LUND UNIVERSITY

Chiral Dynamics 8-13 Sept 2003

$K_{\ell 3}$ at two loops in ChPT

Johan Bijnens

Lund University

Long project of bringing ChPT calculations for three flavour to the two-loop level — G. Amorós, JB, P. Dhonte, P. Talavera

- Chiral Perturbation Theory
- Two Loop Calculations done
- $K \to \pi \ell \nu$ ($K_{\ell 3}$): Definitions and V_{us}
- $f_+(t)$: linearity, ChPT results, fit to CPLEAR, PS E246 data, λ_+
- $f_0(t)$ Main result: $f_-(t) \Rightarrow f_+(0)$
- Conclusions

(Items 3–5:) ChPT works, an example — JB and P.Talavera, hep-ph/0303103

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

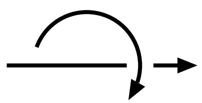
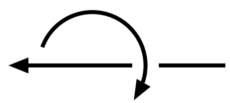
Chiral Symmetry

QCD: 3 light quarks: equal mass can interchange them: $SU(3)_V$

But

$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Can also see that via  $v < c,\ m_q \neq 0 \Longrightarrow$ / $v = c,\ m_q = 0 \not\Longrightarrow$ 

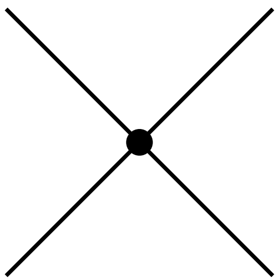
Chiral Perturbation Theory

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$

$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom **and** interaction vanishes at zero momentum

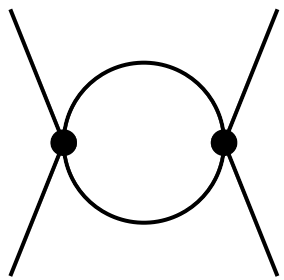
Power counting in momenta:



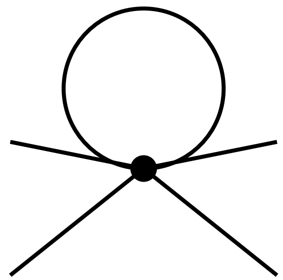
p^2

$1/p^2$

$\int d^4p$ $\quad p^4$



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$



$(p^2)\,(1/p^2)\,p^4 = p^4$

Chiral Perturbation Theory

External currents/masses:

$$m_\pi^2 \sim m_q \langle \bar{q} q \rangle \quad \Longrightarrow \quad m_q \sim p^2$$

$$A_\mu\,,\, W_\mu^\pm \quad \text{in} \quad D_\mu = \partial_\mu - ieA_\mu Q \quad \Longrightarrow \quad A_\mu, W_\mu^\pm \sim p$$

General Papers for two-loops

ChPT basis papers: Gasser Leutwyler 1985

Lagrangians:		
p^2	Long known	two parameters
p^4	Gasser Leutwyler	ten parameters (L_i^r)
p^6	JB, Colangelo, Ecker Fearing, Scherer	90 parameters (C_i^r)

Renormalization and Infinities at p^6: JB, Colangelo, Ecker

Note: At least 3 different renormalization schemes used in actual calculations

Complete Two Loop Calculations Done

Two Flavour

$\gamma\gamma \to \pi^0\pi^0$		Bellucci, Gasser, Sainio
$\gamma\gamma \to \pi^+\pi^-$, F_π, m_π		Bürgi
$\pi\pi$-scattering, F_π, m_π		JB,Colangelo,Ecker,Gasser,Sainio
$F_{V\pi}(t)$, $F_{S\pi}$		JB, Colangelo, Talavera
$\pi \to \ell\nu\gamma$		JB, Talavera

Three Flavour

$\Pi_{VV\pi}$, $\Pi_{VV\eta}$, Π_{VVk}		Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera
$\Pi_{AA\pi}$, $\Pi_{AA\eta}$, F_π, F_η, m_π, m_η		Kambor, Golowich; Amorós, JB, Talavera
Π_{SS}	L_4^r, L_6^r	Moussallam
Π_{VVK}, Π_{AAK}, F_K, m_K		Amorós, JB, Talavera
$K_{\ell 4}$	L_1^r, L_2^r, L_3^r	Amorós, JB, Talavera
F_{π^+}, F_{π^0}, ...,m_{K^+}, m_{K^0} for $m_u \neq m_d$	$L_5^r, L_7^r, L_8^r, m_u/m_d$	Amorós, JB, Talavera
$F_{V\pi}$, F_{VK^+}, F_{VK^0}	L_9^r	Post, Schilcher; JB, Talavera
$K_{\ell 3}$		Post, Schilcher; JB, Talavera
In preparation: $K_{\ell 3}$ for $m_u \neq m_d$		JB, Talavera
In preparation: $F_{S\pi}$, F_{SK}	L_4^r	JB, Dhonte

