresponding higher order counterterms. First examples of this new development are presented in [2] for the case of the quark-mass dependence of the nucleon’s anomalous magnetic moments and in [5] for the study of resonance contributions in dynamical polarizabilities of the nucleon [6].

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# Analysis of the Pion–Kaon Sigma Term

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To parameterize the interaction of the kaon with the non–strange scalar isoscalar source  $\hat{m}(\bar{u}u + \bar{d}d)$ , the scalar formfactor  $\Gamma_K$  of the kaon is defined via

$$\langle K_a(p_2) | \hat{m}(\bar{u}u + \bar{d}d) | K_b(p_1) \rangle = \delta_{ab} \Gamma_K(t) , \quad (1)$$

where  $t = (p_1 - p_2)^2$ . The corresponding sigma term reads  $\sigma_{\pi K} = \Gamma_K(0)/(2M_K)$ . This quantity is of interest e.g. as a building block of the nucleon sigma term, since it is related to the strangeness content of the kaon, etc.

A one–loop calculation in  $SU(3)$  ChPT yields a 16% correction to the tree level result at  $t = 4M_\pi^2$ , which is fairly small as compared to the corresponding corrections to the pion sigma term in  $SU(2)$  ChPT given in [2].

A low–energy theorem [1] gives the decomposition

$$F^2 T_{\pi K}^+(s = u, t = 2M_\pi^2) = \Gamma_K(t = 2M_\pi^2) + \Delta_{\pi K} \quad (2)$$

for the so–called Cheng–Dashen point  $s = u, t = 2M_\pi^2$ .  $T_{\pi K}^+$  is the isospin–even  $\pi K$  scattering amplitude. The so–called remainder  $\Delta_{\pi K}$  vanishes to leading order. The analogue of this low–energy theorem for the  $\pi\pi$  case has been analyzed in [3]. In an  $SU(3)$  framework, the squared decay constant  $F^2$  can be chosen to be  $F_\pi^2$ ,  $F_\pi F_K$ , or  $F_K^2$ . Numerically, the first of these conventions is distinguished by a relative size of the remainder of between 1% and 2%, as against 10% to 20% for the other choices. The smallness of  $\Delta_{\pi K}$  for the  $F_\pi^2$  normalization is due to a complete dropout of any terms proportional to  $M_K$ , which indicates  $SU(2)$  breaking effects. A possible tool to separate quantities undergoing real  $SU(3)$  breaking is heavy kaon ChPT as worked out in [4]. A further investigation is in progress.

A two–loop calculation of  $\Gamma_K$  with dispersive techniques as performed for the pion case in [2] yields corrections to the tree level result of about 5% at the two–pion threshold.

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# Soft pion theorems for partons

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We show that for the processes  $\gamma^*(q)N(p) \rightarrow \pi(k)N(p')$  near the threshold<sup>1</sup> the two limits  $Q^2 \rightarrow \infty$  ( $Q^2 = -q^2$ ) and  $m_\pi \rightarrow 0$  do not commute. For  $Q^2 \ll \Lambda^3/m_\pi$  ( $\Lambda$  is a typical hadronic scale) one can apply classical soft pion theorems by Nambu *et al.* [1] which express the threshold amplitudes of the  $\gamma^*N \rightarrow \pi N'$  reaction in terms of e.m. and axial form factors of the nucleon. In the kinematics where  $Q^2 \gg \Lambda^3/m_\pi$  one can derive new soft pion theorems [2] which also express the threshold amplitudes in terms of the nucleon form factors. The particular expressions depend on the form of the light-cone wave functions (LCWF) of the nucleon, *i.e.* sensitive to the partonic content of the nucleon wave function. If we assume that the dominant component of the nucleon LCWF is symmetric we obtain, for instance, at  $Q^2 \gg \Lambda^3/m_\pi$  we have [2]

$$A(\gamma^*p \rightarrow \pi^0 p)|_{\text{th}} = \frac{1}{3f_\pi} \left( \frac{5}{2} G_{Mp}(q^2) - 4 G_{Mn}(q^2) \right) + O\left(\frac{m_\pi}{\Lambda}\right),$$

which should be contrasted with [1]

$$A(\gamma^*p \rightarrow \pi^0 p)|_{\text{th}} = \frac{g_A}{2f_\pi} G_{Mp}(q^2) + O\left(\frac{m_\pi}{\Lambda}\right),$$

for  $\Lambda^2 \ll Q^2 \ll \Lambda^3/m_\pi$ . The new soft pion theorems for hard processes are in agreement with measurements of  $\gamma^*N \rightarrow \pi N'$  reaction near threshold by E136 collaboration [3]. We also discussed soft pion theorems for other hard exclusive processes, see details in [4].

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# Chiral dynamics in meson baryon, nucleon nucleon and weak $\Lambda N \rightarrow NN$ interactions

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By using the techniques of the unitary extensions of  $\chi PT$  [1] many problems at intermediate energies involving the interaction of mesons and baryons can be tackled [1]. In this talk I reported on a few recent problems which are interconnected and which have been studied recently. On the first hand an extension of the work of [2], where the meson baryon interaction in the strangeness  $S = -1$  sector was studied, has been done in [3], extending the predictions of the chiral model to higher energies. The interesting findings have been that in addition to the  $\Lambda(1405)$  resonance obtained in [2] and [4], two other resonances, the  $\Lambda(1670)$  and the  $\Sigma(1520)$  were also generated dynamically from the lowest order chiral Lagrangian. A remarkable finding is that the  $\Lambda(1670)$  resonance qualifies as a quasibound state of  $K\Xi$ . Similarly, a study has been conducted for the  $\pi N$  interaction in the region of the  $N^*(1535)$  resonance by including the coupled channels of meson baryon plus also the  $\pi\pi N$  channels [5]. It was found that the phase shifts and inelasticities are well reproduced and that the  $\pi\pi N$  channel was essential to reproduce the isospin  $I = 3/2$  sector and the  $\Delta(1620)$  resonance.

