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Chiral Perturbation Theory at Two Loops and the Measurement of V_{us}

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- Chiral Perturbation Theory

JB and P. Talavera, hep-ph/0303103

- Two Loop Calculations done
- $K \rightarrow \pi \ell \nu$ ($K_{\ell 3}$): Definitions and V_{us}
- $f_+(t)$: linearity, ChPT results, fit to CPLEAR data, λ_+
- $f_0(t)$ Main result: $f_-(t) \Rightarrow f_+(0)$
- Conclusions

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

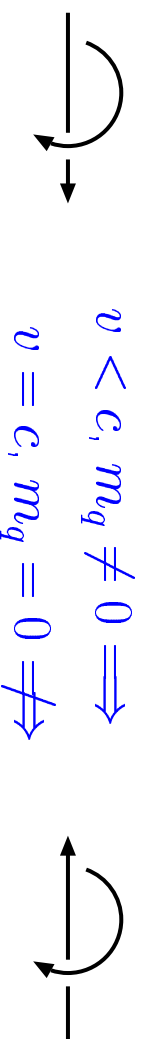
QCD: 3 light quarks: equal mass can interchange them: $SU(3)_V$

But

$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Can also see that via



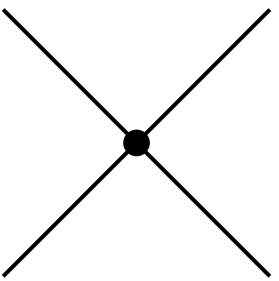
Chiral Perturbation Theory

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$

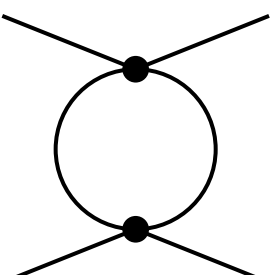
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken \implies 8 massless degrees of freedom **and** interaction vanishes at zero momentum

Power counting in momenta:



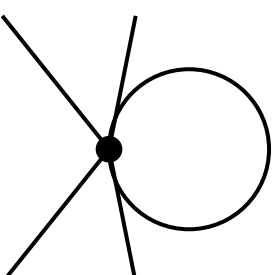
$$p^2$$



$$(p^2)^2 (1/p^2)^2 p^4 = p^4$$



$$1/p^2$$



$$(p^2) (1/p^2) p^4 = p^4$$

$$\int d^4p$$

$$p^4$$

Chiral Perturbation Theory

External currents/masses:

$$m_\pi^2 \sim m_q \langle \bar{q}q \rangle \implies m_q \sim p^2$$

$$A_\mu, W_\mu^\pm \text{ in } D_\mu = \partial_\mu - ieA_\mu Q \implies A_\mu, W_\mu^\pm \sim p$$

General Papers for two-loops

ChPT basis papers: Gasser Leutwyler 1985

$$p^2$$

Long known

two parameters

$$p^4$$

Gasser Leutwyler

ten parameters (L_i^r)

$$p^6$$

JB, Colangelo, Ecker

90 parameters (C_i^r)

Fearing, Scherer

Renormalization and Infinities at p^6 : JB, Colangelo, Ecker

Note: At least 3 different renormalization schemes used in actual calculations

Two Flavour

$$\gamma\gamma \rightarrow \pi^0\pi^0$$

Bellucci, Gasser, Sainio

$$\gamma\gamma \rightarrow \pi^+\pi^-, F_\pi, m_\pi$$

Bürgi

$$\pi\pi\text{-scattering}, F_\pi, m_\pi$$

JB, Colangelo, Ecker, Gasser, Sainio

$$F_{V\pi}(t), F_{S\pi}$$

JB, Colangelo, Talavera

$$\pi \rightarrow l\nu\gamma$$

JB, Talavera

Three Flavour

$$\Pi_{VV\pi}, \Pi_{VV\eta}$$

Kambor, Golowich; Amorós, JB, Talavera

$$\Pi_{AA\pi}, \Pi_{AA\eta}, F_\pi, F_\eta, m_\pi, m_\eta$$

Kambor, Golowich; Amorós, JB, Talavera

$$\Pi_{SS} (L_4^r, L_6^r)$$

Moussallam

$$\Pi_{VVK}, \Pi_{AAK}, F_K, m_K$$

Amorós, JB, Talavera

$$K_{\ell 4}$$

$$(L_1^r, L_2^r, L_3^r)$$

Amorós, JB, Talavera

$$F_{\pi^+}, F_{\pi^0}, \dots, m_{K^+}, m_{K^0} \text{ for } m_u \neq m_d (L_5^r, L_7^r, L_8^r, m_u/m_d)$$

