Exercise 1.2

Convert to natural units. \( c = 2.98792 \times 10^8 \, m/s \)

(a) \( \nu = 10^{-2} \, \text{m} = 10^{-2} \, c = 2.988 \times 10^6 \, m/s \)

(b) \( \rho = 10^{13} \, \text{kg/m}^3 = 10^{13} \times c^2 \, \text{kg/m}^3 = 9 \times 10^{3.5} \, \text{Pa} \)

Pressure is force per area or: \( \text{kg/m}^2 \, \text{s}^{-2} \, \text{m}^2 = \text{kg/m} \, \text{s}^{-2} \)

(c) \( t = 10^{18} \, \text{m} = \frac{10^{18} \, \text{m}}{c} = \frac{1}{3} \times 10^{10} \, \text{s} = 3.33 \times 10^9 \, \text{s} \)

(d) \( u = 1 \, \text{kg/m}^3 = 1 \, \text{kg/m}^3 \, c^2 = 9 \times 10^{16} \, \text{kg/m} \, \text{s}^{-2} \)

Energy: \( \text{kg/m}^2 \, \text{s}^{-2} \), Energy density: \( \text{kg/m} \, \text{s}^{-2} \)

(e) \( a = 10 \, \text{m/s} = 10 \, \text{c}^2 \, \text{m/s} = 9 \times 10^{18} \, \text{m/s}^2 \)
Exerix 1.14

This is an exercise in simultaneity and the Lorentz contraction

\[ \text{Eq. (1.11)} \quad l = l_0 \sqrt{1 - v^2} \]

a) \[ \frac{l}{l_0} = \frac{1}{\sqrt{1 - v^2}} = \frac{1}{\sqrt{0.8^2}} = \frac{1}{0.36} = 0.6 \]

so the length he sees is \( 20 \times 0.6 = 12 \text{ m} \)

b) In the friend/barn frame

barn = 15 m

so \[ \Delta t = \frac{3 \text{ m}}{0.8} = 3.75 \text{ m} \]

So clock diver is at \( \mathbf{x} = (0, 0) \)

but clock is at \( \mathbf{x} = (3.45, 15) \)

\[ \Delta x^2 = -(3.45)^2 + 15^2 = 210.94 \ldots \text{ m}^2 \text{ obviously spacelike} \]

c) \{ Pole is 20 m. \}

\{ Barn is 15 m. \}

\[ \sqrt{1 - v^2} = 15 \text{ m.} \ 0.6 = 0.6 \text{ m.} \]

d) No, obviously not, his pole is longer than the barn.

l) \{ Yes, the clock was closed for friend \}

\{ No, not a runner \}

\[ \Rightarrow \text{ this simply phrased that simultaneity is frame dependent.} \]

f) Spacetime diagrams
The various points:

\[
\begin{align*}
  \{ & t, x \} = 0, 0 \\
  \{ & t, x \} = 0, 12 \\
  \{ & \bar{t}, \bar{x} \} = 3.45, 15 \\
  \{ & \bar{t}, \bar{x} \} = 3.75, 3
\end{align*}
\]

Spacetime event sphere and finds out that back hits wall (if sound velocity very close to c = 1). Future of hitting the back wall.
pole end; door close
pole other end; hit wall

\[\begin{align*}
    t_r &= \gamma (t_g - v x_g) = -43.45 \\
    x_r &= \gamma (x_g - v t_g) = 20
\end{align*}\]
pole when it hits according to friend

worldsheet of barn

future of hitting

the end of the barn

pole when it hits according to survey
(a) We know
\[ x^d = \Lambda^d_q (\tilde{\omega}^r)_q x^r \]
\[ x^d = \Lambda^d_q (\tilde{\omega}^r)_q x^r \]
So putting them together leads to
\[ x^d = \Lambda^d_q (\tilde{\omega}^r)_q \Lambda^q_p (\tilde{\omega}^r)_p x^r = \Lambda^d_p x^p \]

(b) The $\tilde{\omega}$ notation is really matrix multiplication.