In the  $NN$  interaction some progress has also been done by obtaining the contribution of the exchange of two interacting pions in the scalar isoscalar sector [6] using again the techniques of [1]. We find that in addition to a moderate attraction at intermediate distances, one also obtains a repulsion at short distances.

These later findings have also been used in the study of the  $\Lambda N \rightarrow NN$  weak transition in order to make predictions for the nonmesonic decay width of  $\Lambda$  hypernuclei [7] and particularly the ratio of neutron to proton induced  $\Lambda$  decay.

Finally a study of the final state interaction in the  $pp \rightarrow dK^+\bar{K}^0$  and  $pp \rightarrow d\pi\eta$  reactions, investigated in the ANKE experiment of Juelich, has been performed where one finds that the  $K^+\bar{K}^0$  interaction as well as the  $\bar{K}^0 d$  interaction are very important and modify appreciably the invariant mass distributions [8]. By fixing some relevant parameters of the dynamics of the process to some of the mass distributions we could then predict absolute rates for  $\pi\eta$  production which showed clearly the  $a_0(980)$  resonance in the  $\pi\eta$  invariant mass distribution.

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# Perturbative Theories of Nuclear Forces

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There has been recent progress in developing a consistent and converging EFT to describe multi-nucleon systems [1][2][3]. In very recent work it has been shown that chiral perturbation theory in the single-nucleon sector is a special case of a more general ordering of operators [4]. In the presence of more than one nucleon, the momentum expansion is necessarily nonperturbative at some level to accommodate the fine-tuned scales, while the  $m_q$ -expansion remains perturbative. In other words, the perturbative expansion of nuclear forces is an expansion about the chiral limit. In the  $^1S_0$  channel only local operators are treated nonperturbatively, whereas in the  $^3S_1 - ^3D_1$  coupled channels it is necessary to resum the non-local, singular part of OPE which survives in the chiral limit.

Partial higher-order calculations in the  $^3S_1 - ^3D_1$  coupled channels suggest that an expansion about the chiral limit will converge. However, a full NLO calculation is required in order to make a more definite conclusion and to give meaningful predictions for the deuteron binding energy in the chiral limit. The use of the square well as a short-distance regulator has proved valuable in giving analytic formulas for the RG running of the counterterms. However, the necessity of computing processes with external gauge fields suggests use of a regulator that manifestly respects gauge invariance, like dimensional regularization. An intriguing puzzle remains concerning the relationship between square-well regularization and the matching of delta-function interactions to singular potentials. In the short-distance limit one finds that the singular potential wavefunctions vanish. This makes it difficult to understand how a delta-function interaction can modify the physical asymptotic phase.

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# Renormalizing the Lippmann-Schwinger equation for the OPEP

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Since the seminal papers of Weinberg [1], ten years have passed and there is still no consensus on how to organize an EFT for low energy nucleon-nucleon interactions. We follow the general ideas of [2] and consider the two nucleon system interacting through a potential, namely at energies much below the pion mass (but allowing three-momenta to be similar to it), as an EFT on its own. Although the potential can be derived order by order in  $\chi$ PT from the Heavy Baryon Chiral Lagrangian, the calculations in this EFT need not be carried out order by order in  $\chi$ PT anymore. In fact the results of [3] seem to indicate, particularly for the triplet channel, that the OPEP (or at least part of it) should be taken into account at all orders of  $\chi$ PT. If so, the question arises whether the renormalization program, which is usually carried out perturbatively, goes through. We have addressed this question for the OPEP in [4] and obtained the following results:

- The singlet channel is renormalizable.
- The triplet channel is non-renormalizable, unless the coupling constant of a non-local potential is allowed to flow. Even in that case, the renormalized amplitude has undesirable properties (lacks  $^3S_1$ - $^3D_1$  mixing).
- If a suitable piece of the potential in the triplet channel is treated as a perturbation, then it becomes renormalizable at least up to next to leading order.

These results led us to propose a new way to organize the calculation in the triplet channel which, we conjecture, will be renormalizable at any order. We expect the new renormalized perturbative expansion to have better convergence properties than previous proposals as well [5].

