Two-Loop Finite Volume Effects

Karim Ghorbani

PhD Student

Theoretical High Energy Physics Department

Lund University

Advisor : Johan Bijnens

Outline

- QCD at low energy and the applicability of Effective Field Theory
- Dynamical consequences of spontaneous symmetry breakdown
- Chiral lagrangian
- Basic modifications due to finite volume
- Quark condensate at One and Two Loop
- Results

QCD at Low Scale

- α_s grows at low energy such that perturbative expansion (in terms of α_s) is no longer convergent
- Dealing with fundamental d.o.f Is difficult
- The way out is to employ an EFT
  - There exist mass hierarchy in hadronic spectrum
  - Inherites the underlying Symmetry of QCD
  - Irrelevant d.o.f integrated out
  - Instead low energy constant accomodate heavy particle effects

Dynamical Consequences of S.S.B

- When $m_q \rightarrow 0$ QCD exibits $G \equiv SU(N_f)_L \times SU(N_f)_R$
  $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$
- Vacuum manifests only $H \equiv SU(N_f)_V$
  - $< \overline{q}q > \neq 0 \Longrightarrow$ quark condensate

    As order-parameter, has a fundamental importance to study low energy structure of QCD
  - Goldstone theorem implies :

    if $N_f = 3$ then Eight Massless Pseudoscalar Goldstone Particles emerge :

    $\Rightarrow (\pi^+, \pi^-, \pi^o, \eta, K^+, K^-, K^o, \overline{K}^o)$

Effective Chiral Lagrangian

- $\phi^a (a = 1, \cdots, 8)$ Goldstone fields in the coset manifold G/H
- $U(\phi)$ embodies pseudoscalar fields so that $U(\phi) \longrightarrow g_R U(\phi) g_L$

$$U(\phi) = \exp\{i\sqrt{2}\Phi/F\}$$

$$\Phi(x) = \frac{\vec{\lambda}}{\sqrt{2}} \cdot \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^o + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^o \\ K^- & \overline{K}^o & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

$\vec{\lambda}$: Gell-Mann matrices

F : Turns out to be the pion decay constant

Effective Chiral Lagrangian

- We start with writing the most general lagrangian in terms of covariant derevetives of the matrix $U(\phi)$

$$\mathcal{L}_{eff}(U) = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

$$\mathcal{L}_2 = \frac{F^2}{4}\{\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + \chi U^\dagger \rangle\}$$

$$\mathcal{L}_4 = L_1\langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2\langle D_\mu U^\dagger D_\nu U\rangle\langle D^\mu U^\dagger D^\nu U\rangle + \cdots$$

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$$

$\chi = 2B_o(s + ip)$ (s): scalar and (p) : pseudoscalar external fields

With l_μ and r_μ left and right external current

- SCSB Scale : $4\pi F \approx 1.2 GeV$

Chiral Condensate

- Vacuum expectation value is given by this formula

$$<\bar{q}\lambda^a q> = \frac{\delta}{\delta S_a}\exp(iZ)|_{s=diag(m_u,m_d,m_s)}$$

