LUND UNIVERSITY

Chiral Dynamics 8-13 Sept 2003

$K \to 3\pi$ in Chiral Perturbation Theory

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Work done with with Fredrik Persson (now Fredrik Borg) and Pierre Dhonte

Nucl. Phys. B648 (2003) 317-344 [hep-ph/0205341] and work in progress

- ChPT in the nonleptonic mesonic sector
- Lagrangians
- $K \to 3\pi$ Kinematics and Isospin
- Results
- Data and Fits
- Status of the Isospin Breaking Calculations
- Preliminary Numerical Results
- Conclusions

ChPT in the nonleptonic sector

- some earlier work exists: espescially on decays with photons
- Kambor, Missimer, Wyler (KMW) : Constructed the Lagrangian and infinities 1990
- G. Esposito-Farese: Checked Lagrangian and infinities 1991
- KMW : Calculated $K \to 2\pi$ and $K \to 3\pi$ 1991
- Donoghue + Holstein + KMW : clarified the relations between observables 1992
- BUT: explicit formulas never published and were lost (US Mail)
- Kambor, Ecker, Wyler : simplified Lagrangian for octet part 1993
- $K \to 2\pi$ redone: Bijnens, Pallante, Prades 1998
- This work : redo $K \to 3\pi$
- Ecker Isidori Muller Neufeld Pich: Electromagnetic octet Lagrangian plus infinities 2000
- applications to $K \to 2\pi$ (previous talk)
- Isospin breaking in $K \to 3\pi$: work in progress, preliminary results here.

Lagrangians: p^2 and e^2

$$\mathcal{L}_2 = \mathcal{L}_{S2} + \mathcal{L}_{W2} + \mathcal{L}_{E2} \qquad \mathcal{L}_{S2} = \frac{F_0^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$

$$u_\mu = i u^\dagger D_\mu U\, u^\dagger = u_\mu^\dagger\,, \quad u^2 = U\,, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u\,,$$

U contains the Goldstone boson fields

$$U = \exp\left(\frac{i\sqrt{2}}{F_0}M\right)\,, \qquad M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K^0} & \frac{-2}{\sqrt{6}}\eta_8 \end{pmatrix}.$$

Here $\quad \chi = 2B_0 \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$ and $D_\mu U = \partial_\mu U.$

Lagrangians: p^2 and e^2

$$\mathcal{L}_{W2} = C\, F_0^4 \left[G_8 \langle \Delta_{32} u_\mu u^\mu \rangle + G_8' \langle \Delta_{32} \chi_+ \rangle + G_{27} t^{ij,kl} \langle \Delta_{ij} u_\mu \rangle \langle \Delta_{kl} u^\mu \rangle \right] + \text{ h.c.}\,.$$

$$t^{21,13} = t^{13,21} = \frac{1}{3}\,; \quad t^{22,23} = t^{23,22} = -\frac{1}{6}\,; \quad t^{23,33} = t^{33,23} = -\frac{1}{6}\,; \quad t^{23,11} = t^{11,23} = \frac{1}{3}\,,$$

$$\Delta_{ij} \equiv u \lambda_{ij} u^\dagger\,, \quad (\lambda_{ij})_{ab} \equiv \delta_{ia}\,\delta_{jb}\,.$$

C: in the chiral and large N_c limits $G_8 = G_{27} = 1$: $\qquad C = -\dfrac{3}{5}\dfrac{G_F}{\sqrt{2}} V_{ud} V_{us}^*\,.$

Electromagnetic part: $\mathcal{L}_{E2} = e^2 F_0^4 Z \langle \mathcal{Q}_L \mathcal{Q}_R \rangle + \left(e^2 C F_0^6 G_E \langle \Delta_{32} \mathcal{Q}_R \rangle + \text{h.c.} \right)$

$$\mathcal{Q}_L = u Q u^\dagger\,, \quad \mathcal{Q}_R = u^\dagger Q u \ \text{ with } \ Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}$$

Note: F_0 not that well known, fit $C F_0^4 G_8, \ldots$ use *numerics* $F_0 = F_\pi$ to quote $G_8, \ldots$

Lagrangians: p^4 and e^2p^2

$$\mathcal{L}_4 = \mathcal{L}_{S4} + \mathcal{L}_{W4} + \mathcal{L}_{S2E2} + \mathcal{L}_{W2E2}(G8)\,.$$

$\mathcal{L}_{S4}$ values of L_i^r taken from standard Amoros, Bijnens, Talavera p^4 fit