Amorós, JB, Talavera

$$F_{V\pi}, F_{VK^+}, F_{VK^0}$$

$$(L_9^r)$$

Post, Schilcher; JB, Talavera

$$K_{\ell 3}$$

Post, Schilcher; JB, Talavera

In preparation: $K_{\ell 3}$ for $m_u \neq m_d$

JB, Talavera

In preparation: $F_{S\pi}, F_{SK} (L_4^r)$

JB, Dhonte

K_{ℓ3} Definitions

$$\begin{aligned}
 K_{\ell 3}^+ &: & K^+(p) &\rightarrow \pi^0(p')\ell^+(p_\ell)\nu_\ell(p_\nu) \\
 K_{\ell 3}^0 &: & K^0(p) &\rightarrow \pi^-(p')\ell^+(p_\ell)\nu_\ell(p_\nu)
 \end{aligned}$$

$$K_{\ell 3}^+ : \quad T = \frac{G_F}{\sqrt{2}} V_{us}^* \ell^\mu F_\mu^+(p', p)$$

$$\ell^\mu = \bar{u}(p_\nu)\gamma^\mu(1 - \gamma_5)v(p_\ell)$$

$$\begin{aligned}
 F_\mu^+(p', p) &= \langle \pi^0(p') | V_\mu^{4-i5}(0) | K^+(p) \rangle \\
 &= \frac{1}{\sqrt{2}} [(p' + p)_\mu f_+^{K^+\pi^0}(t) + (p - p')_\mu f_-^{K^+\pi^0}(t)]
 \end{aligned}$$

$$\begin{aligned}
 K_{\ell 3}^0 : \quad F_\mu^0(p', p) &= \langle \pi^-(p') | V_\mu^{4-i5}(0) | K^0(p) \rangle \\
 &= (p' + p)_\mu f_+^{K^0\pi^-}(t) + (p - p')_\mu f_-^{K^0\pi^-}(t).
 \end{aligned}$$

Isospin: $f_+^{K^0\pi^-}(t) = f_+^{K^+\pi^0}(t) = f_+(t)$ $f_-^{K^0\pi^-}(t) = f_-^{K^+\pi^0}(t) = f_-(t)$

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor:

$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization:

$$f_{+,0}(t) = f_+(0) \left(1 + \lambda_{+,0} \frac{t}{m_\pi^2} \right)$$

$|V_{us}|$: • Know theoretically $f_+(0) = 1 + \dots$

– Short distance correction to G_F from G_μ Marciano-Sirlin

– Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$

– Isospin Breaking Leuwylter-Roos $\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.022$ In Progress

• Know experimentally $f_+(0)$

– Radiative Corrections: use generalized formfactors Cirigliano et al., hep-ph/0110153

– Parametrize form-factor: is linear enough for $f_+(t)$?

PDG2002:

$$|V_{ud}| = 0.9734 \pm 0.0008$$

$$|V_{us}| = 0.2196 \pm 0.0026$$

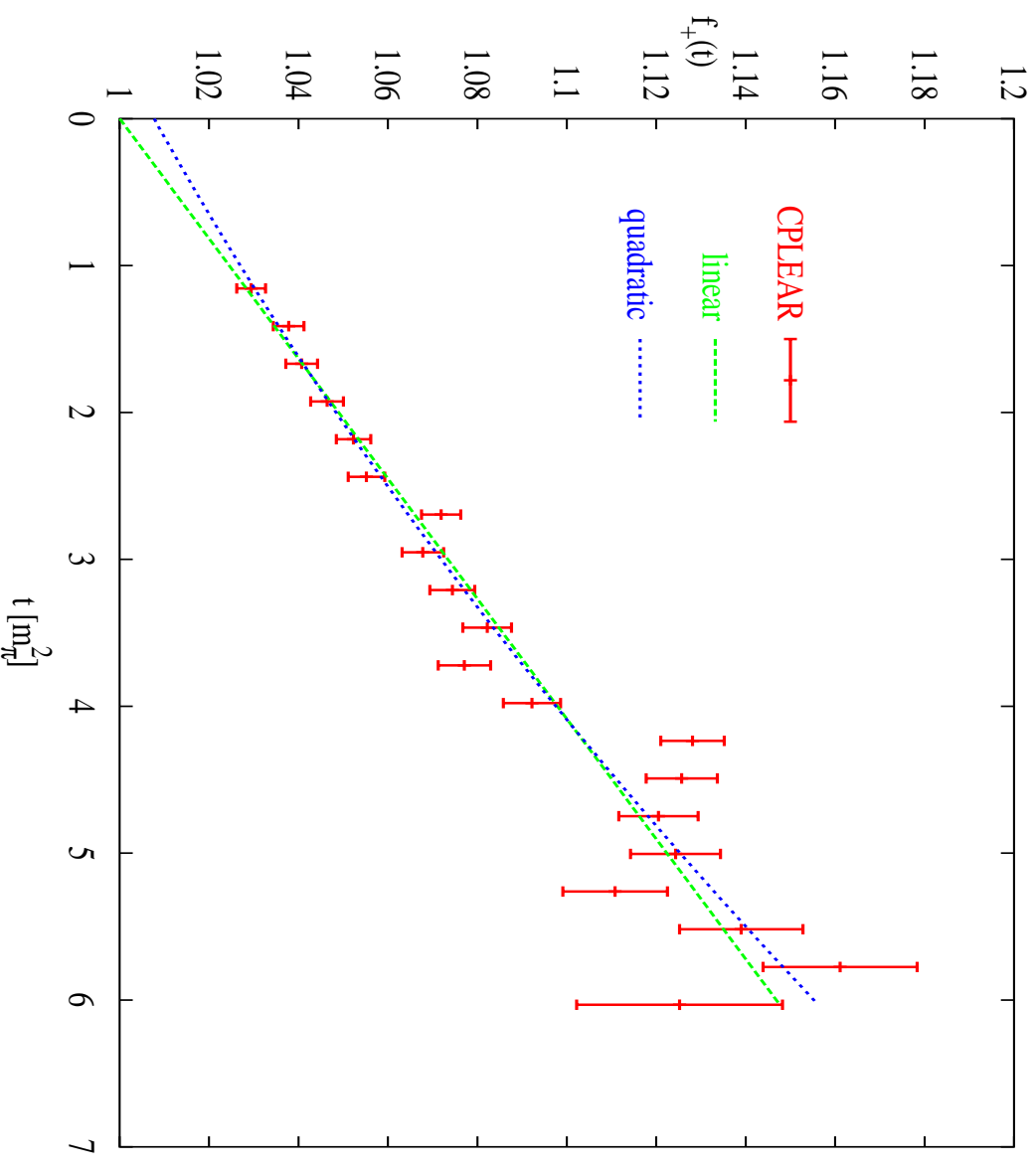
$$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) = 0.9957 \pm 0.0019$$

(V_{ud} from neutron decay only \Rightarrow a bit worse)

Is Linear Enough ?