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# Dispersive Effects in Nucleon Polarisabilities

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A formalism to extract the dynamical nucleon polarisabilities defined via a multipole expansion of the structure amplitudes in nucleon Compton scattering was developed in [1]. In contradistinction to the static polarisabilities, dynamical polarisabilities gauge the response of the internal degrees of freedom of a composed object to an external, real photon field of arbitrary energy. Being energy dependent, they contain information about dispersive effects induced by internal relaxation mechanisms, baryonic resonances and meson production thresholds of the nucleon. Explicit formulae to extract the dynamical electric and magnetic dipole as well as quadrupole polarisabilities from low energy nucleon Compton scattering are presented, and the connection to the definition of static nucleon polarisabilities is discussed. A chiral analysis using iso-scalar dynamical polarisabilities including the  $\Delta(1232)$  as dynamical degree of freedom is performed in [2]. The energy dependence of  $\alpha_{E1}(\omega)$  and  $\beta_{M1}(\omega)$  is well predicted by chiral dynamics. Polarisabilities stem predominantly from charge displacements of the pion cloud around the nucleon and from the  $\Delta$  resonance contribution. However, the counter terms for  $\alpha_{E1}(\omega)$  and  $\beta_{M1}(\omega)$  turn out to be at least 4 times larger than naïve dimensional analysis predicts. By consistently modifying the SSE power counting [3], they are thus promoted to LO, and their size is fixed by fitting to the static polarisabilities measured. Their energy dependence is then a prediction which is in good agreement with a dispersion relation (DR) analysis.  $\alpha_{E2}(\omega)$  is well predicted without free parameters up to 150 MeV, where DR shows the  $E2 N \rightarrow \Delta$  transition to be important.  $\beta_{M2}(\omega)$  lacks overall strength, and the DR curve shows a strong, nearly linear energy dependence which might originate from a strong dia-magnetic quadrupole relaxation.

In Compton scattering on the deuteron at  $\omega = 91$  MeV, dynamical effects are large and cannot be mimicked by taking only the slopes of the polarisabilities at zero energy into account. The dynamical polarisabilities constructed above predict values in agreement with experiment, in contradistinction to traditional analyses where  $\beta_{M1}(0)$  is five times bigger than from extractions at zero energy.

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# PION ELECTROPRODUCTION ON THE DEUTERON NEAR THE TRESHOLD

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In the last years there was a lot of progress in describing low energy physics with the help of chiral perturbation theory. One of them is the description of  $\pi^0$ -electroproduction on a proton. The absence of a neutron target makes the comparison between theory and experiment more complicated: One has to describe the production process on the lightest nucleus, where neutrons are bound. The first calculation of  $\pi^0$ -electroproduction on a deuteron within the framework of HBCHPT was done at  $\pi^0$ -threshold in the static limit[1]. For comparison with experiment the data were extrapolated to the threshold[2]. One obtained no agreement for the longitudinal part. To improve the situation I transformed the single-nucleon contribution into the right frame. Secondly to avoid any extrapolation of the data I calculated the amplitudes above the  $\pi^0$ -threshold, where the data were really measured. The calculation consists of two steps.

The first step is the multipole decomposition. Arenhövel[3] constructed 13 linearly independent spin structures and described the amplitude by 13 structure functions. On the other side, because of the selection rules there are 13 multipole amplitudes for each multipolarity, which do not depend on the angle between the outgoing pion and deuteron. The structure function vector  $F = (F_1, \dots, F_{13})$  can be described in the following way:

$$F = \sum_{L=0}^{\infty} A_L(\cos(\theta))M_L \quad \text{and} \quad M_L = \int_0^1 d \cos(\theta) B_L(\cos(\theta))F, \quad (1)$$

where  $M_L = (M_L^1, \dots, M_L^{13})$  is the multipole vector and  $A_L, B_L$  are matrices that only depend on  $\cos(\theta)$ .

The second step is the calculation of the amplitude. For this purpose I use the hybrid approach first introduced by Weinberg: one calculates the irreducible kernel within the framework of HBCHPT to the order  $q^3$  and convolutes it with phenomenological or chiral wave-functions. To third order in small momenta, the amplitude is finite and a sum of two- and three-body interactions with no undetermined parameters.

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# One Two Three ... Infinity: Nucleons in EFT

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Effective Field Theories (EFTs) provide the only existing framework to derive nuclear structure from QCD. Because the typical momenta of nucleons in the deuteron and in the virtual bound state in the  $^1S_0$   $NN$  channel are smaller than the pion mass, an EFT involving nucleons only (where pions and all other mesons have been integrated in favor of contact interactions) has been developed. This EFT was applied to systems with two [1] and even three [2] nucleons, with much success. An exploratory study of infinite nuclear matter at finite temperature was also conducted with a lattice regulator [3].

In order to bridge the extremes in baryon number, we are now studying halo nuclei. A lower-energy EFT involving contact interactions among nucleons and a core can be formulated [4]. One simple application is  $N$ - $^4\text{He}$  scattering, which is known to display a low-lying  $P_{3/2}$  resonance. The power counting in the case of a shallow  $P$ -wave state is similar to that [1] for  $S$ -wave states relevant in  $NN$  scattering, but has some important differences. In leading order it involves two operators, which can accommodate resonance behavior in addition to a bound state. One indeed finds good agreement with phase-shift analyses. With parameters fitted this way,  $^6\text{He}$  can then be studied as a three-body system using techniques like those of Ref. [2]. Generalization to other halo nuclei is straightforward, once cores with spin are considered.

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# Few-Body Physics in Effective Field Theory

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Effective Field Theory (EFT) provides a powerful framework that exploits a separation of scales in physical systems to perform systematic, model-independent calculations. The long-distance physics is included explicitly, while the corrections from short-distance physics are calculated in an expansion in the ratio of short- and long-distance scales. We apply EFT to three-body systems with short-range interactions and large two-body scattering lengths  $a \gg R$ , where  $R$  is the typical range of the interaction. These systems are interesting because they display universal features such as a logarithmic spectrum of shallow three-body bound states and a discrete scale invariance [1]. While most channels are straightforward, in certain  $S$ -wave channels (e.g. for spinless bosons) a nonderivative, one-parameter three-body force is required at leading order for consistent renormalization. The renormalization group evolution of this three-body force is governed by a limit cycle [2].