$\mathcal{L}_{W4}$ thirteen N_i^r (octet) and twelve D_i^r might contribute

$\mathcal{L}_{S2E2}$ eleven K_i^r can contribute

$\mathcal{L}_{W2E2}$ fourteen Z_i^r can contribute, 27 part not classified

Done here :

G_8, G_{27} and N_i^r, D_i^r fit from isospin conserving calculation
Preliminary results from isospin breaking with $K_i^r = Z_i^r = 0$

$K \longrightarrow 3\pi$ Kinematics and Isospin

$$
\begin{aligned}
K_L(k) &\longrightarrow \pi^0(p_1)\pi^0(p_2)\pi^0(p_3)\,, \quad [A^L_{000}]\,,\\
K_L(k) &\longrightarrow \pi^+(p_1)\pi^-(p_2)\pi^0(p_3)\,, \quad [A^L_{+-0}]\,,\\
K_S(k) &\longrightarrow \pi^+(p_1)\pi^-(p_2)\pi^0(p_3)\,, \quad [A^S_{+-0}]\,,\\
K^+(k) &\longrightarrow \pi^0(p_1)\pi^0(p_2)\pi^+(p_3)\,, \quad [A_{00+}]\,,\\
K^+(k) &\longrightarrow \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\,, \quad [A_{++-}]\,,
\end{aligned}
$$

plus charge conjugate ones

symmetric under $p_1 \leftrightarrow p_2$ except A^S_{+-0} which is odd (CP and/or Bose symmetry)

Kinematics : $s_1 = (k-p_1)^2\,, \quad s_2 = (k-p_2)^2\,, \quad s_3 = (k-p_3)^2\,.$

Dalitzplotvariables : $y = \frac{s_3 - s_0}{m^2_{\pi^+}}\,, \quad x = \frac{s_2 - s_1}{m^2_{\pi^+}}\,, \quad s_0 = \frac{1}{3}(s_1+s_2+s_3)\,.$

Valid also for p^6 expression:

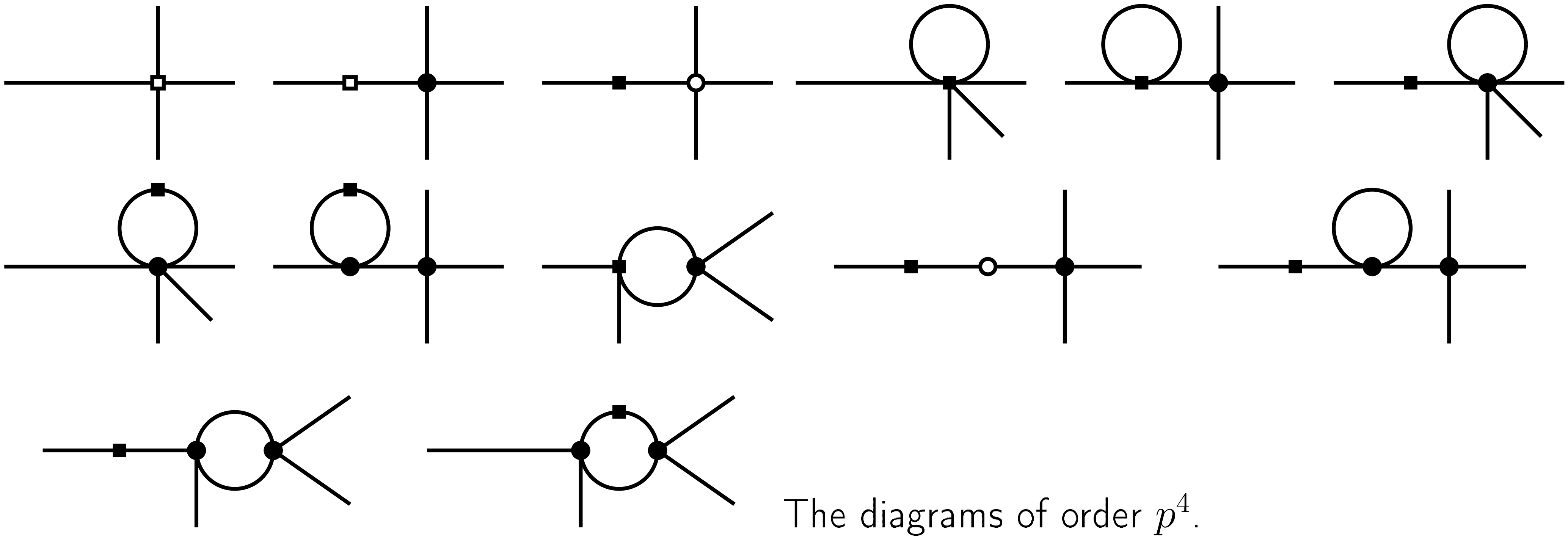
$$\begin{aligned}
A^L_{000}(s_1,s_2,s_3) &= M_0(s_1)+M_0(s_2)+M_0(s_3)\,,\\
A^L_{+-0}(s_1,s_2,s_3) &= M_1(s_3)+M_2(s_1)+M_2(s_2)+M_3(s_1)(s_2-s_3)+M_3(s_2)(s_1-s_3)\,,\\
A^S_{+-0}(s_1,s_2,s_3) &= M_4(s_1)-M_4(s_2)+M_5(s_1)(s_2-s_3)-M_5(s_2)(s_1-s_3)+M_6(s_3)(s_1-s_2)\,,\\
A_{00+}(s_1,s_2,s_3) &= M_7(s_3)+M_8(s_1)+M_8(s_2)+M_9(s_1)(s_2-s_3)+M_9(s_2)(s_1-s_3)\,,\\
A_{++-}(s_1,s_2,s_3) &= M_{10}(s_3)+M_{11}(s_1)+M_{11}(s_2)+M_{12}(s_1)(s_2-s_3)+M_{12}(s_2)(s_1-s_3)\,.
\end{aligned}$$