(c) \[ \tilde{\omega}^r_v = 0.6 \tilde{\omega}^r_y \]
\[ \tilde{\omega}^y_v = 0.8 \tilde{\omega}^y_y \]
\[ \gamma = \frac{1}{3} = 1.25 \]

\[ \Lambda^d_q = \begin{pmatrix}
0.25 & -0.45 & 0 & 0 \\
-0.45 & 1.25 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
\frac{5}{4} & -\frac{3}{4} & 0 & 0 \\
\frac{3}{4} & \frac{5}{4} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \]

\[ \Lambda^d_q = \begin{pmatrix}
1.666... & 0 & -1.333... & 0 \\
0 & 1 & 0 & 0 \\
-1.3333 & 0 & 1.666... & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
\frac{5}{3} & -\frac{2}{3} & 0 & 0 \\
\frac{2}{3} & \frac{5}{3} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \]

\[ \Lambda^d_q = \begin{pmatrix}
2.0833... & -1.25 & -1.3333... & 0 \\
-0.45 & 1.25 & 0 & 0 \\
-1.6666... & 1 & 1.6666... & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
25/12 & -5/4 & -4/3 & 0 \\
-3/4 & 5/4 & 0 & 0 \\
-5/3 & 1 & 5/3 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \]
\[ \Delta \eta^2 = \left( \frac{25}{12} t - \frac{5}{y} \right)^2 \]
\[ + \left( -\frac{3}{4} t + \frac{5}{y} \right)^2 \]
\[ + \left( -\frac{5}{3} t + 2 + \frac{5}{y} \right)^2 \]
\[ + \epsilon^2 \]
\[ = \epsilon^2 \left( \left( \frac{25}{12} \right)^2 + \frac{5}{16} + \frac{25}{9} \right) \]
\[ + \epsilon^2 \left( \frac{y^2}{12} + \frac{y^2}{4} + \frac{y^2}{3} \right) \]
\[ + 2 \epsilon x \left( \frac{5}{12} - \frac{5}{4} - \frac{5}{3} \right) \]
\[ + 2 \epsilon y \left( \frac{5}{12} - \frac{5}{4} - \frac{5}{3} \right) \]
\[ + 2 \epsilon y \left( \frac{5}{12} - \frac{5}{4} - \frac{5}{3} \right) \]

plus multiply and add all of these

(e) The new matrix is by multiplying them in the other order

\[ \begin{pmatrix}
\frac{25}{12} & -\frac{3}{4} & -\frac{5}{3} & 0 \\
-\frac{5}{12} & \frac{5}{4} & 1 & 0 \\
-\frac{1}{4} & 0 & \frac{5}{3} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \]

Lorentz transformations do not commute.
(a) Also done in the lecture.

\[ \Lambda^\mu_\nu \left(-u e^\nu_x\right) = \begin{pmatrix} \gamma & \gamma v & 0 & 0 \\ \gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

So,

\[ \bar{u} \rightarrow \{u^\nu\} : \quad \bar{u}^\mu = \Lambda^\mu_\nu \bar{u}^\nu \rightarrow (\gamma, \gamma v, 0, 0) \]

(b) We first need to derive the general Lorentz transformation.

The spatial part of \( \bar{x} \) which is parallel to \( \bar{v} \) mixes with the time coordinate \( x^0 \) while the part \( \bar{x}^1 \) which is perpendicular to \( \bar{v} \) remains unchanged. So with \( \hat{v} = \frac{\bar{v}}{v^2} \)

Now

\[ \bar{x}^0 = x^0, \quad \hat{v}^0 = \frac{x^0 - \bar{x}^0}{v^2} \]

\[ \bar{x}^1 = x^1 - \frac{\bar{x}^0}{\bar{v}} = \bar{x}^1 - \frac{x^0}{\bar{v}} \]

So,

\[ x^0 = \gamma x^0 - \gamma v \bar{x}^0, \quad \bar{x}^1 = \frac{x^1}{v^2} \]

