Probing deconstructed dimensions with neutrinos

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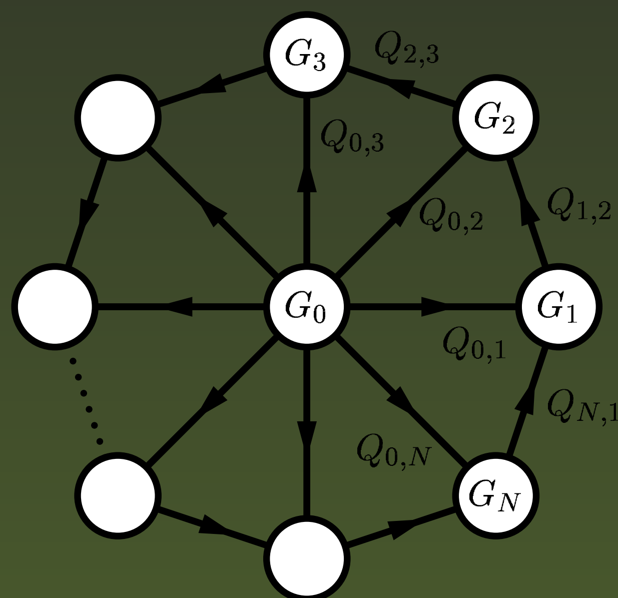
Outline

- Deconstructed $U(1)$ on a disk
- Mixing with KK modes
- Neutrino oscillations
- Summary and outlook

Deconstructed $U(1)$ on a disk

$U(1)^{N+1} = \Pi_{i=0}^{N} U(1)_i$ product gauge group connected by scalar links $\sim (+1, -1)$:



$$\mathcal{L} = -\frac{1}{4}\sum_{i=0}^{N} F_{i\mu\nu}F^{i\mu\nu} + \sum_{i=1}^{N}(|D_\mu Q_{0,i}|^2 + |D_\mu Q_{i,i+1}|^2)$$

Gauge boson mass spectrum for universal values

$g_i = g$, $v =< Q_{0,i} >$, $u =< Q_{i,i+1} >$:

$$M_0^2 = 0 \quad M_n^2 = g^2v^2 + 4g^2u^2 \sin^2 \frac{\pi n}{N} \quad M_N^2 = (N+1)g^2v^2.$$

- gauge boson with mass M_N decouples for $N \to \infty$
- zero mode $\sim U(1)_{\text{diag}}$ + KK spectrum $\sim gun/N \quad (n \ll N)$ raised by gv

The disc theory space have been considered for SUSY breaking and the doublet-triplet splitting problem.

E. Witten, hep-ph/0201018; N. Arkani-Hamed, A.G. Cohen, H. Georgi, JHEP 0207 (2002) 020

Large Lattice Spacings

Scalar Potential

$$V = \sum_{i=1}^{N} [m^2|Q_{0,i}|^2 + M^2|Q_{i,i+1}|^2 + \mu Q_{0,i}Q_{i,i+1}Q^*_{0,i+1} + \frac{1}{2}\lambda_1|Q_{0,i}|^4 + \frac{1}{2}\lambda_2|Q_{i,i+1}|^4 + \cdots + h.c.]$$

- dimensionful quantities m, M, $\mu \in$ UV desert between 1 TeV... M_{Pl}.
- interesting case: hierarchy $m \ll \mu \simeq M$ (links on boundary heavy)

Assume $m^2 < 0$, $\mu < 0$, $M^2 > 0 \rightarrow$ extremum for

$$u \equiv \langle Q_{i,i+1} \rangle \simeq \frac{m^2 \mu}{2[\lambda_1 + (N-1)\lambda_4]M^2 - \mu^2}$$

$$v^2 \equiv \langle Q_{0,i} \rangle^2 \simeq \frac{-m^2}{\lambda_1 + (N-1)\lambda_4}\left(1 + \frac{u\mu}{m^2}\right)$$

Few sites $N \simeq \mathcal{O}(10)$ **F. Bauer, M. Lindner, G. Seidl, JHEP 0405 (2004) 026**

Choosing $m \simeq 1$ TeV and $\mu \simeq M \simeq M_{B-L} \simeq 10^{15}$ GeV $\rightarrow u \simeq (\mu m)^{-1}$.