up to polynomial ambiguity from $s_1+s_2+s_3=\sum_i m_i^2$

Isospin:

$$\begin{aligned}
M_0(s) &= M_1(s)+2M_2(s)\,,\\
2M_7(s)+4M_8(s) &= M_{10}(s)+2M_{11}(s)\,,\\
M_4(s) &= \frac{1}{3}\left(M_7(s)-M_8(s)+M_{10}(s)-M_{11}(s)\right)\\
M_5(s)-M_6(s) &= M_9(s)+M_{12}(s)\,.
\end{aligned}$$

Calculations: isospin limit



The diagrams of order p^4.

Expressions for the $M_i(s)$: see paper

Confirmed by Gamiz, Prades, Scimemi, hep-ph/0305164 (and in progress)
Lashin, hep-ph/0308200 (numerical or analytical not clear)

Only eleven combinations of N_i^r and D_i^r appear: $\tilde{K}_i$

Measurements and amplitude expansion

Inputs

$\left|\frac{A(s_1,s_2,s_3)}{A(s_0,s_0,s_0)}\right|^2 = 1 + gy + hy^2 + kx^2$ for $K_L \to \pi^0\pi^0\pi^0$ $g=0$ and $k=h/3$.

for $K_S \to \pi^+\pi^-\pi^0$ $\lambda = \frac{\int_{y_{min}}^{y_{max}} dy \int_0^{x_{lim}(y)} dx A^{L*}_{+-0} A^S_{+-0}}{\int_{y_{min}}^{y_{max}} dy \int_0^{x_{lim}(y)} dx A^{L*}_{+-0} A^L_{+-0}}$ $A^S_{+-0} = \gamma_S x - \xi_S xy$ all decay rates

Analyze in amplitiude expansion:

$$
\begin{aligned}
A^L_{000} &= 3(\alpha_1+\alpha_3) + 3(\zeta_1 - 2\zeta_3)\left(y^2 + \frac{1}{3}x^2\right)\,,\\
A^L_{+-0} &= (\alpha_1+\alpha_3) - (\beta_1+\beta_3)y + (\zeta_1 - 2\zeta_3)\left(y^2 + \frac{1}{3}x^2\right) + (\xi_1 - 2\xi_3)\left(y^2 - \frac{1}{3}x^2\right)\,,\\
A^S_{+-0} &= \frac{2}{3}\sqrt{3}\,\gamma_3 x - \frac{4}{3}\xi'_3 xy\,,\\
A_{00+} &= \left(-\alpha_1 + \frac{1}{2}\alpha_3\right) + \left(\beta_1 - \frac{1}{2}\beta_3 - \sqrt{3}\gamma_3\right)y + (-\zeta_1 - \zeta_3)\left(y^2 + \frac{1}{3}x^2\right) + (-\xi_1 - \xi_3 - \xi'_3)\left(y^2 - \frac{1}{3}x^2\right)\,,\\
A_{++-} &= (-2\alpha_1 + \alpha_3) + \left(-\beta_1 + \frac{1}{2}\beta_3 - \sqrt{3}\gamma_3\right)y + (-2\zeta_1 - 2\zeta_3)\left(y^2 + \frac{1}{3}x^2\right) + (\xi_1 + \xi_3 - \xi'_3)\left(y^2 - \frac{1}{3}x^2\right)\,.
\end{aligned}
$$

