PARTIALLY QUENCHED CHIRAL PERTURBATION THEORY AT TWO LOOPS

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NNLO (Two Loops) in the Chiral Expansion

- Collaboration with Johan Bijnens and Niclas Danielsson at Lund University;

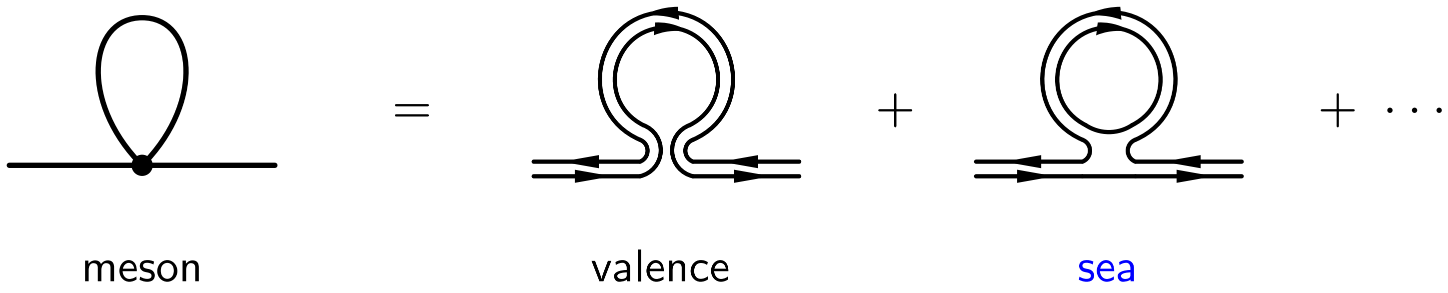
- Calculation of pseudoscalar meson masses and decay constants to NNLO, or $\mathcal{O}(p^6)$ in Partially Quenched Chiral Perturbation Theory (PQχPT). Electromagnetic form factors are also under consideration.

- Numerical implementation for the benefit of the Lattice QCD community; Chiral extrapolations to physical sea quark masses.

- Application to upcoming numerical Lattice QCD simulations with light dynamical sea quarks, ultimate objective: $\longrightarrow$ Determination of QCD low-energy constants!

Quenching of Sea Quark Loops in QCD

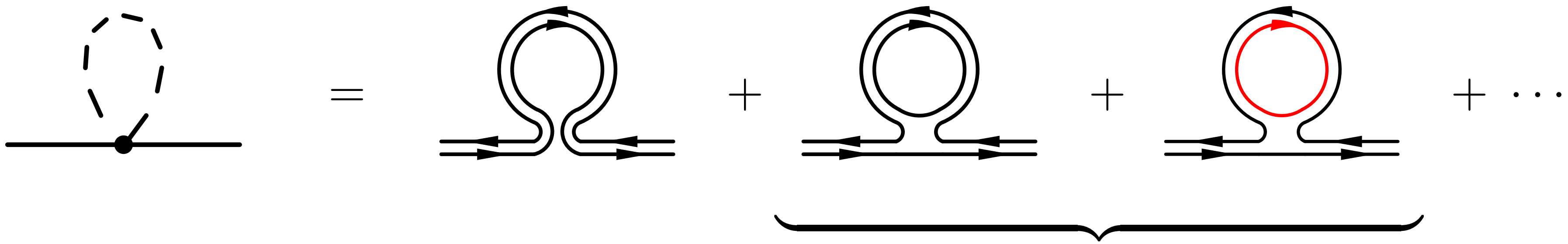
- Valence and sea quark contributions are treated separately in Lattice QCD - no such distinction in χPT.



- Sea quark loops (dynamical fermions) are computationally very demanding; $\longrightarrow$ Quenched (no sea quarks) and partially quenched (heavy sea quarks) simulations attempted.

- Lattice QCD simulations with physical u, d sea quark masses are not (yet?) possible $\longrightarrow$ Partially Quenched χPT applicable if sea quarks are light enough!