In nuclear physics, this EFT has successfully been applied to the neutron-deuteron and  $\Lambda$ -deuteron systems [3]. The variation of the three-body force provides a compelling explanation of the Phillips line.

Especially interesting is the application of this formalism to the physics of cold atoms and Bose-Einstein condensates, where the scattering length  $a$  can be tuned experimentally using Feshbach resonances. As a consequence, the unique  $a$ -dependence predicted by the EFT can be tested. In particular, the EFT predicts logarithmically spaced minima (for  $a > 0$ ) and resonances (for  $a < 0$ ) in the rate for three-body recombination, i.e. when two atoms form a molecule and a third atom balances energy and momentum [4].

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# Application of Chiral Nuclear Forces to Few-Nucleon Systems

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In recent years high precision NN force models have been developed which are related to the OBE picture but modified by purely phenomenological structures. Based on typically 40 parameters they describe the rich set of NN data up to the pion threshold perfectly well. They alone lead to underbinding of nuclear ground states and are insufficient to describe 3N scattering observables well at energies starting around 60 MeV and higher. On the other hand at low energies they work rather well for 3N scattering observables with the outstanding exception of the analysing power  $A_y$  in elastic Nd scattering. Three-nucleon forces are natural candidates to possibly cure these failures. There are various models which, however, are not theoretically linked to the NN potential models. They can be tuned to the  ${}^3H$  binding energy, but they neither cure the  $A_y$ -puzzle nor remove all discrepancies in 3N scattering at the higher energies mentioned. Clearly a more systematic and theoretically founded approach for nuclear forces is needed. Chiral perturbation theory appears to be a promising step in that direction. Based on an effective chiral Lagrangian for pion and nucleon fields one can eliminate in the spirit of the Weinberg power counting the pionic degrees of freedom leading to nuclear force ordered in powers of generic momenta over a mass scale of the order of the  $\rho$ -mass. We applied these chiral forces in the LO, NLO and NNLO's. Up to these orders they are formed out of one - and two-pion exchanges and a string of contact forces with altogether 9 LEC's. At NNLO the resulting predictions based on rigorous Faddeev-Yakubovsky calculations are very similar to the ones of the above mentioned potential models. In the chiral scenario the momenta are cut off (smoothly) between  $\Lambda = 500$  and  $600$  MeV/c and the cut-off dependence shrinks significantly in going from NLO to NNLO. At NNLO the first time three-nucleon forces of three different topologies occur in that systematic approach. They depend on two additional parameters, which allow of course to guarantee the correct  ${}^3H$  binding energy. Predictions for the  ${}^4H$  binding energy and the wealth of 3N scattering observables is in progress. It is further planned to go to NNNLO and to incorporate also electroweak probes, which would significantly improve the theoretical foundation for electroweak many-body currents. There is good reason to hope that the chiral approach to nuclear forces will put nuclear physics on a theoretically better founded basis than hitherto.

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# Nuclear Forces from Chiral EFT: Recent Developments

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Few-nucleon forces at various leading orders in chiral effective field theory derived from the most general chiral invariant Lagrangian for nucleons and pions using the method of unitary transformation [1] are briefly reviewed. To leading order, the NN potential consists of the undisputed one-pion exchange (OPE) and two NN contact interactions. Corrections at NLO stem from the leading chiral TPE and additional contact terms with two derivatives. At NNLO one has to include the subleading TPE which contains  $\pi\pi$ NN interactions with two derivatives (pion mass insertions). The large values of the corresponding coupling constants  $c_{1,3,4}$  obtained in the  $Q^2$  and  $Q^3$  analysis of  $\pi$ N scattering lead at this order to unrealistically strong attraction and, as a consequence, to unphysical deeply bound states [2]. We provide arguments for a reduction of the TPE potential and introduce the NNLO\* version of the NN forces, which is very well suited for few-nucleon calculations. We show that the NN phase shifts are strongly improved at NNLO\* compared to NLO and are mostly rather well reproduced up to  $E_{\text{lab}} = 200$  MeV.

We also establish a link between the NN potential derived in chiral effective field theory and various modern high-precision potentials [3]. To be more specific, we show that the values of the NN contact interactions can be understood from the resonance saturation in terms of well established one-boson exchange models. We also address the issue of the naturalness of the contact terms and consider leading isospin violating effects [4].

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# Chiral dynamics of nuclear matter

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We calculate the equation of state of isospin-symmetric nuclear matter in the 3-loop approximation of chiral perturbation theory [1]. The contributions to the energy per particle  $\bar{E}(k_f)$  from  $1\pi$ - and  $2\pi$ -exchange graphs are ordered in powers of the Fermi momentum  $k_f$  (modulo functions of  $k_f/m_\pi$ ). It is demonstrated that, already at order  $\mathcal{O}(k_f^4)$ ,  $2\pi$ -exchange produces realistic nuclear binding. The underlying saturation mechanism is surprisingly simple (in the chiral limit), namely the combination of an attractive  $k_f^3$ -term and a repulsive  $k_f^4$ -term. The empirical saturation point  $\bar{E}(k_{f0}) \simeq -15.3$  MeV,  $\rho_0 \simeq 0.17$  fm $^{-3}$  and the nuclear compressibility  $K \simeq 255$  MeV are well reproduced at order  $\mathcal{O}(k_f^5)$  with a momentum cut-off of  $\Lambda = 7f_\pi \simeq 0.65$  GeV which parametrizes all necessary short-range dynamics. Next, we calculate the density-dependent asymmetry energy and find  $A(k_{f0}) \simeq 34$  MeV, in very good agreement with the empirical value. The pure neutron matter equation of state is also in fair agreement with sophisticated many-body calculations and a resummation result of effective field theory, for low neutron densities  $\rho_n < 0.25$  fm $^{-3}$ .