Fit of the Amplitude Expansion

quantity	Devlin-Dickey	KMW	Our fit	p^2	$\tilde{K}_i = 0$
$\lvert A_0 \rvert$	0.4687 ± 0.0006	0.4699 ± 0.0012	0.4622 ± 0.0014	input	input
$\lvert A_2 \rvert$	0.0210 ± 0.0001	0.0211 ± 0.0001	0.0212 ± 0.0001	input	input
$\delta_2 - \delta_0$	$(-45.6 \pm 5)^o$	$(-61.5 \pm 4)^o$	$(-58.2 \pm 4)^o$	—	—
α_1	91.4 ± 0.24	91.71 ± 0.32	93.16 ± 0.36	74.0(73.5)	59.4
α_3	-7.14 ± 0.36	-7.36 ± 0.47	-6.72 ± 0.46	$-4.8(4.8)$	-6.5
β_1	-25.83 ± 0.41	-25.68 ± 0.27	-27.06 ± 0.43	$-17.7(16.2)$	-21.9
β_3	-2.48 ± 0.48	-2.43 ± 0.41	-2.22 ± 0.47	$-1.2(1.1)$	-1.0
γ_3	2.51 ± 0.36	2.26 ± 0.23	2.95 ± 0.32	2.3(2.1)	2.5
ζ_1	-0.37 ± 0.11	-0.47 ± 0.15	-0.40 ± 0.19	—	0.26
ζ_3	—	-0.21 ± 0.08	-0.09 ± 0.10	—	-0.01
ξ_1	-1.25 ± 0.12	-1.51 ± 0.30	-1.83 ± 0.30	—	-0.46
ξ_3	—	-0.12 ± 0.17	-0.17 ± 0.16	—	-0.01
ξ'_3	—	-0.21 ± 0.51	-0.56 ± 0.42	—	-0.06
χ^2/DOF	$12.8/3$	$10.3/2$	$5.4/5$	—	—

The χ^2 quoted are for the fits with the then used data. $|A_0|$ and $|A_2|$ are in units of 10^{-6} GeV. $\alpha_1,\ldots,\xi'_3$ are in units of 10^{-8}. Note p^2 and p^4 predictions

Notes and ChPT fit

- New data on K_S branching ratios
- New data on some of the parameters (see other talks)
- The averages have often large scale factors in the PDG averages
- There is no sign of isospin breaking, the fit works very well

Fitted	$\|A_0\|,\ldots,\xi_3'$				experiment
G_8	5.47(2)	5.47(2)	7.24(2.04)	5.49	5.45(2)
G_{27}	0.392(2)	0.392(2)	0.392(2)	0.139	0.392(2)
$10^3\tilde K_1/G_8$	$\equiv 0$	$\equiv 0$	−8.5(7.5)	$\equiv 0$	$\equiv 0$
$10^3\tilde K_2/G_8$	54.7(2.8)	53.6(2.7)	41.3(11.4)	53.7	51.9(3.2)
$10^3\tilde K_3/G_8$	3.0(1.4)	3.5(1.3)	10.0(6.3)	3.2	3.8(1.5)
$10^3\tilde K_4/G_{27}$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 100$	$\equiv 0$
$10^3\tilde K_5/G_{27}$	−54.5(23.4)	−19.8(9.6)	−54.5(23.1)	−49.4	−42.5(16.6)
$10^3\tilde K_6/G_{27}$	−185(114)	$\equiv 0$	−184(113)	−366	−166(113)
$10^3\tilde K_7/G_{27}$	114(46)	45.1(18.2)	114(46)	199	120(32)
χ^2/DOF	12.3/5	14.9/6	11.8/4	11.9/4	26.8/10

In brackets are the MINUIT errors.