On the boundary: sub-mm lattice spacings from mass scales in the UV desert.

Inclusion of Fermions

Placing the SM fermions + three SM singlet neutrinos N_1, N_2 and N_3 on the center:

field	ℓ_α	e^c_α	q_α	u^c_α,d^c_α	N_1, N_2	N_3
$U(1)_0$	+1	-1	-1/3	+1/3	-4	5

- usually N_1, N_2, $N_3 \sim -1$ under gauged $B - L$

R.E. Marshak, R.N. Mohapatra, PLB 91 (1980) 222

- anomaly-free: $(-4)^3 + (-4)^3 + 5^3 = -4 - 4 + 5 = -3$
- Dirac neutrino mass term $\sim \ell_\alpha \epsilon H N_\beta$ forbidden $\rightarrow$ no seesaw for active neutrinos

Breaking of $U(1)_{\rm diag}$ around 1 TeV by SM singlet scalars S residing in the center.

O.C. Anoka, K.S. Babu, I. Gogoladze, NPB 687 (2004) 3

Decoupling of N_1, N_2, N_3 below 1 TeV through renormalizable terms $\sim SN_\alpha N_\beta$

Radiative generation of Majorana neutrino masses $\sim 10^{-2}$ eV with TeV-scale physics. **A. Zee, NPB 263 (1986) 99; K.S. Babu, PLB 203 (1988) 132**

Latticized Bulk Neutrino

Placing on each site (i) on the boundary $\Psi_i \equiv (\nu_{Ri}, \nu^{\bar{c}}_{Ri})^T \sim -1$ under $U(1)_i$.

Wilson action for RH bulk neutrino:

$$\mathcal{S}_{\text{wilson}} = \int d^4x \sum_{n=1}^{N} \nu_{Rn} \left(Q_{n,n+1} \nu^c_{R(n+1)} - \chi_n \nu^c_{Rn} \right) + \text{h.c.}$$

Scalar χ_n with $\langle \chi_n \rangle \equiv u$ elliminates unwanted fermion doublers

Large Dirac neutrino masses $\sim M_{Pl} \nu_{Ri} \nu^c_{Ri}$ forbidden by cyclic symmetry

BC's $(\nu_{R(N+1)}, \nu_{R(N+1)^c}) = \pm(\nu_{R1}, \nu^c_{R1})$ for (un)twisted RH neutrino

C.T. Hill, A.K. Leibovich, PRD 66 (2002) 016006

Neutrino mass spectrum

$$m_n^2 = 4u^2 \sin^2 \frac{\pi(n-1/2)}{N} \quad \text{(twisted)}$$

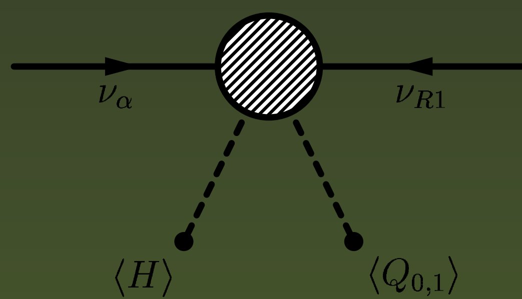
$$m_n^2 = 4u^2 \sin^2 \frac{\pi(n-1)}{N} \quad \text{(untwisted)}$$

In the IR: linear towers of KK excitations with $u \simeq (\mu m)^{-1} \simeq 10^{-1}$ eV masses.

Coupling to Active Neutrinos

"Local" connection of ν_α in the center with ν_{R1} due to cyclic symmetry

$\mathcal{S}_{\text{int}}^{4D} = \int d^4x \frac{Y_\alpha}{M_f} \ell_\alpha \epsilon H Q_{0,1}^* \nu_{R1} +$ h.c.



