Lepton-mixing and neutrinos

The lepton masses are determined from: (CODATA)

e^- : atomic physics : $m_e = .510\ 998\ 910(13)$ MeV/c^2

μ^- : muonic atoms $m_\mu = 105.6583668(38)$ MeV/c^2

notice that these are much better known but m_p is only known to this precision in MeV.

τ^- : threshold behaviour in $e^+e^- \to \tau^+\tau^-$

best precision is now BES in Beijing, China / $\tau \to \pi\pi\pi\nu$ decay

$$m_\tau \simeq 1777.0 \text{ MeV} \pm 0.3 \text{ MeV} \qquad 1776.82 \pm 0.16$$

I will not talk about the leptons very much except in their semileptonic decays / couplings.

Universality has been tested and has been found to hold everywhere with various amounts of precision. Examples of tests are:

$$\left.\begin{array}{l} \mu^- \to e^- \bar{\nu}_e \nu_\mu \\ \tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \\ \tau^- \to e^- \bar{\nu}_e \nu_\tau \end{array}\right\}$$

measurements of all possible couplings (usually known as Michel parameters)

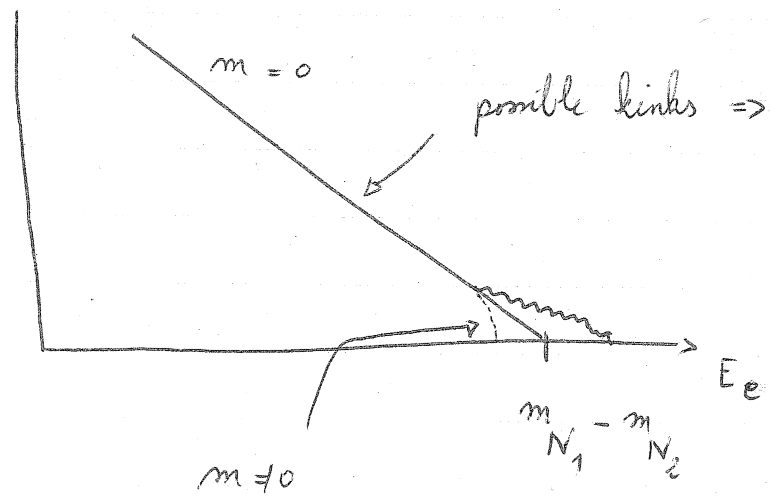
$$W \to e\nu,\ \mu\nu,\ \tau\nu$$
$$Z \to e^+e^-,\ \mu^+\mu^-,\ \tau^+\tau^-$$

So far the indications are that the only difference really is their mass. As examples of some more advanced tests let us now move to the neutrino masses.

Neutrino masses: Direct measurements

i) ν_e : look at β decay spectra near the endpoint

Curie plot



but experimental uncertainties lead to ~~mmm~~ 2 eV $\lesssim 0.?$ 2013

present upper limits are : $m_{\nu_e} \lesssim$ ~~1.5 eV~~ ~~$\lesssim 3$ eV~~

but experiment gives $m_\nu^2 < 0$ ~~[illegible]~~

-0.6 ± 1.9

from the SN 1987A : is far away and all neutrinos arrived together

=> $m_\nu \lesssim 18$ eV (some claim up 5 eV)

ii) ν_μ $m \lesssim 0.~~27~~$ 19 MeV

from studies of $\mu^- \to e^- \nu \bar{\nu}$ e^- spectrum

but best limit from $\pi^+ \to \mu^+ \nu$ + measure μ^+ momentum

iii) ν_τ $m \lesssim$ ~~27~~ 18 MeV

studies of $\tau \to \begin{matrix} 6\pi \\ 5\pi \\ 3\pi \end{matrix} \nu$ decays

iv) e^- majorana mass :

from neutrinoless $\beta\beta$ decay

$m_\nu < $ ~~1.1~~ 0.48 eV

notice $2\beta 2\nu$ measures lifetimes of about $10^{18} - 10^{20}$ years

only proton decay has done better.