$\tilde K_8 = \cdots = \tilde K_{11} = 0$ suppressed by m_π^2/m_K^2

γ_3 via large $\tilde K_6$ source of large numbers

dependence on $\tilde K_1$ and $\tilde K_4$

ChPT fit continued

Decay	Quantity		S	Ref.	CHPT fit
$K_L \to \pi^0\pi^0\pi^0$	h	-0.0033 ± 0.0013		E731	
	h	-0.0061 ± 0.0010		NA48	
	h	-0.00506 ± 0.00135	1.7	average	-0.0072
$K_L \to \pi^+\pi^-\pi^0$	g	0.678 ± 0.008	1.5	CPLEAR,PDG	0.677
	h	0.076 ± 0.006		CPLEAR,PDG	0.085
	k	0.0099 ± 0.0015		CPLEAR,PDG	0.0055
$K_S \to \pi^+\pi^-\pi^0$	Re(λ)	0.0316 ± 0.0062		CPLEAR,Zou	0.0359
	Im(λ)	-0.0088 ± 0.0068		CPLEAR,Zou	-0.003
	γ_S	$(3.3 \pm 0.5) \cdot 10^{-8}$		CPLEAR	$3.4 \cdot 10^{-8}$
	ξ_S	$(0.4 \pm 0.8) \cdot 10^{-8}$		CPLEAR	$-0.2 \cdot 10^{-8}$
$K^\pm \to \pi^0\pi^0\pi^\pm$	g	0.652 ± 0.031	2.7	Batusov,Bolotov,PDG	0.638
	h	0.057 ± 0.018	1.4	Batusov,Bolotov,PDG	0.074
	k	0.0197 ± 0.0054		Batusov,PDG	0.0045
$K^+ \to \pi^+\pi^+\pi^-$	g	-0.2154 ± 0.0035	1.4	PDG	-0.216
	h	0.012 ± 0.008	1.4	PDG	0.012
	k	-0.0101 ± 0.0034	2.1	PDG	-0.0052
$K^- \to \pi^-\pi^-\pi^+$	g	-0.217 ± 0.007	2.5	PDG	
	h	0.010 ± 0.006		PDG	
	k	-0.0084 ± 0.0019	2.1	PDG	

S: PDG Scale factor

PDG 2000 edition

Note: fit well but curvatures might be a problem

- Data ?
- Higher order ChPT ?
- Isospin Breaking ?
- Radiative Corrections ?

Isospin Breaking Calculation: Status

work with Fredrik Borg

Reproduced known results on decay constants and masses, to order p^4, $(m_u - m_d)p^2$ and e^2p^2 extended to all orders in $m_u - m_d$ (numerically irrelevant)

Finished $K \to 3\pi$ amplitudes to order p^4, $(m_u - m_d)p^2$ and e^2p^2 excluding photon loops also have all orders in $m_u - m_d$ (numerically irrelevant)

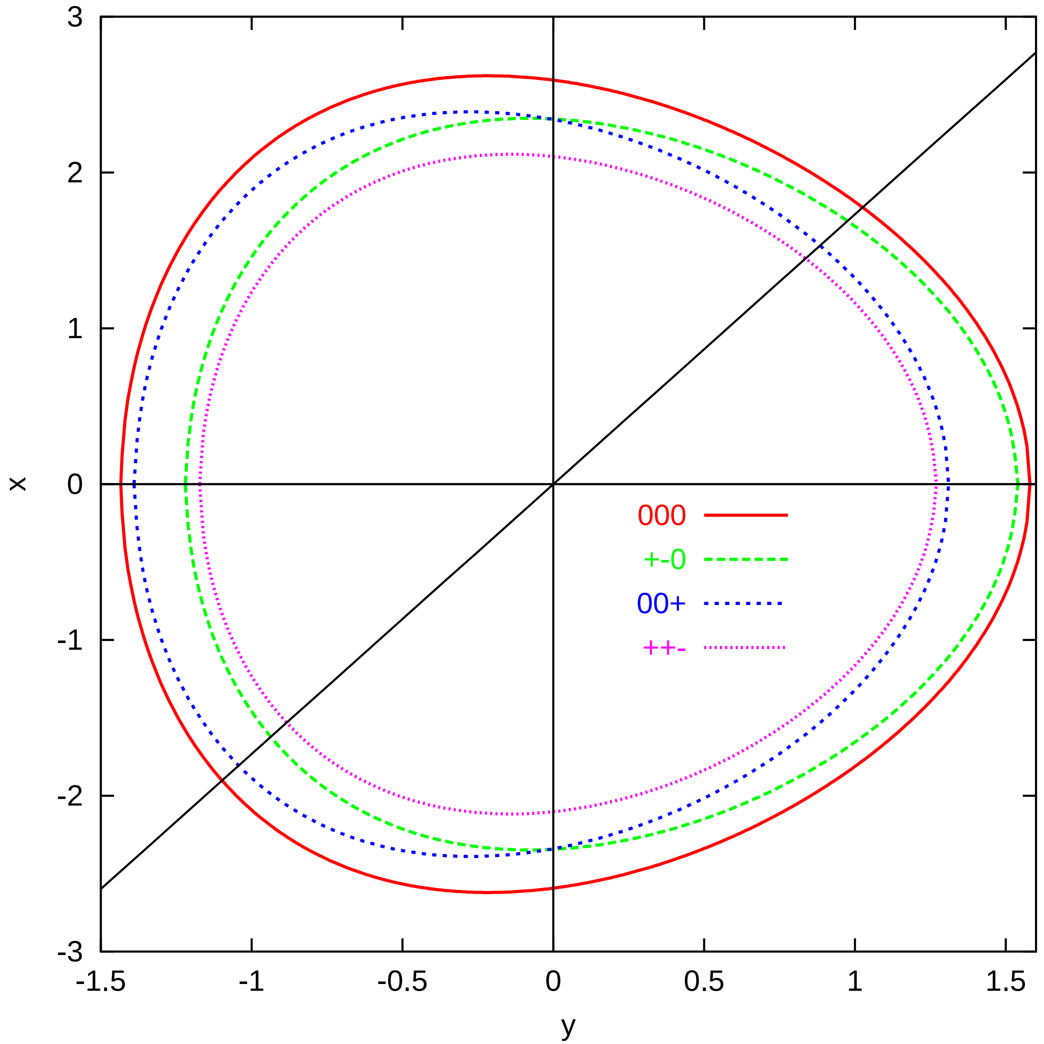
Infinities cancel for the parts where subtraction is known. (e^2G_{27} not known)