$$\exp(iZ) =< 0|T\exp[i\int d^4x\mathcal{L}_{ext}(x)]|0 >$$

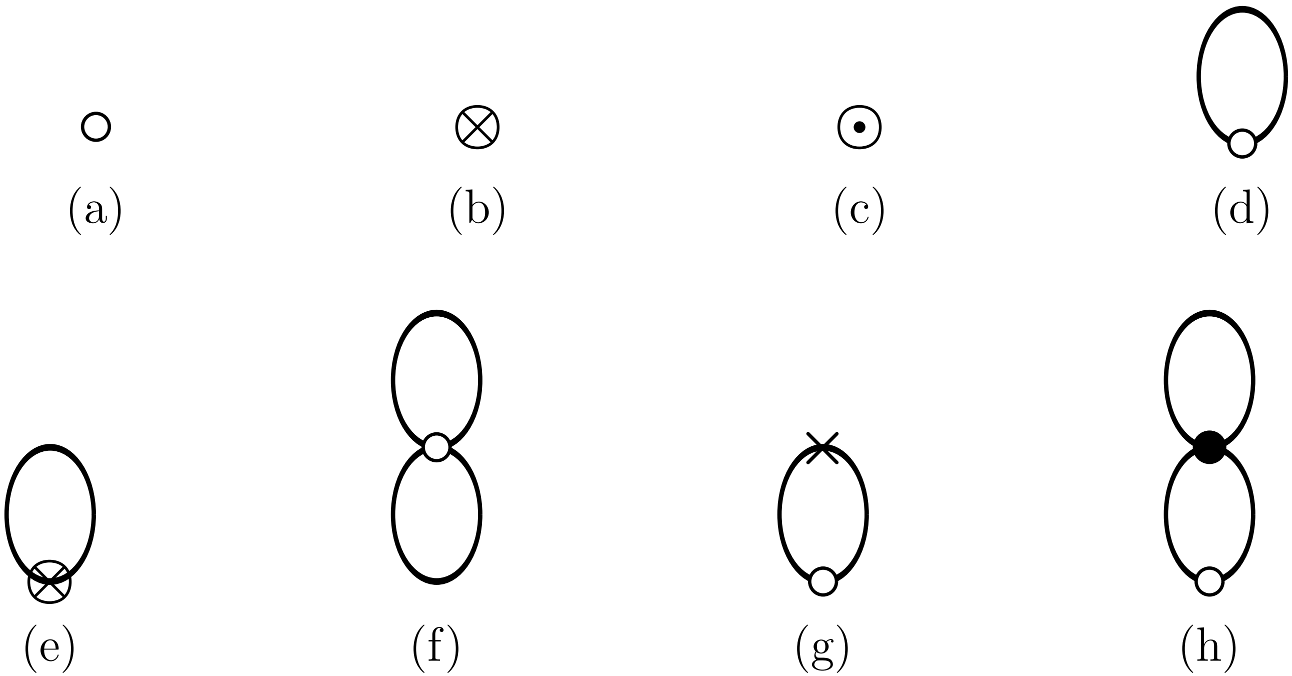
$(a = 1, \cdots, 8)$and S Scalar external fields include quark masses)

- To calculate V.E.V (Includes loop diagrams) Take into considation:
    - Weinberg's Power Counting Scheme:

$$D = 2L + 2 + \sum_d N_d(d-2)$$

    - Dimensional Regularization Scheme at arbitrary scale μ
    - Renormalization after adding counter-terms

Relevant Feynman diagrams up to order p^6



$\circ$ p^2 insertion of $\overline{q}q$
$\otimes$ p^4 insertion of $\overline{q}q$
$\odot$ p^6 insertion of $\overline{q}q$
$\bullet$ p^2 vertex
$\times$ p^4 vertex

Chiral Condensate in infinite volume

- Non-strange current at order P^4

  $< 0|\overline{q}q|0 >= \frac{1}{F_0^2}\{\frac{3}{2}\overline{A}(m_\pi^2)+\overline{A}(m_K^2)+\frac{1}{6}\overline{A}(m_\eta^2)+4m_\pi^2(4L_6^r+4L_8^r+H_2^r)\}$

  $\overline{A}(m^2) = -\frac{m_1^2}{16\pi^2}\ln(m_1^2)$

- Full calculation to order P^6
  G. Amoros, J. Bijnens, P. Talavera (2000)

Transition to Finite Volume

- To study the finite volume in ChPT framework

  Gasser and Leutwyler (1987, 1988)

  - The system is put into a finite box L_1, L_2, L_3
  - Imposing the boundrary condition on the fields $\phi(\vec{X}) = \phi(\vec{X} + L)$
  - Momentum becomes discrete: $\vec{P} = \frac{2\pi}{L}\vec{n}$ $\vec{n} = (n_1, n_2, n_3)$ $n_1, n_2, n_3 \, \epsilon Z$
  - Consequently pseudo goldston boson's propagator becomes periodic : $G_L(\vec{x}, t) = \sum_{\vec{n}} G_\infty(\vec{x} + \vec{n}L, t)$
  - $\frac{1}{(2\pi)^d} \int d^d p \longrightarrow \frac{1}{V} \sum_{\vec{p}}$
  - ChPT validity (p-regime)is limited to $L \gg \frac{1}{2F} \sim 1fm$

