



## $K_{\ell 3}$ at two loops in ChPT

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Long project of bringing ChPT calculations for three flavour to the two-loop level

G. Amorós, JB, P. Dhonte, P. Talavera

- Chiral Perturbation Theory
- Two Loop Calculations done
- $K \rightarrow \pi \ell \nu$  ( $K_{\ell 3}$ ): Definitions and  $V_{us}$
- $f_+(t)$ : linearity, ChPT results, fit to CPLEAR, PS E246 data,  $\lambda_+$
- $f_0(t)$  Main result:  $f_-(t) \Rightarrow f_+(0)$
- Conclusions

} ChPT works,  
an example  
JB and P. Talavera,  
hep-ph/0303103

# Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so  $M_\rho$  or higher depending on the channel

## Chiral Symmetry

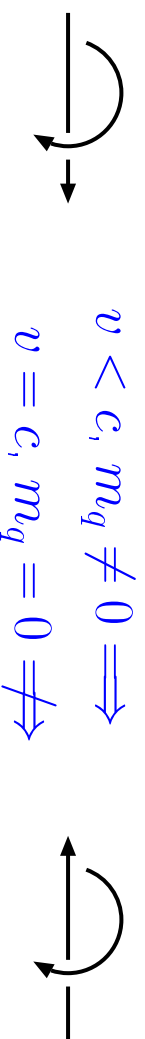
QCD: 3 light quarks: equal mass can interchange them:  $SU(3)_V$

But

$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$$

So if  $m_q = 0$  then  $SU(3)_L \times SU(3)_R$ .

Can also see that via



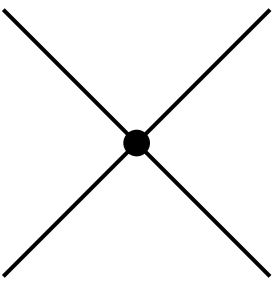
# Chiral Perturbation Theory

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$

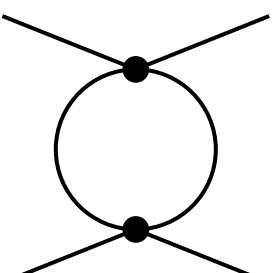
$SU(3)_L \times SU(3)_R$  broken spontaneously to  $SU(3)_V$

8 generators broken  $\implies$  8 massless degrees of freedom **and** interaction vanishes at zero momentum

Power counting in momenta:



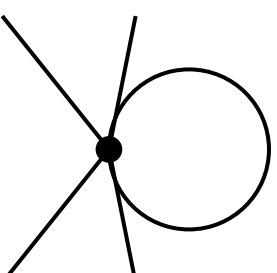
$$p^2$$



$$(p^2)^2 (1/p^2)^2 p^4 = p^4$$



$$1/p^2$$



$$(p^2) (1/p^2) p^4 = p^4$$

$$\int d^4 p$$

$$p^4$$

# Chiral Perturbation Theory

External currents/masses:

$$m_\pi^2 \sim m_q \langle \bar{q}q \rangle \implies m_q \sim p^2$$
$$A_\mu, W_\mu^\pm \text{ in } D_\mu = \partial_\mu - ieA_\mu Q \implies A_\mu, W_\mu^\pm \sim p$$

## General Papers for two-loops

ChPT basis papers: Gasser Leutwyler 1985

$p^2$	Long known	two parameters
$p^4$	Gasser Leutwyler	ten parameters ( $L_i^r$ )
$p^6$	JB, Colangelo, Ecker Fearing, Scherer	90 parameters ( $C_i^r$ )

