Let us now go systematically through the decays.

\[ \begin{align*}
\pi & \rightarrow \mu \nu \\
K & \rightarrow \mu \nu
\end{align*} \]

\[ F_K = \frac{1}{F_\pi} \left( 1 - \frac{1}{2} \mu_K - \frac{3}{4} \mu_{\eta'} + \frac{5}{4} \mu_\pi + 4 \left( m_\Lambda - m^3 \right) \frac{B_\rho}{F_\pi} \right) \]

\[ \mu_i = \frac{m_{\pi}^2}{2 \pi^2 F_\pi^2} \log \frac{m_i^2}{m^2} \]

\[ \frac{F_K}{F_\pi} \rightarrow L_5^2(\mu) \]

\[ \begin{align*}
\pi & \rightarrow \mu \nu \\
K & \rightarrow \mu \nu
\end{align*} \]

Here there are 2 types of contributions:

\( H(\rho) \rightarrow \ell^+(p_\ell) \nu(\bar{p}_\nu) \gamma(q) \)

\[ a) \quad M^+ \rightarrow \nu W \rightarrow \ell^+ \]

Bremstrahlung: must be there

Structure dependent part

\[ H_{\mu\nu} T = +i G_F e \varepsilon_{\mu\nu\rho\sigma} \frac{V^*_\mu}{m_\mu} H^{\nu\rho} \rightarrow \gamma(1+\delta_5) e \]

\[ H^{\mu\nu} = -i \varepsilon^{\mu\nu\rho\sigma} q_\mu p_\sigma - A(q \cdot W \gamma^{\nu\rho} - W^{\nu\rho}) \]
$V$ can be related to $\pi^0 \to \gamma \gamma$
A can be related to $L_{\gamma + L_{\mu}}$ and in fact serves to
determine it.
From $\pi^+ \to l^- \nu l  l$ we can then predict $K^+ \to l^- \nu l  l$
The $\gamma$ can be off-shell, i.e. it is $e^+e^-$ or $\mu^+\mu^-$
the extra piece can to $\mathcal{O}(p^4)$ be fully expressed in terms
of $F_V^\pi(q^2)$
So it is fully predicted.
Agreement with experiment is good
(but not with PDG values)
The expressions are rather long.

\[
\begin{align*}
\pi^+ & \to \pi^0 \ e^+ e^- \\
K^+ & \to \pi^0 \ (e^+ e^-) \ (\mu^+ \nu^-) \\
K^0 & \to \pi^0 \ (e^+ e^-) \ (\mu^+ \nu^-)
\end{align*}
\]

These have the general structure
\[
A = \frac{G_F}{\sqrt{2}} \ V_{u3}^* \ \overline{\tau} \ \gamma^\mu \ (1 + \gamma_5) \ e^- \ \frac{<\pi^0 | V_\mu | K^+>}{{\rho}' \rho} \\
\]
\[
\frac{1}{\sqrt{2}} \left[ \left( \rho^+ + \rho^- \right) S_+ + \left( \rho^+ - \rho^- \right) S_- \right]
\]

$s_+$ and $s_-$ depend on $(p_\mu + p_\nu)^2$
and can be calculated using the CHPT formalism.
PCAC gives $\xi_+ = 1$

$$\xi_+ \approx 0$$

at next to leading order a good approximation is

$$\xi_+(t) = \xi_+(0) \left[ 1 + \lambda_+ \frac{t}{m_{\pi^+}^2} \right]$$

$$\xi_-(t) = \xi_-(0) = \xi_+(0) (\lambda_0 - \lambda_+) \frac{m_{\pi^-}^2 - m_{\pi^0}^2}{m_{\pi^+}^2}$$

with $\lambda_+ \approx 0.031$ ($0.030 \pm 0.002$) $K^+ \rightarrow \pi^0 e^+ \nu$  

$\lambda_0 \approx 0.017$ ($0.015 \pm 0.004$) in $K^+ \rightarrow \pi^0 \mu^+ \nu$ decay

$\xi_-(t)$ only contributes $\propto m_{\pi^0}^2$, so is negligible in $K^+ \rightarrow \pi^0 \mu^+ \nu$ decay

there are small corrections from $\xi_+(0) \approx 1$ (vector symmetry)

$$K_{\ell 3 \gamma} : K^+ \rightarrow \pi^0 \ell^+ \nu \gamma$$

lots of new form factors (so in total)

all calculated + quite nontrivial results

good tests of chiral symmetry

$$K_{\ell 4} \text{ decay}$$

$K \rightarrow \pi\pi e^+ \nu, \mu^+ \nu$

4 form factors per decay; all known to next to leading order

+ some to higher orders