Preliminary numerical studies in progress

Photon Loops

Diagrams programmed and infinities cancel, infrared and collinear divergence treatment under study.

Isospin Breaking Calculation: Preliminary Numerics

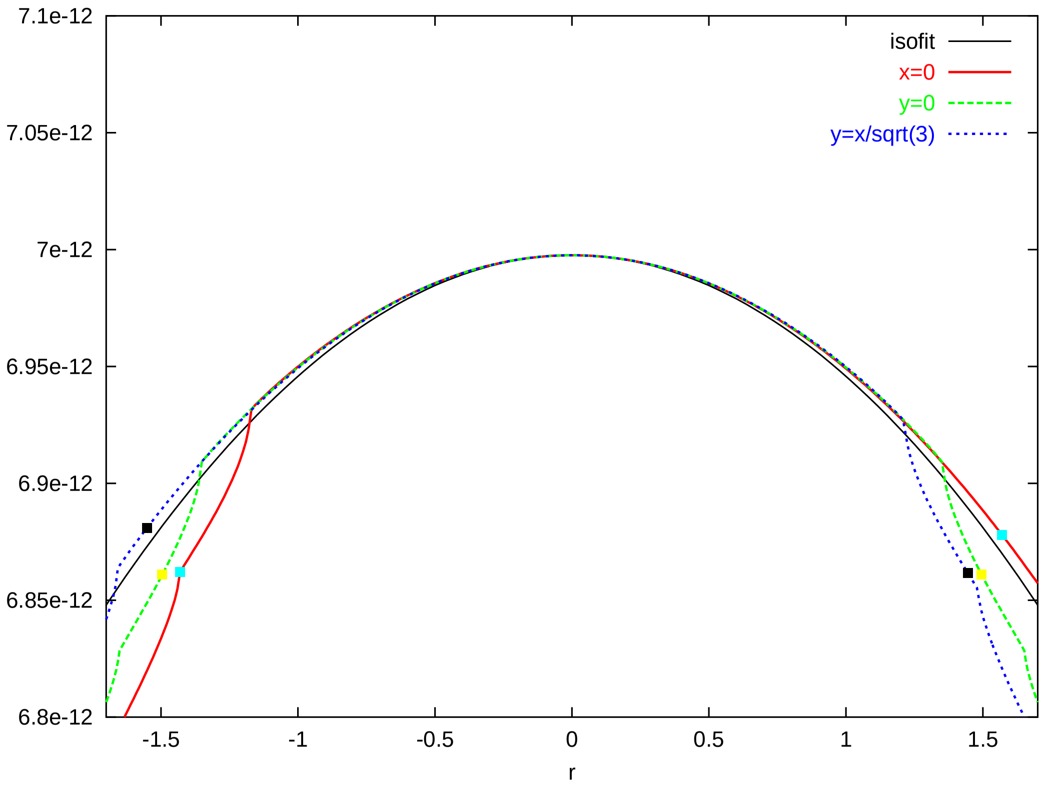