$K_{\ell 3}$ Definitions

$K^+_{\ell 3}$: $$K^+(p) \rightarrow \pi^0(p')\ell^+(p_\ell)\nu_\ell(p_\nu)$$

$K^0_{\ell 3}$: $$K^0(p) \rightarrow \pi^-(p')\ell^+(p_\ell)\nu_\ell(p_\nu)$$

$K^+_{\ell 3}$:

$$\begin{aligned} T &= \frac{G_F}{\sqrt{2}} V^\star_{us} \ell^\mu F_\mu{}^+(p',p) \\ \ell^\mu &= \bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_\ell) \\ F_\mu{}^+(p',p) &= < \pi^0(p') \mid V_\mu^{4-i5}(0) \mid K^+(p) > \\ &= \frac{1}{\sqrt{2}}[(p'+p)_\mu f_+^{K^+\pi^0}(t) + (p-p')_\mu f_-^{K^+\pi^0}(t)] \end{aligned}$$

$K^0_{\ell 3}$:

$$\begin{aligned} F_\mu{}^0(p',p) &= < \pi^-(p') \mid V_\mu^{4-i5}(0) \mid K^0(p) > \\ &= (p'+p)_\mu f_+^{K^0\pi^-}(t) + (p-p')_\mu f_-^{K^0\pi^-}(t). \end{aligned}$$

Isospin: $f_+^{K^0\pi^-}(t) = f_+^{K^+\pi^0}(t) = f_+(t)$ $\qquad$ $f_-^{K^0\pi^-}(t) = f_-^{K^+\pi^0}(t) = f_-(t)$

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor: $$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization: $$f_{+,0}(t) = f_+(0)\left(1 + \lambda_{+,0}\frac{t}{m_\pi^2}\right)$$

$|V_{us}|$:

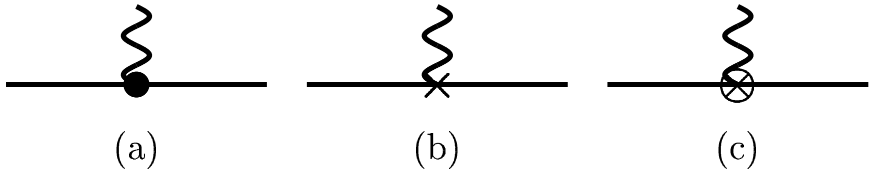
- Know theoretically $f_+(0) = 1 + \cdots$
  - Short distance correction to G_F from G_μ Marciano-Sirlin
  - Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$
  - Isospin Breaking Leuwyler-Roos $\dfrac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.022$ In Progress
- Know experimentally $f_+(0)$
  - Radiative Corrections: use generalized formfactors Cirigliano et al., hep-ph/0110153
  - Parametrize form-factor: is linear enough for $f_+(t)$?

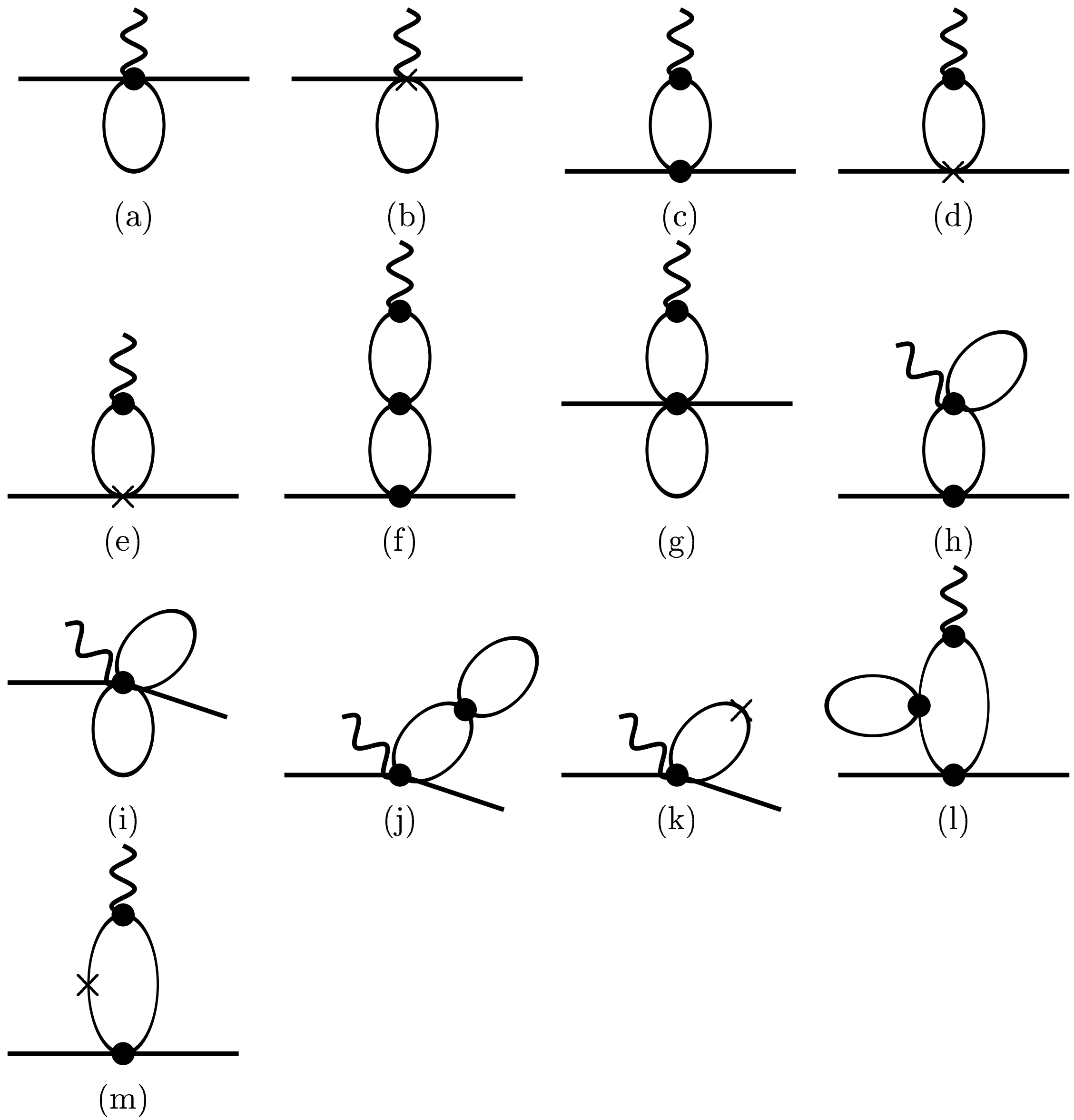
PDG2002: $|V_{ud}| = 0.9734 \pm 0.0008$ $\quad |V_{us}| = 0.2196 \pm 0.0026$

