hep-lat/0012005 Münster, Walzl
hep-lat/0009001
hep-lat/0509046 Christine Davies

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- the yearly lattice ~~[illegible]~~ meeting : big very specialized
- INT Seattle has had a school 2012
  2007
- les Houches 2009 , 1997

Books : I. Montvay - G. Münster
H. Rothe
J. Smit
Gattringer & Lang

- QCD: . asymptotically free : can prove in perturbation theory
  - . infrared slavery / confinement: automatically involves strong coupling: perturbation theory does not work
  - . there are effects beyond perturbation theory
    - bound states
    - instantons & other classical configurations
      $\Rightarrow$ eg Baryon number violation in the Standard Model
    - monopoles
- Need a non perturbative definition of QFT

use the path integral

$$\langle x' | e^{-iHT} | x \rangle = \int \underbrace{\mathcal{D}x}_{\text{all paths } x(t)} \, e^{iS}$$

$$S = \int_0^T dt \underbrace{\left\{ \frac{m}{2} \left(\frac{dx}{dt} \right)^2 - V(x) \right\}}_{\mathcal{L}} \qquad \mathcal{D}x = \prod_t x_t$$

QFT

$$\langle 0 | T(\phi(x_1) \dots \phi(x_n)) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}\phi \, \phi(x_1) \dots \phi(x_n) e^{iS}$$

$$Z = \int \mathcal{D}\phi \, e^{iS} \qquad S = \int d^d x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4 + \dots \right]$$

- $$e^{iHT} \to e^{-H\tau}$$

$$e^{-H\tau} = \sum_i e^{-E_i \tau} |i\rangle\langle i|$$

so for long τ the ground state always dominates

$$\langle O_1(\tau_1)\, O_2(\tau_2)\, O_3(\tau_3) \rangle$$

$$e^{-H(\tau_1-\tau_2)} \qquad e^{-H(\tau_2-\tau_3)}$$

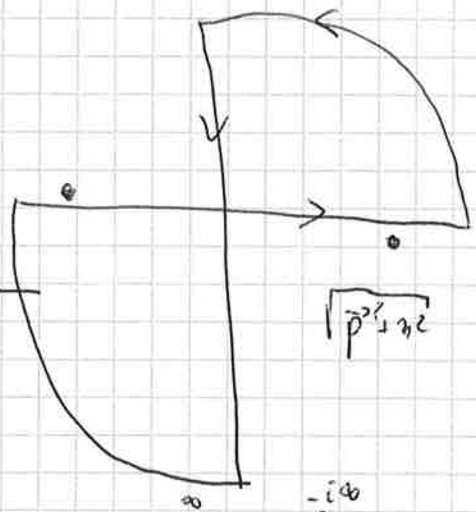
$$= \langle 0 | O_1 | i \rangle\langle i | \, e^{-E_i(\tau_1-\tau_2)} \, e^{-E_j(\tau_2-\tau_3)} \, O_2 | j \rangle\langle j | O_3 | 0 \rangle$$

$\Rightarrow$ comment about excited states difficult ρ versus 2π

This way you can get at 3 point functions, ...

- $$\Delta_F^E = \int \frac{d^4p}{(2\pi)^4} \frac{e^{i p\cdot x}}{p^2+m_0^2}$$

$$\Delta_F^M = i \int \frac{d^4p}{(2\pi)^4} \frac{e^{-i p\cdot x}}{p^2-m_0^2+i\varepsilon}$$



$$\oint_C \frac{1}{p^2-m_0^2+i\varepsilon} = 0 \Rightarrow \int_{-\infty}^{\infty} dp^0 + \int_{i\infty}^{-i\infty} dp^0 = 0$$

$$\Rightarrow \int_{-\infty}^{\infty} dp^0 = i\int_{-\infty}^{\infty} dp_4$$

- lattice $\quad \mathcal{D}\phi = \prod_x d\phi(x)$

$$x_\mu = a\, n_\mu \qquad n_\mu \in \mathbb{Z}$$

$$\partial_\mu \phi \to \frac{1}{a}\left(\phi(x + a\hat{\mu}) - \phi(x)\right)$$

$$\int d^4x \to \sum_x a^4$$

Fourier transform: $\tilde{\phi}(p) = \sum_x a^4 \, e^{-ip\cdot x} \phi(x)$

$p' = p + \frac{2\pi \vec{m}}{a}$ is the same

$\Rightarrow$ Brillouin zone $\qquad -\frac{\pi}{a} < p_\mu \leq \frac{\pi}{a}$: Cut off!