v) Cosmology $m_\nu \lesssim 1$ eV structure formation

Neutrino masses & mixings

If the neutrinos had mass and the mass-eigenstates would be different we would observe neutrino oscillations. This is the equivalent of 2 waves of different frequencies interfering.
Let us look at an escample:
at $t=0$ we have ν_e neutrinos only and assume that they mix with the ν_τ.*
Mass eigenstates are $\nu_{1,2}$ with mass m_1 and m_2.
and

$$|\nu_e\rangle = c_\theta |\nu_1\rangle + s_\theta |\nu_2\rangle$$
$$|\nu_\tau\rangle = -s_\theta |\nu_1\rangle + c_\theta |\nu_2\rangle$$

$\Rightarrow$

$$|\nu_e(t)\rangle = c_\theta\, e^{-iE_1 t} |\nu_1\rangle + s_\theta |\nu_2\rangle e^{-iE_2 t}$$

$$= |\nu_e\rangle \left(c_\theta^2\, e^{-iE_1 t} + s_\theta^2\, e^{-iE_2 t} \right) + |\nu_\tau\rangle\; c_\theta s_\theta \left(e^{-iE_1 t} - e^{-iE_2 t} \right)$$

now $E_i = \sqrt{k^2 + m_i^2} \simeq k + \frac{m_i^2}{2k} + \dots$
and relativistically $t \simeq L$ the distance

$$P_{\nu_\tau} = (s_\theta c_\theta)^2 \left| e^{i \frac{E_1+E_2}{2} t} \left(e^{i\frac{\Delta E}{2} t} - e^{-i\frac{\Delta E}{2} t} \right) \right|^2$$

$$= \sin^2 2\theta \, \sin^2\left(\frac{\Delta E\, t}{2}\right) \simeq \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4k}\right)$$

now experiments are done over a few distances and a few energies.
$\Rightarrow$ $\sin^2$ modulation leads to a rather funny shape of the limits

* The reason to take ν_τ is because mixing with ν_μ is escperimentally very restricted.

The MSW effect.

in vacuum we have

$$i \frac{d}{dt}\begin{pmatrix}\nu_e \\ \nu_\tau\end{pmatrix} = \begin{pmatrix}c_\theta & s_\theta \\ -s_\theta & c_\theta\end{pmatrix}\begin{pmatrix}E_1 & 0 \\ 0 & E_2\end{pmatrix}\begin{pmatrix}c_\theta & -s_\theta \\ s_\theta & c_\theta\end{pmatrix}\begin{pmatrix}\nu_e \\ \nu_\tau\end{pmatrix}$$

in matter

$$+ \begin{pmatrix}\sqrt{2}\, G_F N_e & 0 \\ 0 & 0\end{pmatrix}\begin{pmatrix}\nu_e \\ \nu_\tau\end{pmatrix} = H_{tot}\begin{pmatrix}\nu_e \\ \nu_\tau\end{pmatrix}$$

$$= \begin{pmatrix}c_\theta^2 E_1 + s_\theta^2 E_2 + \sqrt{2} G_F N_e & c_\theta s_\theta(-E_1+E_2) \\ c_\theta s_\theta(-E_1+E_2) & s_\theta^2 E_1 + c_\theta^2 E_2\end{pmatrix}\begin{pmatrix}\nu_e \\ \nu_\tau\end{pmatrix}$$

$$E_i = \sqrt{m_i^2 + k^2} \simeq k + \frac{m_i^2}{2k}$$

so we pull out $\quad \bar{H} = 1 \times \quad k + \frac{m_1^2+m_2^2}{4k} + \frac{\sqrt{2}\, G_F N_e}{2}$

This term we can drop since it is diagonal and identical.

$$\Delta m^2 = m_2^2 - m_1^2$$

$$= \begin{pmatrix}\frac{\Delta m^2}{4k}\left(-c_\theta^2 + s_\theta^2\right) + \frac{G_F N_e}{\sqrt{2}} & 2 c_\theta s_\theta \frac{\Delta m^2}{4k} \\ 2 c_\theta s_\theta \frac{\Delta m^2}{4k} & \frac{\Delta m^2}{4k}\left(c_\theta^2 - s_\theta^2\right) - \frac{G_F N_e}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\nu_e \\ \nu_\tau\end{pmatrix}$$