Renormalization and Infinities at  $p^6$ : JB, Colangelo, Ecker

**Note:** At least 3 different renormalization schemes used in actual calculations

## Two Flavour

$$\gamma\gamma \rightarrow \pi^0\pi^0$$

Bellucci, Gasser, Sainio

$$\gamma\gamma \rightarrow \pi^+\pi^-, F_\pi, m_\pi$$

Bürgi

$$\pi\pi\text{-scattering}, F_\pi, m_\pi$$

JB, Colangelo, Ecker, Gasser, Sainio

$$F_{V\pi}(t), F_{S\pi}$$

JB, Colangelo, Talavera

$$\pi \rightarrow \ell\nu\gamma$$

JB, Talavera

## Three Flavour

$$\Pi_{VV\pi}, \Pi_{VV\eta}, \Pi_{VVk}$$

Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera

$$\Pi_{AA\pi}, \Pi_{AA\eta}, F_\pi, F_\eta, m_\pi, m_\eta$$

Kambor, Golowich; Amorós, JB, Talavera

$$\Pi_{SS}$$

$$L_4^r, L_6^r$$

Moussallam

$$\Pi_{VVK}, \Pi_{AAK}, F_K, m_K$$

Amorós, JB, Talavera

$$K_{\ell 4}$$

$$L_1^r, L_2^r, L_3^r$$

Amorós, JB, Talavera

$$F_{\pi^+}, F_{\pi^0}, \dots, m_{K^+}, m_{K^0} \text{ for } m_u \neq m_d$$

$$L_5^r, L_7^r, L_8^r, m_u/m_d$$

Amorós, JB, Talavera

$$F_{V\pi}, F_{VK^+}, F_{VK^0}$$

$$L_9^r$$

Post, Schilcher; JB, Talavera

$$K_{\ell 3}$$

Post, Schilcher; JB, Talavera

In preparation:  $K_{\ell 3}$  for  $m_u \neq m_d$

JB, Talavera

In preparation:  $F_{S\pi}, F_{SK}$

$$L_4^r$$

JB, Dhonte

## $K_{\ell 3}$ Definitions

$$\begin{aligned}
 K_{\ell 3}^+ &: & K^+(p) &\rightarrow \pi^0(p')\ell^+(p_\ell)\nu_\ell(p_\nu) \\
 K_{\ell 3}^0 &: & K^0(p) &\rightarrow \pi^-(p')\ell^+(p_\ell)\nu_\ell(p_\nu)
 \end{aligned}$$

$$K_{\ell 3}^+ : \quad T = \frac{G_F}{\sqrt{2}} V_{us}^* \ell^\mu F_\mu^+(p', p)$$

$$\ell^\mu = \bar{u}(p_\nu)\gamma^\mu(1 - \gamma_5)v(p_\ell)$$

$$\begin{aligned}
 F_\mu^+(p', p) &= \langle \pi^0(p') | V_\mu^{4-i5}(0) | K^+(p) \rangle \\
 &= \frac{1}{\sqrt{2}} [(p' + p)_\mu f_+^{K^+\pi^0}(t) + (p - p')_\mu f_-^{K^+\pi^0}(t)]
 \end{aligned}$$

$$\begin{aligned}