- Finite lattice + periodic

$$\phi(x + aL_\mu \hat{\mu}) = \phi(x)$$

momenta discretised: $\quad p_\mu = \frac{2\pi}{a} \frac{l_\mu}{L_\mu} \qquad l_\mu$ integer

L_μ size of lattice

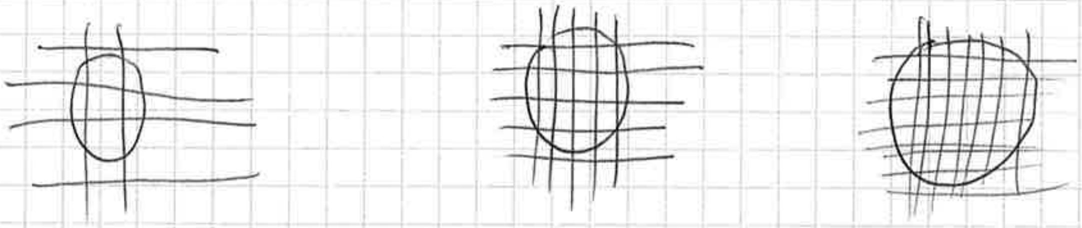
- lattice regulator + finite volume:

Lorentz symmetry + translations $\Rightarrow$ discrete symmetry

= Poincaré group

presumably restored for $L_\mu \to \infty$; $a \to 0$
(if in the same phase & universality class)

- $a \to 0$ continuum limit



correlation lengths on the lattice should diverge
$\Rightarrow$ critical point in statistical mechanics
= continuum limit of a QFT

Note: must exist this limit

- typical sizes at present: 64×32^3, 128×64^3 or variations
16, 24, 48 also used
- improved algorithms: make effects due to nonzero a smaller
- renormalisation $Z(a)$, $m^2(a)$, $\lambda(a)$ such that continuum limit exists
  - $\Rightarrow$ can be done perturbatively
  - $\Rightarrow$ but also nonperturbatively: must choose which physical quantities
  - $\Rightarrow$ divergences a^4, a^2, $\ln a$: can be painful in the numerics.

Gauge theory $\qquad \partial_\mu - i g A_\mu$

<u>But</u> ∂_μ is a <u>non local object</u>

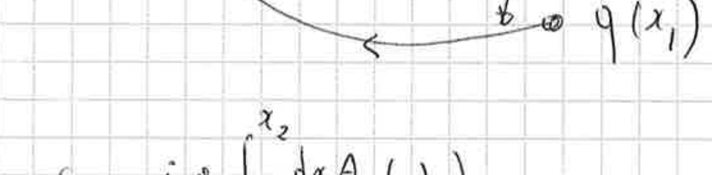
<u>2 options</u> take continuum Lagrangian + discretize
$\Rightarrow$ problem is continuum limit
$\begin{cases} \text{gauge invariance only restored at } a=0 \\ \text{but needed for renormalisibility} \end{cases}$

$\Rightarrow$ have gauge invariance for finite a

Wilson : link variables

Continuum : path ordered exponential

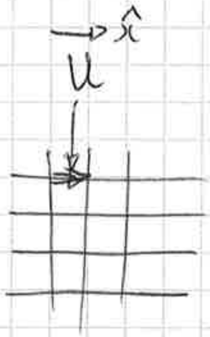
$$\bar{q}(x_2) \; P \; e^{-ig\int_{x_1}^{x_2} dx \, A_\mu(x)} \; q(x_1)$$



$$U = P\left(e^{-ig\int_{x_1}^{x_2} dx\, A_\mu(x)}\right) \rightarrow \quad U(x_2)\, U\, U^\dagger(x_1)$$

$$A_\mu(x) \rightarrow \quad U(x) A_\mu U(x)^\dagger - \frac{i}{g} U^\dagger \partial_\mu U$$

