"Nonaccelerator" experiments (Ch 14)

There are 2 ways to search for physics beyond the standard model. One is to search for new particles / interactions by going to very high energy and producing them directly.

Another method is to measure at low energies at a very high precision so we can then extract "virtual" effects from a very high mass.

Now this is a whole range:

- nuclear β decay $\Rightarrow$ majorana mass ν_e limits
- μ^- decay, μ^- capture
  $\Rightarrow$ look for processes where lepton number is not conserved or the type of lepton changes

$\mu^- \to e^- \gamma$	< 1.2 ~~10^{-11}~~	2.4×10^{-12}
$\tau^- \to \mu^- \gamma$	$4.4 < 10^{-8}$	
$\tau^- \to \mu^+ \mu^- \mu^-$	$2.1 < 10^{-8}$	
$\mu^- \to e^+ e^- e^-$	$BR < 1.0\, 10^{-12}$	
$K^0 \to \mu e$	4.7×10^{-12}	
$K^+ \to \pi^+ \mu^+ e^-$	1.3×10^{-11}	
$B^0 \to \mu e$	6.4×10^{-8}	
$B^0 \to \tau \mu$	2.2×10^{-5}	

here some incredibly small branching ratio limits have been set and measured

proton decay $p \to e^+ \pi^0$ $\tau > 8.2 \times 10^{33}$ years

universe is $13 \cdot 10^9$ years

$\beta\beta$ ^{76}Ge $\tau > 1.9 \times 10^{25}$ years

Chapter 23 Quark Masses

Hadron masses are well defined ⇒ can measure directly

Constituent quark mass : $m_p \simeq m_n \simeq 3\, m_q$, $m_\rho \simeq m_q + m_{\bar{q}}$ $\Big\} \Rightarrow$ $M_u \simeq M_d \simeq 300\,\text{MeV}$, $M_s \simeq 500\,\text{MeV}$

(current) quark masses :
- not directly measurable
- well defined as a parameter in the QCD lagrangian (but note running)

Hadron masses have also electromagnetic effects

$m_p < m_n$ — electromagnetic > 0

$m_u < m_d$

$m_{K^+} < m_{K^0}$ — sme

$m_{\pi^+} > m_{\pi^0}$ — essentially all electromagnetic

Chapter 22 : Mixing

We had given masses to the leptons/quarks via

$$- g_{eij} \bar{L}_i \Phi e_{Rj} - g^*_{eij} \bar{e}_{Rj} \Phi^\dagger L_i$$

$$- g_{dij} \bar{Q}_i \Phi d_{Rj} - g^*_{dij} \bar{d}_{Rj} \Phi^\dagger Q_i$$

$$+ g_{uij} \bar{Q}_i \tilde{\Phi} u_{Rj} + g^*_{uij} \bar{u}_{Rj} \tilde{\Phi}^\dagger Q_j$$

complex matrices!

$$L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \qquad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \tilde{\Phi} = -i \tau_2 \Phi$$

Remember that Dirac and Colour indices are hidden here

But in reality we have 3 of each

$$L_1 = \begin{pmatrix} \nu'_{eL} \\ e'_L \end{pmatrix} \qquad L_2 = \begin{pmatrix} \nu'_{\mu L} \\ \mu'_L \end{pmatrix} \qquad L_3 = \begin{pmatrix} \nu'_{\tau L} \\ \tau'_L \end{pmatrix}$$

and same for right handed

$$Q_1 = \begin{pmatrix} u'_L \\ d'_L \end{pmatrix} \qquad Q_2 = \begin{pmatrix} c'_L \\ s'_L \end{pmatrix} \qquad Q_3 = \begin{pmatrix} t'_L \\ s'_L \end{pmatrix}$$

$u_1 = u'$
$u_2 = c'$
$u_3 = t'$
etc.