 K_{\ell 3}^0 : \quad F_\mu^0(p', p) &= \langle \pi^-(p') | V_\mu^{4-i5}(0) | K^0(p) \rangle \\
 &= (p' + p)_\mu f_+^{K^0\pi^-}(t) + (p - p')_\mu f_-^{K^0\pi^-}(t).
 \end{aligned}$$

**Isospin:**

$$\begin{aligned}
 f_+^{K^0\pi^-}(t) &= f_+^{K^+\pi^0}(t) = f_+(t) & f_-^{K^0\pi^-}(t) &= f_-^{K^+\pi^0}(t) = f_-(t)
 \end{aligned}$$

## $K_{\ell 3}$ Definitions and $V_{us}$

Scalar formfactor:

$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization:

$$f_{+,0}(t) = f_+(0) \left( 1 + \lambda_{+,0} \frac{t}{m_\pi^2} \right)$$

$|V_{us}|$ : • Know theoretically  $f_+(0) = 1 + \dots$

– Short distance correction to  $G_F$  from  $G_\mu$  **Marciano-Sirlin**

– **Ademollo-Gatto-Behrends-Sirlin theorem**:  $(m_s - \hat{m})^2$

– Isospin Breaking **Leuwylter-Roos**  $\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.022$  **In Progress**

• Know experimentally  $f_+(0)$

– Radiative Corrections: use generalized formfactors **Cirigliano et al.**, hep-ph/0110153

– **Parametrize form-factor: is linear enough for  $f_+(t)$ ?**

**PDG2002:**

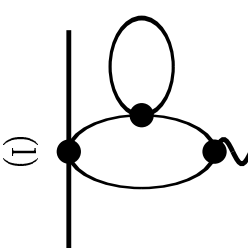
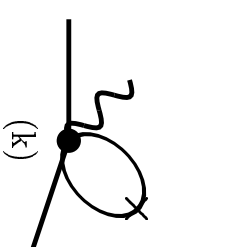
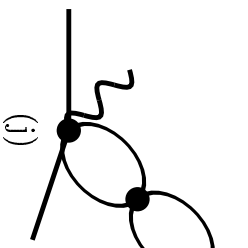
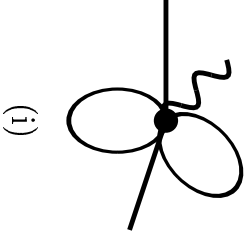
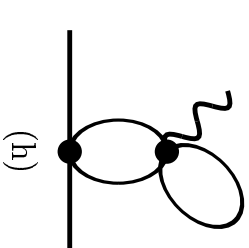
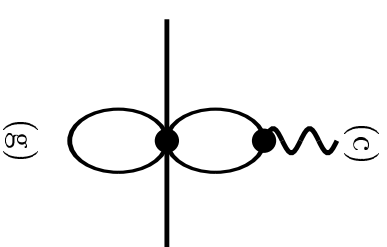
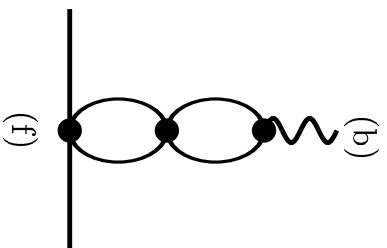
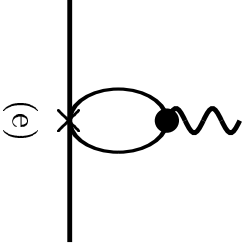
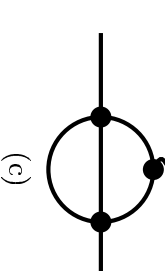
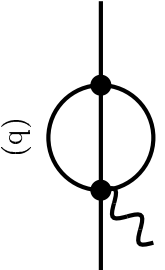
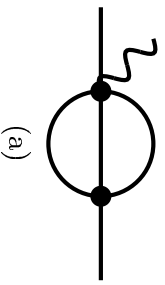
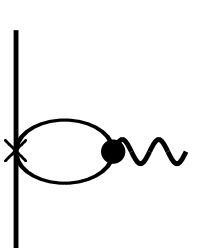
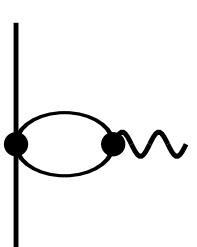
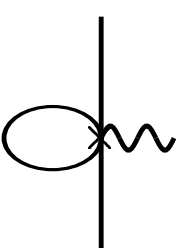
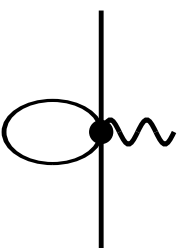
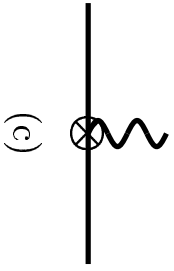
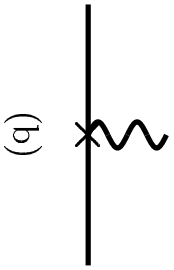
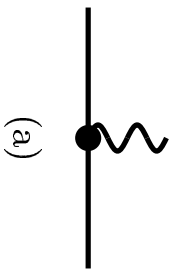
$$|V_{ud}| = 0.9734 \pm 0.0008$$