$\rightarrow$ Dirac mass term between ν_α and lowest lying KK-neutrino

$M_{D_\alpha} = Y_\alpha \langle H \rangle v / (\sqrt{N} M_f) \simeq 10^{-3}$ eV

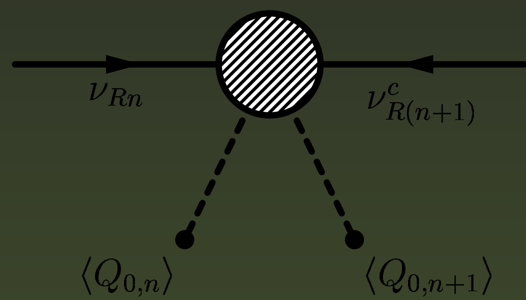
Compared to 5D ADD scenario:

$\mathcal{S}_{\text{int}}^{5D} = \int d^4x \frac{Y_\alpha^D}{\sqrt{M_*}} \ell_\alpha \epsilon H(x) \nu_R(x, y = 0)$

$\rightarrow$ 4D Dirac-type coupling to zero mode $\sim Y^D_\alpha \ell_\alpha \epsilon H \nu_{0R} M_* / M_{Pl}$

- similar suppression of couplings for coarse latticizations $\sqrt{N} \sim \mathcal{O}(1 - 10)$
- volume factor (5D) vs. lattice spacing v^{-1} compared to Planck length M_f^{-1} (4D)

Unwanted operators

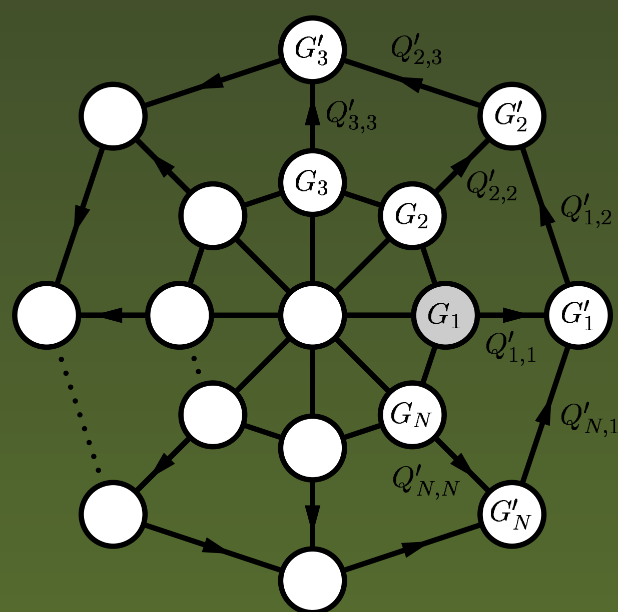


$$\mathcal{S}_{\rm dim5} = \int d^4x \sum_{n=1}^{N} \frac{Y_n}{M_f} Q^*_{0,n} Q_{0,n+1} \nu_{Rn} \nu^c_{R(n-1)} + \text{h.c.}$$

$\rightarrow$ small Dirac masses $\sim 10^{-2}$ eV between ν_{Rn} and $\nu^c_{R(n+1)}$ for $M_f \simeq 10^{17}$ GeV

Switched off in spider web theory space:

- SM on site (1) RH neutrino on outer circle
- links with masses $\sim 10^{12}$ GeV and $\sim 10^{2}$ TeV
- unwanted dimension 6 operators suppressed by $\sim 10^{-2}$ TeV
- $\langle Q'_{i,i+1} \rangle \sim (\mu m)^{-1}$



Mixing with KK modes

Neutrino mass and mixing terms (one active neutrino ν)

$\mathcal{L}_m^\nu = m_\nu \nu\nu + \sqrt{N} m_D \nu \nu_{R1} + u\nu_{RN}(T\nu_{R1}^c - \nu_{RN}^c) +$
$u\sum_{n=1}^{N-1} \nu_{Rn}(\nu_{R(n+1)}^c - \nu_{Rn}^c)$+h.c.

where $m_\nu \simeq 10^{-2}$ eV, $\sqrt{N} mD = M_f^{-1}\langle H\rangle\langle Q_{0,1}\rangle \simeq 10^{-2}$ eV and $u \simeq (\mu m)^{-1}$

