THE 7/11 RULE AND AN ESTIMATE OF m_ρ

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The two-point functions for the vector and axial currents $V_\mu = \overline{d}\gamma_\mu u$ and $A_\mu = \overline{d}\gamma_\mu\gamma_5 u$ contain information on the hadronic resonances ρ, ρ', a_1, *etc.* While it has thus far not been possible to calculate these two-point functions from QCD, their behavior is constrained at low energy by Chiral Perturbation Theory (ChPT), at high energy by the OPE, and at all scales by large N_c, in which limit the two-point functions are saturated by an infinite tower of narrow resonances:

$$\frac{1}{\pi}\mathrm{Im}\,\Pi_V(t) = \sum_{n=1}^{\infty} 2f_n^2 m_n^2 \delta(t-m_n^2)\,, \tag{1}$$

with $m_1 = m_\rho$, $f_1 = f_\rho$ the EM decay constant of the ρ. Similarly, the axial two-point function is a tower of resonances starting with π and a_1 in large N_c. The above-mentioned constraints lead to relations between these resonance parameters, ChPT parameters and condensates.

Since eq. (1) contains infinitely many parameters, we need additional input, which we will take to be the LMD *ansatz*,[1]

$$\frac{1}{\pi}\mathrm{Im}\,\Pi_V(t) = 2f_\rho^2 m_\rho^2 \delta(t-m_\rho^2) + \left(\frac{4}{3}\frac{N_c}{(4\pi)^2} + \dots\right)\theta(t-s_0)\,, \tag{2}$$

where the second term is obtained from large-N_c QCD perturbation theory (pQCD), starting from a scale s_0. A similar *ansatz* is used in the axial channel, with the same s_0, motivated by the fact that pQCD does not break chiral symmetry. The parameter s_0 is a parameter to be determined. Clearly, one needs $s_0 > m_\rho^2, m_{a_1}^2$ for consistency. More explicit resonances can be put in.

Considering $\Pi_{V,A}$ for large space-like Q^2 term by term in the (large-N_c) OPE, one obtains (from the first two terms) the well-known Weinberg sum rules, and from the four-fermion condensate the new relation (in the chiral limit, and to leading order in α_s)[2]

$$1 - \frac{3}{4}\frac{(4\pi)^4}{N_c^2}f_\rho^4 = -\frac{7}{11}\left(\left(1-\frac{f_\pi^2}{f_\rho^2 m_\rho^2}\right)^{-1} - \frac{3}{4}\frac{(4\pi)^4}{N_c^2}f_\rho^4\right)\,, \tag{3}$$

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the "7/11" rule. From the first Weinberg sum rule, $f_\rho^2 m_\rho^2 > f_\pi^2$, from which it follows that $f_\rho > 0.15$, just from the fact that the rhs of eq. (3) has to be negative. If we use also the relation[3] $f_\rho^2 m_\rho^2 = 2f_\pi^2$, we find, using eq. (3) and the Weinberg sum rules, that $m_\rho/f_\pi = 8.8$, $m_{a_1}/m_\rho = 1.4$, and $f_\rho = 2f_{a_1} = 0.16$, in remarkable agreement with experiment. We find that $\sqrt{s_0} = 4\pi f_\pi > m_{a_1}$, hence it is not unreasonable to use pQCD above $\sqrt{s_0}$. There are perturbative corrections to the 7/11 rule, unlike the Weinberg sum rules but, to the order we are working, the rule does not depend on any condensates. Furthermore the phenomenological results are robust to $O(\alpha_s)$ corrections.

For completeness, we also considered two-point functions for the scalar and pseudo-scalar densities, to the same order in the OPE. The idea is to use the gluon and four-fermion condensates predicted in this approach in the (axial-)vector sector, and see what we learn in the (pseudo-)scalar sector. In this case, there are two instead of one subtractions for each two-point function, because of the lack of current conservation, and we end up with only four instead of six equations, with five parameters, the scalar mass m_S, $m_{\pi'}$, the amplitudes c_m, d_m replacing f_{ρ,a_1}, and $\hat{s}_0$, equivalent, but not equal, to s_0. We supplemented the four equations with $L_8 = c_m^2/2m_S^2 - d_m^2/2m_{\pi'}^2 = 0.0009(3)$.[3]

Our findings are as follows. With the condensates from the vector sector, we find no solutions with $\sqrt{\hat{s}_0} > m_{\pi'}$, essentially because of a clash in the equations for $\hat{s}_0$ involving dimension-six operators. Instead, we eliminated this quantity (*i.e.* the terms originating from the pQCD part of our *ansatz* for the (pseudo-)scalar two-point functions) from the equations involving the four-fermion condensate, took $m_{\pi'} = 1300$ MeV as input, and found $c_m = 41$ MeV, $d_m = 27$ MeV, $m_S = 860$ MeV, $\sqrt{\hat{s}_0} = 2.2$ GeV. This solution is very insensitive to the gluon condensate, α_s, and $m_{\pi'}$, and $700 < m_S < 1200$ (MeV) for $12 > 10^4 L_8 > 6$. Hence, for the central value of L_8, phenomenological results are rather insensitive to our input, and one always finds a scalar mass of order $800 - 900$ MeV in the chiral and large-N_c limits. Our result for c_m is in good agreement with other estimates.[4] Using our results as additional input in the analysis of Peris *et al.*,[1] good predictions for $L_{1,2,3,5,8,9,10}$ are obtained if a Nambu–Jona-Lasinio-type model is assumed. We conclude with noting that the fact that we do not find a "perfect" solution for the *combined* vector and scalar sectors suggests that other resonances may have to be taken into account.

References

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