$$|V_{us}| = 0.2196 \pm 0.0026$$

$$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) = \mathbf{0.9957 \pm 0.0019}$$

( $V_{ud}$  from neutron decay only  $\Rightarrow$  a bit worse)

# $K_{l3}$ Diagrams



- :  $p^2$  vertex
- X :  $p^4$  vertex
- ⊗ :  $p^6$  vertex

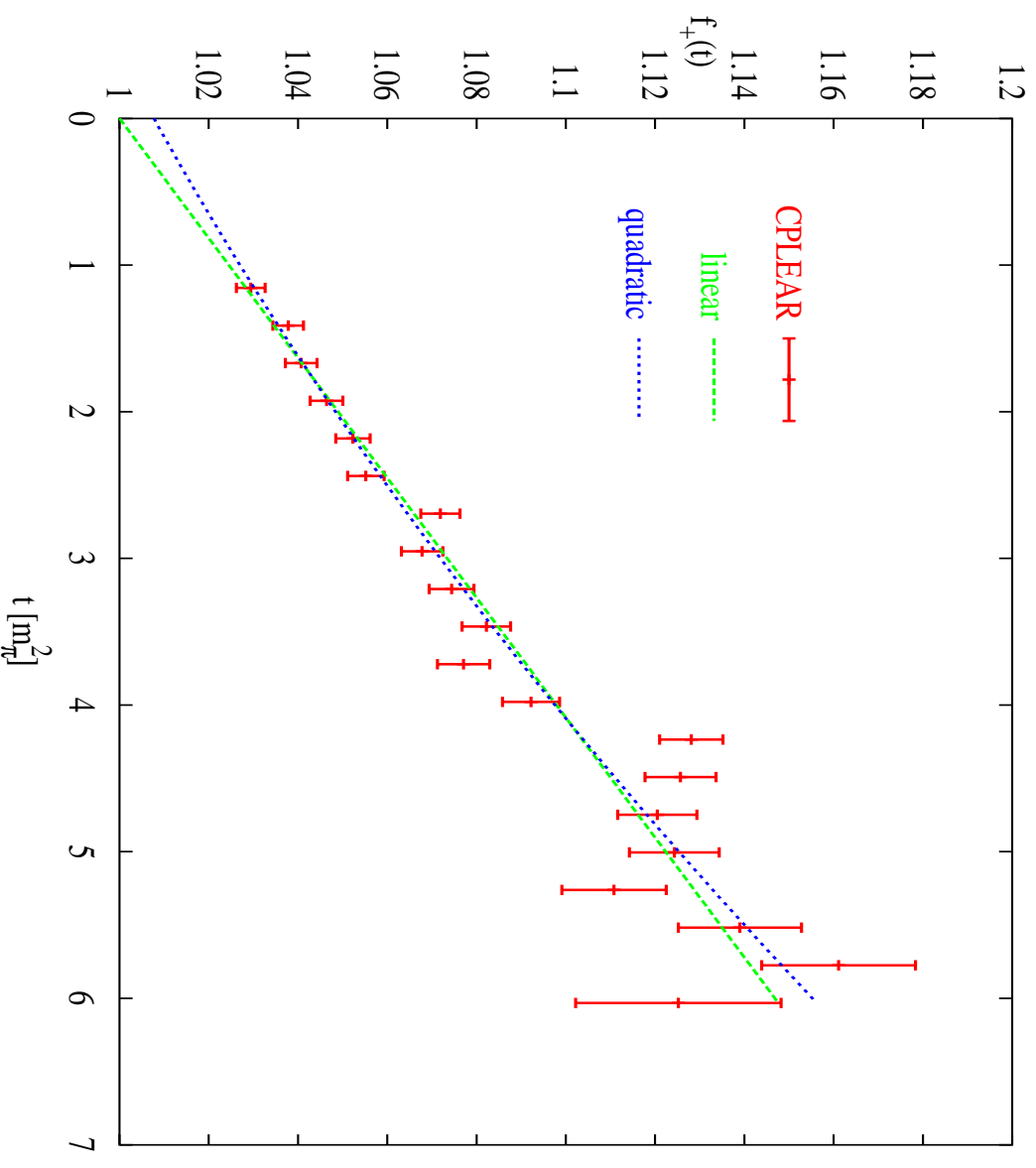




## Is Linear Enough ?

$$f_+(t) = a_+ \left( 1 + \lambda_+ \frac{t}{m_{\pi^+}^2} + c_+ t^2 \right)$$

$a_+$	$\lambda_+$	$c_+$ [GeV <sup>-4</sup> ]
$\equiv 1$	$0.0245 \pm 0.0006$	$\equiv 0$
$1.000 \pm 0.004$	$0.0245 \pm 0.0015$	$\equiv 0$
$\equiv 1$	$0.0238 \pm 0.0017$	$0.5 \pm 1.2$
$1.008 \pm 0.009$	$0.0181 \pm 0.0068$	$2.8 \pm 2.8$



Curvature IS Important at 1% Level!!

# $f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4} (-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

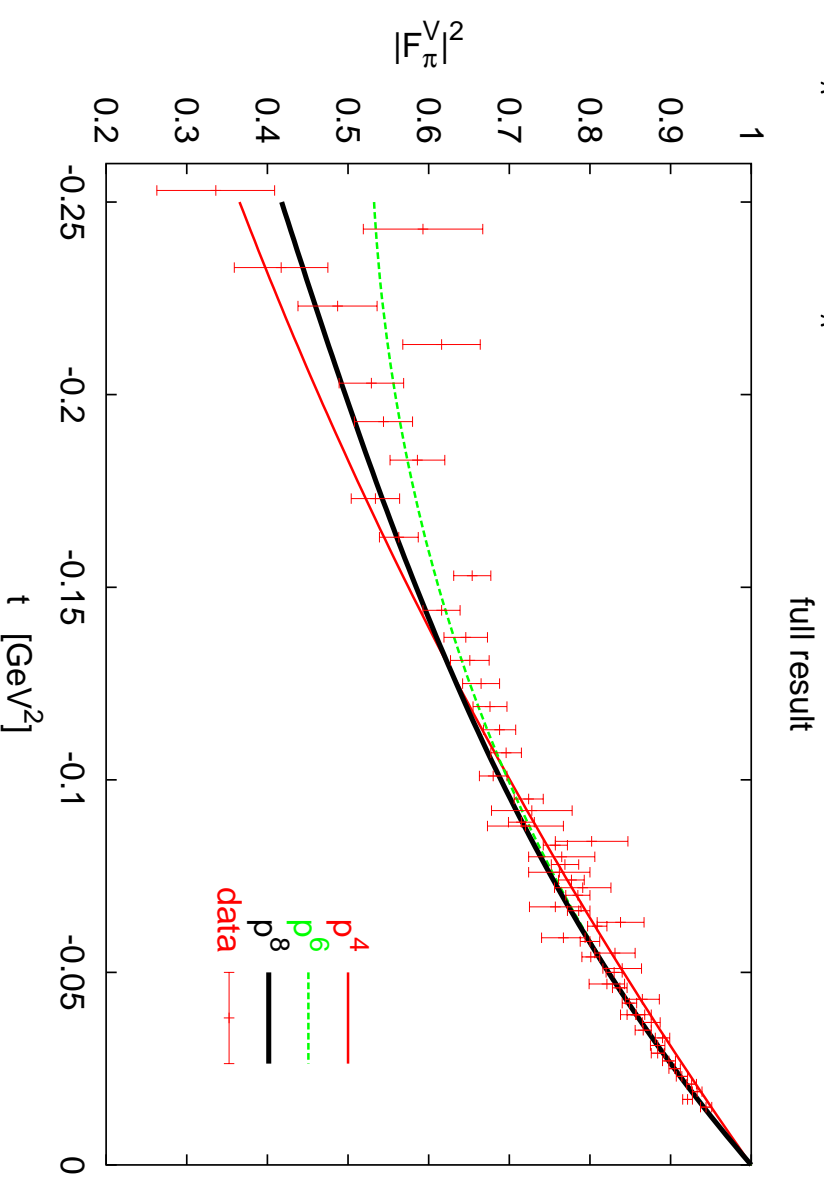
Pion electromagnetic Form factor:

JB, Talavera

$$L_9^r = 0.00593 \pm 0.00043$$

$$-4C_{88}^r + 4C_{90}^r = 0.00022 \pm 0.00002$$

$$\text{VMD: } R_{+1}^{K\pi} \approx -4 \cdot 10^{-5} \text{ GeV}^2$$



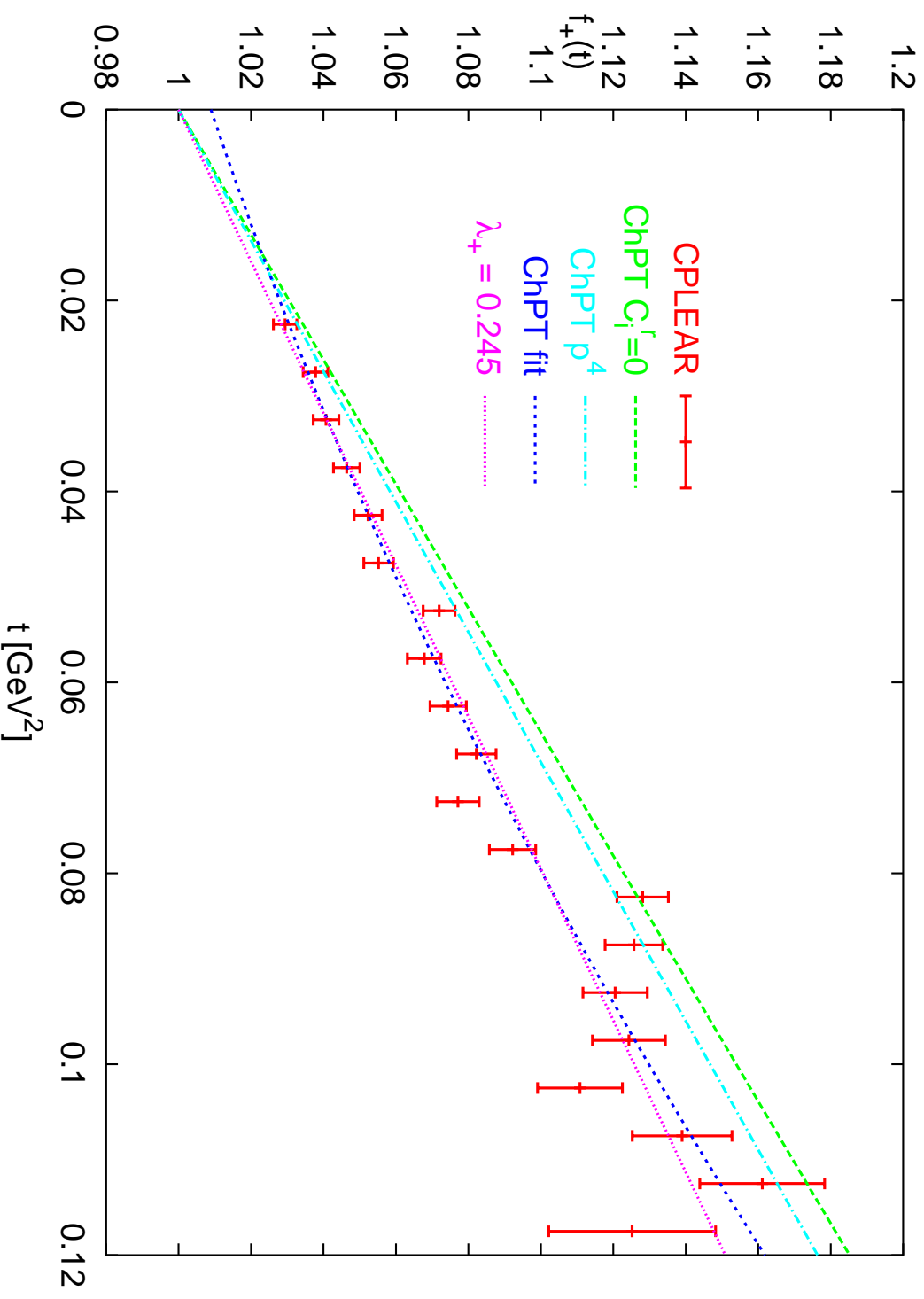
## ChPT fit to $f_+(t)$

$$\Rightarrow R_{+1}^{K\pi} = -(4.7 \pm 0.5) 10^{-5} \text{ GeV}^2$$

$$(c_+ = 3.2 \text{ GeV}^{-4})$$

$$\Rightarrow a_+ = 1.009 \pm 0.004$$

$$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$$



## ChPT fit to $f_+(t)$

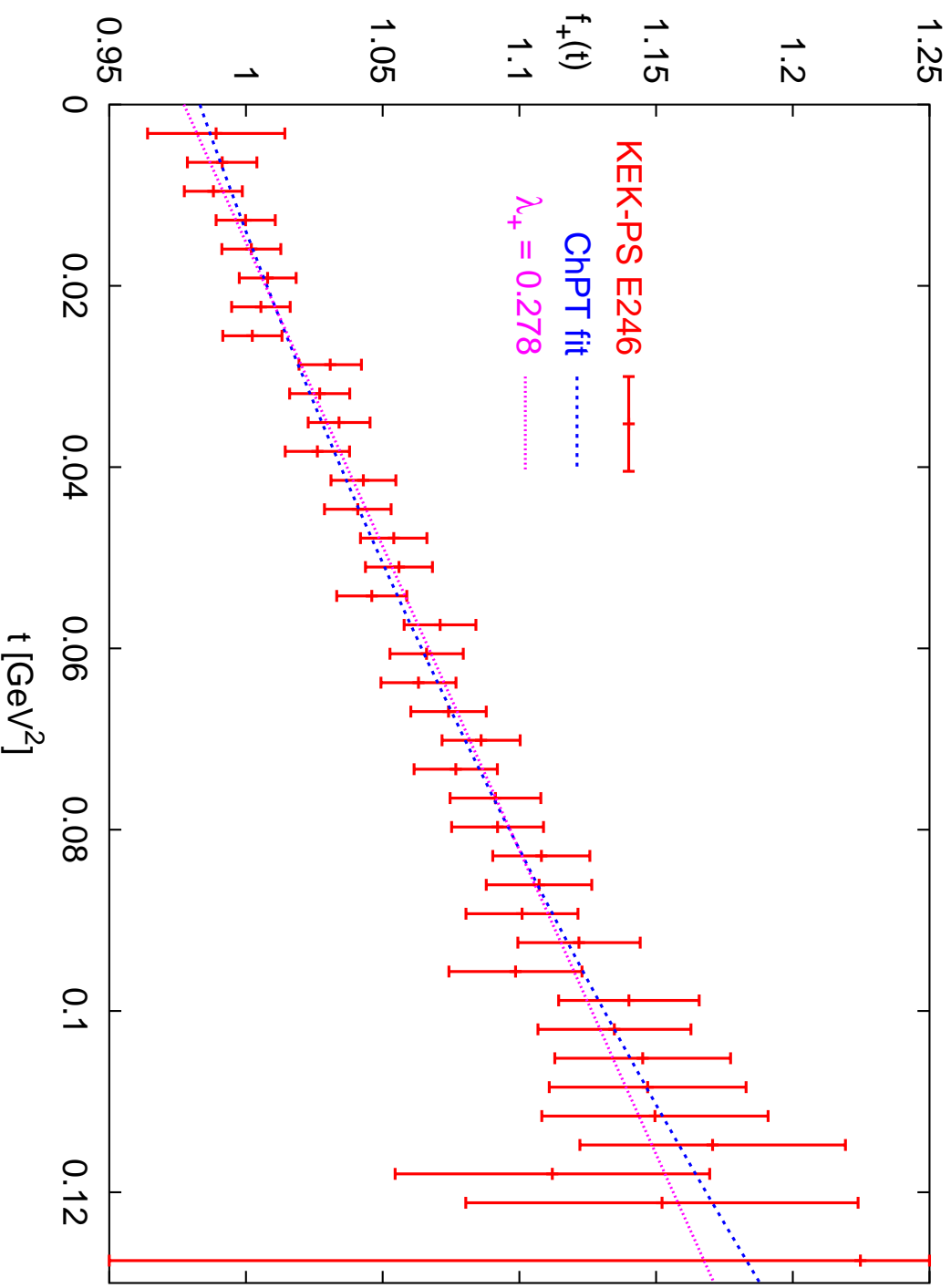
$$\Rightarrow R_{+1}^{K\pi} =$$

$$-2.5 \cdot 10^{-5} \text{ GeV}^2$$

$$(c_+ = 3.2 \text{ GeV}^{-4})$$

$$\Rightarrow a_+ = 1.006$$

$$\Rightarrow \lambda_+ = 0.0214 \pm 0.0018$$



