Pion-Pion Phase-Shifts and the Value of Quark-Antiquark Condensate in the Chiral Limit

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1. The chiral symmetry of massless QCD ($m_u = m_d = m_s = 0$) is spontaneously broken, what implies the existence of eight Goldstone bosons coupled to the quark currents, with strength F that, in the chiral limit, is of order 100 MeV. One may ask what is the size of the quark condensate parameter B_0, defined by $F^2 B_0 = -\langle \bar{q}q \rangle_0$.

In the standard Chiral Perturbation Theory (ChPT) [1] B_0 is assumed to be as large as 1 GeV. On the other hand, by considering the possibility of a low value for the quark condensate, some authors [2] have developed the Generalized Chiral Perturbation Theory (GChPT). The differences between the two approaches become more important for example when including loop corrections to the total pion-pion amplitude. The general form of the amplitude up to one loop can be written in terms of two collections of ChPT low energy constants, namely α and β and two free parameters λ_1 and λ_2, as in Ref. 2, which is identical to the expression in Eq. 2.1 of Ref. 3, obtained long ago by Sá Borges in the context of the Unitarization Program of Current Algebra (UPCA), if one takes α and β equal to one. The constants α and β can be written in terms of several low energy constants (LECs), such as B_0 as well as other ones coming from higher orders and loop contributions. In the standard case, α and β are kept very close to one, while in GChPT α can change say up to 4 and β up to 1.3.

2. In the present exercise we will consider the phenomenological consequences of the two approaches to ChPT by analysing pion-pion phase-shifts. The constants λ_1 and λ_2 are used for to fit to the phase shifts as follow. For each fixed choice of α and β within the ranges mentioned above, we perform the fit of P-wave phase-shift, which determines the combination $\lambda_2 - \lambda_1$. This value is then introduced in the expression of isospin zero S-wave in order to determine the remaining parameter by fitting experimental data. Isospin two amplitude does not depend on any new parameter. For each set of α and β, we could evaluate the quality of the fit as well as calculate the values of the S-wave scattering lengths, shown in the Table.

In figure 1 and 2 we present the one-parameter fits of respectively P- and S-wave phase-shifts resulting from different choices of α and β. The quality of

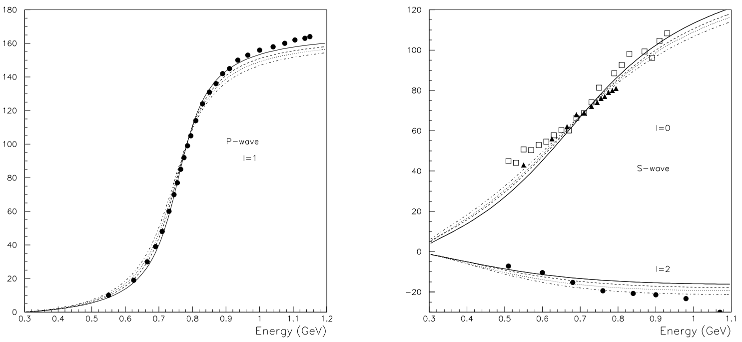

Figure 1: S- and P-wave pion-pion phase-shift, as functions of center of mass energy, in GeV.

α	β	curve	$\mathbf{a}_0^0$	$\mathbf{a}_0^2$	λ_1	λ_2	χ^2(%)
1.0	1.0	solid	.1642	-.0451	.00752	-.00653	3.1
2.0	1.1	dashed	.2249	-.0362	.00849	-.00642	3.3
2.0	1.2	dotted	.2408	-.0414	.00798	-.00657	14
3.0	1.3	dot-dashed	.3060	-.0323	.01090	-.00620	30

fits becomes poorer as α and β get different from one. On the other hand, the threshold behaviour shows discrepancies, mainly for S-waves.

3. As a summary, our main concern was to verify whether pion-pion phase-shifts could distinguish between the large and low value of the quark condensate. In order to achieve it, we have fitted experimental data using the expressions following from the two scenarios. We have obtained that the χ^2 of the fit corresponding to the Standard ChPT amplitude is smaller than those corresponding to any set of parameters of the generalized approach. On the other hand, we realized that the scattering lengths vary a lot even under the constraint of the ρ mass and width, suggesting that our approach is not suitable to predict such observables. Nevertheless, only accurate measurements of S-wave scattering lengths will allow any conclusion about the validity of each approach and, as a consequence, an indication about the size of quark condensate.

1. J. Gasser and H. Leutwyler, *Ann. Phys.* **158** (1984) 142.
2. M. Knecht and J. Stern, preprint hep-ph/9411253. M. Knecht, B. Moussallam and J. Stern, preprint hep-ph/9411259.
3. J. Sá Borges, *Phys. Lett. B* **149** (1984) 21.