CHIRAL SYMMETRY RESTORATION WITH N_F AND THE SCALAR MESONS

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Motivated by the possible vicinity of a chiral phase transition, the evolution of the order parameters $<\bar{u}u>$ and F_π from chiral $SU(2)$ to chiral $SU(3)$ is considered. Combining CHPT with sum rules we relate this evolution to the physics of the scalar resonances. A significant chiral restoration is found, in particular for $<\bar{u}u>$.

QCD is expected to undergo phase transitions if one varies its fermion content. This can be seen already in the perturbative regime: with $N_c = 3$, if $N_f > 16.5$ the theory is no longer asymptotically free. Decreasing N_f below 16.5 one first encounters the "conformal phase" then, for $N_f \leq N_f^{crit}$ the phase which displays confinement and, assuming the N_f fermions to be massless, the spontaneous breaking of the $SU(N_f) \times SU(N_f)$ chiral symmetry. The exact value of N_f^{crit} is not yet known, but recent lattice simulations[1] suggest that N_f^{crit} could be not much larger than $N_f = 3$, which is the number of light flavours in actual QCD. If this is true, one should observe a significant decrease of the chiral order parameters in going from chiral $SU(2)$ to chiral $SU(3)$. Here, I will summarize recent work[2] which shows how to relate this evolution to experiment. Consider the two basic order parameters

$$<\bar{u}u>_{N_f} \quad \text{and} \quad (F_\pi)_{N_f}, \tag{1}$$

where the subscript N_f means that the masses of the N_f lightest quarks are extrapolated to zero, while the masses of the remaining quarks are kept at their physical values. Chiral perturbation theory enables one to express the evolution from $N_f = 2$ to $N_f = 3$ as an expansion in powers of m_K/Λ, in terms of low-energy coupling constants (LEC's). At chiral order p^4 two LEC's are involved L_4 and L_6 and at $O(p^6)$ there appears C_{16} and the combination $C_{20} + 3C_{21}$ in the nomenclature of refs.[3] and [4] respectively.

Unfortunately, little is known about these LEC's which were usually simply assumed to be suppressed, according to large N_c arguments[3]. These arguments might fail here, because $N_c \to \infty$ implies $N_f^{crit} \to \infty$. More information can be obtained by considering scalar form-factors, i.e. matrix elements between the vacuum and $\pi\pi$ or $K\bar{K}$ states of the scalar currents $S_u = \bar{u}u + \bar{d}d$ and $S_s = \bar{s}s$. Contrary to the vector (or axial-vector) form-factors, the scalar ones are not directly measurable, but a method for constructing them, in an

Table 1. For a given ratio of the successive terms in the expansion of $\Pi_6(0)$, the values of the LEC's $L_6^r(m_\eta)$ and the $C_{20}^r(\mu) + 3C_{21}^r(\mu)$ are computed for $\mu = m_\eta$ (in brackets, $\mu = m_\rho$). The last column shows the successive terms in the expansion of $\langle\bar{u}u\rangle_{SU(2)}/\langle\bar{u}u\rangle_{SU(3)} - 1$.

$O(p^6)/O(p^4)$	$10^3\,L_6$	$10^5\,(C_{20}+3C_{21})$	$\langle\bar{u}u\rangle_{SU(2)}/\langle\bar{u}u\rangle_{SU(3)} - 1$
0.10	0.73 [0.56]	0.29 [-0.02]	0.85 + 0.14
0.30	0.64 [0.47]	0.37 [0.06]	0.76 + 0.18
0.50	0.57 [0.41]	0.43 [0.12]	0.69 + 0.21
1	0.46 [0.30]	0.52 [0.21]	0.58 + 0.26

energy range $E \leq 1$ GeV, using input from $\pi\pi$ scattering data in the S-wave (which brings in the physics of the scalar mesons), was proposed some time ago[5]. From these form–factors, one can then construct (in the same energy range) the spectral function associated with the correlator $\Pi_6(p^2)$ of $S_u(x)$ and $S_s(0)$, it can be shown that this spectral function satisfies a super-convergence relation. At chiral order p^4, these considerations are sufficient to determine the LEC's L_4 and L_6[2]. However, it is important to consider $O(p^6)$ corrections. At this order, it is necessary to make an assumption concerning the rate of convergence of the chiral expansion. Results are shown in table 1 in which we have assumed that the ratio of successive terms in the expansion of $\Pi_6(0)$ lies between 10% and 100%. We find a strong decrease of the chiral condensate with N_f by approximately a factor of two. A decrease of F_π is also found, of the order of 20%. These results reflect the tendency of the light scalar mesons to violate the OZI rule. This discussion shows that this puzzling feature of the scalars is linked to chiral symmetry (and its restoration). If the chiral condensate is too small when $N_f = 3$, the conventional chiral expansion will break down. This situation is discussed in ref.[6].

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