Wilson : use $U \in G$ directly as the variable



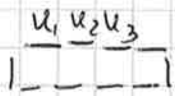
$$U(x_2, x_1) \to U(x_2)\, U(x_1, x_1)\, U^\dagger(x_1)$$

link variable $\quad x_2 = x_1 + a\hat{\mu}$

also $\quad U(x, \hat{\mu})$

Lagrangian + other gauge invariant objects

lines or curves of links : Wilson lines/ loops



$\mathrm{tr}\left(U_1 U_2 \dots U_3\right)$ is gauge invariant

Polyakov ; 't Hooft lines : use the periodic boundary conditions to connect them

Use Plaquette as basic action 

$$\mathrm{tr}\left(U_1 U_2 U_3 U_4\right) \to \frac{-1}{4} F_{\mu\nu} F^{\mu\nu}$$

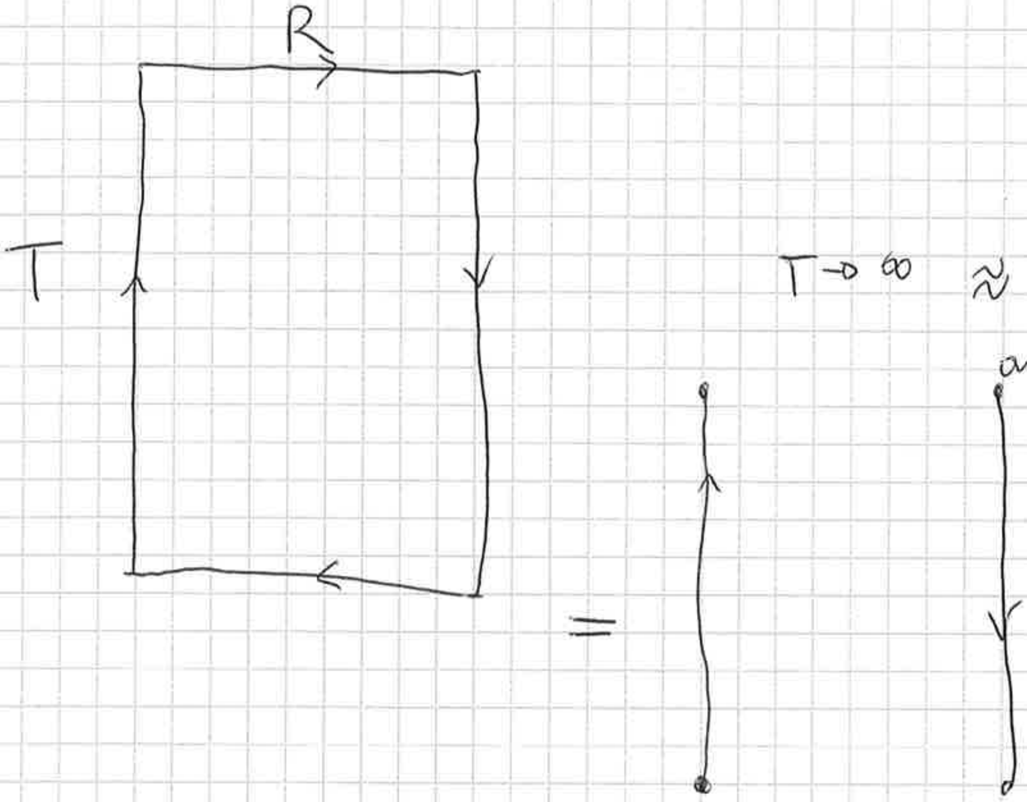
$\mu\nu$ orientation of the plaquette

Integration : Haar measure
$$\begin{cases} \int_G dU\, 1 = 1 \\ \int_G dU\, f(U) = \int_G dU\, f(UV) \end{cases}$$

V is any group element

is defined for compact Lie groups

Wilson loop:



