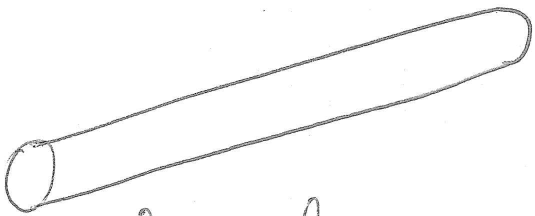
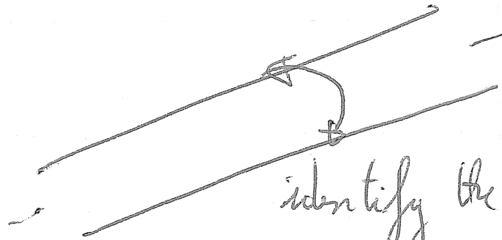
Extra dimensions and superstrings

1 + 1 + 1 dimensional examples
(time) (space) (compactified space)

a) Kaluza Klein type

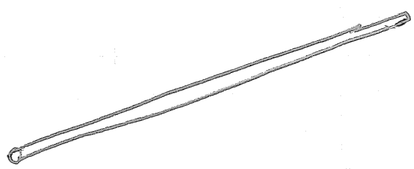


or



identify the 2 parts

or seen from far away (difficult when you're inside I know)



$$\mathcal{L} = \int d^3x \left(\partial_\rho \phi^* \partial^\rho \phi \right)$$

$\rho = 0, 1, 2$

$\mu = 0, 1$

But $\phi(t, x, y) = \phi(t, x, y+L)$

so we can write

$$\phi(t,x,y) = \frac{1}{\sqrt{L}} \sum_{n=0}^{\infty} e^{i \frac{2\pi n}{L} y} \phi_n(t,x)$$

and
$$\phi_n(t,x) = \int_0^L dy \frac{1}{\sqrt{L}} e^{-i \frac{2\pi n}{L}} \phi(t,x,y)$$

We plug that in $\mathcal{L}$ and get

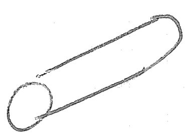
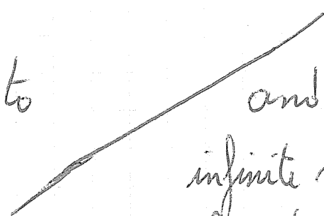
$$\int d^3x \; \partial_\rho \phi^* \partial^\rho \phi$$

$$= \int d^2x \int_0^L \frac{1}{L} \sum_{n,m} \Big[\partial_\mu \phi_n^* \partial^\mu \phi_m \; e^{-i\frac{2\pi}{L}(m-n)}$$

$$+ \phi_n^* \frac{-i2\pi n}{L} \frac{i2\pi}{L} m \, \phi_m \times e^{-i\frac{2\pi}{L}(m-n)}$$

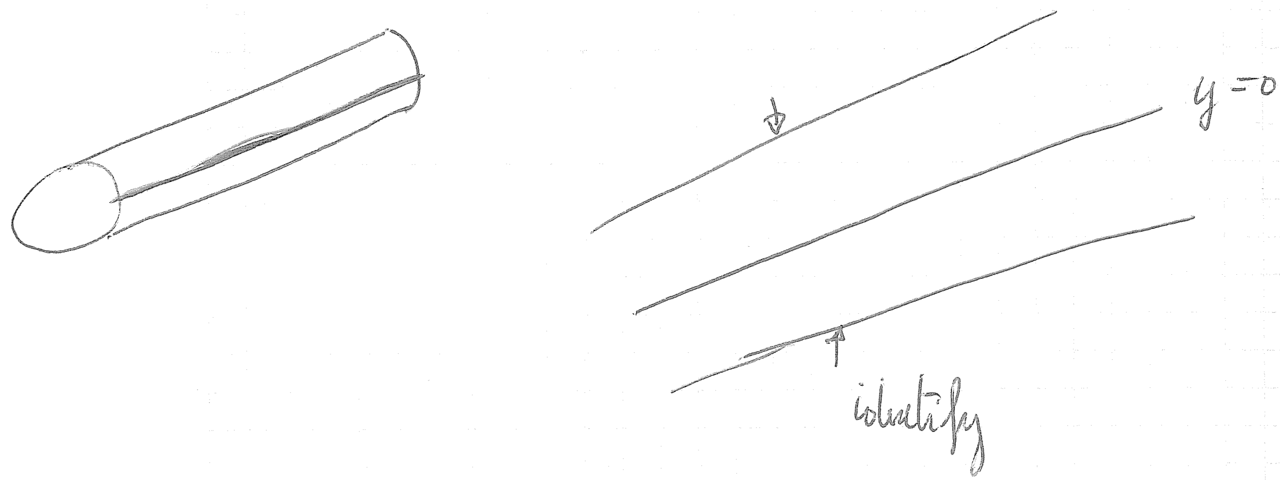
from $g_{yy} = -1$!