(\( \bar{x}^0 \)) = \bar{x}^0
\[ \overline{\dot{x}}_{\parallel} = -\gamma^2 \dot{x} + \gamma \ddot{x} \]

\[ a = \gamma x_0 - \gamma v \dot{x}_0 \]

\[ x^\tau = (\overline{\dot{x}} + \overline{x}) \]

\[ \overline{x} = \gamma x_0 - \gamma v \dot{x}_0 \]

\[ x^\tau = \left( \frac{x^\tau}{v} \right) \]

\[ x^\tau = -\gamma v \dot{x} + \delta^i \dot{v}_i + (\gamma - 1) \frac{\dot{v}^i}{v^2} x^i \]

\[ \bar{\mu} = \left( \begin{array}{c} \gamma & -\gamma v x & -\gamma v y & -\delta v^2 \\ -\gamma v x & 1 + d \frac{x^2}{v^2} & d \frac{v x y}{v^2} & d \frac{v^2 x}{v^2} \\ -\gamma v y & d \frac{v y}{v^2} & 1 + d \frac{v^2 y}{v^2} & d \frac{v^2 y}{v^2} \\ -\delta v^2 & d \frac{v^2 x}{v^2} & d \frac{v^2 y}{v^2} & 1 + d \frac{v^2}{v^2} \end{array} \right) \]

\[ \text{will } d = \gamma - 1 \]

Putting \( \overline{x} \) to \( -\overline{x} \) and using the same argument as in (a) we thus get \( U \rightarrow (\gamma, \gamma \frac{\overline{x}}{\bar{v}}) \)
(c) \( v^t = \frac{u^t}{u^0} \)

(d) \( (2, 1, 1, 1) \) \( \xrightarrow{\mathcal{N}} \) \( \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right) \)
null coordinates or light cone coordinates
\[ v = t + x \]
\[ u = t - x \]
\[ \frac{x}{c} = \frac{1}{2} (u + v) \]
\[ x = \frac{1}{2} (-u + v) \]

(a) \( \vec{e}_u \) connects \( \{1, 0, 0, 0, 0\} \) to \( (\frac{1}{2}, \frac{1}{2}, 0, 0, 0) \)
and \( \{0, 0, 0, 0, 0\} \) to \( (0, 0, 0, 0, 0) \)

\( \vec{e}_v \) connects \( \{0, 1, 0, 0, 0\} \) to \( (\frac{1}{2}, \frac{1}{2}, 0, 0, 0) \)
\( \{0, 0, 0, 0, 0\} \) to \( (0, 0, 0, 0, 0) \)

So
\[ \vec{e}_u = \frac{1}{2} \vec{e}_t - \frac{1}{2} \vec{e}_x \]
\[ \vec{e}_v = \frac{1}{2} \vec{e}_t + \frac{1}{2} \vec{e}_x \]

(b) Obviously since \( \vec{e}_x \) and \( \vec{e}_t \) are themselves linear combinations of \( \vec{e}_u \) and \( \vec{e}_v \)
we can simply rewrite all linear combinations \( \Rightarrow \) it is a good basis

Alternatively the matrix connecting them this determinant \( \neq 0 \)

(c) \[ g_{uu} = \vec{e}_u \cdot \vec{e}_u = \frac{1}{4} (\vec{e}_t \cdot \vec{e}_t + \vec{e}_x \cdot \vec{e}_x) = 0 \]
\[ g_{vv} = \vec{e}_v \cdot \vec{e}_v = \frac{1}{4} (\vec{e}_t \cdot \vec{e}_t + \vec{e}_x \cdot \vec{e}_x) = 0 \]
\[ g_{uv} = \vec{e}_u \cdot \vec{e}_v = \frac{1}{4} (\vec{e}_t \cdot \vec{e}_t - \vec{e}_x \cdot \vec{e}_x) = -\frac{1}{2} \]
the remainder is rather easy.

(d) In (c) I already showed that \( \vec{e}_u \cdot \vec{e}_u = 0 \), so they are null vectors, are on the light-cone, and \( \vec{e}_u \cdot \vec{e}_v = -\frac{1}{2} \neq 0 \), so not orthogonal.