$$\mathcal{L}_{GF} = \sum_{j=1,2,3} \Big[\bar{L}_j i \gamma_\mu D^\mu L_j + \bar{Q}_j i \gamma_\mu D^\mu Q_j + \bar{u}_{Rj} i \gamma_\mu D^\mu u_{Rj} + \bar{d}_{Rj} i \gamma_\mu D^\mu d_{Rj} + \bar{e}_{Rj} \gamma_\mu D^\mu e_{Rj}$$

- weak eigenstates or interaction eigenstates (cf $W^3_\mu, B_\mu \Leftrightarrow A_\mu, Z_\mu$)
- So add to the Yukawa terms generation indices

- Now as vectors, matrices in generation space!

- $$\Phi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} \qquad \tilde{\Phi}^{*} = \begin{pmatrix} -\frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix}$$

- Mass term for down quark looks like + Higgs interaction

$$-\left(\frac{v+H}{\sqrt{2}}\right)\left(\bar{d}'_{Li}\, g_{dij}\, d'_{Rj} + \bar{d}'_{Rj}\, g^{*}_{dij}\, d'_{Li}\right)$$

+ same for up quarks, charged leptons

- For any complex matrix M we can find unitary matrices U, V such that

$$U M V^{\dagger} = \begin{pmatrix} \ddots \end{pmatrix} \quad \text{diagonal + real + positive}$$

$$U_d\, g_d\, V_d^{\dagger} = M_d = \begin{pmatrix} \lambda_d & & \\ & \lambda_s & \\ & & \lambda_b \end{pmatrix}$$

Now set $D'_L = \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix} = U_d^{\dagger} \underbrace{\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}}_{D_L} \qquad \underbrace{\begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix}}_{D'_R} = V_d^{\dagger} \underbrace{\begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}}_{D_R}$

So the masses + Higgs are

$$-\left(\frac{v+H}{\sqrt{2}}\right)\left(\bar{D}'_L\, g_d\, D'_R + \bar{D}'_R\, g_d^{\dagger}\, D'_L\right)$$

$$= -\left(\frac{v+H}{\sqrt{2}}\right)\left(\bar{D}_L \underbrace{U_d\, g_d\, V_d^{\dagger}}_{\begin{pmatrix} \lambda_d & & \\ & \lambda_s & \\ & & \lambda_b \end{pmatrix}} D_R + \bar{D}_R \underbrace{V_d\, g_d^{\dagger}\, U_d^{\dagger}}_{\begin{pmatrix} \lambda_d & & \\ & \lambda_s & \\ & & \lambda_b \end{pmatrix}} D_L\right)$$

But the W_μ has different particles

$$-\frac{g_2}{\sqrt{2}} W^+_\mu \left(\bar{u}'_{Li} \gamma^\mu d'_{Li}\right) - \frac{g_2}{\sqrt{2}} W^{-}_\mu \left(\bar{d}'_{Li} \gamma^\mu u'_{Li}\right)$$

$$= -\frac{g_2}{\sqrt{2}} W^+_\mu \bar{U}'_L \gamma^\mu D'_L + \dots$$

$$= -\frac{g_2}{\sqrt{2}} W^+_\mu \bar{U}_L U_u \gamma^\mu U_d^\dagger D_L + \dots$$

$$= \frac{-g_2}{\sqrt{2}} W^+_\mu \bar{U}_L \gamma^\mu \underbrace{U_u U_d^\dagger}_{V_{CKM}} D_L + \dots$$