- $\mathcal{L}_m^\nu \to (2N+1)^2$ mass matrix M with “heavy” KK modes: $u \ll m_\nu, \sqrt{N} m_D$
- diagonlization $MM^\dagger \to V^T M M^\dagger V^*$ in perturbation theory
- using small expansion parameter $\epsilon \equiv \sqrt{N} m_D / u \ll 1$

$T = -1$ N even mixing angles from 1st row:

$V_{1i} = (1, \epsilon A_1, \ldots, \epsilon A_{N/2}, \epsilon B_1, \ldots, \epsilon B_{N/2})$ where

$$A_n = \frac{m_\nu}{u\sqrt{8N}[sin^2 \frac{(n-1/2)\pi}{N} - \frac{\lambda}{4}}$$

$$B_n = \frac{sin \frac{(n-1/2)\pi}{N}}{\sqrt{2N}[\sin^2 \frac{(n-1/2)\pi}{N} - \frac{\lambda}{4}]}$$

Neutrino Oscillations

G.R. Dvali, A.Y. Smirnov, NPB 563 (1999) 63; R.N. Mohapatra, S. Nandi, A. Perez-Lorenzana, PLB 466 (1999) 115

Flavor eigenstate

$$|\nu_f\rangle = \frac{1}{K}\left(|\nu\rangle + \epsilon\sum_{n=1}^{N/2} A_n|\hat{\nu}_n\rangle + \epsilon\sum_{n=1}^{N/2} B_n|\nu_n\rangle\right)$$

From $|\langle\nu_f|\nu_f\rangle|^2 = 1$: normalization constant

$$K^2 = 1 + \epsilon^2\sum_{n=1}^{N/2}(A_n^2 + B_n^2)$$

Transition survival probability: $P_{ff} \equiv P(\nu_f \to \nu_f) \equiv |\langle\nu_f|\nu_f(t)\rangle|^2$