(e) \( \delta u \) the gradient one form of \( u \) is using \( u = \frac{1}{2} t - x \)

\[ \delta u = \frac{1}{2} \delta t - \delta x \]

\[ \delta v = \frac{1}{2} \delta t + \delta x \]

\[ g(\vec{e}_u, \vec{e}_u) = g(\vec{e}_u, \vec{e}_t) \delta t + g(\vec{e}_u, \vec{e}_x) \delta x = -\frac{1}{2} \delta t + \frac{1}{2} \delta x = -\frac{1}{2} \delta u \]

\[ g(\vec{e}_v, \vec{e}_v) = g(\vec{e}_v, \vec{e}_t) \delta t + g(\vec{e}_v, \vec{e}_x) \delta x = -\frac{1}{2} \delta t + \frac{1}{2} \delta x = -\frac{1}{2} \delta u \]
\( \vec{U} \rightarrow (1 + t^2, t^2, \sqrt{t}, 0) \)

\( \vec{D} \rightarrow (x, 5tx, \sqrt{t}, 0) \)

\( \mathbf{g} = x^2 + t^2 - y^2 \)

(a) \( \vec{U} \cdot \vec{U} = -(1 + t^2)^2 + t^4 + 2t^2 = -1 \)

so it is suitable as a velocity field.

\( \vec{U} \cdot \vec{D} = -x(1 + t^2) + 5t^3x + 2t^2 \)

\( \vec{D} \cdot \vec{D} = -x^2 + 2\sqrt{5}t^2x^2 + 2t^2 \)

so it is NOT suitable as a velocity field.

(b) \( \vec{\omega} \) (spatial velocity)

\( \vec{U} = (\gamma, \gamma \vec{\omega}) \)

\( \mathbf{r} \cdot \vec{\omega} = \left( \frac{t^2}{1 + t^2}, \frac{\sqrt{t}}{1 + t^2}, 0 \right) \)

\( \mathbf{g}^2 = \frac{t^4 + 2t^2}{(1 + t^2)^2} \)

\( \leq 1 \)

\( t \rightarrow 0 \) then \( \vec{\omega} = 0 \)

\( t \rightarrow \infty \) then \( \vec{\omega} = (1, 0, 0) \)

(c) \( \vec{U}_d \rightarrow (1 + t^2, t^2, \sqrt{t}, 0) \)

(d) \( \vec{U}_d, \rho = \begin{pmatrix} 2t & 0 & 0 & 0 & 0 \\ 2t & 0 & 0 & 0 & 0 \\ \sqrt{t} & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 & 0 \end{pmatrix} \)
(e) \[ U_d^4 U_4^{(4)} = - (1 + t^2)^2 t + 2 t^3 + 12 \sqrt{2} t + 0 \]

\[ \beta = 0 \]

\[ \beta = 1, 2, 3 \text{ obviously zero} \]

(f) \[ \mathcal{D}^\beta \, U_4^{(4)} = \frac{\partial \mathcal{X}}{\partial t} + \frac{\partial (5t \mathcal{X})}{\partial x} + \frac{\partial (\sqrt{2} t)}{\partial y} + \frac{\partial (\mathcal{O})}{\partial z} \]

\[ = 5t \]

(g) \[ \left( U_4^{d, \mathcal{D}^\beta} \right)_\beta = + \frac{2}{\partial t} \left( (1 + t^2) x \right) + \frac{2}{\partial x} \left( 5t^3 x \right) + \frac{2}{\partial y} \left( 5 \sqrt{2} t^2 x \right) + \frac{2}{\partial z} \left( 2 \mathcal{O} \right) \]

\[ d = 0 \]

\[ = 2t x + 5 t^3 \]

\[ d = 1 \]

\[ \left\{ \begin{array}{l}
= \frac{2}{\partial t} \left( t^2 x \right) + \frac{2}{\partial x} \left( 5 t^3 x \right) + 0 \\
= 2t x + 5 t^3 \\
= \frac{2}{\partial t} \left( \sqrt{2} t x \right) + \frac{2}{\partial x} \left( 5 \sqrt{2} t^2 x \right) + 0 \\
= \sqrt{2} x + 5 \sqrt{2} t^2 \\
= 0
\end{array} \right. \]

\[ d = 2 \]

\[ \left\{ \begin{array}{l}
= \frac{2}{\partial t} \left( t^2 x \right) + \frac{2}{\partial x} \left( 5 t^3 x \right) + 0 \\
= 2t x + 5 t^3 \\
= \sqrt{2} x + 5 \sqrt{2} t^2 \\
= 0
\end{array} \right. \]

\[ d = 3 \]