$$= \int d^2x \sum_{n=-\infty}^{+\infty} \left[\partial_\mu \phi_n^* \partial^\mu \phi_n - \left(\frac{2\pi n}{L}\right)^2 \phi_n^* \phi_n \right]$$

So  and one ϕ is equivalent to  and an infinite number of ϕ_n

For energies $\ll \frac{2\pi}{L}$ we only keep the mass-less sector

b) Large extra dimension type



2 different types of objects exist

$$\begin{cases} \psi(t,x,y) = \psi(t,x)\,\delta(y) \\ \phi(t,x,y) \end{cases}$$

($y=0$: brane ; $y \neq 0$ bulk)

$$\begin{cases} \psi \text{ lives only on the brane} \\ \phi \text{ lives in the bulk} \end{cases}$$

string theory produces objects that actually have this type of behaviour

As long as you only have ψ type of experiments you cannot see the extra dimension, even with energies $\gg \frac{2\pi}{L}$

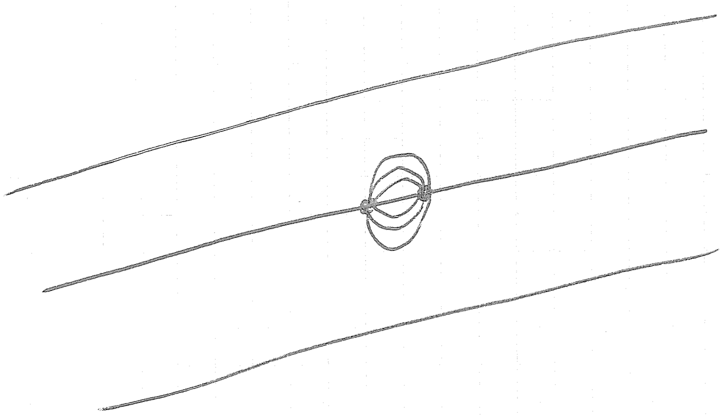
Why do you want this?

Hierarchy problem: $\frac{e^2}{r} \sim G_N \frac{E^2}{r}$

requires $E \sim 10^{19}$ GeV (M_{Planck})

in $3+n$ spatial dimensions: $V \sim \frac{1}{r^{1+n}}$

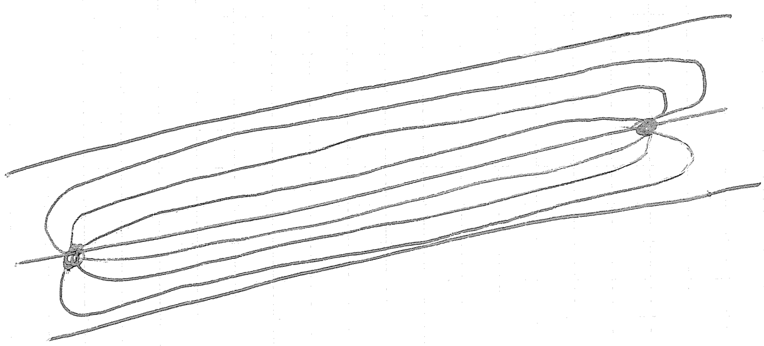
So



$V(r) \sim \frac{1}{r^{1+n}}$

(well $\log r$ for 2 dim)

but



$V(r) \sim \frac{1}{r}$

(well $\sim r$ for 1 dim)