## Values for $\lambda_+$

Process	Ref.	$\lambda_+$	$\lambda_0$
$K_{\mu,3}^+$	PDG2002	$0.033 \pm 0.010$	$0.004 \pm 0.009$
$K_{\ell,3}^+$	PDG2002 $\mu e$	$0.0282 \pm 0.0027$	$0.013 \pm 0.005$
$K_{\mu,3}^0$	PDG2002	$0.033 \pm 0.005$	$0.027 \pm 0.006$
$K_{\ell,3}^0$	PDG2002 $\mu e$	$0.0300 \pm 0.0020$	$0.030 \pm 0.005$
$K_{e,3}^0$	PDG2002	$0.0291 \pm 0.0018$	–
$K_{e,3}^+$	PDG2002	$0.0278 \pm 0.0019$	–
$K_{e,3}^+$	ISTRA	$0.0293 \pm 0.0025$	–
$K_{\mu,3}^+$	ISTRA	$0.0321 \pm 0.0045$	$0.0209 \pm 0.0045$
$K_{e,3}^+$	KEK	$0.0278 \pm 0.0023$	–
$K_{e,3}^0$	KTev	$0.02748 \pm 0.00084$	–

LINEAR

ASSUMED

$\mu e$  means that lepton universality has been used in the measurement.