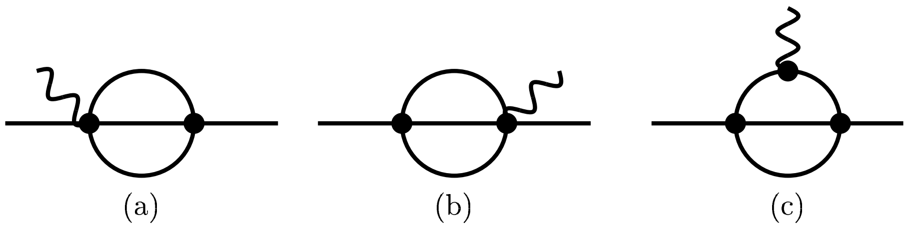
$$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) = 0.9957 \pm 0.0019$$

(V_{ud} from neutron decay only $\Rightarrow$ a bit worse)

$K_{\ell 3}$ Diagrams







● : p^2 vertex

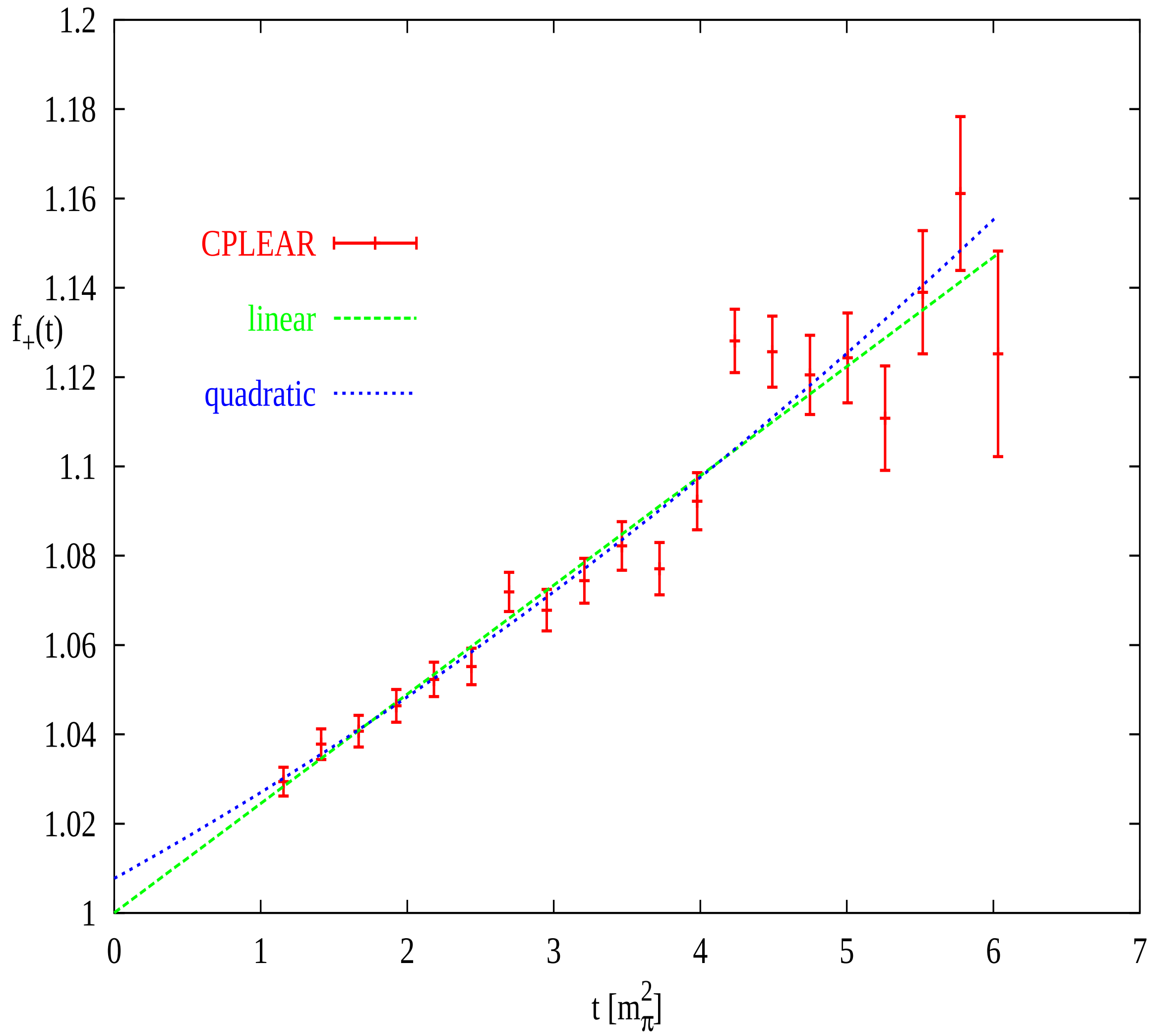
×: p^4 vertex

⊗: p^6 vertex

Is Linear Enough ?

$$f_+(t) = a_+\left(1 + \lambda_+\frac{t}{m^2_{\pi^+}} + c_+ t^2\right)$$