So $V(r) = " \frac{M_{Pl}^{-n-2}}{r^{1+n}} "$ and $\frac{1}{M_{Pl}^{n} L^{n} r}$

$r \ll L$ $\qquad$ $r \gg L$

So $\left(M_{Pl}^{(4)}\right)^2 \approx \underbrace{M_{Pl}^{n} L^{n}}\, M_{Pl}^{2}$

We know $M_{Pl} \gtrsim$ few 100 GeV $\qquad$ this allows to put the hierarchy here

For one extra dimension:

$$\frac{1}{p^2} \sim \frac{1}{r}$$

(here Fouriertransform in the usual dimensions)

$$\frac{1}{p^2 - m^2} \sim \frac{e^{-mr}}{r}$$

So you get an extra factor compared to only one

$$\sum_{n=0}^{\infty} \frac{e^{-nr\frac{2\pi}{L}}}{} = \frac{1}{1-e^{-r2\pi/L}}$$

$$\approx 1 \quad \text{for} \quad r \gg L$$

$$\approx \frac{L}{r2\pi} \quad \text{for} \quad r \ll L$$

What did Kaluza and Klein do?

Start with General Relativity in 5 dimensions and let's look at the gravity matter interaction

$$\mathcal{L} = \int d^5x \sqrt{-g} \left(\partial_\rho \phi^* \partial^\rho \phi \right) \sim$$

$\nu, \mu = 0, \ldots, 3$

$\sigma, \rho = 0, \ldots, 5$

$$g_{\rho\sigma} = \eta_{\rho\sigma} + h_{\rho\sigma}$$

$$g_{\rho\sigma} : \quad g_{\mu\nu} \, , \; A_\mu = g_{\mu 5} \, , \; \psi = g_{55}$$

and expand all fields in the fifth dimension (w is the x^5 coordinate)

$$g_{\mu\nu} = \frac{1}{\sqrt{L}} \sum_n g_{\mu\nu}^{(n)} e^{i \frac{2\pi}{L} w}$$

$$\phi = \frac{1}{\sqrt{L}} \sum_n \phi^{(n)} e^{i \frac{2\pi}{L} w}$$

$$A_\mu = \frac{1}{\sqrt{L}} \sum_n A_\mu^{(n)} e^{i \frac{2\pi}{L} w}$$

$$\psi = \frac{1}{\sqrt{L}} \sum_n \psi^{(n)} e^{i \frac{2\pi}{L} w}$$

$A_\mu^{(0)}$ behaves like a photon and $\phi^{(n)}$ like a charged particle with charge n

Notice : · zero mass sector : simple field theory
· lots of scalars : possible sources for Higgs
· gauge theories and symmetries out of nothing

- One can choose the extra dimensions in a much more complicated fashion to generate ~~bigger~~ gauge groups
- The extra structure can have funny topological aspects as well and singularities

so lots of possibilities.

Extra dimensions and gravity: missing momentum type of measurements and other limits

a) Direct tests of $\frac{1}{r}$ at short distances:

$\frac{1}{r}$ has now been tested down to about 40 μm

(gravity is really weak for small masses and they need to be close to reach those distances: so cannot be big)

b) How does gravity generate missing momentum?

look eg to $e^+e^- \to g\gamma$

$$\sigma \sim G_N e^2 E^2$$

really really small until $E^2 \sim M_{Pl}^2$

But if extra dimensions: for 1 of size L

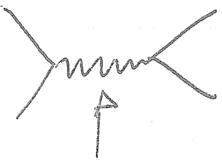
about $E/\frac{2\pi}{L}$ different gravitons can be radiated each with a cross-section of order $G_N e^2 E^2$

and for n extra dimensions the factor is

$$\left(\frac{E}{2\pi/L}\right)^n$$

which can be enormous $(1\,fm)^{-1} \simeq 200$ MeV

(c)