Time evolved state

$$|\nu_f(t)\rangle =$$

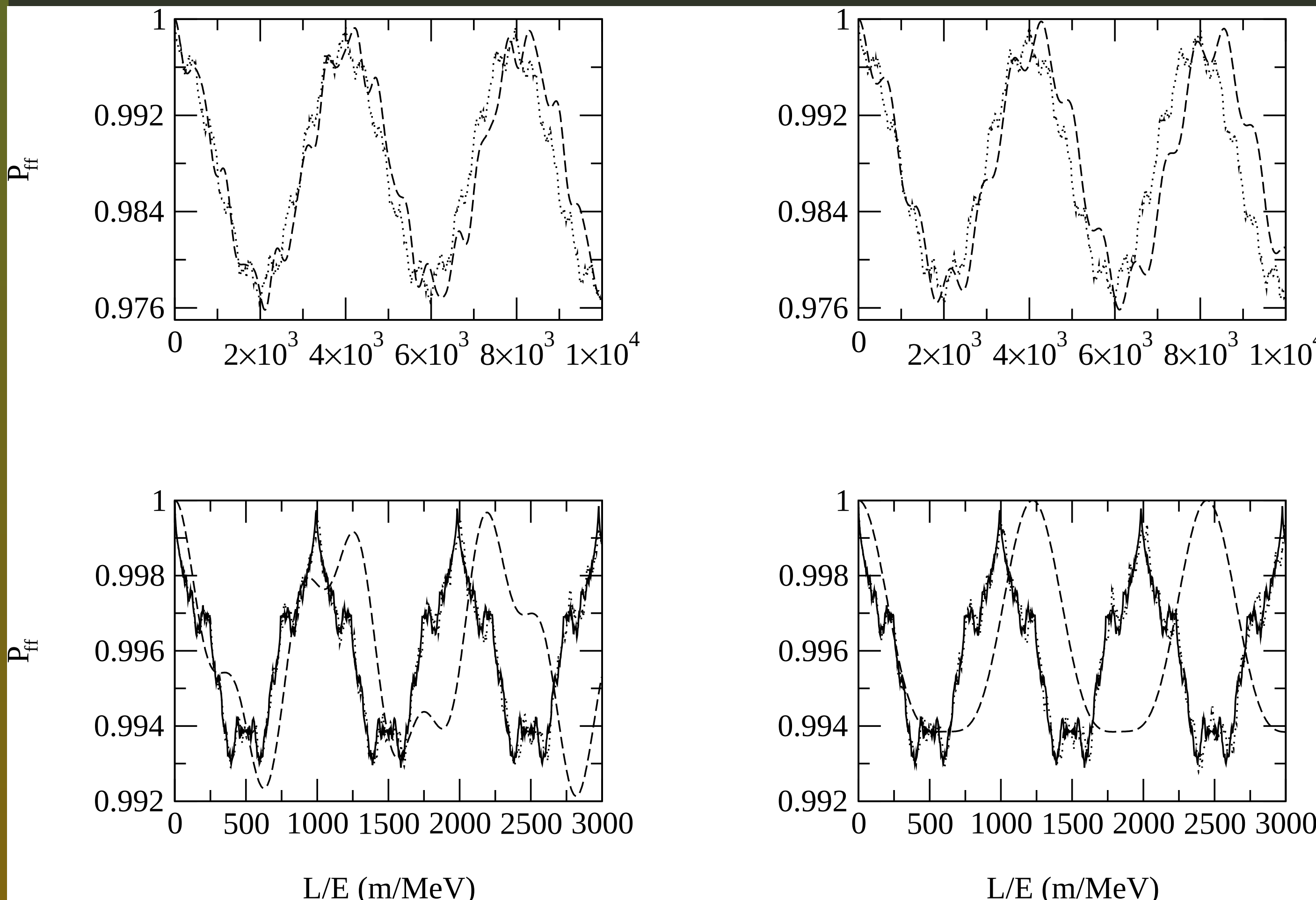
$$\frac{1}{K}\mathrm{e}^{-\mathrm{i}\frac{\left(Nm_D^2+m_\nu^2\right)t}{2E}}\left(|\nu\rangle + \epsilon\sum_{n=1}^{N/2} A_n\mathrm{e}^{\mathrm{i}\phi_n}|\hat{\nu}_n\rangle + \epsilon\sum_{n=1}^{N/2} B_n\mathrm{e}^{\mathrm{i}\phi_n}|\nu_n\rangle\right)$$