(h) \[ U_d^4 \left( U_4^{d, \mathcal{D}^\beta} \right)_\beta = - (1 + t^2) \left( 2t x + 5 t^3 \right) \]

\[ + t^2 \left( 2t x + 5 t^3 \right) \]

\[ + \sqrt{2} t \left( \sqrt{2} x + 5 \sqrt{2} t^2 \right) \]

\[ = \mathcal{X} \left( - 2t - 2 t^3 + 2 t \right) + 5t \left( - 1 - 2 t^2 - t^4 + t^4 + 2 t^2 \right) \]

\[ = - 5t \]
\[ U_\alpha (U_\beta \c D^\alpha)_{,\beta} = \underbrace{U_\alpha U_\beta \c D^\alpha}_{= 0 \text{ (e)}} + U_\alpha U_\beta \c D^\beta_{,\beta} = - \c D^\beta_{,\beta} \]

\[(e) \quad g_{,\alpha} \rightarrow (2t, 2x, -2y, 0)\]
\[g_{,\beta} \rightarrow (-2t, 2x, -2y, 0)\]  
These are the components of the vector gradient.

\[(j) \quad \nabla_{\vec{u}} g = U_\alpha g_{,\alpha} = 4t (1 + t^2) + 2xt - 2\sqrt{t} y\]

\[\nabla_{\vec{u}} \bar{D} : U_\alpha \bar{D}^\alpha \]
\[\bar{D}^\alpha_{,\beta} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 5x & 5t & 0 & 0 \\ \sqrt{t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\]

\[\nabla_{\vec{u}} \bar{D} \rightarrow (t^2, 5x (1 + t^2) + 5t^3, \sqrt{t} (1 + t^2), 0)\]

\[\nabla_{\bar{D}} g = D_\alpha g_{,\alpha} = 2tx + 10tx^2 - 2\sqrt{t} y t\]

\[\nabla_{\bar{D}} \bar{u} : D_\alpha \bar{u}^\alpha_{,\alpha}\]
\[\nabla_{\bar{D}} \bar{u} \rightarrow (2tx, 2t^2 x, \sqrt{t} x, 0)\]
Solution 1.24. It is not possible for one inertial frame to jerk back and forth with respect to another. The "paradox" comes from the tacit and erroneous assumption that the cylinder remains rigid when the frogs hit the ends. This cannot be true since the elastic waves that inform one end of the tube that the other has been hit must propagate along the walls of the tube at less than the speed of light. In the freely-falling frame of the cylinder, the two ends are driven outward by the impact of the frogs. Tension waves fly from each end of the cylinder toward the other. Not until the waves meet at the center of the tube do they discover the existence of each other. Then they pass each other and counteract each other's effects, pulling the tube back to its original shape. The tube then pulsates back and forth in its fundamental mode of vibration. During this pulsation the frogs hit the ends of the tube time and again, each time changing the amplitude and phase of the pulsation.

From another freely-falling frame (perhaps instantaneously comoving with onlooking birds), the cylinder rushes by at speed \( \beta \), and the frogs do not hit the ends of the tube simultaneously, the pulsations of the two ends will be out of phase with each other, but the overall picture will not be much different. In particular, the center of the tube (which is at rest in the inertial frame of the cylinder) will not jerk back and forth in any other inertial frame. The accompanying spacetime diagrams illustrate the phenomena.
A less destructive variant of the runner and the lam

A runner runs in the xy plane with velocity $\vec{v} = (0.8, 0.3, 0)$ and holds a pole in the $x$-direction of length $L = 0.9\ m$. He is at the origin at $t = 0$.

There is a wall on the $x$-axis with an open door from $x=0$ to $x=1\ m$. So he can clearly pass through the door with width $L = 1\ m$.

In the runner's frame his stick is longer $1.652\ m$ and the door is smaller $0.6\ m$. How does the stick get through the door from the point of view of the runner?

- The stick in frame 0 (door at rest) has endpoints at

$$\begin{align*}
\vec{x}_1 \rightarrow (t, 0^x, 0^y, t, 0) \quad \text{and} \\
\vec{x}_2 \rightarrow (t, 0^x, t + l, 0^y, t, 0)
\end{align*}$$

The runner himself is at $\vec{x}_1$.