- A first step toward Partially Quenched theories is to remove the contributions with disconnected quark lines from the meson loops:

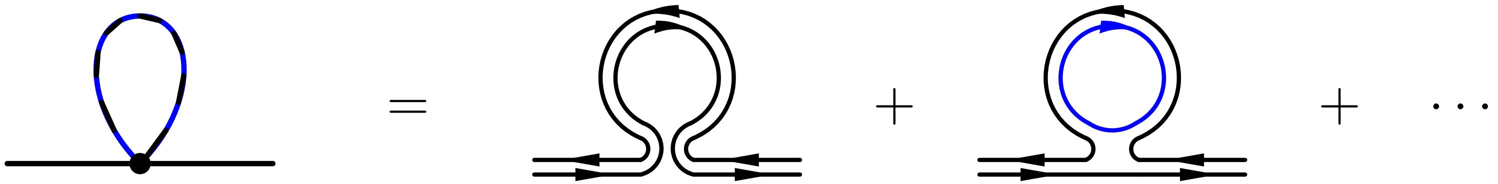


- Manual intervention at the level of each Feynman diagram is a possible approach; $\longrightarrow$ Systematic treatment preferrable!

- Introduce bosonic ghost quarks in order to cancel out all sea quark contributions from χPT; $\longrightarrow$ Supersymmetric formulation[1] of quenched χPT.

[1] C.W. Bernard and M.F.L. Golterman, Phys.Rev. **D49** (1994) 486

- Addition of explicit sea quark fields with independent, arbitrary masses to quenched χPT; $\longrightarrow$ Partially Quenched χPT.



- PQχPT is a systematic Lagrangian framework[2] where all necessary cancellations are carried out automatically, for any given set of Feynman diagrams.

- Low-energy constants of PQχPT are linear combinations of the unquenched ones; $\longrightarrow$ Direct information about the physical world from partially quenched Lattice QCD data!

[2] S. Sharpe and N. Shoresh, Phys.Rev. **D64** (2001) 114510, Phys.Rev. **D62** (2000) 094503

Calculation of Meson Masses to NNLO

- The physical squared masses $M^2_{\rm phys} = M^2$ are calculated from the pole position of the resummed propagator,

$$M^2 = M_0^2 + \Sigma(M^2).$$

- Up to NNLO, in terms of the lowest order mass M_0 and the self-energy contribution Σ, the masses of the pseudoscalar mesons are given by

$$M^2_{\rm phys} \quad = \quad M_0^2 + \Sigma_4(M_0^2) + \underbrace{\Sigma_4(M_0^2) \left. \frac{\partial \Sigma_4(p^2)}{\partial p^2} \right|_{M_0^2} + \Sigma_6(M_0^2)}_{\mathcal{O}(p^6)\ \text{contribution}} + \mathcal{O}(p^8).$$

- The self-energy Σ_4 represents the NLO (one-loop) mass shift.

Calculation of Decay Constants to NNLO

- The decay constants F^a are calculated from the matrix element of the axial current operator,