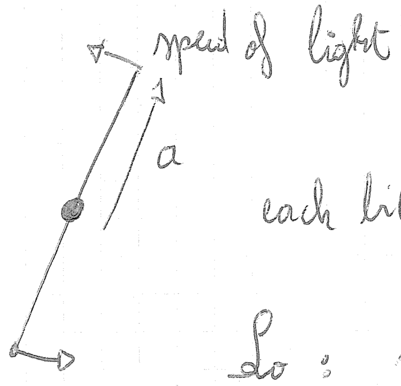
again the weak graviton exchange gets strongly enhanced by the number of gravitons that can be exchanged

b) Well if gravity gets strong: black holes, —
(lots of speculations; but little really known)

Strings

Why do strings excite people so much?

60's Chew - Low - Frautschi $M^2 \sim J$



each bit has velocity $v = \frac{rc}{a}$

So: $$m = \int_0^a \frac{K\,dr}{\sqrt{1 - r^2/a^2}}$$

$r = xa$

$$= Ka \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$= \frac{\pi}{2} K a$$

$$J = \int_0^a \frac{r/a\,K\,r\,dr}{\sqrt{1 - r^2/a^2}}$$

$$= K a^2 \int_0^1 \frac{x^2}{\sqrt{1-x^2}}\,dx$$

$$= \frac{\pi}{4} K a^2$$

$$m^2 = \pi K J$$

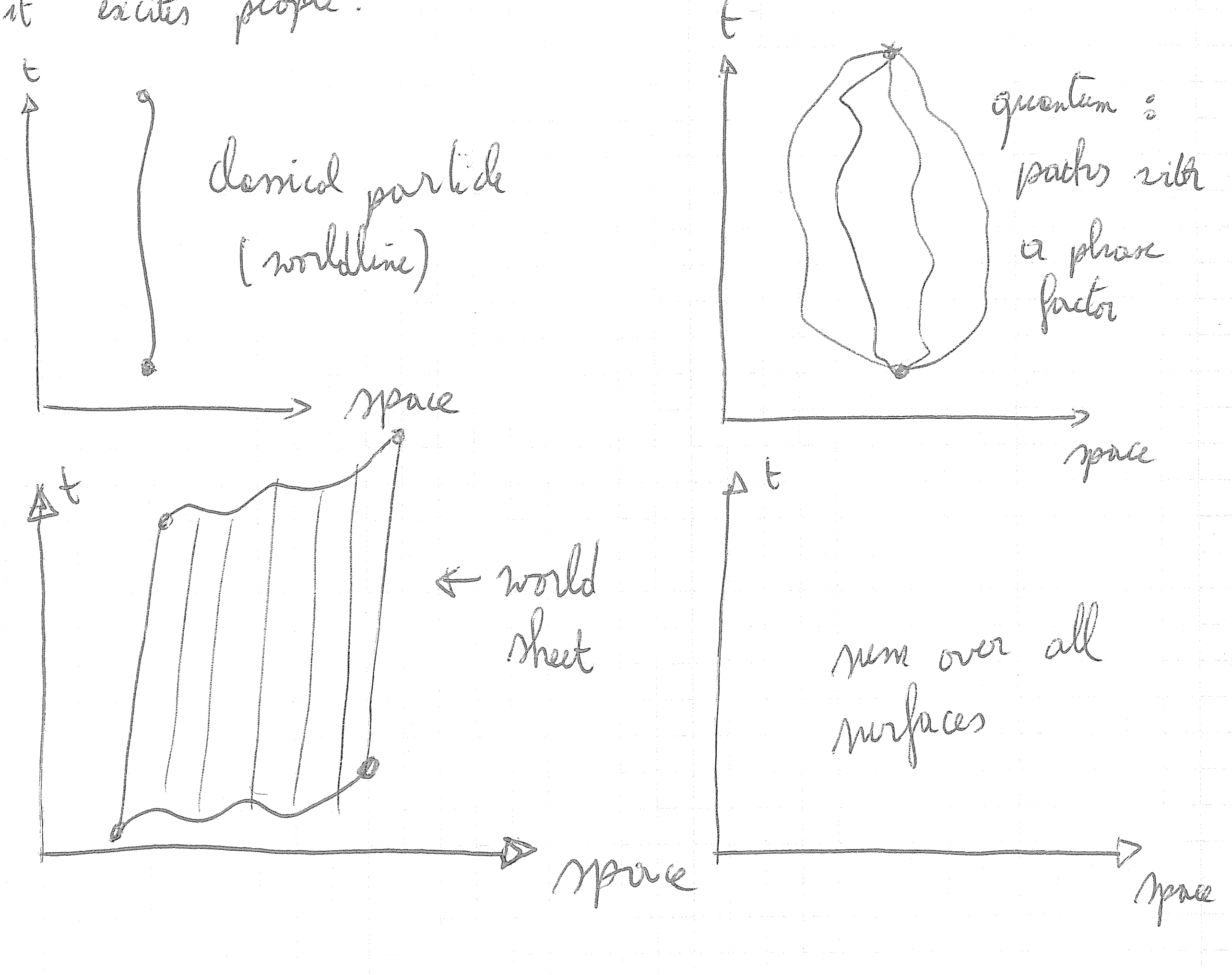
seen in the hadron spectrum

now qualitatively assumed from QCD

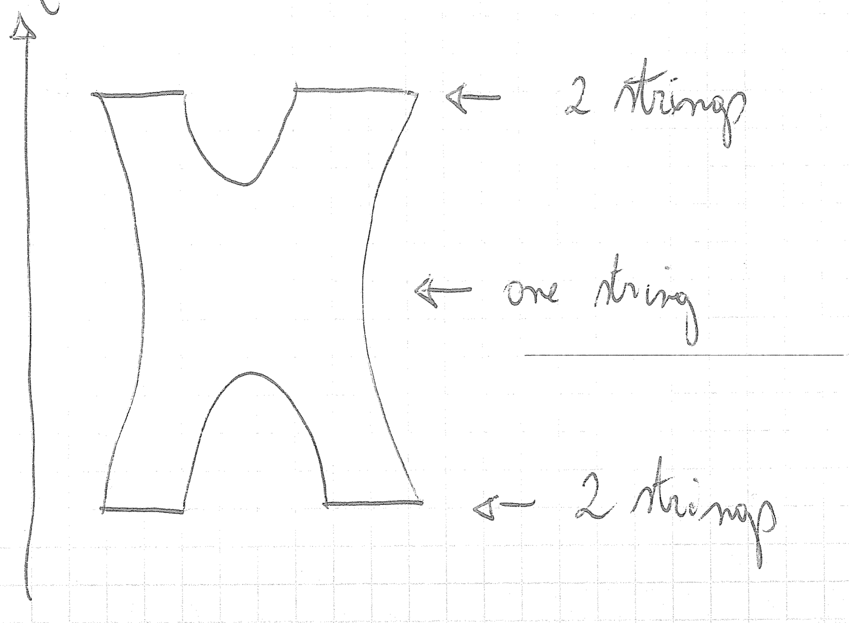
People started working on the mathematics of strings:

many problems

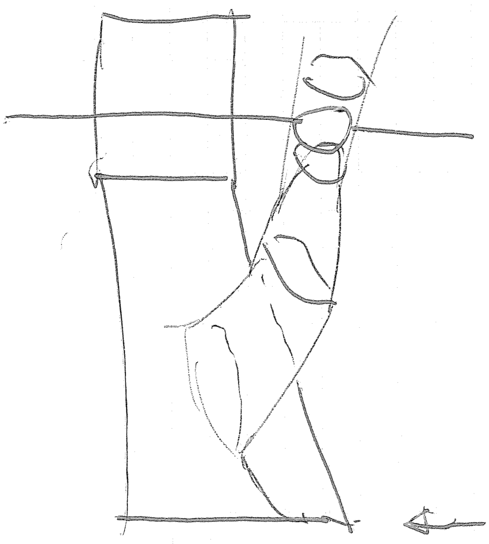
Let me give an indication of what the problems are but also why it excites people.