Calibbo - Kobayashi - Maskawa

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

is a unitary matrix

$$V^\dagger_{CKM} V_{CKM} = \left(U_u U_d^\dagger\right)^\dagger U_u U_d^\dagger = U_d U_u^\dagger U_u U_d^\dagger = U_d U_d^\dagger = \mathbb{1}$$

and $\quad -\frac{v+H}{\sqrt{2}} \lambda_d \left(\bar{d}_L d_R + \bar{d}_R d_L \right) - \frac{v+H}{\sqrt{2}} \lambda_s \left(\bar{s}_L s_R + \bar{s}_R s_L \right)$

$$- \frac{v+H}{\sqrt{2}} \lambda_b \left(\bar{b}_L b_R + \bar{b}_R b_L \right)$$

The unprimed ones: mass eigenstates

And we do the same for up quarks + charged leptons
(⇒ neutrino oscillations / masses later)

$$D'_L = U_d^\dagger D_L \qquad D'_R = V_d^\dagger D_R$$

$$U'_L = U_u^\dagger U_L \qquad U'_R = V_u^\dagger U_R \qquad U_L = \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix}$$

. What does this do to the interactions?

eg γ: $\quad -\frac{1}{3} e A_\mu \; \bar{d}'_{Li} \gamma^\mu d'_{Li} \quad + \dots$

$$= -\frac{1}{3} e A_\mu \; \bar{D}'_L \gamma^\mu D'_L \quad + \dots$$

$$= -\frac{1}{3} e A_\mu \; \bar{D}_L U_d \gamma^\mu U_d^\dagger D_L + \dots$$

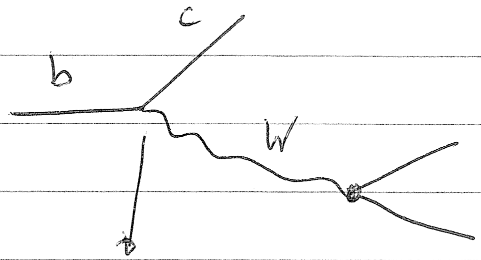
so $U_d U_d^\dagger = 1$ (remember different types of indices ⇒ commute)

$$= -\frac{1}{3} e A_\mu \; \bar{D}_L \gamma^\mu D_L + \dots$$

$$= -\frac{1}{3} e A_\mu \left(\bar{d}_L \gamma^\mu d_L + \bar{s}_L \gamma^\mu s_L + \bar{b}_L \gamma^\mu b_L \right) + \dots$$

If you think about it: same for gluon, Z_μ since it has $\bar{\psi}'_f \gamma^\mu \psi'_f$ as well with the same ψ on both sides

. No flavour changing neutral currents ! (qualification: at tree level)



$$\frac{g_2}{\sqrt{2}} V_{cb} \; \bar{c} \gamma_\mu P_L b$$

so just like muon decay but must sum over all particles

$$\Gamma_b = 9 \, \frac{V_{cb}^2 G_F^2 M_b^5}{192 \pi^3}$$

$$\frac{\Gamma_b}{\Gamma_\tau} = \frac{9}{5} V_{cb}^2 \frac{M_b^5}{M_\tau^5}$$

$9 = 3 \times \cos^2\theta_c$ ud *
$+ 3 \times \cancel{\cos}^2\theta_c$ cs
$+ 3 \times \sin^2\theta_c$ cd
$+ 3 \times \cos^2\theta_c$ us
$+ 1 \times$ $e\nu$ *
$+ 1 \times$ $\mu\nu$ *
$+ 1 \times$ $\tau\nu$

for τ only

and $\cos^2\theta_c \simeq 1$ so 5!

This is how you measure V_{cb} (also $\dfrac{\Gamma(b \to c e \nu_e)}{\Gamma(\tau \to \nu_\tau e \nu_e)} = 3 V_{cb}^2 \dfrac{M_b^5}{M_\tau^5}$

let's look at 2×2 case

$$V = e^{i\delta}\begin{pmatrix} \cos\theta\, e^{i\alpha} & \sin\theta\, e^{i\gamma} \\ -\sin\theta\, e^{-i\gamma} & \cos\theta\, e^{-i\alpha} \end{pmatrix}$$

~~but~~ $d_L \to e^{-i\delta} d_L \qquad d_R \to e^{-i\delta} d_R$

$s_L \to e^{-i\delta} s_L \qquad s_R \to e^{-i\delta} s_R$

let's do one more thing

$$u_L \to e^{i\alpha_u} u_L \qquad c_L \to e^{i\alpha_c} c_L \qquad d_L \to e^{i\alpha_d} d_L \qquad s_L \to e^{i\alpha_s} s_L$$