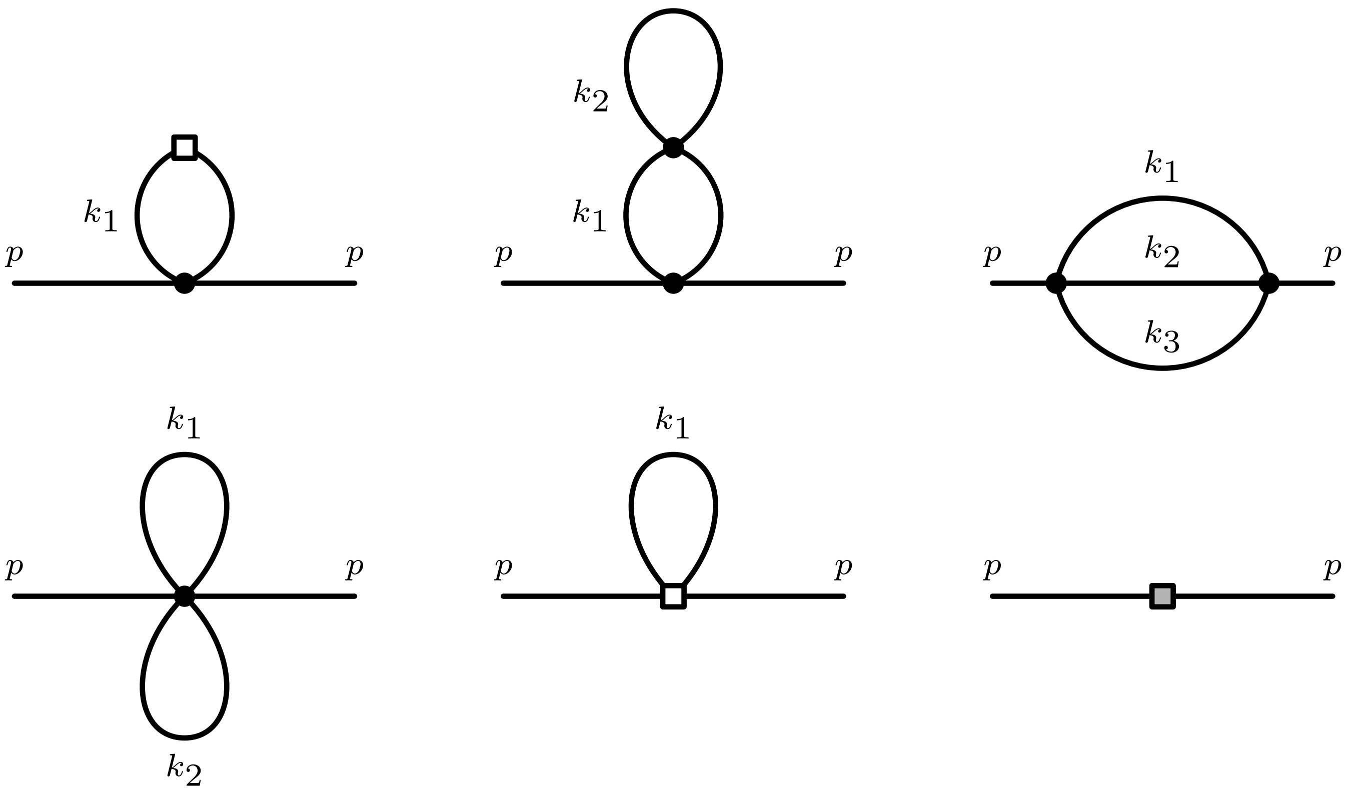
$$\langle 0|A^a_\mu(0)|\phi^a(p)\rangle = i\sqrt{2}\,p_\mu\,F^a,$$

- Up to NNLO, in terms of the lowest order result $F_2 = F_0$ and the self-energy contributions Σ from the wavefunction renormalization, one obtains

$$F_{\mathrm{phys}} = \underbrace{F_0 + F_4(M_0^2) + F_0\,\frac{\partial\Sigma_4(p^2)}{2\,\partial p^2}\bigg|_{M_0^2}}_{\mathcal{O}(p^4)\ \mathrm{contribution}} + F_0\,\frac{3}{8}\left(\frac{\partial\Sigma_4(p^2)}{\partial p^2}\bigg|_{M_0^2}\right)^2$$

$$+ \quad F_0\,\frac{\partial\Sigma_6(p^2)}{2\,\partial p^2}\bigg|_{M_0^2} + F_4(M_0^2)\,\frac{\partial\Sigma_4(p^2)}{2\,\partial p^2}\bigg|_{M_0^2} + F_6(M_0^2) \; + \; \mathcal{O}(p^8).$$

- At NNLO, the following diagrams contribute to $-i\Sigma$:



- Black dots, open squares and shaded squares denote vertices of the $\mathcal{O}(p^2)$, $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ Lagrangians, respectively.

