Homework set 3, FYTN04, autumn 2014

Due: Friday 28 November 2014, 10.15

1. Consider a collision $A+B \rightarrow C+D$ in its center of mass frame. Calculate $E_C$, $E_D$ och $|p_C|$ as functions of the CM energy and the masses (which are assumed non-negligible).

2. (9.1 in book, with clarification/addition) Suppose a fourth family exists, with a complete set of particles including a fourth neutrino $\nu_4$ of mass similar to the other neutrinos, and gauge interactions identical to those of the other families. What would the branching ratio be for $Z \rightarrow \nu_4\overline{\nu}_4$? What about the branching ratio for $W^+ \rightarrow L_4^+\nu_4$, where $L_4$ is the charged lepton of the family? At first assume that top and the fourth generation quarks are too heavy to be produced, and later consider what would have changed if instead they had been very light.

3. (9.2 in book) Suppose we want to measure the cross section for colliding $\nu_\mu$ with right-handed electrons, $e_R$, in order to determine $\sin^2 \theta_W$. (i) How does the cross section vary with $\theta_W$ at fixed $g_2$? (ii) Estimate the cross section assuming $m_Z^2 \gg s \gg m_e^2$. Proceed by (a) drawing any diagrams that allow the scattering to occur, using the rules of Chapter 7; (b) put in the vertex factors and propagators; (c) make approximations as described in section 9.3 to obtain the matrix element; (d) go from the matrix element to the cross section using the results of the beginning of this chapter. How does the answer to (i) change if $e$ is fixed rather than $g_2$?

4. (10.1 in book) Suppose in a collision two oppositely charged particles emerge into the detector with four-momenta measured to be $p_1 \approx (43; 0, 0, 43)$ and $p_2 \approx (48; 0, 0, -48)$ with units in GeV. How would you interpret the event?

5. (10.3 in book, with simplification/clarification) Estimate the number of $Z^0$’s produced and detected at the old $p\overline{p}$ collider at CERN, with $\sqrt{s} = 630$ GeV. Rather than fully repeating the calculation of sections 10.1–10.4, just estimate the ratios of $Z^0$ to $W^\pm$ at each stage of the argument. For the constituent cross section, it would be a good approximation to use the branching ratio numbers of Chapter 9. Assume that only $e^+e^-$ and $\mu^+\mu^-$ ($e\nu$ and $\mu\nu$ for the $W$) final states are observed. The numerical challenge is to estimate how much the integral $I$ of eq. (10.12) is changed by the difference between the $W$ and $Z$ masses. To simplify the task, you are at this particular stage of the calculation allowed to neglect the mass difference. (But remember to get prefactors right.)
6. Complement the study in the lectures of the three-body phase space in muon decay. (With the neglect of electron and neutrino masses.)
   (a) Calculate $\cos \theta_{qk}$.
   (b) Derive the limits of the allowed triangular area of the $(q = |q|, k = |k|)$ plane.

7. (12.1 in book) Suppose an accelerator has been constructed that provides a one TeV proton beam. Now it is necessary to decide how to utilize it. Four possibilities are:
   (a) hit a fixed target,
   (b) collide it with a 50 GeV electron beam to study $ep$ collisions,
   (c) collide it with another proton beam, also accelerated to one TeV, and
   (d) collide it with an antiproton beam made from a source of $\bar{p}$'s (antiprotons are harder to obtain than protons, of course).
   What is the maximum available energy for production of new particles in each case? What other considerations might be important in making a decision?

8. (13.2 in book) Suppose $Z^0$'s are produced in the process $q + \bar{q} \rightarrow Z^0 + g$.
   What fraction of $Z^0$'s would not appear in the detector? How could such events be recognized anyhow?