# WHAT $\pi\pi$-SCATTERING DOES NOT TELL US ABOUT $\bar{Q}Q$ CONDENSATE

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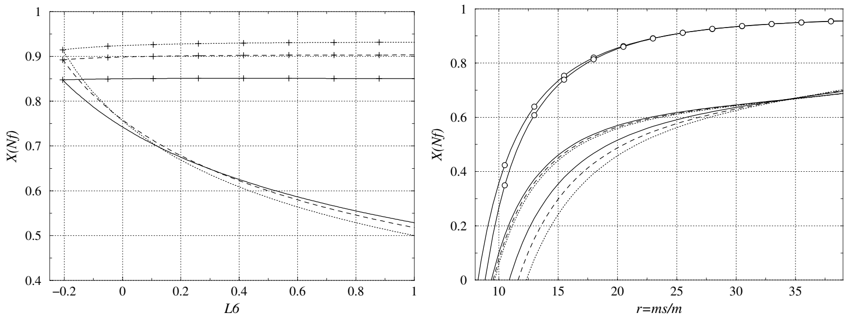
$\chi$PT with 3 light flavors (u,d,s) [1] involves the three-flavor condensate $\Sigma(3) = -\lim_{m_u,m_d,m_s \to 0} \langle \bar{u}u \rangle$ . It is convenient to express this order parameter in appropriate physical units:

$$X(3) = \frac{m_u + m_d}{F_\pi^2 M_\pi^2} \Sigma(3). \tag{1}$$

The ratio $X(3)$ measures the deviation from Gell-Mann,Oakes, Renner (GOR) relation $X(3) = 1$ and it can be, in principle, inferred from low-energy observables (see [2] and references therein). If, on the other hand, one brings $m_s$ to its physical value keeping u and d massless, one gets the two-flavor condensate $\Sigma(2) = -\lim_{m_u,m_d \to 0} \langle \bar{u}u \rangle_{m_s = physical}$ and the two-flavor GOR ratio $X(2) = (m_u + m_d) F_\pi^{-2} M_\pi^{-2} \Sigma(2)$ which can, in principle differ from $\Sigma(3)$ and $X(3)$ , (on general grounds, one expects $X(2) > X(3)$.) The $N_f$ - dependence of the condensate is an OZI-rule violating (large $N_c$ suppressed) effect reflecting vacuum fluctuations of light $\bar{q}q$ pairs. In $\chi$PT the latter are described by the low-energy constant $L_6^r(\mu)$ which is not known [1,2]. As for other Zweig-rule violating effects, the splitting $X(2) - X(3)$ has often been supposed to be marginal. It has been shown , however, that given the pseudoscalar masses and decay constants, chiral Ward identities imply a strong correlation between $X(N_f)$ and the precise value of $L_6(M_\rho)$ [3]: unless $L_6(M_\rho)$ is in a narrow band around $-0.26 \times 10^{-3}$, vacuum fluctuations significantly suppress $X(3)$ compared to $X(2)$ and to 1 , no matter how large is the quark mass ratio $r = m_s/m$. (For $r < 20$ , both $X(2)$ and $X(3)$ are suppressed.) This result is summarized in the first Figure showing $X(3)$ and $X(2)$ (lines with crosses) as a function of$10^{-3} L_6(M_\rho)$ for three values of $r = m_s/m$: $r = 20$ (solid lines), $r = 25$ (dashed lines) and $r = 30$ (dotted lines).

The actual importance of vacuum fluctuations and their effect on $X(3)$ can be estimated using QCD chiral sum rule [4] for the connected two-point function $\Pi \sim \langle \bar{u}u(x) \bar{s}s(y) \rangle$ at $m = 0, m_s \neq 0$. This large $N_c$ suppressed correlator defines both $L_6$ and the splitting $X(2) - X(3)$. $Im\Pi(s)$ collects contributions of scalar-isosacalar states that couple both to $\bar{u}u + \bar{d}d$ and to $\bar{s}s$, i.e. violate the OZI and ideal mixing rules. At low energies ($\sqrt{s} < 1.2 GeV$), $Im\Pi(s)$ can be contructed [4] solving Omnes-Muskhelishvili equations with




the multi-channel $\pi\pi$ T-matrix as a input. The dominant effect arises from the $f_0(980)$ resonance. At higher energies $s > s_0$, $\sqrt{s_0} = 1.5 - 1.7GeV$, the sum rule is controlled using QCD-OPE. Intermediate energy contribution $(1.2GeV < \sqrt{s} < \sqrt{s_0})$ is bounded from both sides. Examples of resulting bounds for $X(3)$ as a function of $r = m_s/m$ are represented in the second Figure by the solid, dashed and dotted lines for $s_0 = 1.5; 1.6$ and$1.7GeV$ respectively. The allowed range for $X(2)$ is also plotted (lines with open circles). An important suppression of $X(3)$ is apparent in the whole range of r. It is compatible with the results of refs.[2] and [4].

$X(2)$ can be extracted from accurate **low energy $\pi\pi$ scattering experiments**, providing a valuable information about $r = m_s/m$ (see Fig.) . However, $\pi\pi$ scattering alone can hardly distinguish between the component of $X(2)$ due to vacuum fluctuations ($\sim m_s$) and the contribution of genuine massless three-flavor condensate. Even if $X(2) \sim 1$ and $r \sim 25$, $X(3)$ could still be tiny requiring a reordering of the 3- flavor S$\chi$PT.

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## References


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