a_+	λ_+	c_+ [GeV^{-4}]
$\equiv 1$	0.0245 ± 0.0006	$\equiv 0$
1.000 ± 0.004	0.0245 ± 0.0015	$\equiv 0$
$\equiv 1$	0.0238 ± 0.0017	0.5 ± 1.2
1.008 ± 0.009	0.0181 ± 0.0068	2.8 ± 2.8



Curvature IS Important at 1% Level!!

$f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4}(-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

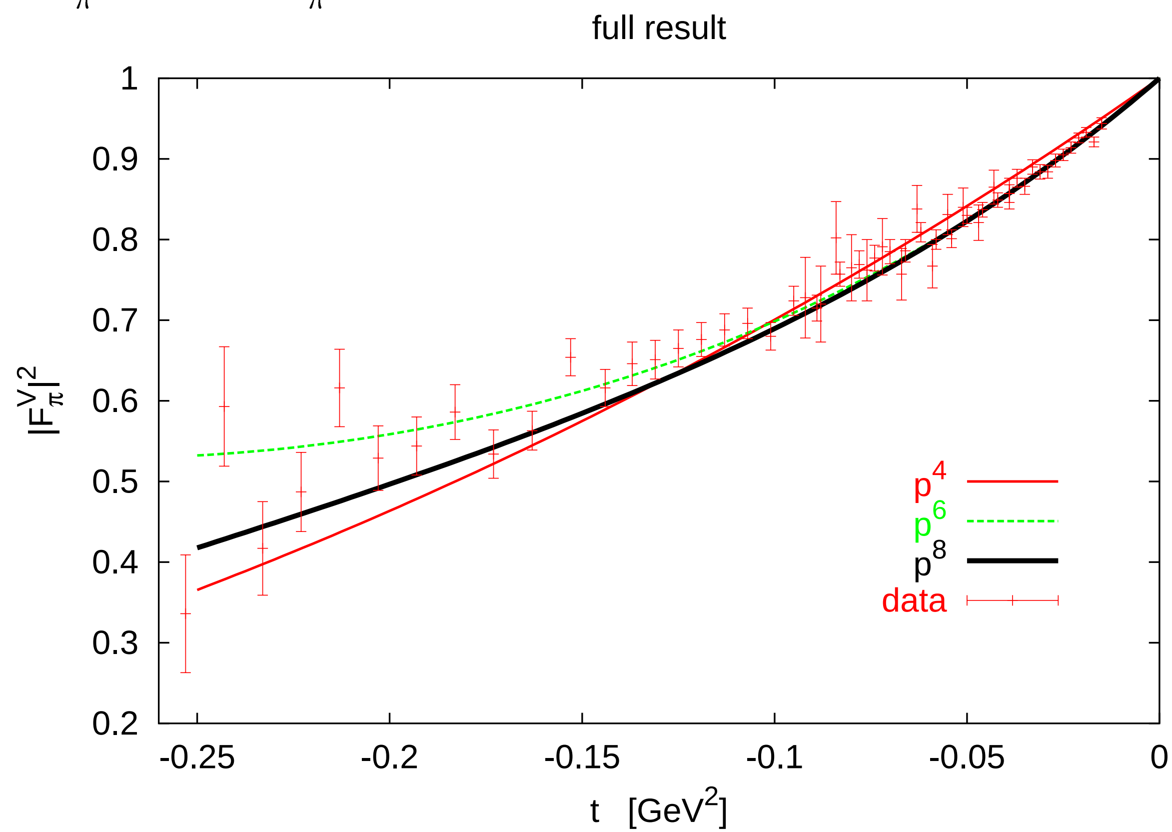
Pion electromagnetic Form factor:

JB, Talavera

$L_9^r = 0.00593 \pm 0.00043$

$-4C_{88}^r + 4C_{90}^r = 0.00022 \pm 0.00002$

VMD: $R_{+1}^{K\pi} \approx -4\ 10^{-5}$ GeV^2



ChPT fit to $f_+(t)$

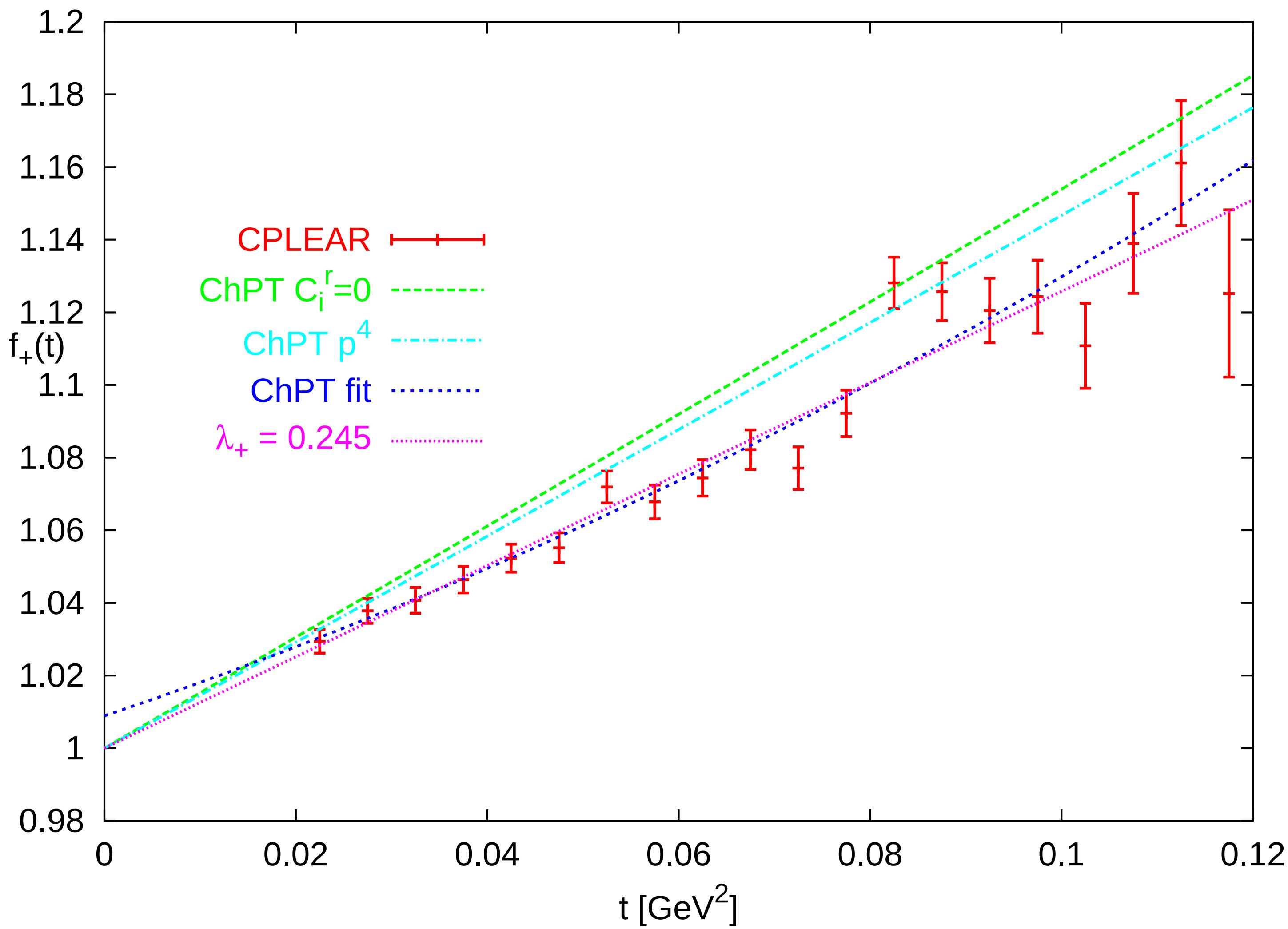
$\Rightarrow R_{+1}^{K\pi} =$

$-(4.7 \pm 0.5)\ 10^{-5}$ GeV^2

$(c_+ = 3.2$ GeV$^{-4})$

$\Rightarrow a_+ = 1.009 \pm 0.004$

$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$



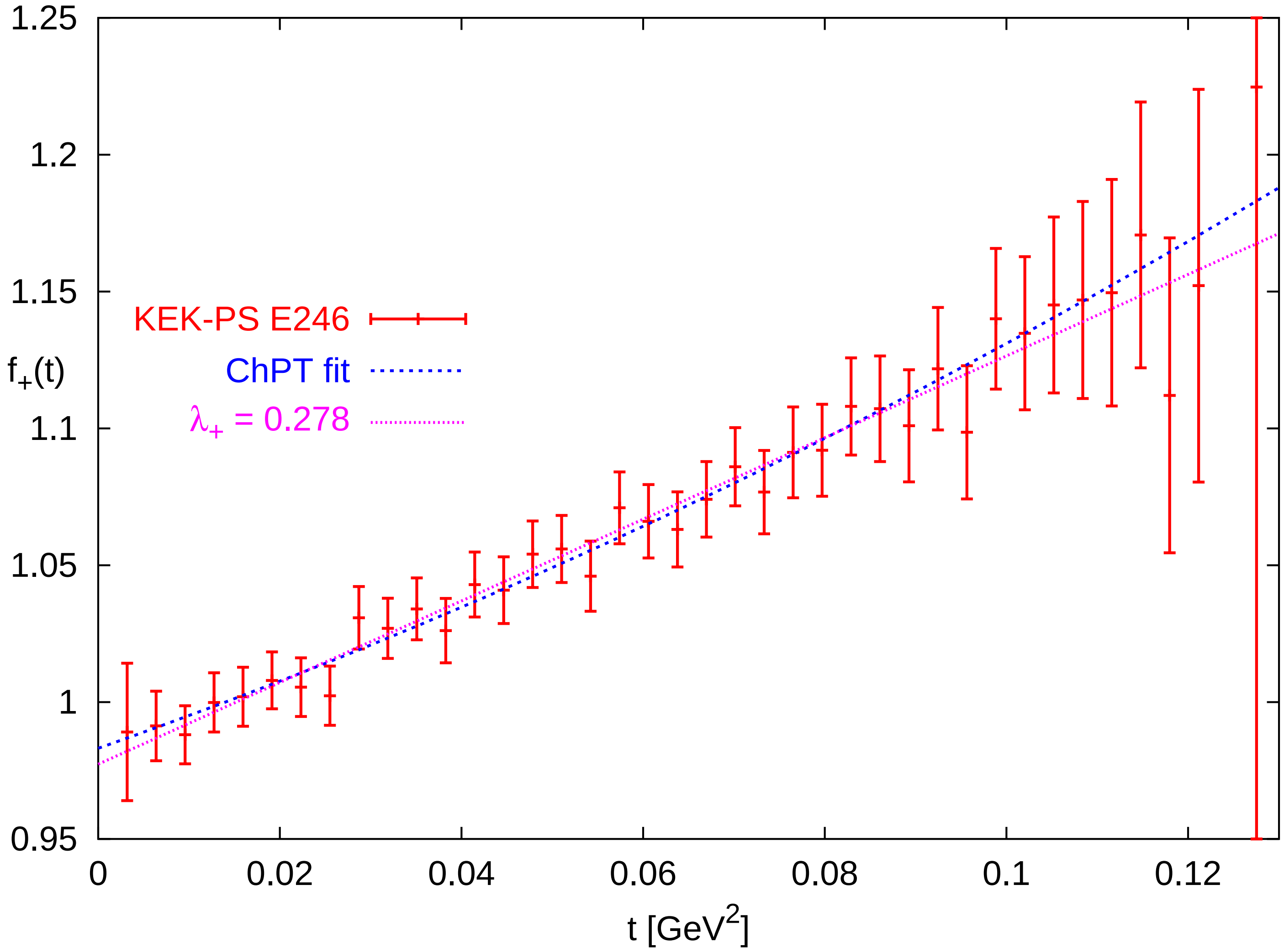
ChPT fit to $f_+(t)$

$\Rightarrow R_{+1}^{K\pi} =$
$-2.5\ 10^{-5}\ \text{GeV}^2$

$\left(c_+ = 3.2\ \text{GeV}^{-4}\right)$

$\Rightarrow a_+ = 1.006$

$\Rightarrow \lambda_+ = 0.0214 \pm 0.0018$



Values for λ_+

Process	Ref.	λ_+	λ_0
$K^+_{\mu3}$	PDG2002	0.033 ± 0.010	0.004 ± 0.009
$K^+_{\ell3}$	PDG2002μe	0.0282 ± 0.0027	0.013 ± 0.005
$K^0_{\mu3}$	PDG2002	0.033 ± 0.005	0.027 ± 0.006
$K^0_{\ell3}$	PDG2002μe	0.0300 ± 0.0020	0.030 ± 0.005
K^0_{e3}	PDG2002	0.0291 ± 0.0018	–
K^+_{e3}	PDG2002	0.0278 ± 0.0019	–
K^+_{e3}	ISTRA	0.0293 ± 0.0025	–
$K^+_{\mu3}$	ISTRA	0.0321 ± 0.0045	0.0209 ± 0.0045
K^+_{e3}	KEK	0.0278 ± 0.0023	–
K^0_{e3}	KTeV	0.02748 ± 0.00084	–

LINEAR

ASSUMED

μe means that lepton universality has been used in the measurement.