this is identical to the vacuum case if we define

$$\frac{\Delta m_M^2}{4k} \sin 2\theta_M = \frac{\Delta m^2}{4k} \sin 2\theta$$

$$\frac{\Delta m^2}{4k} \cos 2\theta - \frac{G_F N_e}{\sqrt{2}} = \frac{\Delta m_M^2}{4k} \cos 2\theta_M$$

or $$\Delta m^2_M = \Delta m^2 \frac{\sin 2\theta}{\sin 2\theta_M}$$

and $$\mathrm{tg}\, 2\theta_M = \frac{\sin 2\theta}{\cos 2\theta - \dfrac{2\sqrt{2}\, k\, N_e G_F}{\Delta m^2}}$$

this can lead to a very large enhancement for θ_M compared to θ if

$$\cos 2\theta \simeq \frac{2\sqrt{2}\, k N_e\, G_F}{\Delta m^2}$$

eg for water $N_e \simeq 1000 \dfrac{kg}{m^3} \, \dfrac{1}{1.7 \times 10^{-27}\, kg} \, \dfrac{1}{2} \simeq 3 \times 10^{29}\ m^{-3}$

$$\uparrow$$
$$N_e \simeq \tfrac{1}{2}(N_p + N_n)$$

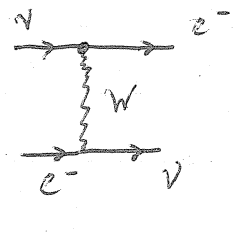
$$k = \frac{1\ \mathrm{MeV}}{6.6 \times 10^{-22}\ \mathrm{MeV\ sec} \times 3 \times 10^{8}\ m/\mathrm{sec}} \simeq 5 \times 10^{12}\ m^{-1}$$

$$\Delta m^2 \simeq 3 \times 10^{29} \times 5 \times 10^{12} \times 3\ \ 1.2 \times 10^{-5}\ (2 \cdot 10^{-13})^4\ \cancel{m^{-4}}^{\,0}\ \underbrace{\mathrm{GeV}^{-2}}_{10^{-18}\, eV^{-2}}\ \underbrace{\mathrm{MeV}^4}_{10^{24}\, eV^4}\ \cancel{m^{4}}^{\,0}$$

$$\simeq 900 \times 10^{\cancel{29}+\cancel{12}-5-\cancel{52}-\cancel{18}+\cancel{24}}\ eV^2 \quad (+1\ \ -6)$$

$$\simeq 10^{-7}\ eV^2$$

Neutrino oscillations in matter or the MSW effect:
in matter there is an additional difference between the neutrinos since normal matter contains lots of electrons but no muons or taus.
The neutral current interaction is the same for all ~~electrons~~ neutrinos but in addition the charged current can contribute to electron neutrinos via



scattering of the electrons.
This enhancing of mixing can be very large (see appendix). Maximal enhancement is obtained for

$$\cos 2\theta \, \Delta m^2 = 2\sqrt{2}\, k\, G_F N_e$$

which for very small θ's and a density of water is obtained for
$\Delta m^2 \simeq 10^{-7}\, eV^2$.
Since in the sun N_e varies it is possible to obtain this enhancement for a range of Δm^2.

For the solar neutrinos N_e varies so one has to integrate the equations with the possibility that $\Delta m_N^2 = 0$ are the levels cross.
What is the general signal we want to discuss:

The spectrum of the sun consist mainly of 3 components:
^8B 0 to 15 MeV
^7Be a sharp peak 860 keV
main pp cycle 0 -> 450 keV : determined by the luminosity of the sun.

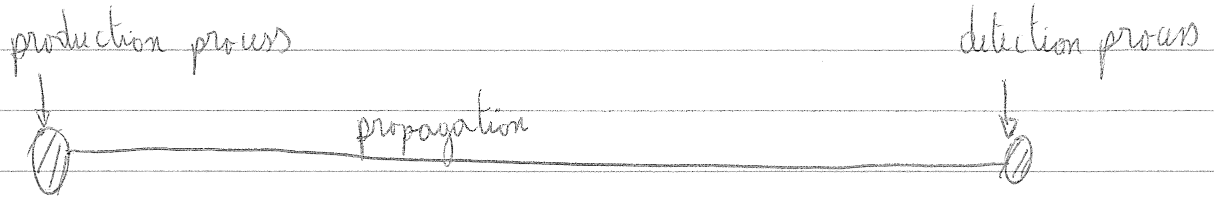
Kamiokande sees ^8B at $\sim 60\% \pm 8$
Homestake sees $^8B + {}^7Be$ at $\sim 30\%$
Gallex + Sage see $pp + {}^7Be + {}^8B$ at $\sim 70\%$

The solution is: let pp through
fully suppress ^7Be
suppress ^8B partly
(Remember $\sigma \sim E^2$ so small high energy fluxes give a large part of the cross-section.)