Ours:  $\lambda_+ = 0.0170 \pm 0.0015$

This value comes from ChPT via

$$\lambda_+ = 0.0283 (p^4) + 0.0011 (\text{loops } p^6) - 0.0124(C_i^r).$$

$$f_0(t)$$

Main Result: 
$$f_0(t) = 1 - \frac{8}{F_\pi^4} (C_{12}^{rr} + C_{34}^{rr}) (m_K^2 - m_\pi^2)^2$$

$$+ 8 \frac{t}{F_\pi^4} (2C_{12}^{rr} + C_{34}^{rr}) (m_K^2 + m_\pi^2) + \frac{t}{m_K^2 - m_\pi^2} (F_K/F_\pi - 1) - \frac{8}{F_\pi^4} t^2 C_{12}^{rr} + \bar{\Delta}(t) + \Delta(0).$$

$\bar{\Delta}(t)$  and  $\Delta(0)$  contain **NO**  $C_i^{rr}$  and only depend on the  $L_i^r$  at order  $p^6$

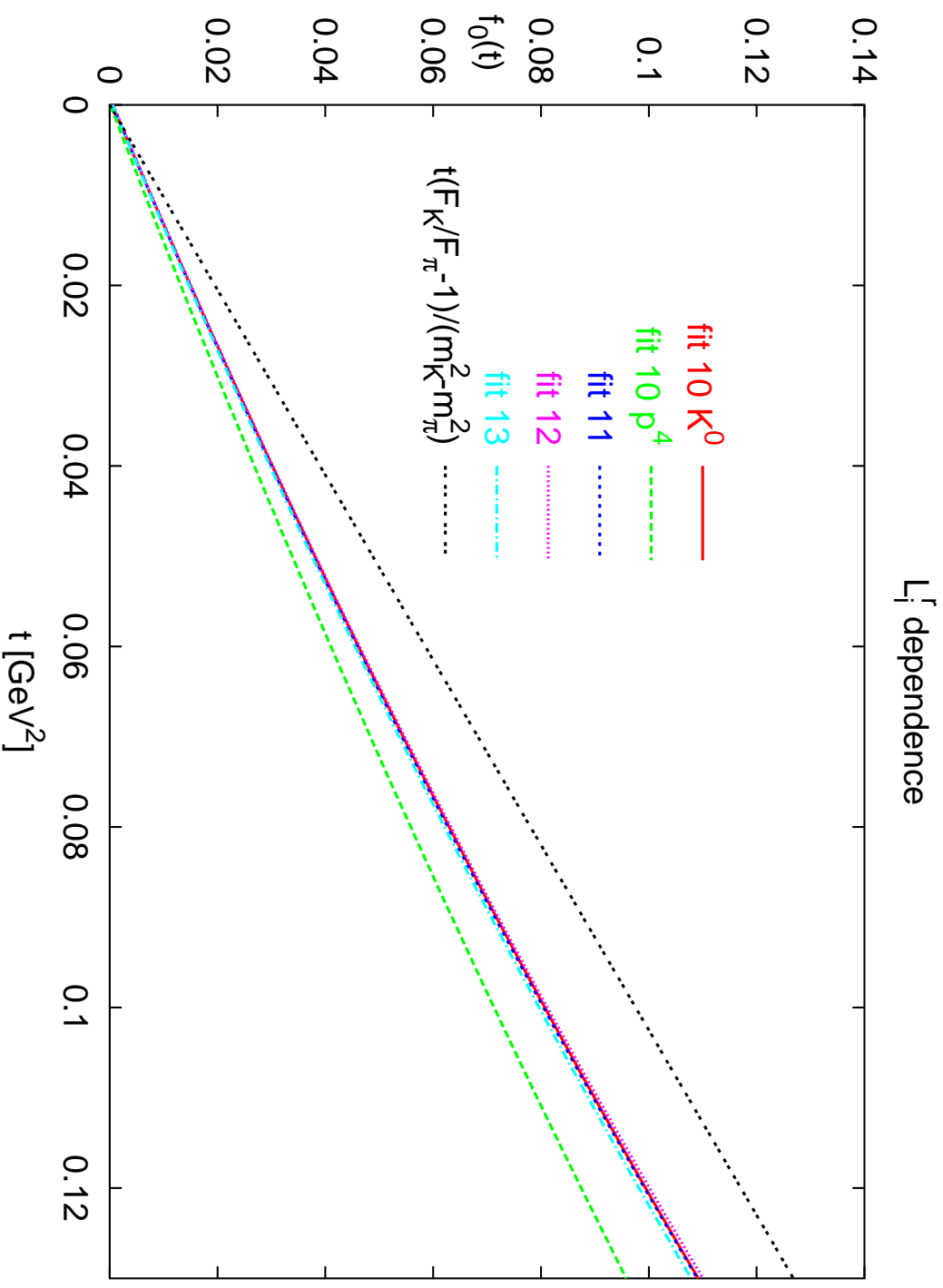
$\implies$

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r].$$

## $\bar{\Delta}(t)$ and $\Delta(0)$

$f_0(t) - f_0(0)$  for  $C_i^r = 0$



	$K_{\ell 3}^0$	$K_{\ell 3}^+$
$p^4$	-0.022266	-0.022275
$p^6$ loops only	0.01130	0.01104
$p^6-L_i$ fit 10	0.00332	0.00320
$p^6-L_i$ fit 11	0.00375	0.00355
$p^6-L_i$ fit 12	0.00216	0.00189
$p^6-L_i$ fit 13	0.00539	0.00526
$p^6-L_i p^4$ fit	0.00891	0.00863