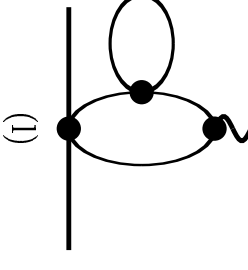
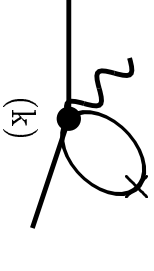
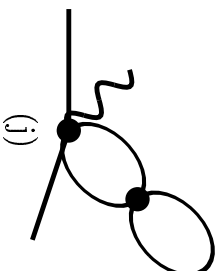
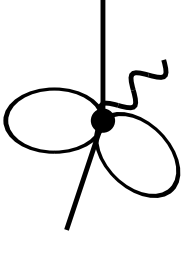
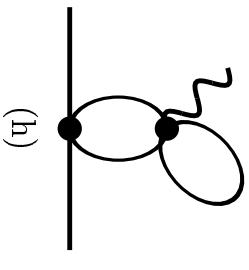
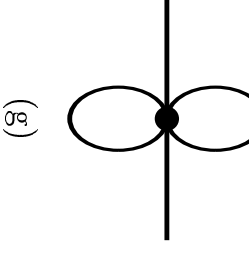
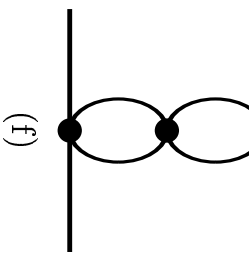
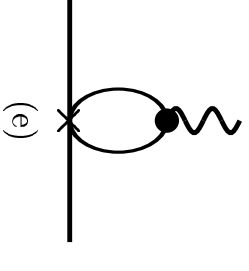
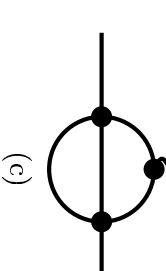
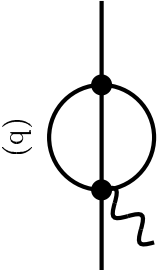
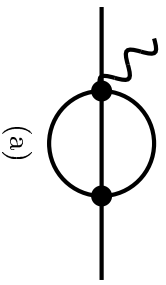
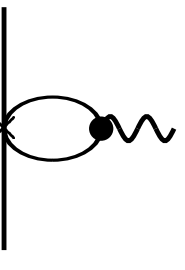
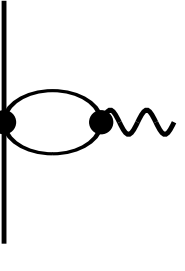
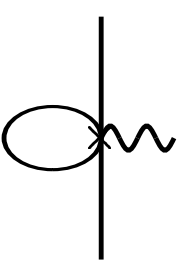
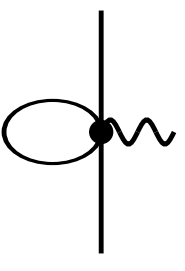
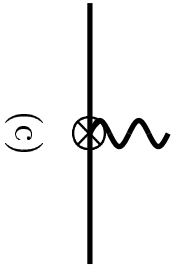
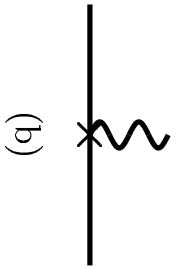
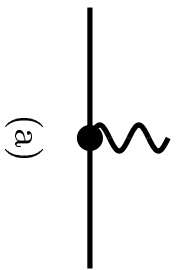
$$f_+(t) = a_+ \left(1 + \lambda_+ \frac{t}{m_{\pi^+}^2} + c_+ t^2 \right)$$

a_+	λ_+	c_+ [GeV ⁻⁴]
$\equiv 1$	0.0245 ± 0.0006	$\equiv 0$
1.000 ± 0.004	0.0245 ± 0.0015	$\equiv 0$
$\equiv 1$	0.0238 ± 0.0017	0.5 ± 1.2
1.008 ± 0.009	0.0181 ± 0.0068	2.8 ± 2.8



Curvature IS Important at 1% Level!!

$K_{\mu 3}$ Diagrams



- : p^2 vertex
- X : p^4 vertex
- ⊗ : p^6 vertex



$f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4} (-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