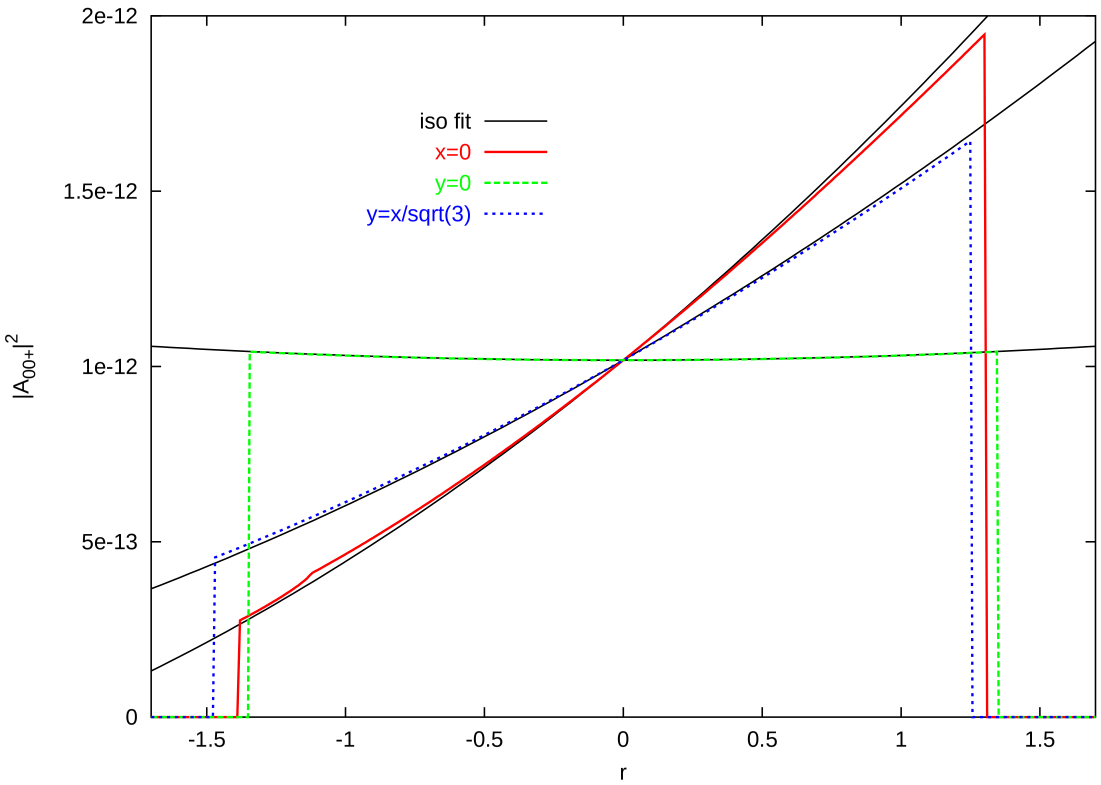
Isospin Breaking in phase space:
already included, very important

In amplitudes: estimate by varying masses: small

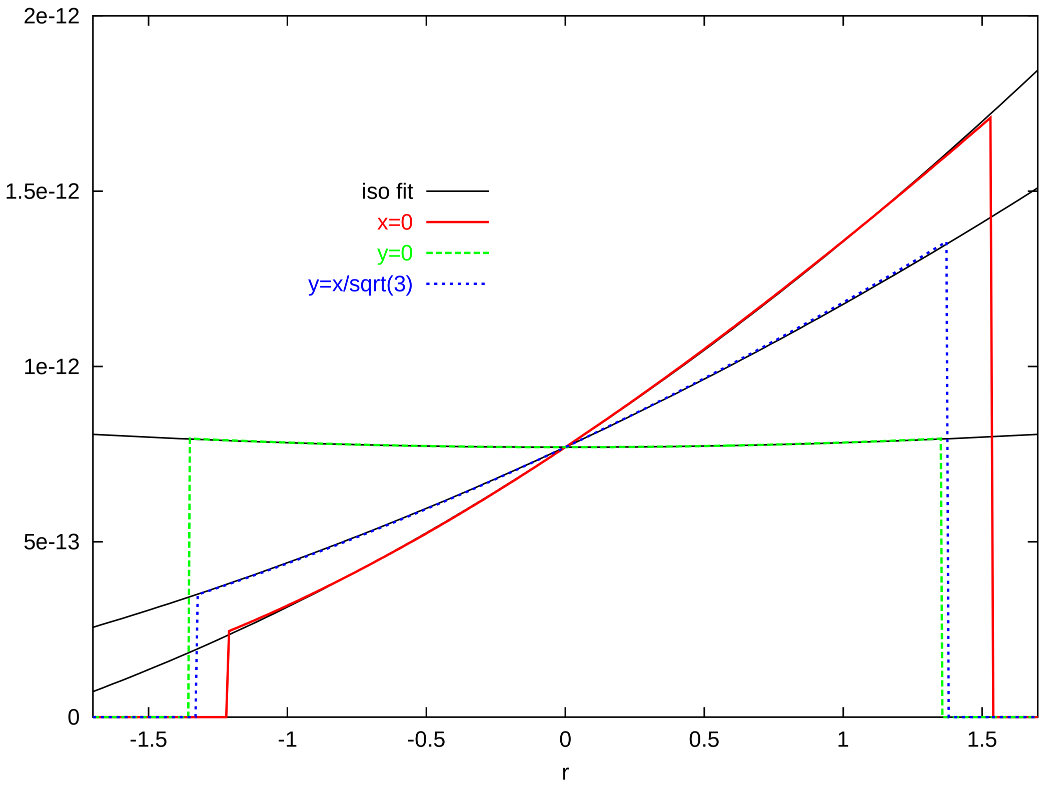
Thresholds inside phase space: large effects ?