Technical Issues and Numerical Results

- Expressions in NNLO PQχPT are very large; $\longrightarrow$ Heavy use of `FORM`[3].

- The raw `FORM` output is ugly, bloated and inefficient and contains $\sim 10^4$ terms; $\longrightarrow$ Efficient simplification with `MAPLE` to about 350 terms!

- The NNLO expressions for F and M^2 depend on several low-energy constants: F_0, B_0 of $\mathcal{O}(p^2)$, the L_i of $\mathcal{O}(p^4)$, and the K_i of $\mathcal{O}(p^6)$.

- Quark mass dependence through "squared masses" $\chi_i = 2\,B_0\,m_i$.

- Numerical results given in terms of the relative corrections Δ_i, defined as $M^2 = M_0^2\,(1+\Delta_M)$ and $F = F_0\,(1+\Delta_F)$.

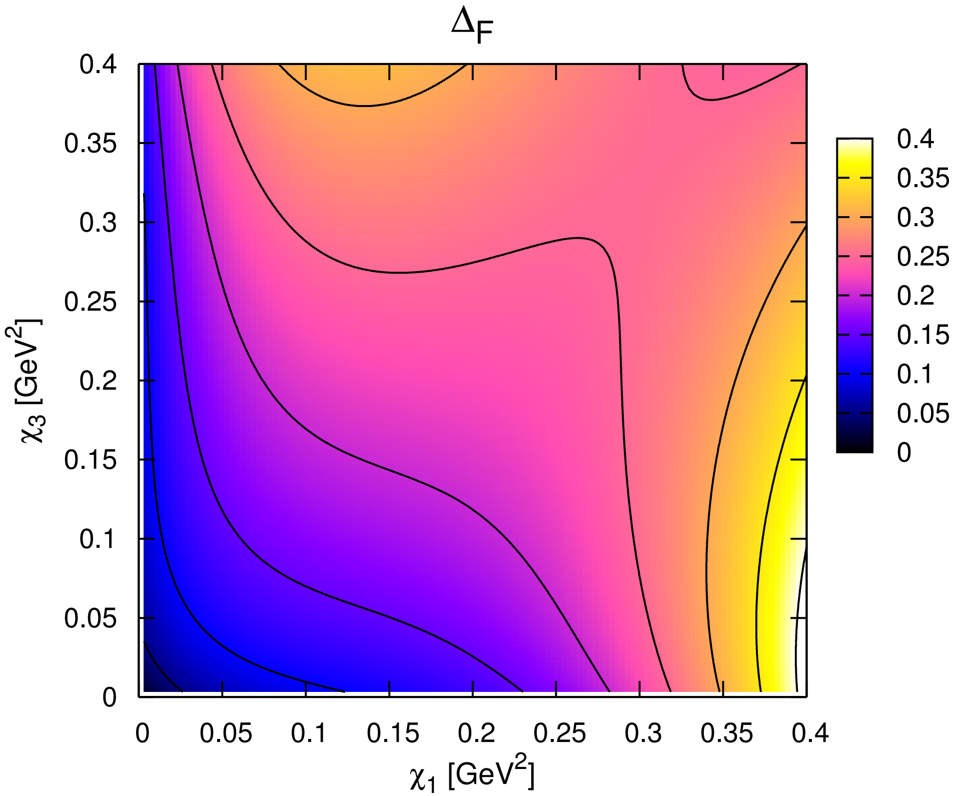
[3] J. Vermaseren, `http://www.nikhef.nl/~form`

- NNLO contribution to the decay constant Δ_F, defined as $F = F_0\,(1 + \Delta_F)$:

- Calculated for two sea quark flavors, $n_{\rm sea} = 2$.

- Plotted for degenerate sea quark masses χ_3.

- Unquenched χPT recovered along the diagonal!