In the same framework we evaluate the momentum and density dependent complex single particle potential  $U(p, k_f) + iW(p, k_f)$  [2].  $1\pi$ - and  $2\pi$ -exchange give rise to a potential depth (for a nucleon at the bottom of the Fermi sea) of  $U(0, k_{f0}) = -53.2$  MeV, in agreement with the shell model potential. The momentum dependence of the real part  $U(p, k_{f0})$  is non-monotonic and can be translated into a mean effective nucleon mass of  $\bar{M}^* \simeq 0.82M$ . The imaginary part  $W(p, k_f)$  is generated entirely by iterated  $1\pi$ -exchange. The half width of a nucleon hole with  $p = 0$  comes out as  $W(0, k_{f0}) = 29.7$  MeV. The basic theorems of Hugenholtz-Van-Hove and Luttinger are satisfied in our calculation.

We also consider nuclear matter finite temperature  $T$ . The free energy per particle  $\bar{F}(\rho, T)$  is obtained from the energy density functional by inserting a Fermi-Dirac distribution for the density of states. The calculated pressure isotherms indicate a liquid-gas phase transition at  $T_c \simeq 26$  MeV and  $\rho_c \simeq 0.09$  fm $^{-3}$ .

Finally, we calculate the nuclear spin-orbit strength from the spin-dependent part of the interaction energy  $\Sigma_{spin} = \frac{i}{2} \vec{\sigma} \cdot (\vec{q} \times \vec{p}) U_{ls}(p, k_f)$  of a nucleon scattering off weakly inhomogeneous nuclear matter. We find from iterated  $1\pi$ -exchange  $U_{ls}(0, k_{f0}) = 35.1$  MeVfm $^2$  in perfect agreement with the shell model value.

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# Chiral Perturbation Theory at Finite Density. Beyond the Mean Field Approach

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An explicit expression for the generating functional of two-flavor low-energy QCD with external sources in the presence of non-vanishing nucleon densities has been derived recently [1]. Within this approach we derive [2] power counting rules for the calculation of in-medium pion properties. We develop the so-called standard rules for residual nucleon energies of the order of the pion mass and a modified scheme (non-standard counting) for vanishing residual nucleon energies. We also establish the different scales for the range of applicability of this perturbative expansion, which are  $\sqrt{6}\pi f_\pi \simeq 0.7$  GeV for the standard and  $6\pi^2 f_\pi^2/2m_N \simeq 0.27$  GeV for non-standard counting, respectively. We have performed a systematic analysis of n-point in-medium Green functions up to and including next-to-leading order when the standard rules apply. These include the in-medium contributions to quark condensates, pion propagators, pion masses and couplings of the axial-vector, vector and pseudoscalar currents to pions. In particular, we find a mass shift for negatively charged pions in heavy nuclei,  $\Delta M_{\pi^-} = (18 \pm 5)$  MeV, that agrees with recent determinations from deeply bound pionic <sup>207</sup>Pb. We also show that the unique role of  $f_\pi$  as order parameter of chiral symmetry breaking in vacuum corresponds to  $f_t$  in symmetric nuclear matter. The latter is the coupling of a pion at rest to the temporal component of the axial-vector current. In addition, we have established the absence of in-medium renormalization in the  $\pi^0 \rightarrow \gamma\gamma$  decay amplitude up to the same order. The study of  $\pi\pi$  scattering requires the use of the non-standard counting and the calculation is done at leading order. Even at that order we establish new contributions not considered so far. We also point towards further possible improvements of this scheme and touch upon its relation to more conventional many-body approaches.

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# Color superconductivity in high-density QCD

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At high quark density, it is expected that a condensate of Cooper pairs of quarks will form, spontaneously breaking the  $SU(3)$  color gauge symmetry. This is known as color superconductivity. Since the QCD interaction between quarks is strongly attractive, the pairing is expected to be characterized by a large gap, in the range of tens to hundreds of MeV. For recent reviews, see the introduction to this miniproceedings.

In matter above nuclear density, the quark chemical potential  $\mu \gtrsim 400$  MeV, which is greater than the “bare” mass of the strange quark but less than that of the charm quark, so it is reasonable to consider 3 flavor quark matter. This leads to a “color-flavor locked” (CFL) pattern of quark pairing [1], which has many interesting properties. It breaks chiral symmetry without a  $\langle \bar{q}q \rangle$  condensate, causes the photon to mix with one of the gluons, and may be continuously connected to lower density hyperonic matter (“quark hadron continuity”).

A compact star is the best place to look for CFL quark matter in nature, since it is dense and cold. Recently there has been work on the expected structure of such a star [2]. The two possibilities are (a) a sharp interface between a CFL core and a nuclear matter mantle, and (b) a layer of mixed phase, consisting of positively charged nuclear matter mixed with negatively charged CFL quark matter. In Ref. [2] we found that the sharp interface would be favored if the surface tension of the nuclear-CFL interface  $\sigma \gtrsim 40$  MeV/fm.