Extra notes for neutrino (and other) oscillations

1) We do the oscillations in the rest frame and use the same $\vec{k}$ but a different mass: what about energy conservation?

One has to keep in mind that what one really does is something like



At the production and detection process the relevant (at least for neutrinos) states are the flavour eigenstates if the detection/production involves a charged lepton.

$$|\nu_l\rangle = \sum_i U^*_{li}\,|\nu_i\rangle \qquad l = e, \mu, \tau$$

The propagation is most easily described in the mass eigenstate basis. Then of course everything is really wave packets (as in your field theory course)

So all oscillations involve really

$$\langle \nu^f_{l'}|\ \text{propagation}\ |\nu^i_l\rangle = |(U^\dagger)_{il'}\ \text{diagonal propagation}_{ii}\ U^*_{li}$$

and if Majorana one has to add the antineutrino eigenstates as well

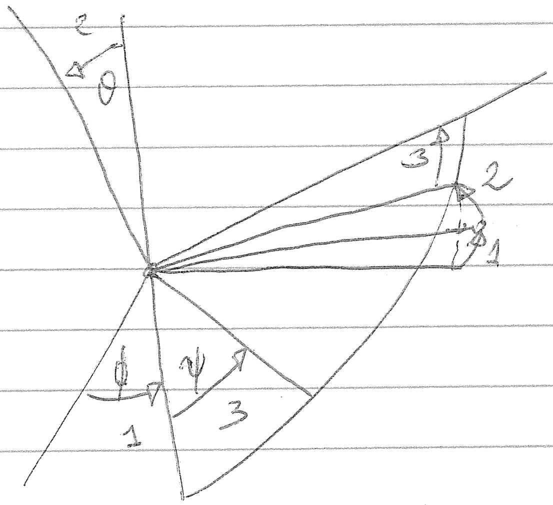
2) To understand neutrino experiments at very low energies: $\sigma \sim E^2 G_F^2$ so increases fast!

- type and energy of incoming beam
- type of detection
- length of propagation

3) Understanding the angles and phases

All the choices are Euler angle variants

The most common Euler angle convention



$$\begin{pmatrix} x''' \\ y''' \\ z''' \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi & \\ -\sin\psi & \cos\psi & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & \cos\theta & \sin\theta \\ & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi & \\ -\sin\phi & \cos\phi & \\ & & 1 \end{pmatrix}$$

Note that clearly many different choices are possible

PDG choices: (both for quarks & leptons)

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{cp}} \\ -s_{12}c_{23}-c_{12}s_{13}s_{23}e^{i\delta_{cp}} & c_{12}c_{23}-s_{12}s_{13}s_{23}e^{i\delta_{cp}} & c_{13}s_{23} \\ s_{12}s_{23}-c_{12}s_{13}c_{23}e^{i\delta_{cp}} & -c_{12}s_{23}-s_{12}s_{13}e^{i\delta_{cp}} & c_{13}c_{23} \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & & \\ & e^{i\frac{\alpha_{21}}{2}} & \\ & & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

Representative data PDG 2012

	neutrinos 1209.3023	quarks	
θ_{12}	33.3 ± 0.8 (2.4)%	13.023 ± 0.040 (3%)	$s_{12} = \lambda$
θ_{23}	40.0 ± 2 or 50.0 ± 1.3 (4%)	2.32 ± 0.07 (3%)	$s_{23} = A\lambda^2$
θ_{13}	8.6 ± 0.5 (6%)	0.20 ± 0.01 (5%)	
δ	not really known	-69 ± 4 (6%)	

Unitarity : quarks:

top row	0.9999 ± 0.0006
2nd	1.067 ± 0.047
~~1st~~ column	1.002 ± 0.005
2nd column	1.065 ± 0.046

Note:

$$\begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{-i\delta} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$