Interactions

Take several field ϕ_i, ψ_i, F_i

$$\delta\phi_i = \varepsilon\psi_i \qquad \delta\phi_i^* = \varepsilon^\dagger\psi_i^\dagger$$

$$\delta\psi = -i\left(\sigma^\mu\varepsilon^\dagger\right)\partial_\mu\phi_i + \varepsilon F_i$$

$$\delta\psi^\dagger = i\left(\varepsilon\sigma^\mu\right)\partial_\mu\phi_i^* + \varepsilon^\dagger F_i^*$$

$$\delta F_i = -i\,\varepsilon^\dagger\bar\sigma^\mu\partial_\mu\psi_i$$

$$\delta F_i^* = i\,\partial_\mu\psi^\dagger\sigma^\mu\varepsilon$$

$$\mathcal{L} = -\partial_\mu\phi_i^*\partial^\mu\phi_i + i\,\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_i + F_i^* F_i$$

Can I add interactions? dimensions or $\phi : 1$

$\psi : 3/2$

$F : 2$

- Can have $x^{ij}F_iF_j$, $F_i^*F_j^*$ ($F_iF_j^*$ would interfere with)

$W^i F_i$

+ hermition conjugates

$W^{ij}\psi_i\psi_j$

remember that $\psi_i^\dagger\psi_j$ cannot be made Lorentz invariant; only via $\psi_i^\dagger\bar\sigma^\mu\psi_j$

and adding ∂^μ is the term we already have in the kinetic terms

Interactions of a chiral multiplet

$$\mathcal{L}_{free} = -\partial_\mu \phi_i^* \partial^\mu \phi_i + i\, \psi_i^\dagger \bar\sigma^\mu \partial_\mu \psi_i + F_i^* F_i$$

$$\delta \phi_i = \varepsilon \psi_i \qquad \delta \psi_i = (-i\, \sigma^\mu \varepsilon^\dagger)\,\partial_\mu \phi_i + \varepsilon F_i$$

$$\delta \phi_i^* = \varepsilon^\dagger \psi_i^\dagger$$

$$\delta F_i = -i\, \varepsilon^\dagger \bar\sigma^\mu \partial_\mu \psi_i$$

$$\delta F_i^* = i\, \partial_\mu \psi^\dagger \bar\sigma^\mu \varepsilon$$

$$U(\phi_i, \phi_j^*) \rightarrow \underbrace{\frac{\partial U}{\partial \phi_i}\, \varepsilon \psi_i}_{\phi \phi^* \varepsilon \psi_i} + \dots$$

cannot be produced by the others

$$\left.\begin{array}{l} F_i F_j \\ F_i^* F_j^* \end{array}\right\} \rightarrow F_i\, (-i)\, \varepsilon^\dagger \bar\sigma^\mu \partial_\mu \psi_i$$

cannot be produced by the other

$$F_i\, W^i(\phi, \phi^*) + \text{h.c.} \rightarrow -i\, \varepsilon^\dagger \bar\sigma^\mu \partial_\mu \psi_i\, W^i(\phi,\phi^*) + F_i\, \varepsilon \psi_j \frac{\partial W^i}{\partial \phi_j} + F_i \varepsilon^\dagger \psi_j^\dagger \frac{\partial W^i}{\partial \phi_j^*}$$

$$\psi_i \psi_j\, W^{ij}(\phi,\phi^*) + \psi_i^\dagger \psi_j^\dagger \rightarrow -i\, 2\, \psi_i \sigma^\mu \varepsilon^\dagger \partial_\mu \phi_j\, W^{ij}(\phi,\phi^*)$$

$$+ \psi_i \psi_j \frac{\partial W^{ij}}{\partial \phi^k} \varepsilon \psi_k \;(1) + \psi_i \psi_j \frac{\partial W^{ij}}{\partial \phi^{*k}} \varepsilon^\dagger \psi_k^\dagger \;(2)$$

$$+ \text{h.c.}$$

$$+ 2\, \psi_i \varepsilon\, F_j\, W^{ij}(\phi,\phi^*)$$

(1) + Fierzing $\dfrac{\partial W^{ij}}{\partial \phi^k}$ must be fully symmetric in $i\, j\, k$

(2) $\dfrac{\partial W^{ij}}{\partial \phi_k^*} = 0$

$$U(\phi_i, \phi_j^*) \Rightarrow \frac{\partial U}{\partial \phi_i} \varepsilon \psi_i + \frac{\partial U}{\partial \phi_i^*} \varepsilon^+ \psi_i^+$$

$F_i F_j^*$: mixes with the kinetic terms

$$F_i F_j x^{ij} \Rightarrow 2 x^{ij} F_j (-i) \varepsilon^+ \bar\sigma^\mu \partial_\mu \psi$$

$$F_i W^i(\phi_j, \phi_k^*) \Rightarrow W^i (-i) \varepsilon^+ \bar\sigma^\mu \partial_\mu \psi_i + F_i \frac{\partial W^i}{\partial \phi_j} \varepsilon \psi$$

$$+ F_i \frac{\partial W^i}{\partial \phi_j^*} \varepsilon^+ \psi_j^+$$

$$\psi_i \psi_j W^{ij}(\phi, \phi^*) \Rightarrow 2$$

Gauge supermultiplet

	A_μ	λ	D
on-shell	2	2	0
off-shell	3	4	1

dim	A^a	1
	λ^a	3/2
	D^a	2

$$\delta A^a_\mu = \left((\varepsilon^\dagger \bar{\sigma}_\mu \lambda)^a + \lambda^{a\dagger} \bar{\sigma}_\mu \varepsilon \right) \times \frac{-1}{\sqrt{2}}$$

$$\delta \lambda^a = \frac{i}{2\sqrt{2}} (\sigma^\mu \bar{\sigma}^\nu \varepsilon) F^a_{\mu\nu} + \frac{1}{\sqrt{2}} \varepsilon D^a$$

$$\delta D^a = \frac{i}{\sqrt{2}} \left(-\varepsilon^\dagger \bar{\sigma}^\mu \nabla_\mu \lambda^a + \nabla_\mu \lambda^{a\dagger} \bar{\sigma}^\mu \varepsilon \right)$$

overall factor fixed such that $[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}] X^a = \left(-\varepsilon_2 \sigma^\mu \varepsilon_1^\dagger + \varepsilon_1 \sigma^\mu \varepsilon_2^\dagger \right) i \nabla_\mu X^a$

Note:
- covariant derivatives
- $\Rightarrow$ nonlinear
- can add (even more) auxiliary fields to make it linear

Can make $\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i \lambda^a \bar{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2} D^a D^a$

supersymmetric in this way.

Couple gauge & chiral multiplets

$$\Rightarrow \quad \delta F_i = -i\, \varepsilon^\dagger \bar{\sigma}^\mu \nabla_\mu \psi_i + \sqrt{2}\, g\, (T^a \phi)_i\, \lambda^{a\dagger}$$

& we need extra interactions with λ^a D^a as well

$$-\sqrt{2}\, g\, (\phi^* T^a \psi)\, \lambda^a - \sqrt{2}\, g\, \lambda^{\dagger a} (\psi^\dagger T^a \phi) + g\, (\phi^* T^a \phi)\, D^a$$

are dim ≤ 4

$W^i (T^a \phi_i)$ must be zero $\Rightarrow$ also needed for gauge invariance

$\Rightarrow$ scalar interactions fixed!