The next step is to find observable phenomena that indicate the internal structure of the star. The sharp interface creates a discontinuity in the density by a factor of two, so as well as affecting the mass-radius relation it may lead to signatures in the gravitational waves emitted in the merger of two such stars, which are supposed to be detectable by future gravitational wave detectors such as LIGO II. The mixed phase has structure on the scale of a few fm, which is expected to reduce the mean free path of neutrinos, leading to possible signatures in the time profile of neutrinos emitted during the supernova that creates the compact star. Work on these topics is in progress.

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# Crystalline Color Superconductivity

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At asymptotic quark number densities, the ground state of QCD is expected to be the color-flavor locked (CFL) phase, in which all quarks participate in BCS pairing [1]. Below some critical density determined by quark mass differences, CFL is disrupted, leaving some species of quarks with differing Fermi momenta unpaired. It is natural to ask whether there is some generalization of the BCS ansatz in which pairing between two species of quarks persists even once their Fermi momenta differ. Crystalline color superconductivity is the answer to this question. The idea is that it may be favorable for quarks with differing Fermi momenta to form pairs whose momenta are *not* equal in magnitude and opposite in sign [2,3]. Condensates of this sort spontaneously break translational and rotational invariance, leading to gaps which vary in a crystalline pattern. The favored crystal structure can be determined by fixing the coefficients in a Ginzburg-Landau EFT; this is the subject of ongoing work. The phonons of this phase can be analyzed using EFT methods [4]. The window in parameter space (or in density) in which crystalline color superconductivity arises is narrow in toy models, but this phase is generic in QCD [5]. If in some shell within the quark matter core of a neutron star the quark number densities are such that crystalline color superconductivity arises, rotational vortices may be pinned in this shell, making it a locus for glitch phenomena.

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# The Prefactor of the Color-Superconducting Gap

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The magnitude of the color-superconducting gap  $\phi$  in cold, dense quark matter is computed via a gap equation. At the Fermi surface and for  $g \ll 1$  ( $g$  is the QCD coupling constant),

$$\phi = b \mu \exp\left(-\frac{c}{g}\right) [1 + O(g)] , \quad (1)$$

where  $\mu$  is the quark chemical potential,  $c = 3\pi^2/\sqrt{2}$ , and  $b = 512\pi^4 [2/(N_f g^2)]^{5/2} \times \exp[-(\pi^2 + 4)/8]$ ,  $N_f$  is the number of degenerate quark flavors. The constant  $c$  is determined by almost static magnetic gluon exchange [1], which is a long-range interaction in QCD. The constant  $b$  is determined by static electric and non-static magnetic gluon exchange [2], as well as by contributions from the quark self-energy [3,4].  $O(g)$  corrections to the constant  $b$  arise for instance from non-static electric gluons [2], vertex corrections [3], the color-Meissner effect [5], and from the finite lifetime of quasi-particles off the Fermi surface [6]. At this order, there exists also an apparent gauge dependence of the gap equation in mean-field approximation [7]. The effect of the running of the coupling constant [8] on  $b$  is unclear at present.

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# Effective Theories for QCD at high density

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The effective lagrangian description [1] of two phases of QCD at high density is presented. We describe the CFL phase, realized at very high density in the case of three massless quarks, and the LOFF phase, a possible crystalline state of QCD matter at intermediate density. In both cases we construct the relevant low-energy effective lagrangian and show how to evaluate perturbatively the couplings. These calculations have been performed by using the idea of gapped quasi-fermionic excitations close to the Fermi surface [2]. We couple these excitations to the Nambu Goldstone bosons (NGB) and evaluate the corresponding relevant Green functions, obtaining the couplings. The main advantage of this description is in the fact that close to the Fermi surface the physics reduces to infinite copies of a two-dimensional problem. For the CFL phase we evaluate the NGB decay constants and their velocity (being at finite density, Lorentz invariance is broken). We also evaluate the gluon self-energy showing that the gluon physical masses are not of order  $g_s\mu$  ( $\mu$  being the chemical potential) but rather of the order of the gap  $\Delta$ . This effect is due to a very large wave function renormalization effect of order  $g_s\mu/\Delta$ . Finally we discuss the LOFF phase. In this case translational and rotational invariance are broken, but we show that only one physical NGB appears, the phonon. On the basis of the effective lagrangian approach, the phonon shows a peculiar anisotropic dispersion relation. Work in progress [3] about the evaluation of the anisotropy is also presented.

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# Dense QCD

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QCD at asymptotic densities exhibits a color superconducting state with color-flavor locking (CFL) and parametrically large gaps thanks to the long-range character of the colormagnetic interaction [1]. At non-asymptotic densities, QCD may crystallize in either an insulating or a superconducting phase [2]. The CFL phase supports light Goldstone modes in the form of particle-particle and hole-hole excitations with anomalous radiative decays. Very dense QCD is ideally suited for an effective Lagrangian approach with coefficient constants calculable from first principles [3]. One way to investigate the physical nature of this phase as well as the general character of the phase diagram in QCD is to quantify the amount of electromagnetic emissivities. Although these emissivities are strictly calculable at asymptotic densities, it is always intriguing to extrapolate them at few times that of normal nuclear matter. For large superconducting gaps as suggested by both perturbative and nonperturbative estimates, the dilepton and photon rates from the CFL phase at temperatures of the order of  $T = 80$  MeV are comparable to the one extrapolated from the low density hadronic phase [4].