$T \to \infty$ $\approx$ static quark anti quark

large time $\propto e^{-T\,V(R)}$

$\Rightarrow$ measure potential

$\Rightarrow$ confinement give $V(R) \propto R$ for large R

$\Rightarrow$ Wilson loop follows area law

$$S_W = -\sum_p \frac{\beta}{N} \, \mathrm{Re}\left(\mathrm{Tr}\left(U(p)\right)\right)$$

sum over plaquettes

(subtract the constant part sometimes)

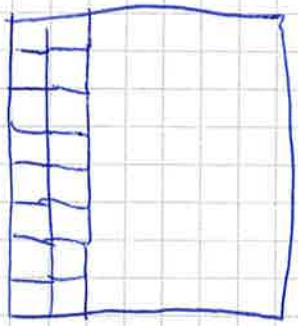
$$\Rightarrow \int d^4x \, \frac{\beta g_0^2}{8N} F_{\mu\nu} F^{\mu\nu} \qquad U \approx e^{-i g_0 a A}$$

$$\Rightarrow \beta = \frac{2N}{g_0^2}$$

$$\int \mathcal{D}U \; e^{-S_W}$$

gauge fixing <u>not</u> needed

$$e^{\frac{\beta}{N} \mathrm{Re}\,(\mathrm{Tr}\, U)} = 1 + \beta + \dots$$



$$\int dU\, U = 0$$

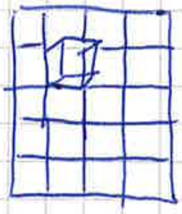
each U must be cancelled by a U^{+}

$$\propto \beta^{RT} = e^{(\ln\beta)\, RT}$$

so $V(R) \propto -(\ln\beta)\, R$ or

strong coupling $\Rightarrow$ linear confinement

NLO



$+ \dots$

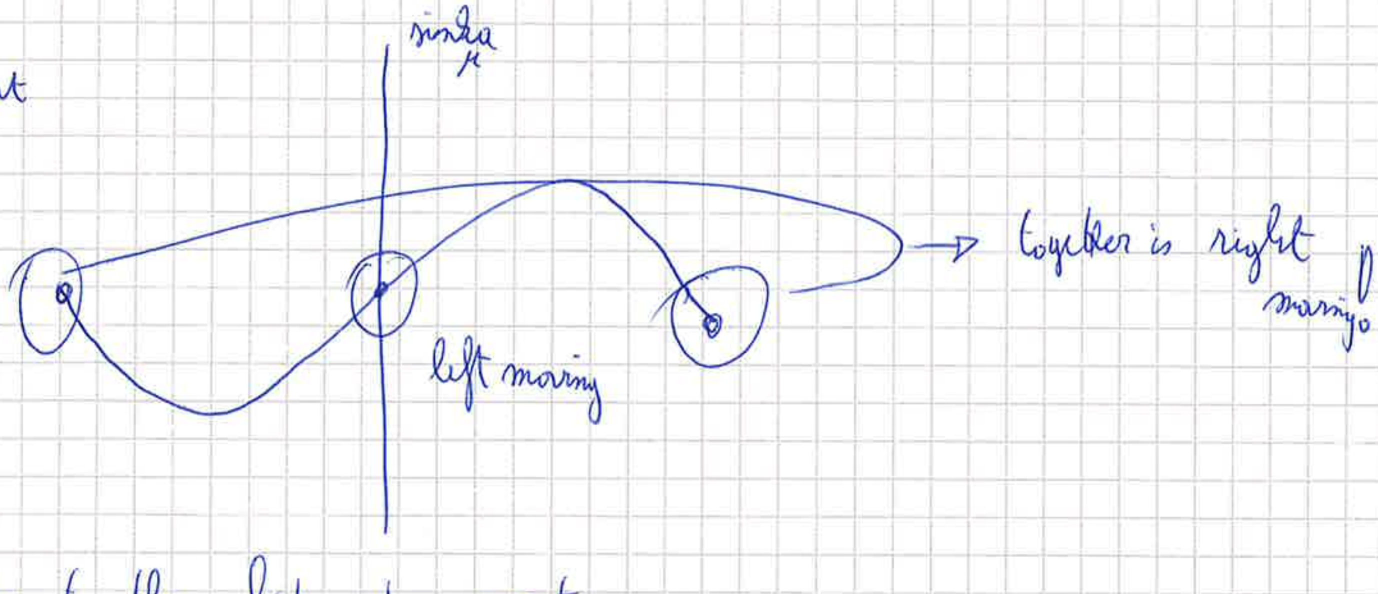
Fermion doubling

Kinetic term is linear $\frac{1}{2a}\left(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*\right)$ or its equivalent

$$\int_{-\frac{\pi}{a}}^{\pi/a} dk \quad \phi^*(k) \sum_\mu \frac{1}{a} \sin(k_\mu a) \quad \phi(-k)$$

small k part $\Rightarrow$ k_μ

but



due to the first order nature

$$\partial_\mu \phi^* \partial^\mu \phi \quad \rightarrow$$

$$\int_{-\frac{\pi}{a}}^{\pi/a} dk \quad \phi^*(-k)\, \phi(k) \quad k^2$$

This becomes a right handed degree $\Rightarrow$ problems with chiral theories