Ours: $\lambda_+ = 0.0170 \pm 0.0015$

This value comes from ChPT via

$$\lambda_+ = 0.0283\ (p^4) \quad + 0.0011\ (\text{loops } p^6) \quad - 0.0124(C_i^r)\,.$$

$f_0(t)$

Main Result:

$$
\begin{aligned}
f_0(t) \;=\; & 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 \\
& + 8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}\left(F_K/F_\pi - 1\right) \\
& - \frac{8}{F_\pi^4}t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
\end{aligned}
$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO C_i^r and only depend on the L_i^r at order p^6

$\Longrightarrow$

All needed parameters can be determined experimentally

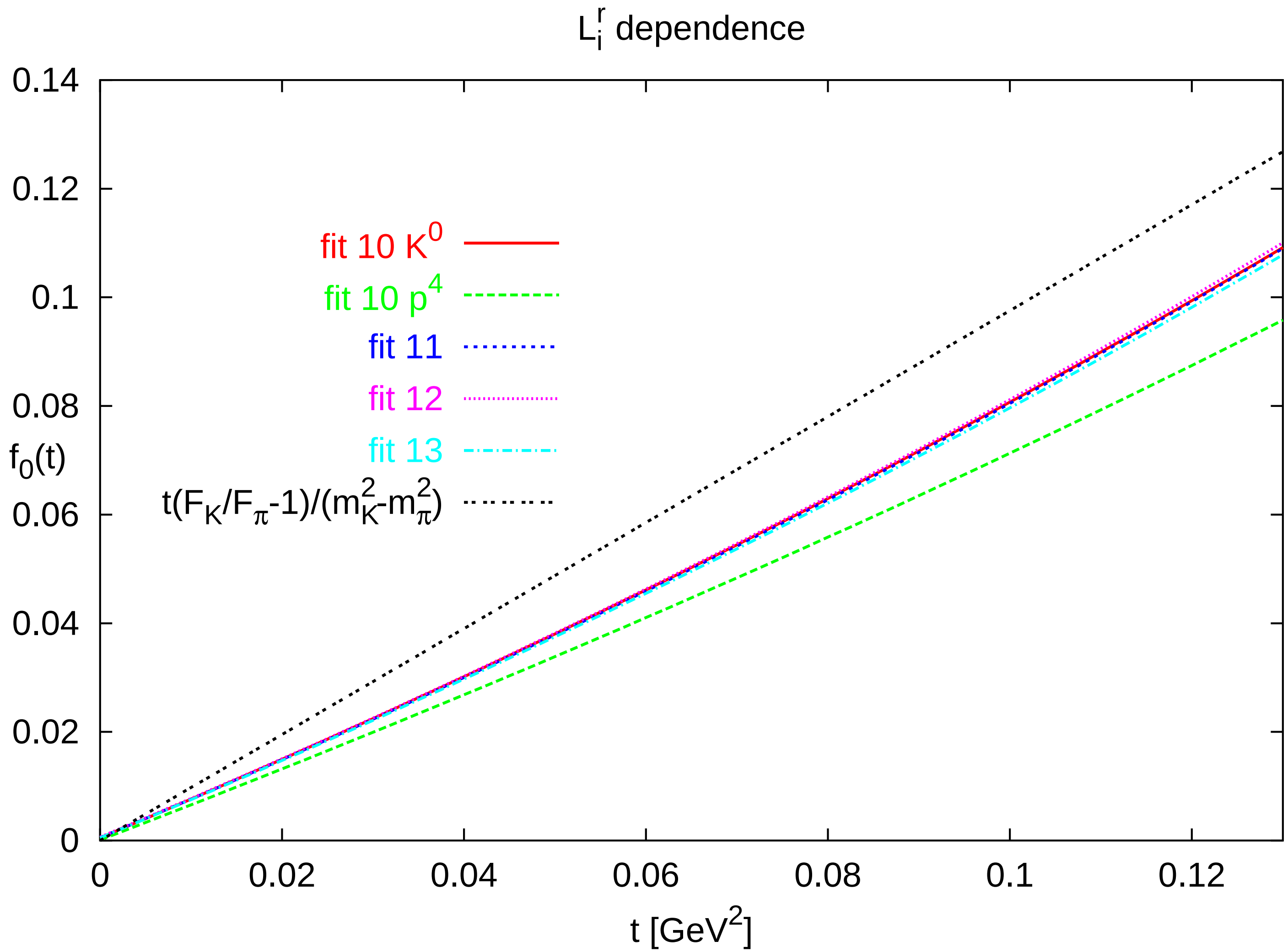
$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r]\,.$$

$\overline{\Delta}(t)$ and $\Delta(0)$

$f_0(t) - f_0(0)$ for $C_i^r = 0$

	$K^0_{\ell 3}$	$K^+_{\ell 3}$
p^4	-0.02266	-0.02275
p^6 loops only	0.01130	0.01104
p^6-L_i fit 10	0.00332	0.00320
p^6-L_i fit 11	0.00375	0.00355
p^6-L_i fit 12	0.00216	0.00189
p^6-L_i fit 13	0.00539	0.00526
p^6-L_i p^4 fit	0.00891	0.00863