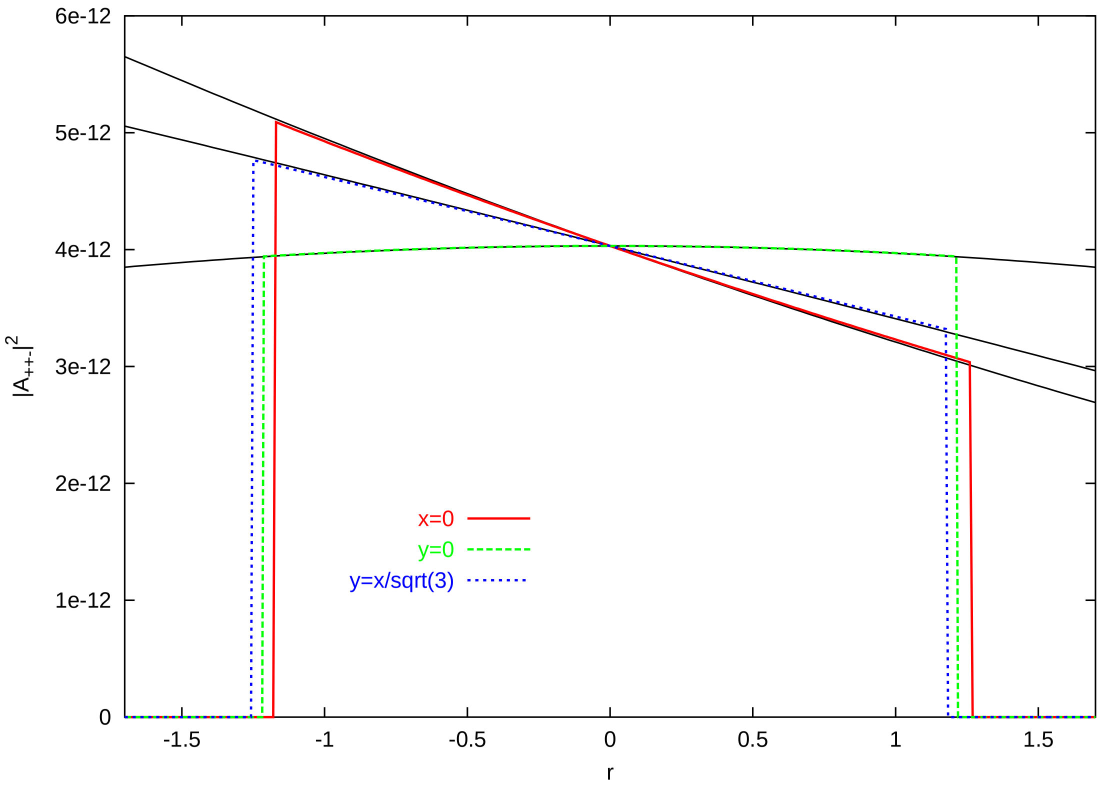
Isospin Breaking Calculation: Preliminary Numerics





Isospin Breaking Calculation: Preliminary Numerics





Conclusions and Work in Progress

- $K \to 3\pi$ now calculated to p^4 in isospin limit
- Fit to experiment performed
- Dependence on 11+2 parameters but cannot all be disentangled
- Discrepancy (?) with curvatures
- Quark Mass and *Local* Electromagnetic Effects seem rather small
- Photonic loops under investigation (When Fredrik returns from parental leave of absence)

- CP violating assymmetries can be worked out: Gamiz, Prades, Scimemi: only Δg_C not strongly dependent on $\tilde{K}_i$
- Phases at higher order might be important