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# Color Superfluidity in Two Color QCD on the Lattice

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We analyze the eigenvalue spectrum of the staggered Dirac matrix in two-color QCD at nonzero chemical potential  $\mu$  when the eigenvalues become complex. The quasi-zero modes and their role for chiral symmetry breaking and the deconfinement transition are examined, see Figure 1. The density  $\rho(y)$  is used to estimate a value for the chiral condensate by applying the Banks-Casher relation (which originally was derived for real eigenvalues appearing in pairs of opposite sign). We further employed the standard definition of the Green function by inverting the fermionic matrix with a noisy source and by computing its eigenvalues exactly, respectively, to get the condensate. An analogous analysis is performed for the spectrum of the Gor'kov representation of the fermionic action yielding the diquark condensate [1].

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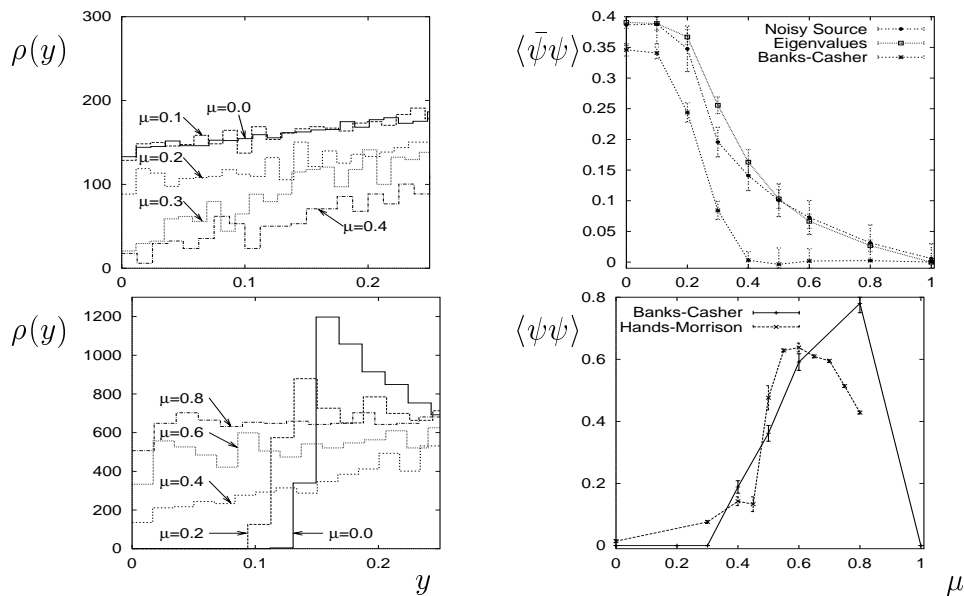


Figure 1: Upper plots: Density  $\rho(y)$  of small eigenvalues of the Dirac operator for two-color QCD on a  $6^4$  lattice from  $\mu = 0$  to 0.4 (left). The loss of quasi-zero modes is accompanied by a vanishing of the chiral condensate. Chiral condensate extracted by different methods (right). Lower plots: Similar for the “Gor’kov operator”.

# Baryons in Partially Quenched and Quenched QCD

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At this point in time it is not possible to perform lattice simulations of hadronic matrix elements at the physical values of the light quark masses,  $m_q$ . In order to make some connection between lattice simulations and nature an extrapolation in the quark masses from those used in the simulations down to their physical values is required. Many of us at this meeting have spent a significant fraction of our lives computing the  $m_q$ -dependence of hadronic observables in chiral perturbation theory ( $\chi$ PT). However, the results of these calculations can only be used to extrapolate the results of unquenched simulations, and unfortunately, these simulations are performed at sufficiently large values of  $m_q$  that the convergence of the chiral expansion is questionable, and most probably absent.

During the past decade significant progress has been made in understanding how to make QCD predictions from partially quenched [1] simulations. The global flavor symmetry group structure for partially quenched QCD (PQQCD) with three valence quarks, three ghost quarks and three sea-quarks, in the chiral limit is  $SU(6|3)_L \otimes SU(6|3)_R \otimes U(1)$  where  $SU(m|n)$  is a graded lie-algebra. The lowest-lying octet baryons are included into this framework by embedding them into a **240** dimensional irreducible representation of  $SU(6|3)$  [2,3,4], and their properties are computed systematically in an expansion of  $m_{\text{val}}/\Lambda_\chi$ ,  $m_{\text{sea}}/\Lambda_\chi$  and  $p/\Lambda_\chi$ . One interesting aspect of this theory is that the extension of electromagnetic, and weak charges from QCD to PQQCD is not unique. Different choices correspond to different weighting's of disconnected quark diagrams. At the meeting I presented results obtained by Chen and myself [4] for the baryon masses, magnetic moments and matrix elements of the isovector twist-2 operators.

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# Spectrum of the QCD Dirac operator in a finite volume

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Many hadronic properties depend sensitively on the spectrum of the QCD Dirac operator which we would like to compute in a finite volume  $V$ . In this case, three scales are important: the magnitude of the smallest Dirac eigenvalues,  $\lambda_{\min} \sim \pi/V\Sigma$  (with  $\Sigma = |\langle \bar{\psi}\psi \rangle|$ ), the Thouless energy,  $E_c \sim f_\pi^2/\Sigma\sqrt{V}$ , and the rho mass,  $m_\rho$ . Below  $m_\rho$ , QCD can be described by an effective chiral theory. For energies below  $E_c$ , the kinetic terms in the effective Lagrangian can be neglected, but the zero-momentum modes of the Goldstone fields must be treated nonperturbatively [1]. This is equivalent to chiral random matrix theory [2].

