Homework set 5, FYTN04, autumn 2016

Due: Wednesday 14 December 2016, 10.15

1. a) The measured τ lepton lifetime is 2.90×10^{-13} s, known to an accuracy of about 0.4%. How does that compare with the prediction of the standard model? Remember the calculation of τ_μ in the lectures, and the number of channels.
b) QCD corrections modify the decay width to quarks roughly by a factor $1+\alpha_s/\pi$. Use the data above to estimate $\alpha_s(M_\tau^2)$.

2. (a) Show that any coupling strength $\alpha_i(Q^2)$ (at least in the Standard Model) satisfy an equation of the form

$$\frac{\partial \alpha_i(Q^2)}{\partial \left[\ln(Q^2/\mu^2)\right]} = B\alpha_i^2 \ .$$

What does B depend on?

(b) If Λ_{QCD} is measured to be 225 MeV, what is α_3 at $Q^2 = M_\Upsilon^2$ and at M_Z^2? (Reminder: Υ is the $b\bar{b}$ state equivalent to J/ψ of $c\bar{c}$. See Appendix C for mass.)

3. The α coupling of QED increases with Q^2. Disregarding the rest of the standard model, it becomes singular at some scale. This phenomenon is called a Landau pole. Estimate the energy scale of the pole. As extremes, either include only the electron in the theory, or three full generations of fermions. You may simplify by assuming that all fermions have the electron mass and that $\alpha(m_e^2) = 1/137$. Compare with the Planck energy, $E_{\rm Pl} = 1/\sqrt{G_{\rm N}}$, where $G_{\rm N}$ is Newton's gravitational constant.

4. Show that the state of two π^0's has even CP. Which of the K_L and K_S could decay to $\pi^0\pi^0$?

5. Explain why the decay $D^0 \to K^-\pi^+$ is allowed, while $D^0 \to K^+\pi^-$ is suppressed. Derive an expected ratio of branching ratios, based on the coupling structure alone. Compare with data, where the branching ratio for the former is observed to be $3.89 \cdot 10^{-2}$ and for the latter $1.48 \cdot 10^{-4}$.

6. The $B^0 - \overline{B}^0$ and $B_s^0 - \overline{B}_s^0$ meson systems oscillate, similarly to the $K^0 - \overline{K}^0$ one. The oscillations are slower, however, in the sense that the oscillation times are not too dissimilar from the decay times. Therefore it is meaningful to ask whether a particle produced e.g. as a B^0 remains as that at a later time, or has oscillated to an $\overline{B}^0$ one. This has been studied e.g. at LEP1. The B mesons produced there take most of the energy of their jet, so $E_B \approx 40$ GeV.
a) Calculate the typical distance a B^0 meson travels before it decays and compare this with the $B^0 - \overline{B}^0$ oscillation length. Some numbers: $m_B = 5.279$ GeV, $\Delta m_B = 0.464 \times 10^{12} \hbar \mathrm{s}^{-1}$, and $\tau = 1.56 \times 10^{-12}$ s. The decay rates of the two states is the same.
b) Repeat the exercise above for the B_s^0 system. Here $m_{B_s^0} = 5.366$ GeV, $\Delta m_{B_s^0} = 17.8 \times 10^{12} \hbar \mathrm{s}^{-1}$ and $\tau = 1.47 \times 10^{-12}$ s.
c) For each of the two particles, calculate how often an original B^0/B_s^0 decays as a $\overline{B}^0/\overline{B}_s^0$, integrated over all times.
d) Propose a suitable experimental signature for the presence of decays of the "wrong" particle.

7. The supernova SN1987A was detected as a pulse of neutrinos of about 5–10 MeV arriving within a 10 s window, from a distance of about 170 000 light years. The pulse length is consistent with the expectations in supernova models, but could alternatively be used to set limits on the ν_e mass. Roughly what would those limits be?