with phases and mass-squared eigenvalues

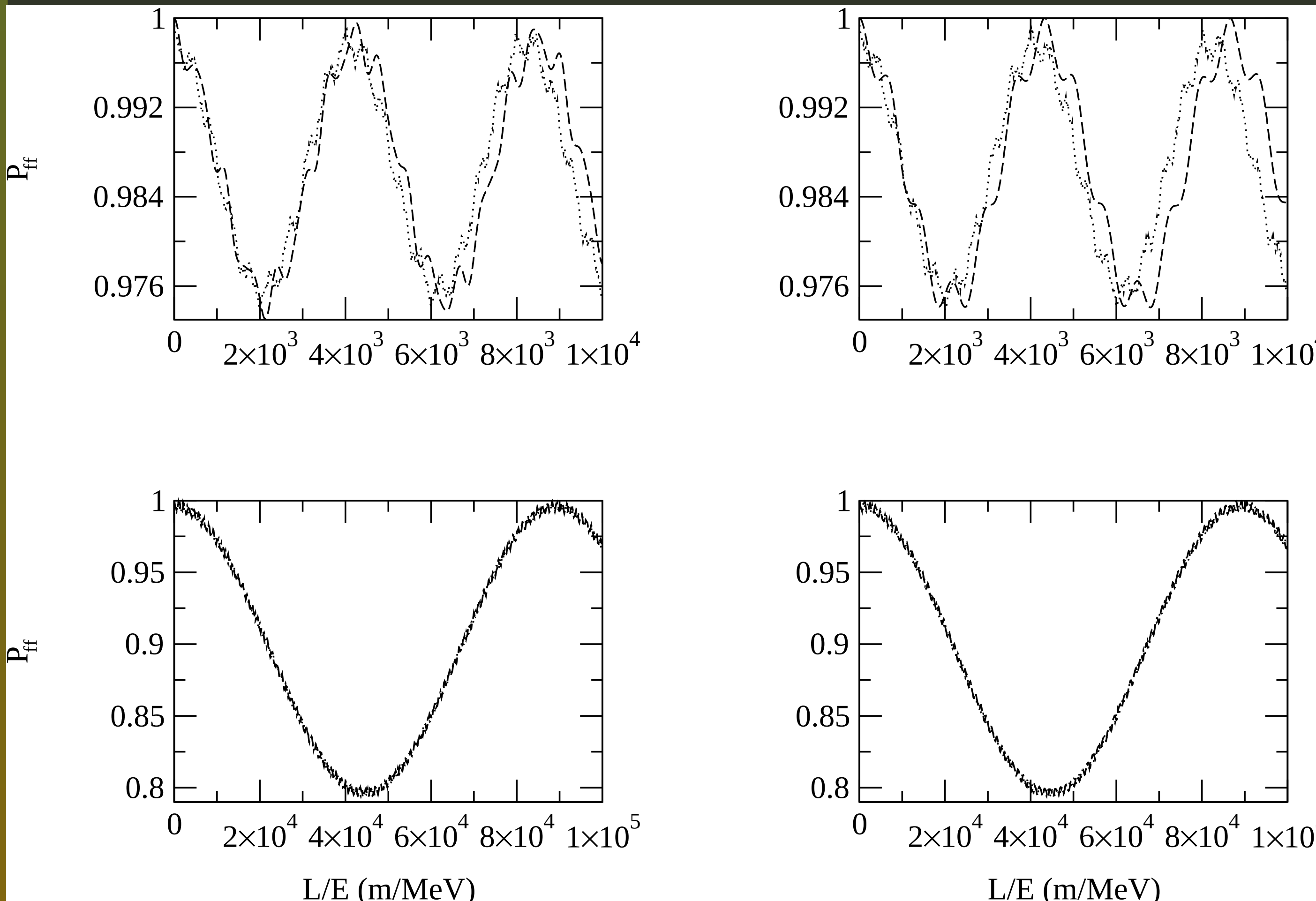
$$\phi_n = \frac{(Nm_D^2 + m_\nu^2 - m_n^2)t}{2E} \quad \text{and} \quad m_n^2 = 4u^2 \sin^2 \frac{(n-1/2)\pi}{N}$$