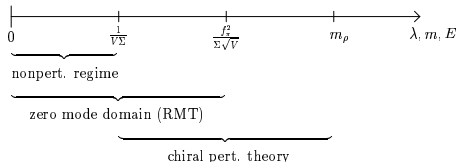


Fig. 1. Applicability of chiral RMT and standard chiral perturbation theory in a finite volume. Note the overlap region of the two theories, also visible in Fig. 2.

We have computed a number of quantities in chiral RMT, chiral perturbation theory and lattice simulations to confirm this theoretical picture [3], see Fig. 2.

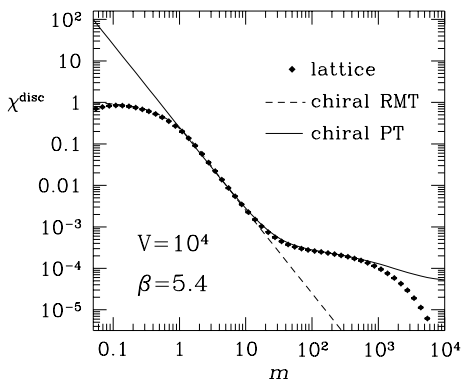


Fig. 2. The disconnected scalar susceptibility as a function of the valence quark mass from a quenched SU(3) simulation using the staggered Dirac operator. The theoretical curves are from chiral RMT and from a finite-volume chiral perturbation theory applicable for staggered fermions at moderate to strong coupling, see Ref. [3]. Our calculations nicely confirm the theoretical expectation of Fig. 2.

I would like to thank M. Göckeler, H. Hehl, P.E.L. Rakow, and A. Schäfer for a fruitful collaboration on these topics.

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# Topological lumps near the QCD phase transition

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During the last 20 years a quite comprehensive picture of instantons in QCD has been developed [1]. Of particular importance is the understanding of chiral symmetry breaking: A fluid of weakly interacting instantons produces a non-vanishing density of eigenvalues near the origin. This spectral density is related to the chiral condensate through the Banks-Casher relation [2]. When increasing the temperature above its critical value  $T_c$  chiral symmetry is restored. At this transition the topological lumps are expected to form tightly bound molecules of instantons and anti-instantons. These molecules do no longer produce eigenvalues near the origin, the spectrum develops a gap and chiral symmetry is restored.

In a series of articles [3] we analyzed evidence for chiral symmetry breaking through instantons from ab-initio calculations on the lattice. We studied eigenvalues and eigenvectors of an approximate Ginsparg-Wilson fermion [4] developed in [5]. The eigenvectors are known to trace the topological lumps of the underlying gauge fields and thus can be used to analyze the instanton structure. In particular we focussed on the localization and local chirality [6] of the eigenvectors.

We show that the most localized states can be found near the origin ( $T < T_c$ ) respectively near the edge of the spectral gap ( $T > T_c$ ). Below  $T_c$  the eigenstates are chiral, i.e. the underlying instantons are relatively unperturbed. Above  $T_c$  the states have lost their local chirality and one can no longer speak of intact instantons. Our findings support the picture of chiral symmetry breaking through instantons and the mechanism for chiral symmetry restoration above  $T_c$ .

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# Direct Determination of Flavor Singlet Masses from the Lattice

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The  $\eta'$ -mass is believed to be related to the  $U_A(1)$  anomaly and hence bears important witness of the topological structure of the QCD vacuum, as suggested by the Witten-Veneziano formula that holds in the large  $N_c$  limit of the theory. So far, flavor singlet mesons have been rather elusive to the lattice approach because of large fluctuations encountered in the so-called OZI-rule suppressed diagrams.

We present first evidence[1,2] from two-flavor QCD for a *direct signal* of such flavor singlet meson masses, by applying

- spectral methods to the OZI-rule forbidden piece and
- ground state projection techniques to the OZI-rule allowed contribution

of the singlet meson propagator.

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# Calculating $K \rightarrow \pi\pi$ decay amplitudes beyond the leading-order chiral expansion

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The calculation of non-leptonic decays of hadrons from lattice QCD presents a very challenging problem, mainly because of the formulation of the theory in Euclidean space [1]. The finite-volume techniques developed in Refs. [2,3] offer a possibility of tackling  $K \rightarrow \pi\pi$  decays. These techniques do not rely on any expansion, and yield results of exponential precision in the volume, hence they will eventually give very accurate estimates for  $\langle K|\mathcal{O}|\pi\pi\rangle$ . Nevertheless, implementing these finite-volume techniques is computationally demanding.

Traditionally, lattice calculations for  $K \rightarrow \pi\pi$  amplitudes have been done, following the proposal in [4], by computing matrix elements of the kind  $\langle K|\mathcal{O}|\pi\rangle$  then relating them to  $\langle K|\mathcal{O}|\pi\pi\rangle$  via the lowest-order chiral expansion. In this approach, the chiral corrections are completely missing, and it is very difficult to study these corrections.

In this talk, a novel approach to the calculation of  $K \rightarrow \pi\pi$  amplitudes at the next-to-leading order chiral expansion is presented. The basic idea is to compute matrix elements of the kind  $\langle K|\mathcal{O}|\pi\pi\rangle$  directly on the lattice, allowing energy-momentum injections via the weak operator. More details can be found in Refs. [5,6].

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