But



Strings always have interactions in a quantum world



Mathematics : particle $x^\mu(\tau)$

string $x^\mu(\sigma, \tau)$

now do a Fourier series in σ

either closed string $\sigma : 0 \to 2\pi$

or open string σ also finite range

so roughly

$$x^\mu(\sigma,\tau) = \sum_n x^\mu_{(n)}(\tau)\, e^{-i n \sigma}$$

Then quantise

$x^\mu(\tau)$ is a bosonic particle

lots of trouble

- bosonic string $D = 26$ needed
  and there is always a tachyon $m^2 < 0$

"otherwise Lorentz invariance has anomalies"

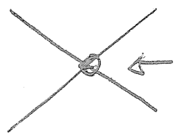
- closed string : massless spin 2 particle
  (not seen as a hadronic state)
- also allow fermions $\psi(\sigma, \tau)$

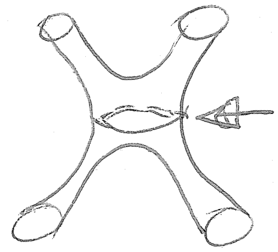
3 types found in $D = 10$: type I (open)

closed } type IIA : periodic
type IIB : antiperiodic

Scherk - Schwarz: massless spin 2 ⇒ graviton
maybe is the fundamental theory

ent



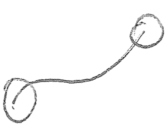


softer: maybe a finite theory

(I actually don't know for sure if it's proven, many claims ...)

D = 10 superstring: no tachyon

II A, II B: seem fine but no gauge interactions



← can hang charges there: SO(n) groups

(Chan - Paton factors)

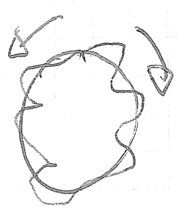
trouble: anomalies : SO(32) OK Green - Schwarz

(first real string revolution)

look in zero mass sector: $E_8 \times E_8$ should be OK too

modes travelling left and right can be different

some D=26 bosonic ; some D=10 fermionic



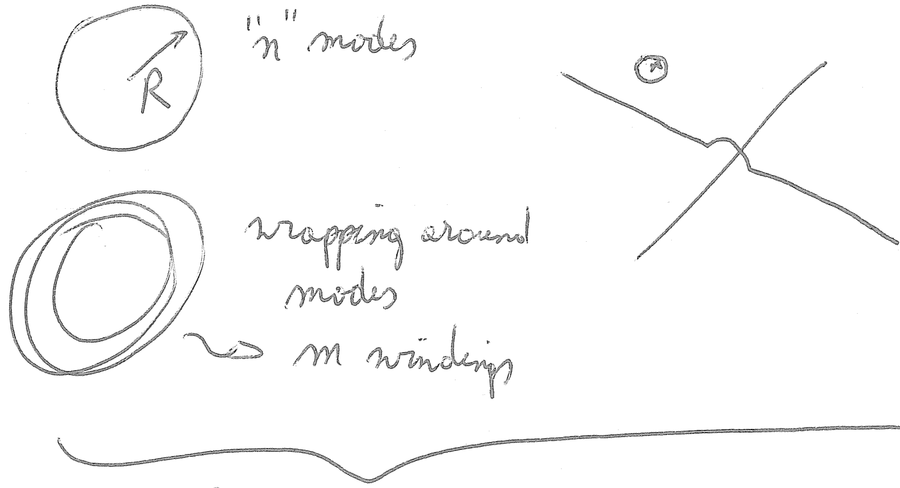
⇒ heterotic string : $\begin{cases} E_8 \times E_8 \\ SO(32) \end{cases}$