Nielsen Ninomiya theorem

$$\begin{cases} \cdot \text{ finite range} \\ \cdot \ \gamma_5 \text{ symmetry} \end{cases} \Rightarrow \text{doublers}$$

Way out : · add a Wilson term : $\bar{\psi}\,\partial^2\psi$

$\Rightarrow$ doublers heavy

$\Rightarrow$ chiral symmetry restored

· make range of ∂ infinite

· Ginsparg-Wilson fermions $\gamma_5 D + D\gamma_5 = D\gamma_5 D$

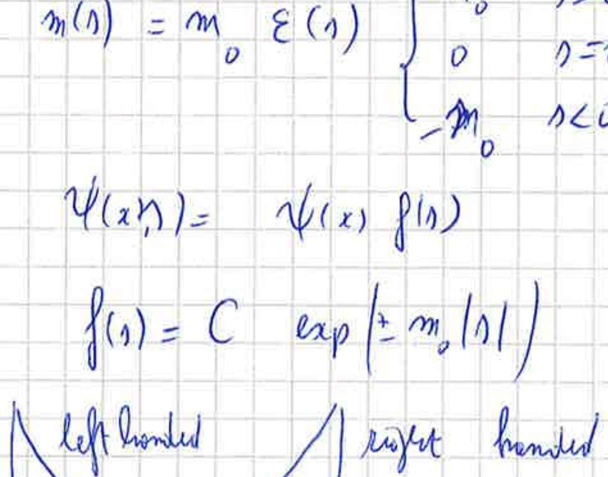
· Domain Wall fermions / Overlap fermions

$$S = \int d^4x\, ds\ \bar{\psi}(x,s)\left[\underbrace{\gamma_A \partial_A}_{5\,\text{dim}} - m(s)\right]\psi(x,s)$$

$$m(s) = m_0\,\varepsilon(s) \begin{cases} m_0 & s>0 \\ 0 & s=0 \\ -m_0 & s<0 \end{cases}$$

$$\psi(x,s) = \psi(x)\,f(s)$$

$$f(s) = C\ \exp\left(\pm m_0 |s|\right)$$



Fermions => Grassmann variables

action quadratic $\int \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \; e^{i\int d^4x \, \bar{\psi} \not{D} \psi}$

$\propto \det \not{D}$

but $\det \not{D}$ is a very non local object

=> difficult.

needs evaluating many times

=> quenched

=> partially quenched

- importance sampling ; Metropolis + other Monte Carlo methods
- eye graphs ; calculating propagators
-

- QCD is confining => need nonperturbative methods

- path integral
- Euclidean space / Wick rotation

- parallel transporter ≃ link variables
- Plaquettes, Wilson lines / loops, confinement
- fermions & doubling

- strong coupling expansion

- explain how observables are measured
- noise sampling, ...
- quenched, partially quenched ,