PION AND KAON VECTOR FORM FACTORS: A CHIRAL UNITARY APPROACH

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1 Unitarization

Using an appropriate unitarization method we take into account the final state interaction corrections to the tree level amplitudes calculated from lowest order χPT [1] and from the inclusion of explicit resonance fields in a chiral symmetry fashion as given in ref. [2]. A similar procedure has already been used in the scalar sector to describe the scalar form factor associated with the strange-change scalar current $\overline{u}s$ in ref. [3].

We will work in the isospin limit, with $|\pi\pi\rangle$ and $|K\bar{K}\rangle$ states (and the ρ resonance) in the $I=1$ channel, and we will use matrix notation, labeling pions with 1 and kaons with 2. In the $I=0$ case we only have kaons (and the ω and ϕ resonances). Starting from the unitarity of the S-matrix and the introduction of the electromagnetic meson form factor $F_{MM'}(s)$:

$$\langle\gamma(q)|T|M(p)M'(p')\rangle = e\epsilon_\mu(p-p')^\mu F_{MM'}(s) \tag{1}$$

with $q^2=s$, e the modulus of the electron charge and ϵ_μ the photon polarization vector, we can arrive at the expression:

$$F^I(s) = \left[1+\widetilde{Q}(s)^{-1}\cdot K^I(s)\cdot\widetilde{Q}(s)\cdot g^I(s)\right]^{-1}\cdot R^I(s) \tag{2}$$

where $\widetilde{Q}_{ij}(s)=p_i(s)\delta_{ij}$ and $K^I(s)$ is the matrix collecting the tree level scattering amplitudes[a] between definite $\pi\pi$ and $K\bar{K}$ isospin states. $F^I(s)$ is the column matrix $F^I(s)_i = F^I_i(s)$; $R^I(s)$ is a vector made up by functions without any cut and $g^I(s)$ is the diagonal matrix given by the loop with two meson propagators (see ref. [4]).

[a] Derived from lowest order χPT plus s-channel vector resonance exchange contributions [2] corresponding to the transition $i\to j$.

The expression for the P-wave amplitudes $T^I(s)$ that we have used to go from eq.(1) to eq.(2) is the one provided by the N/D method adapted to the chiral framework (see ref. [4])

In the large N_c limit loop physics is supressed. Thus, in this limit $F^I(s) = R_{N_c^{leading}}(s) = F_t^I(s)$, where $F_t^I(s)$ is the tree level form factor.[b]

This allows us to write:

$$F^I(s) = \left[1 + \widetilde{Q}(s)^{-1} \cdot K^I(s) \cdot \widetilde{Q}(s) \cdot g^I(s)\right]^{-1} \cdot \left[F_t^I(s) + R^I_{subleading}(s)\right] \quad (3)$$

with $R^I_{subleading}(s)$ being $\mathcal{O}(N_c^{-1})$. If we require that the vector form factor of eq. (3) vanishes when $s \to \infty$ as is suggested by the experiments, we find that the subleading part of $R^I(s)$, which at first can be an arbitrary polynomial, must be a constant. One can fix the values of d_i^I (parameters appearing in the g_i^I functions, see ref.[4]) and $R^{I=0}_{subleading}(s)$ by matching our results to those of one loop χPT, and the bare masses of the resonances by the requirements that the moduli of the scattering amplitudes have a maximum for $\sqrt{s} = M^{physical}_{resonance}$. We take $R^{I=1}_{subleading}(s) = 0$ in order to constrain further our approach, and then all the parameters are fixed.

2 Results and conclusions.

If we compare our results with the experiment we see that we can describe in a very precise way the pion and kaon vector form factors and the P-wave $\pi\pi$ phase shifts up to about s=1.44 GeV^2 (even for higher energies in the case of phase shifts). If we compare our results with those of χPT, we see that the resummation of our scheme leads to a much better agreement with the two loop χPT pion vector form factor than with the one loop one. The resonance regions are also well reproduced.

References

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[b] The tree level form factors are also evaluated using the lowest order χPT Lagrangian [1] plus the chiral resonance Lagrangian [2].