$$\Delta(0) = -0.0080$$

$$\pm 0.0057 \text{ [loops]}$$

$$\pm 0.0028 [L_i^r]$$

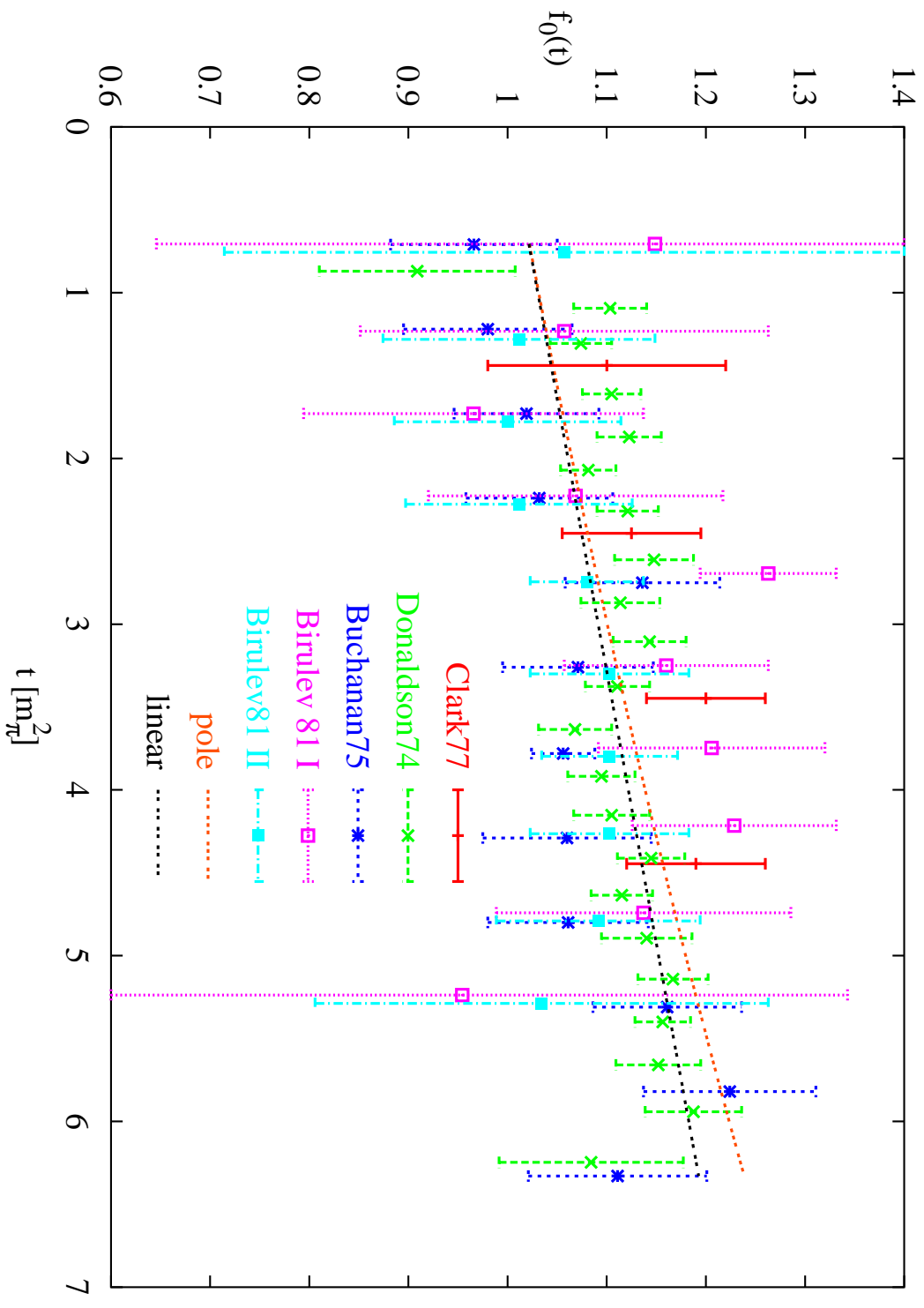
# $f_0(t)$ curvature plus estimates

$$C_{12}^{rr}|_{SMD} = -\frac{F_\pi^4}{8m_S^2} \approx -1.0 \cdot 10^{-5}$$

$$\Lambda_0 = 0.009 \pm 0.010 \implies$$

$$2C_{12}^{rr} + C_{34}^{rr} = (-1 \pm 1.7) \cdot 10^{-5}$$

$$f_+(0)|_{C_i^{rr}} \approx 0.0 \pm 0.1$$





## Conclusions and Work in Progress

- Small numeric disagreement with *Post-Schlicher*: under investigation
- $C_{12}^{rr}$  from curvature in pion scalar formfactors: JB, Dhonte
- Isospin breaking contribution will be known to the same precision JB, Talavera
- Need precision measurements of  $f_+(t)$  and  $f_0(t)$
- Do not neglect curvature in the analysis
- ChPT can predict the needed curvatures