$T = -1$ and N even

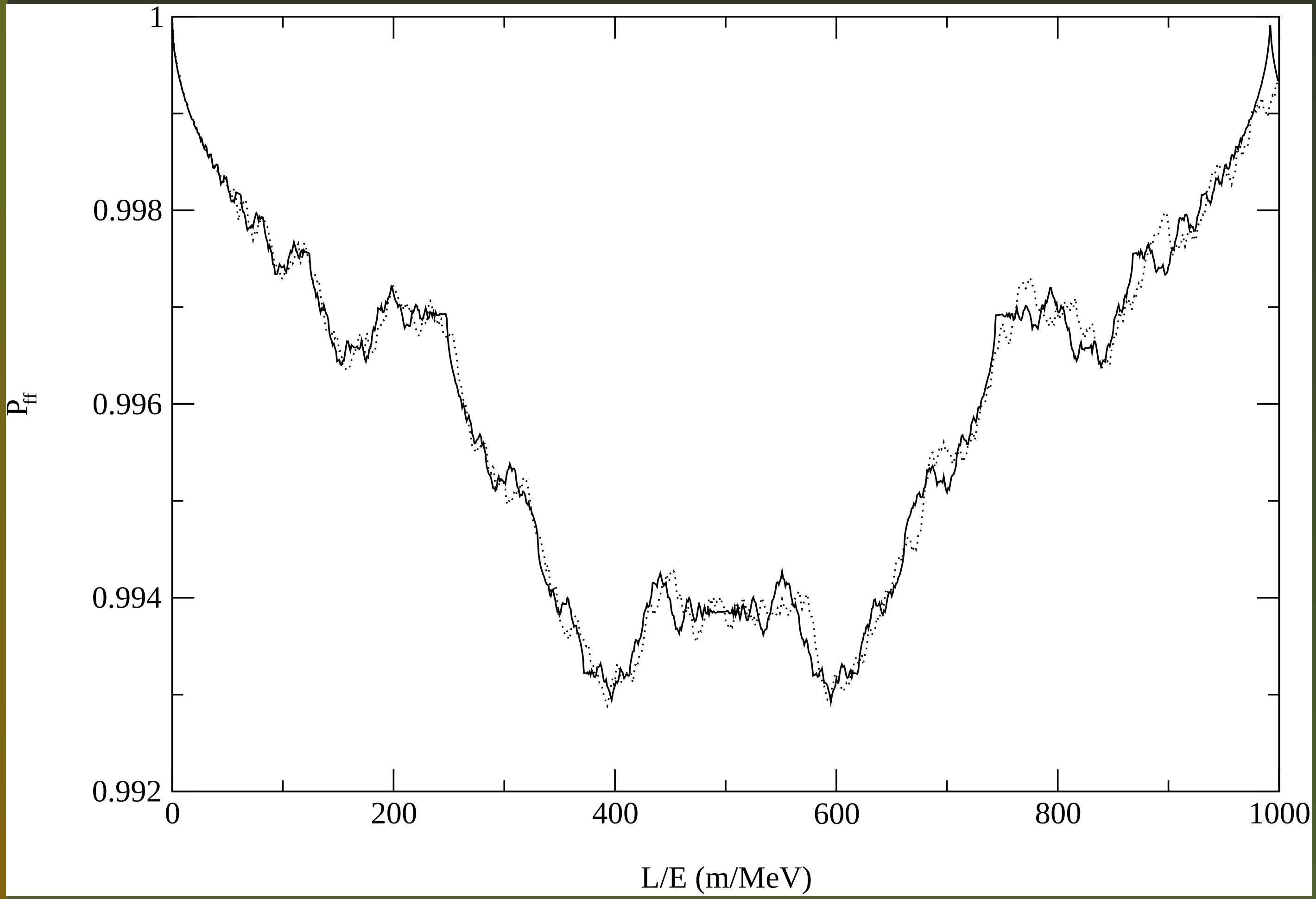
$$P_{ff} = \frac{1}{K^4} \left| 1 + \epsilon^2 \sum_{n=1}^{N/2} (A_n^2 + B_n^2) \mathrm{e}^{\mathrm{i}\phi_n} \right|^2$$



Neutrino transition survival probability P_{ff} for $m_\nu = 0$. Upper left panel: T=-1 and N odd for N=55(dashes) and N=55 (dots). Upper right panel: T=-1 and N even for N=4 (dashes) and N=44 (dots). Lower left panel: T=1 and N odd for N=5 (dashes) and N=55 (dots). Lower right panel: T=1 and N even for N=4 (dashes) and N=44 (dots)



Neutrino transition survival probability P_{ff} for $m_\nu \neq 0$. Upper left panel: T=-1 and N odd for N=5 (dashes) and N=55 (dots). Upper right panel: T=-1 and N even for N=4 (dashes) and N=44 (dots). Lower left panel: T=1 and N odd for N=5 (dashes) and N=55 (dots). Lower right panel: T=1 and N even for N=4 (dashes) and N=44 (dots).



Amplification of P_{ff}, comparing the continuum model (solid) with N=55 and T=1 (dots)

Summary and Outlook

- Probing deconstruction with neutrino oscillations
- Model for sub-mm lattice spacings from UV desert between TeV and Planck scale
- Possible subleading corrections to standard three-neutrino oscillations
- Test in (future) $\bar{\nu}_e$ or ν_e precision disappearence epxeriments like KamLAND, Borexino, Double-CHOOZ,...
- BBN constraints can be avoided by primordial lepton asymmery or low reheating temperatur

Reference: T. Hällgren, T. Ohlsson, G. Seidl, JHEP **0502** (2995) 049, hep-ph/0411312