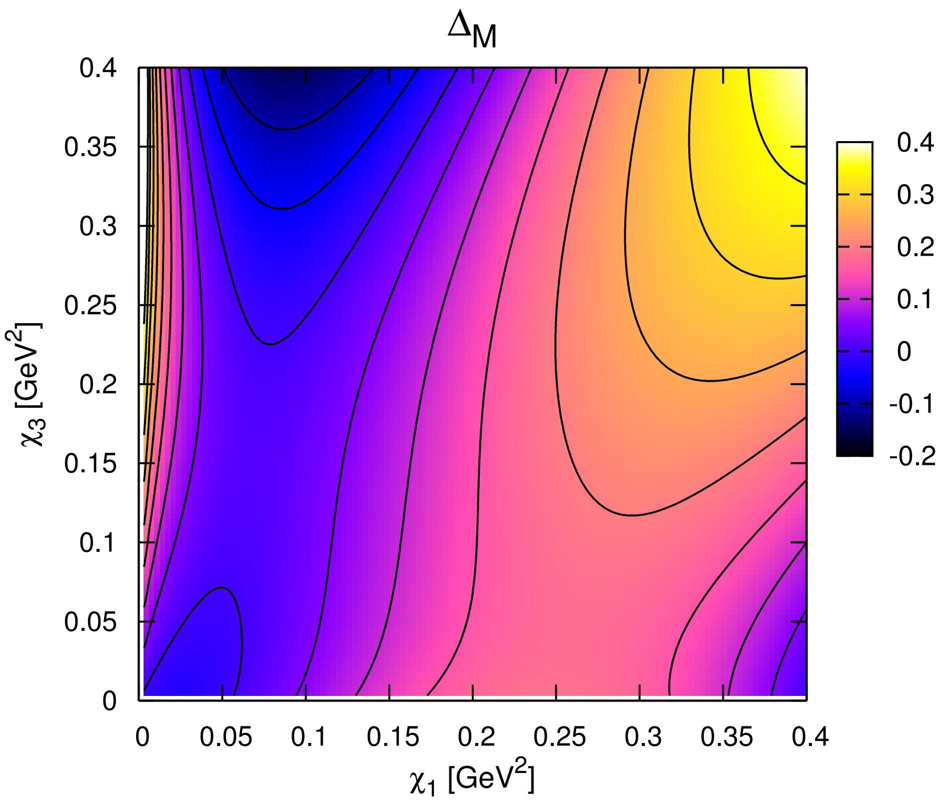


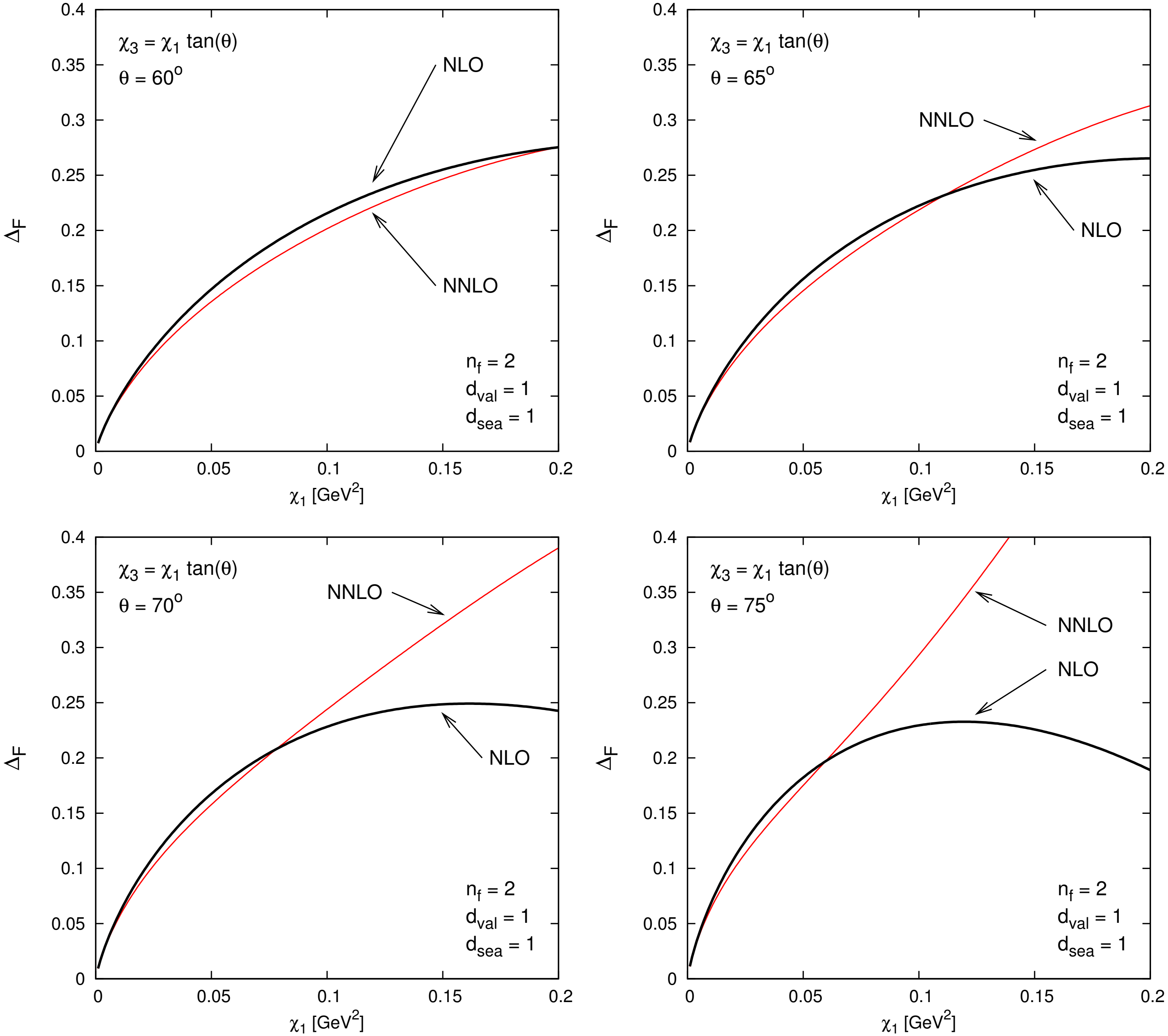
- NNLO contribution to the meson mass Δ_M, defined as $M^2 = M_0^2\,(1 + \Delta_M)$:

