Solution of the conundrum

The problem

An accelerated charge radiates EM waves. We on Earth are not at rest, but are actually accelerated by its surface. Do the charges we have here then actually radiate EM waves?

A freely falling observer is not able to detect the gravitational field, when considering the events close to her worldline. This implies, that in suitably chosen coordinates, i.e. the locally flat coordinates, the observer has the flat metric and its derivatives are equal to zero on her worldline. Otherwise she would be able to detect the gravitational field by launching test particles and measuring Christoffel symbols (actually you need a bit of a generalization of the locally flat coordinate system called Fermi coordinates, see e.g. section 13.6 in the book by Misner, Thorne and Wheeler)

Now, as the Christoffel symbols are zero, all the covariant derivatives become simple derivatives, and Maxwell’s equations work exactly as they would in the special relativistic case, note that this is only true near the observer.

Should the free-falling observer fall through the Earth (as a thought experiment), then, when passing through the charge, she would detect the radiation, which is characteristic of accelerated charges in special relativity. Should she make a Lorenz transformation to the locally static, yet inertial, observer, the EM-radiation would still be there, because Lorenz transformations cannot cancel the EM-radiation. Since this is true for any moment, the static inertial observer on the Earth would be constantly measuring the flux from the charge.

Note, though, that Lorentz transformations are allowed only between inertial (freely falling) observers, so, strictly speaking, the above described applies to the on-Earth observers, which are not accelerated (that is, who will fall through the Earth’s surface a few moments later). In a more careful treatment of accelerated observers (by considering explicitly Maxwell’s equations), whose world line coincides with that of the charge, they would not be able to detect the radiation from the charge in their vicinity. However, they would be able to detect radiation, being sufficiently separated from the charge (i.e. if their world line was far enough away from the charge).

This means, that the charges actually do radiate. However, the effect is so minuscule, that it has never been measured. The energy for this radiation comes from the surface of the Earth, which accelerates the charge.