ELECTROMAGNETIC CORRECTIONS TO $K \to \pi\pi$ AND DISPERSIVE MATCHING

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I report on work done in collaboration with J.F.Donoghue and E.Golowich on electromagnetic corrections to $K \to \pi\pi$ decays [1,2].

In the analysis of $K \to \pi\pi$ decays, the $\Delta I = 1/2$ enhancement provides an amplification mechanism for isospin breaking (IB) effects. This circumstance makes IB corrections to $\Delta I \geq 3/2$ amplitudes of the natural size of about 10%. In this contribution I report on some issues arising in the analysis of electromagnetic (EM) effects [3].

In presence of EM interactions, the kaon partial decay width Γ_n (where $n = +-, 00, +0$) can be written as:

$$\Gamma_n(\omega) \;=\; \frac{\Phi_n}{2\sqrt{s_n}}\, |\overline{\mathcal{A}}_n|^2\, G_n(\omega) \quad . \tag{1}$$

We indicate with ω its dependence on how much of the $n+$ soft γ channel is included in the measured rate. $\Phi_n/(2\sqrt{s_n})$ is a kinematical factor, $\overline{\mathcal{A}}_n$ is the infrared finite amplitude - defined by standard procedure [4,5] - and $G_n(\omega)$ is the infrared factor associated with the combined effect of *virtual* and *real* soft photons. $G_n(\omega)$ has to be calculated with the same prescriptions on soft photons cuts used in the measurement of Γ_n. Since the PDG [6] data come with no reference to this, no precise inclusion of radiative corrections is possible at present [2]. The data analysis at KLOE will hopefully fill this gap [7].

The IR finite amplitudes can then be parameterized as follows:

$$\begin{aligned}
\overline{\mathcal{A}}_{+-} &= (A_0 + \delta A_0)\, e^{i(\delta_0 + \gamma_0)} + \frac{1}{\sqrt{2}}\,(A_2 + \delta A_2)\, e^{i(\delta_2+\gamma_2)} \ , \\
\overline{\mathcal{A}}_{00} &= (A_0 + \delta A_0)\, e^{i(\delta_0 + \gamma_0)} - \sqrt{2}\,(A_2 + \delta A_2)\, e^{i(\delta_2+\gamma_2)} \ , \\
\overline{\mathcal{A}}_{+0} &= \frac{3}{2}\left(A_2 + \delta A_2^{+}\right) e^{i(\delta_2+\gamma_2')} \ ,
\end{aligned} \tag{2}$$

in terms of shifts δA_I, γ_I to the original isospin amplitudes A_I and strong rescattering phases δ_I, entering the usual parameterization as required by Watson's theorem. In general $\delta A_2 \neq \delta A_2^{+}$, due to a $\Delta I = 5/2$ amplitude induced by electromagnetism.

We have calculated δA_I within an approach inspired by dispersive theory [1], where the EM amplitude is written as follows:

$$A_{f\,i}(p_{\rm ext}) = \frac{e^2}{2} \int \frac{d^4 q}{(2\pi)^4} \, D_{\mu\nu}(q) \; A_{f\,i}^{\mu\nu}(q, p_{\rm ext}) = \frac{\alpha}{4\pi} \int dQ^2 \;\, A_{f\,i}(Q^2, p_{\rm ext}) \; . \tag{3}$$

$D_{\mu\nu}$ is the photon propagator, $A_{fi}^{\mu\nu}$ is the amplitude for $\gamma(q)\, i(p_i) \to \gamma(q)\, f(p_f)$, and the second form is obtained upon integration over the angular variables. Model independent information on the low Q^2 region of the integrand is provided by chiral symmetry through the tool of chiral lagrangians. The OPE is used to investigate the high Q^2 behavior. We then interpolate between the two regions using the assumption that low lying resonances play a dominant role in the intermediate Q^2 region. The matching procedure provides also an estimate of the theoretical uncertainty. Our results are:

$$\begin{aligned} \delta A_0/A_0 &= (0.46 \pm 0.13)\%; \\ \delta A_2/A_2 &= (6.0 \pm 2.5)\%; \\ \delta A_2^+/A_2 &= -(3.3 \pm 3.6)\%. \end{aligned} \tag{4}$$

We have also analyzed the dynamical content of the phases γ_I through unitarity relations in presence of IB interactions [2]. We find that γ_2 is enhanced by the large ratio A_0/A_2 and is related to EM corrections to $\pi\pi$ scattering [8] at energy equal to the kaon mass. A leading order estimate of these effects gives $\gamma_2 = 3.1^o$, which already amounts to 40% of the strong phase shift δ_2 at $s = M_K^2$. Sizeable higher order corrections $\gamma_2^{(e^2 p^2)}$ cannot be excluded and require a full analysis of $\pi\pi$ scattering at this order.
The implications of this work on the direct CP violation quantity ϵ'/ϵ are discussed in Ref. [2].

References

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