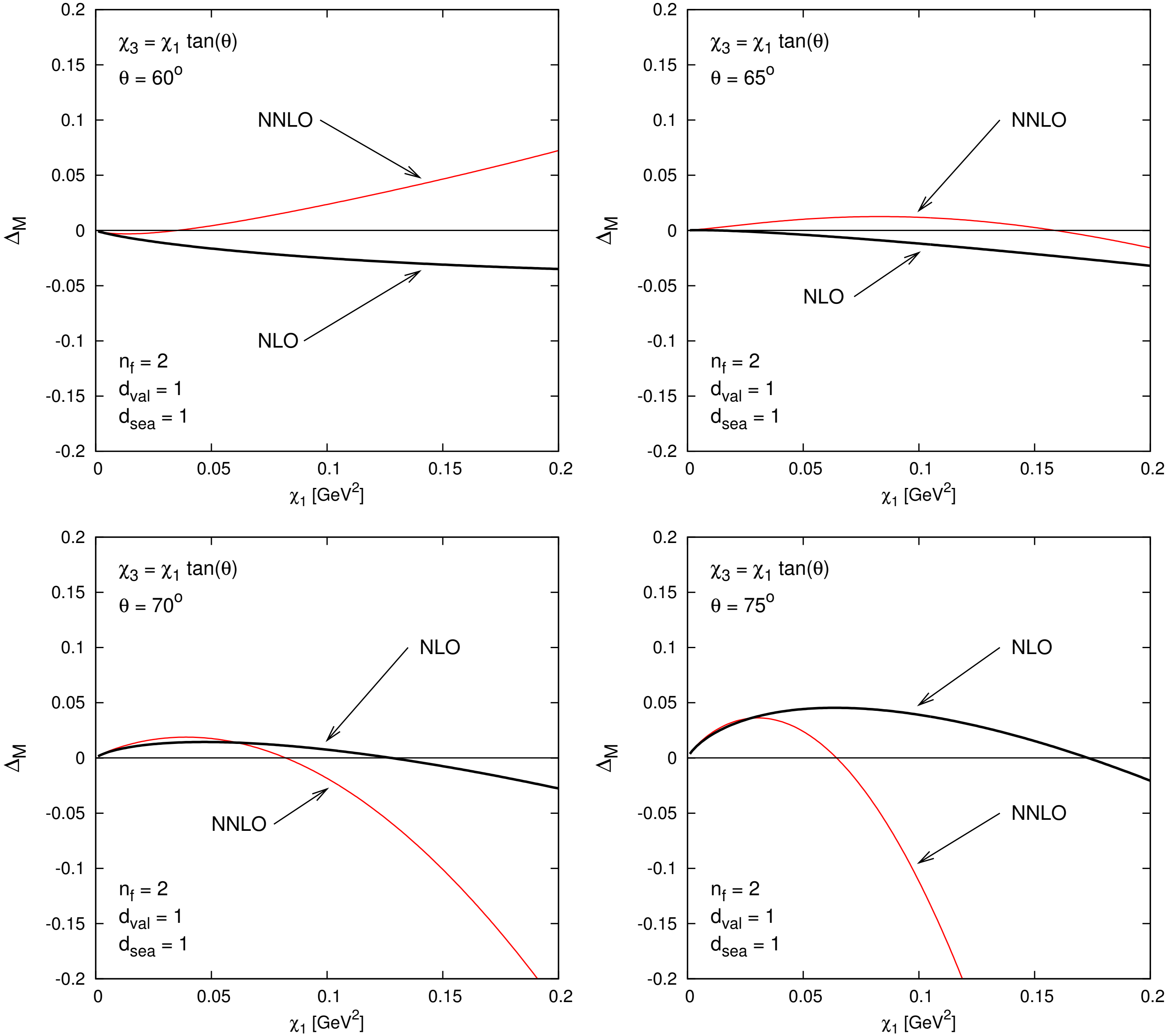
- Note the divergence of the quenched chiral logarithms as $\chi_1 \rightarrow 0$ for a constant sea quark mass.

- Calculated for two sea quark flavors, $n_{\text{sea}} = 2$.

- Plotted for degenerate sea quark masses χ_3.




χ3 = χ1 tan(θ)
θ = 60°
NLO
NNLO
nf = 2
dval = 1
dsea = 1
χ3 = χ1 tan(θ)
θ = 65°
NNLO
NLO
nf = 2
dval = 1
dsea = 1
χ3 = χ1 tan(θ)
θ = 70°
NNLO
NLO
nf = 2
dval = 1
dsea = 1
χ3 = χ1 tan(θ)
θ = 75°
NNLO
NLO
nf = 2
dval = 1
dsea = 1
ΔF
χ1 [GeV2]



χ3 = χ1 tan(θ)
θ = 60°
NNLO
NLO
nf = 2
dval = 1
dsea = 1
θ = 65°
θ = 70°
θ = 75°
ΔM
χ1 [GeV2]

Published Papers in PQχPT:

- Phys.Rev.**D70**:111503 (2004), hep-lat/0406017,
  "Pseudoscalar meson mass to two loops in three-flavor partially quenched chiral perturbation theory"
- Phys.Rev.**D71**:094502 (2005), hep-lat/0501014,
  "Decay constants of pseudoscalar mesons to two loops in three-flavor partially quenched chiral perturbation theory"

To appear shortly:

- Meson masses and decay constants in two-flavor PQχPT,
- Meson masses in three-flavor PQχPT, for nondegenerate sea quarks.

Planned future calculations (depending on interest):

- Electromagnetic form factors in PQχPT, properties of neutral pseudoscalars.