Homework set 4, FYTN04, autumn 2016

Due: Wednesday 7 December 2016, 10.15

1. Calculate, approximately, the branching ratio for Higgs decay $h^0 \to \tau^+\tau^-$ if
(a) $M_h = 8$ GeV;
(b) $M_h = 15$ GeV;
(c) $M_h = 126$ GeV; and
(d) $M_h = 250$ GeV.

2. In $e^+e^- \to \gamma^*/Z^0 \to q\bar{q}$, i.e. annihilation to quark pairs, the average number of charged particles is about 8 at 10 GeV, and at 30 GeV it has increased to 13.
(a) What would one expect at LEP1, 91 GeV, and LEP2, 200 GeV?
(Hint: recall the approximately logarithmic increase introduced in the lectures.)
(b) At LEP2 energies, a competing process is W^+W^- pair production. What average number of charged particles should one expect there, again assuming that the W's decay to quark pairs?
(c) What sources of error could one imagine in the predictions above?
(d) Roughly how big is the region over which the particle production process occurs at LEP1 energies, in space–time and in rapidity?

3. Empirically, it has been found that vector and pseudoscalar mesons obey the mass relation

$$m_{\mathrm{V}}^2 - m_{\mathrm{PS}}^2 \approx \text{constant}$$

when comparing different pairs of the same flavour content.
(a) Explore how well the relation holds for (ρ, π), (K^*, K), (D^*, D) and (B^*, B).
(Hint: mass tables can be accessed at `http://pdg.lbl.gov`.)
(b) Motivate the validity of this mass relation from the mass equation with hyperfine splitting described in the lectures.

4. Interpret the following relations satisfied by parton distribution functions,
(a) $\int_0^1 x\,\mathrm{d}x \sum_{i=q,\bar{q},g} f_i(x) = 1$,
(b) $\int_0^1 \mathrm{d}x\,(f_{u/p}(x) - f_{\bar{u}/p}(x)) = 2$,
(c) $\int_0^1 \mathrm{d}x\,(f_{s/p}(x) - f_{\bar{s}/p}(x)) = 0$.

5. When two hadrons collide at high energies, the quarks in them can collide and produce a lepton pair ($\mu^+\mu^-$ or e^+e^-) via an intermediate virtual photon. (Similarly to the graph for Z^0 production.) This is called a Drell-Yan process.
(a) Convince yourself that $\hat{s} = x_1 x_2 s$, like for W and Z production, and that the cross section is approximately given by

$$\sigma = \sum_{q,\bar{q}} \int \mathrm{d}x_1 \mathrm{d}x_2 f_{q/A}(x_1) f_{\bar{q}/B}(x_2) \hat{\sigma}(q\bar{q} \to \mu^+\mu^-) \ .$$

(b) Argue that the partonic cross section is approximately

$$\hat{\sigma} \sim \frac{\pi \alpha^2 Q_q^2}{\hat{s}} \ ,$$

where Q_q is the electric charge of the quark q in units of e.
(c) Consider two cases for incoming beam, (i) π^+ and (ii) π^-, on a carbon target. Carbon is isoscalar, i.e. contains as many u as d quarks. Show that the ratio

$$R = \frac{\sigma(\pi^+ \mathrm{C} \to \mu^+\mu^- + \text{anything})}{\sigma(\pi^- \mathrm{C} \to \mu^+\mu^- + \text{anything})}$$

should be near 1 when $\hat{s}/s$ is small, and that R should approach $1/4$ for $\hat{s}/s \to 1$.
(d) What is the ratio of the Drell-Yan cross section to produce a $\mu^+\mu^-$ pair to that to produce an e^+e^- pair?