- The door is stationary in this frame and has endpoints

$$\begin{align*}
\vec{x}_3 \rightarrow (t, 0, 0, 0) \\
\vec{x}_4 \rightarrow (t, L, 0, 0)
\end{align*}$$

When we vary $t$ we get the worldlines for the ends of the door and the stick.

The door is stationary and the stick moves as shown in Fig. 1.
We now want the worldlines but in the coordinate system of the runner $\overline{O}$.

We have that $\overline{x}^\mu = \Lambda^\mu_\nu x^\nu$ with

$$
\Lambda^\mu_\nu = \begin{pmatrix}
\gamma & -\gamma v^x & -\gamma v^y & -\gamma v^z \\
-\gamma v^x & 1 + A v^x v^x & A v^x v^y & A v^x v^z \\
-\gamma v^y & A v^x v^y & 1 + A v^y v^y & A v^y v^z \\
-\gamma v^z & A v^x v^z & A v^y v^z & 1 + A v^z v^z
\end{pmatrix}
$$

with $A = \frac{\gamma - 1}{v^2}$,

$$
\gamma = \frac{1}{\sqrt{1 - v^2}},
\quad
v^2 = (v^x)^2 + (v^y)^2 + (v^z)^2
$$

We can now rewrite the worldlines in the $\overline{O}$ coordinate and get

$$
\overline{x_1^x} \rightarrow \left( \frac{t}{\gamma}, 0, 0, 0 \right)
$$

$$
\overline{x_2^x} \rightarrow \gamma \left( \frac{t}{\gamma} - \gamma v^x l, \gamma l (1 + A v^x v^x), \gamma A v^x v^y, 0 \right)
$$

It is more natural to use $\overline{E}$ to parametrize the worldlines in the $\overline{O}$ frame. So we set the time coordinates equal to $\overline{E}$ for all the points (Note that this results in different times for the different $\overline{x_i^x}$).

So we get

$$
\overline{x_1^x} \rightarrow \left( \overline{E}, 0, 0, 0 \right)
$$

$$
\overline{x_2^x} \rightarrow \left( \overline{E}, \gamma l (1 + A v^x v^x), \gamma A v^x v^y, 0 \right)
$$

For $\overline{x_1^x}$ $\overline{E} = \frac{t}{\gamma}$; for $\overline{x_2^x}$ $\overline{E} = \frac{t}{\gamma} - \gamma v^x l$.
The runner is at the $F$ axis as expected and his stick is standing still in his frame also as expected. The length of the stick in his frame can be calculated and (after some simplifications)

$$l = l \sqrt{1 - \gamma^2 v^2} = 1.652 \ldots \text{ m}$$

Doing the same for the door

$$\vec{x}_3 \rightarrow \left( \gamma t, -\gamma v^x t, -\gamma v^y t, 0 \right)$$

$$\vec{x}_4 \rightarrow \left( \gamma t - \gamma v^x L, -\gamma v^x t + L (1 + A v^x v^x), -\gamma v^y t + L A v^x v^y, 0 \right)$$

The more natural parametrization of the world lines is in terms of $F$. For $\vec{x}_3$ choose $F = \gamma t$ and we get

$$\vec{x}_3 \rightarrow \left( F, -v^x F, -v^y F, 0 \right)$$

(note that it indeed goes back with $-\vec{F}$)

For $\vec{x}_4$ choose $F = \gamma t - \gamma v^x L$ and we get

$$\vec{x}_4 \rightarrow \left( F, -v^x F + L (1 + A v^x v^x - \gamma v^x v^x), -v^y F + L (A - \gamma) v^x v^y, 0 \right)$$

Note that the door and the stick are not parallel to the $\vec{x}$ axis at equal $F$.

The length of the door is

$$L = l \sqrt{1 - v^x v^x} = 0.6 \text{ m}$$

How the stick gets through the door is shown in Fig. 2. Here the door moves but the stick stands still.

Note that Lorentz contraction and time dilatation is clearly visible in all cases.
door frame, \( t = -5, \ldots, 5 \)
runner frame, $\tilde{t} = -5, \ldots, 5$