Anyway lots of excitement final theory ——

But ~1990 very many four dimensional consistent compactifications found but there were many more revolutions

Dualities: Is there a shortest distance

- T duality $R \Leftrightarrow 1/R$ seems a symmetry



different limits but same results

- S duality $g \leftrightarrow 1/g$

several other dualities found which related the various theories: maybe one center: M theory

AdS/CFT (and related subjects)

string theories in 5D $\Leftrightarrow$ gauge theories in 4D

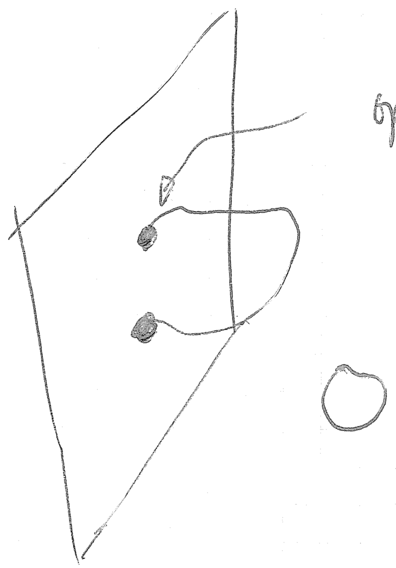
Quantum gravity Strings seem to contain enough degrees of freedom to hold black hole entropy

(holography: all info is in the surfaces)

Well: particles → strings → branes
↑ (general name for all)
even more trouble trying to construct a consistent theory (Duff, ...)

But: **String theory contains branes**

These are solutions of string theory (or of some low energy approximation)

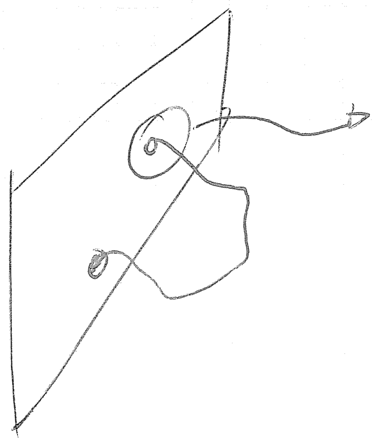


open strings must end on a p dimensional brane, rest only closed strings

Dp for Dirichlet boundary conditions

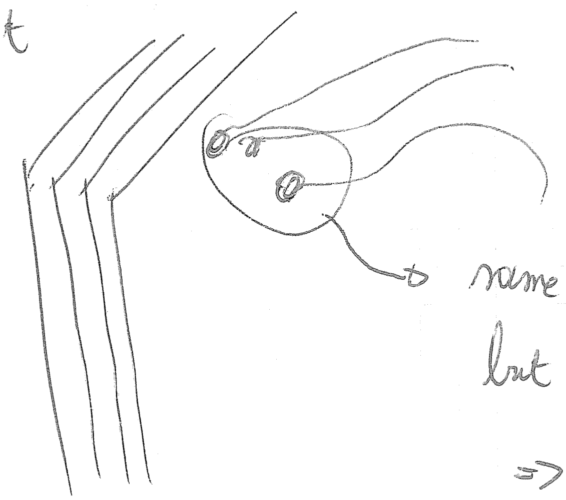
This allows for many funny constructions (i.e. all the stuff with shapes of branes, topology...) but provides the "large extra dimension scenario".

Also provides us with new ways to provide gauge groups on the lower dimensions.



→ really behaves as a U(1) because of the phase freedom

But solutions with more branes "on top" of each other also exist



→ same 4dimensional space time point but different branes

=> due to the interchanging a U(N) symmetry surfaces.

But:
- no realistic model exist
- there are so many possibilities => landscape
  (universe tries all: we just live in the one we can live in or the anthropic principle)
- no experimental evidence
- no "final" formulation exists

http://superstringtheory.com