Chiral Logs Reshape

- Closer look at propagator new shape :
  - A typical chiral log as a tadpole correction

$$log(\frac{m^2}{\mu^2}) \longrightarrow log(\frac{m^2}{\mu^2}) - \sum_{k=1}^{\infty} \frac{4m(k)}{\sqrt{k}\lambda} K_1(\sqrt{k}\lambda)$$

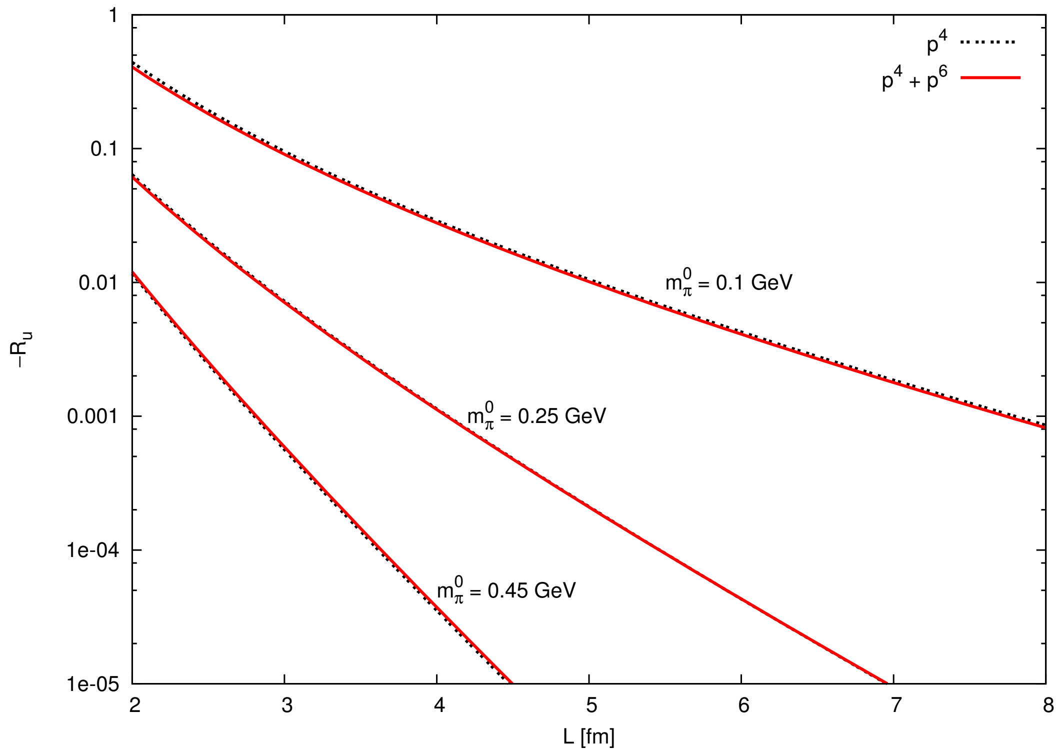
$\lambda = M_\pi L$ and $\vec{n}^2 = k$

  - For large value of λ $K_1(\sqrt{k}\lambda) \longrightarrow exp(-\sqrt{k}\lambda)$
  - Only the first several terms are numerically significant
  - Two-loop finite size effect needs this replacement :

$$B(m^2)_\infty \longrightarrow B(m^2)_\infty + \sum_{k=1}^{\infty} 2m(k) K_0(\sqrt{k}\lambda)$$

$$B(m^2)_\infty = -\frac{1}{16\pi^2} - \frac{1}{16\pi^2} log(\frac{m^2}{\mu^2})$$

Result



Relative finite volume corrections to the quark condensate:

$$R_u = \frac{<\overline{u}u>_L - <\overline{u}u>}{<\overline{u}u>}$$