$\Delta(0) = -0.0080$
± 0.0057 [loops]
± 0.0028 $[L_i^r]$

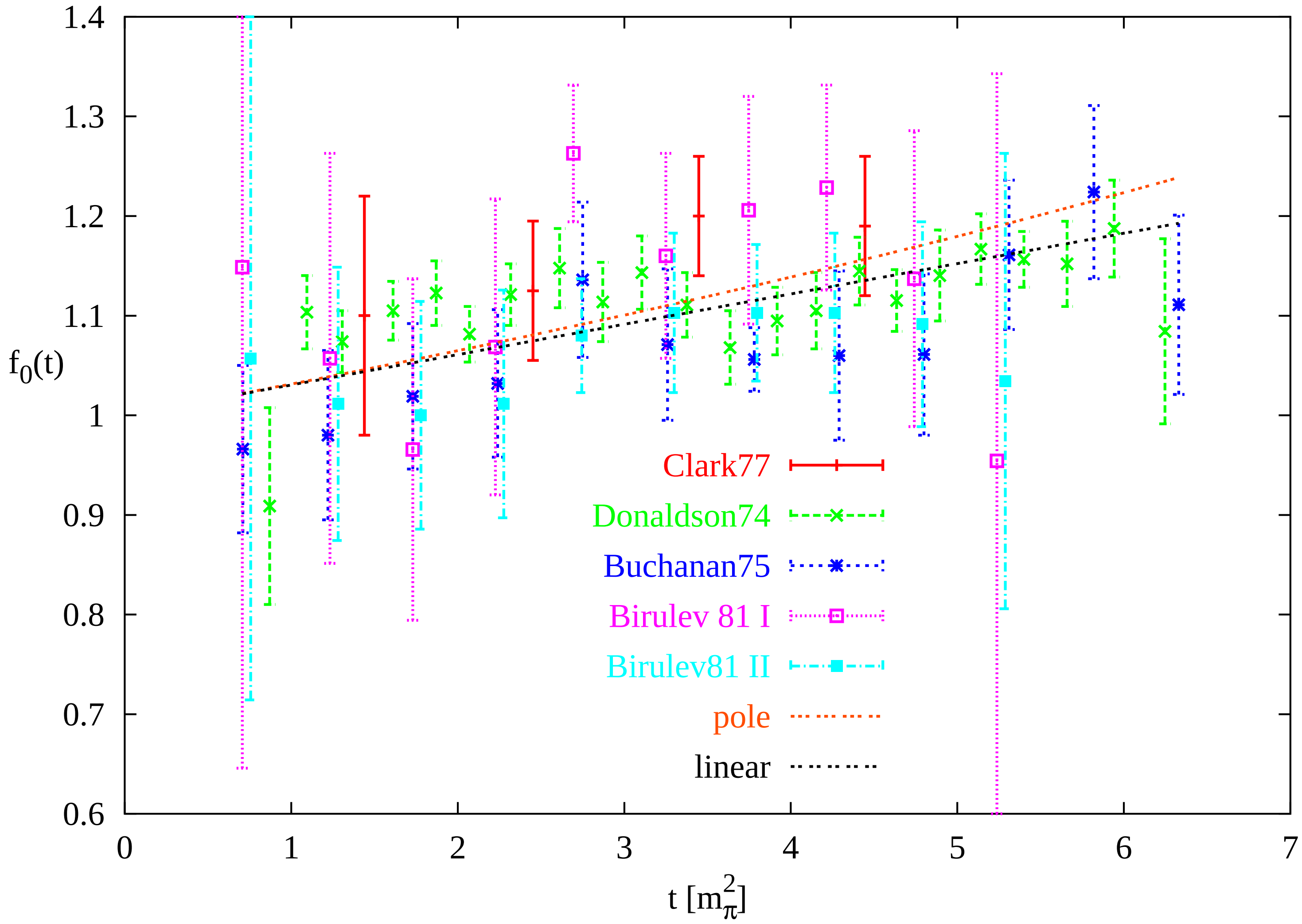


$f_0(t)$ curvature plus estimates

$$C^r_{12}|_{SMD} = -\frac{F_\pi^4}{8m_S^2} \approx -1.0\ 10^{-5}$$

$$\lambda_0 = 0.009 \pm 0.010 \implies$$
$$2C^r_{12} + C^r_{34} = (-1 \pm 1.7)\ 10^{-5}$$

$$f_+(0)|_{C_i^r} \approx 0.0 \pm 0.1$$



Conclusions and Work in Progress

- Small numeric disagreement with Post-Schilcher: under investigation
- C_{12}^r from curvature in pion scalar formfactors: JB, Dhonte
- Isospin breaking contribution will be known to the same precision JB, Talavera
- Need precision measurements of $f_+(t)$ and $f_0(t)$
- Do not neglect curvature in the analysis
- ChPT can predict the needed curvatures