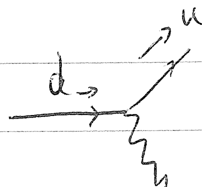
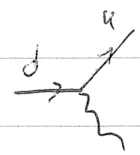
$$u_R \to \;''\; u_R$$

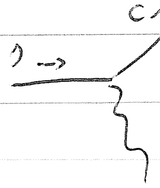
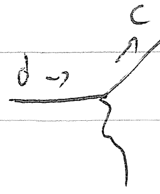
$\Rightarrow$ mass terms are $\bar{u}_L u_R + \dots$ do not change

but $V \to \begin{pmatrix} e^{-i\alpha_u} & \\ & e^{-i\alpha_c} \end{pmatrix} V \begin{pmatrix} e^{i\alpha_d} & \\ & e^{i\alpha_s} \end{pmatrix}$

$$\begin{cases} 0 = -\alpha_u + \alpha_d + \delta + \alpha \\ 0 = -\alpha_u + \alpha_s + \delta + \gamma \\ 0 = -\alpha_c + \alpha_d + \delta - \gamma \\ 0 = -\alpha_c + \alpha_s + \delta - \alpha \end{cases} \qquad \begin{cases} \alpha_d = -\delta - \alpha + \alpha_u \\ \alpha_s = -\delta - \gamma + \alpha_u \\ \alpha_c = -\alpha - \gamma + \alpha_u \\ \alpha_c = -\alpha - \gamma + \alpha_u \end{cases}$$

so a solution is allowed

and  $\frac{g_2}{\sqrt{2}} \cos\theta_c\, \gamma^\mu P_L$ or  $\frac{g_2}{\sqrt{2}} V_{ud}\, \gamma^\mu P_L$

 $\frac{g_2}{\sqrt{2}} \cos\theta_c$ or  $-\frac{g_2 \sin\theta_c}{\sqrt{2}} \gamma^\mu P_L = \frac{g_2}{\sqrt{2}} V_{cd}\, \gamma_\mu P_L$

etc. $\Rightarrow$ generations can change into each other via the weak interaction

Ch. 24 CP violation

Meson $P = (-1)^{L+1}$ L orbital angular momentum

S spin

$C = (-1)^{L+S}$

$CP(K^0) = +\bar{K}^0$ $C(K^0) = -\bar{K^0}$

$CP(\pi^0) = -\pi^0$

$K_L = \frac{1}{\sqrt{2}}(K_0 - \bar{K}_0) \rightarrow 3\pi^0$ but not $2\pi^0$ so lives a lot longer

$K_S = \frac{1}{\sqrt{2}}(K_0 + \bar{K}_0) \rightarrow 2\pi^0$ $3\pi^0$

Hopson: $K_L \rightarrow 3\pi^0$ was seen in early sixties: CP violation!

$\sim 10^{-3}$

$m_L - m_S = (3.483 \pm 0.006) \times 10^{-12}$ MeV

$m_{K^0} = 497.614 \pm 0.02$ MeV

later

$$\frac{\Gamma(K_L \to \pi^- e^+ \nu) - \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_L \to \pi^- e^+ \nu) + \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)} = 0.00332 \pm 0.00006$$

Now measured in very many decays in K, B_s, B_d system!

$\cancel{CP}$ in SM

in field theory: CPT is always a good symmetry

T $\quad e^{-iEt} \to e^{iEt}$ "complex conjugate"

need a real complex coupling constant
(not one I can get rid of as earlier today)

Only place in SM is V_{CKM} with 3 generations

$\Rightarrow$ one phase left (Kobayashi - Maskawa Nobel Prize 2008)

$\Rightarrow$ predicted the need for a third generation!