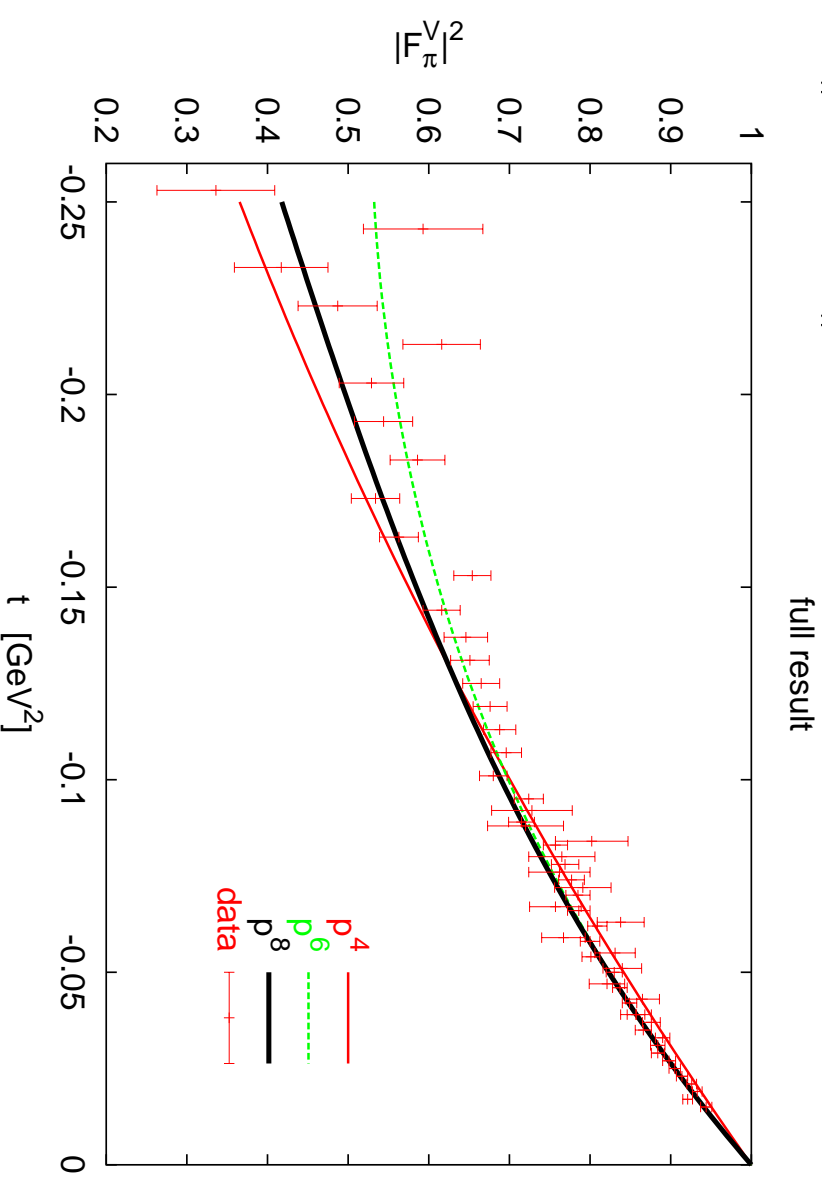
Pion electromagnetic Form factor:

JB, Talavera

$$L_9^r = 0.00593 \pm 0.00043$$

$$-4C_{88}^r + 4C_{90}^r = 0.00022 \pm 0.00002$$

$$\text{VMD: } R_{+1}^{K\pi} \approx -4 \cdot 10^{-5} \text{ GeV}^2$$



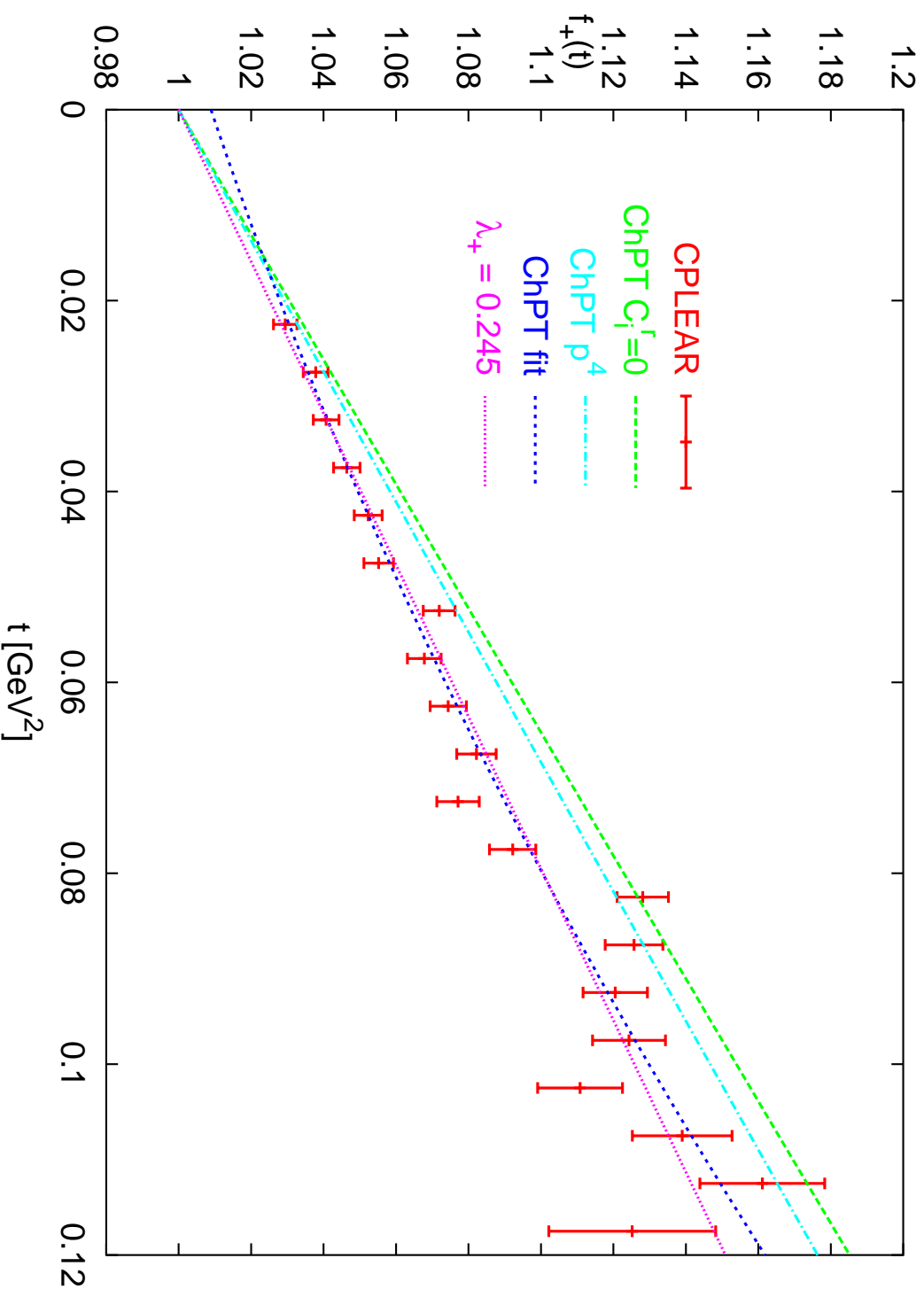
ChPT fit to $f_+(t)$

$$\Rightarrow R_{+1}^{K\pi} = -(4.7 \pm 0.5) 10^{-5} \text{ GeV}^2$$

$$(c_+ = 3.2 \text{ GeV}^{-4})$$

$$\Rightarrow a_+ = 1.009 \pm 0.004$$

$$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$$



Values for λ_+

Process	Ref.	λ_+	λ_0
$K_{\mu,3}^+$	PDG2002	0.033 ± 0.010	0.004 ± 0.009
$K_{\ell,3}^+$	PDG2002 μe	0.0282 ± 0.0027	0.013 ± 0.005
$K_{\mu,3}^0$	PDG2002	0.033 ± 0.005	0.027 ± 0.006
$K_{\ell,3}^0$	PDG2002 μe	0.0300 ± 0.0020	0.030 ± 0.005
$K_{e,3}^0$	PDG2002	0.0291 ± 0.0018	–
$K_{e,3}^+$	PDG2002	0.0278 ± 0.0019	–
$K_{e,3}^+$	ISTRA	0.0293 ± 0.0025	–
$K_{\mu,3}^+$	ISTRA	0.0321 ± 0.0045	0.0209 ± 0.0045
$K_{e,3}^+$	KEK	0.0278 ± 0.0023	–
$K_{e,3}^0$	KTev	0.02748 ± 0.00084	–

LINEAR

ASSUMED

μe means that lepton universality has been used in the measurement.

Ours: $\lambda_+ = 0.0170 \pm 0.0015$

This value comes from the ChPT via

$$\lambda_+ = 0.0283 (p^4) + 0.0011 (\text{loops } p^6) - 0.0124(C_i^r).$$

$$f_0(t)$$

Main Result:

$$f_0(t) = 1 - \frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 + 8 \frac{t}{F_\pi^4} (2C_{12}^r + C_{34}^r) (m_K^2 + m_\pi^2) + \frac{t}{m_K^2 - m_\pi^2} (F_K/F_\pi - 1) - \frac{8}{F_\pi^4} t^2 C_{12}^r + \bar{\Delta}(t) + \Delta(0).$$

$\bar{\Delta}(t)$ and $\Delta(0)$ contain **NO** C_i^r and only depend on the L_i^r at order p^6
 \implies

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r].$$

$\bar{\Delta}(t)$ and $\Delta(0)$

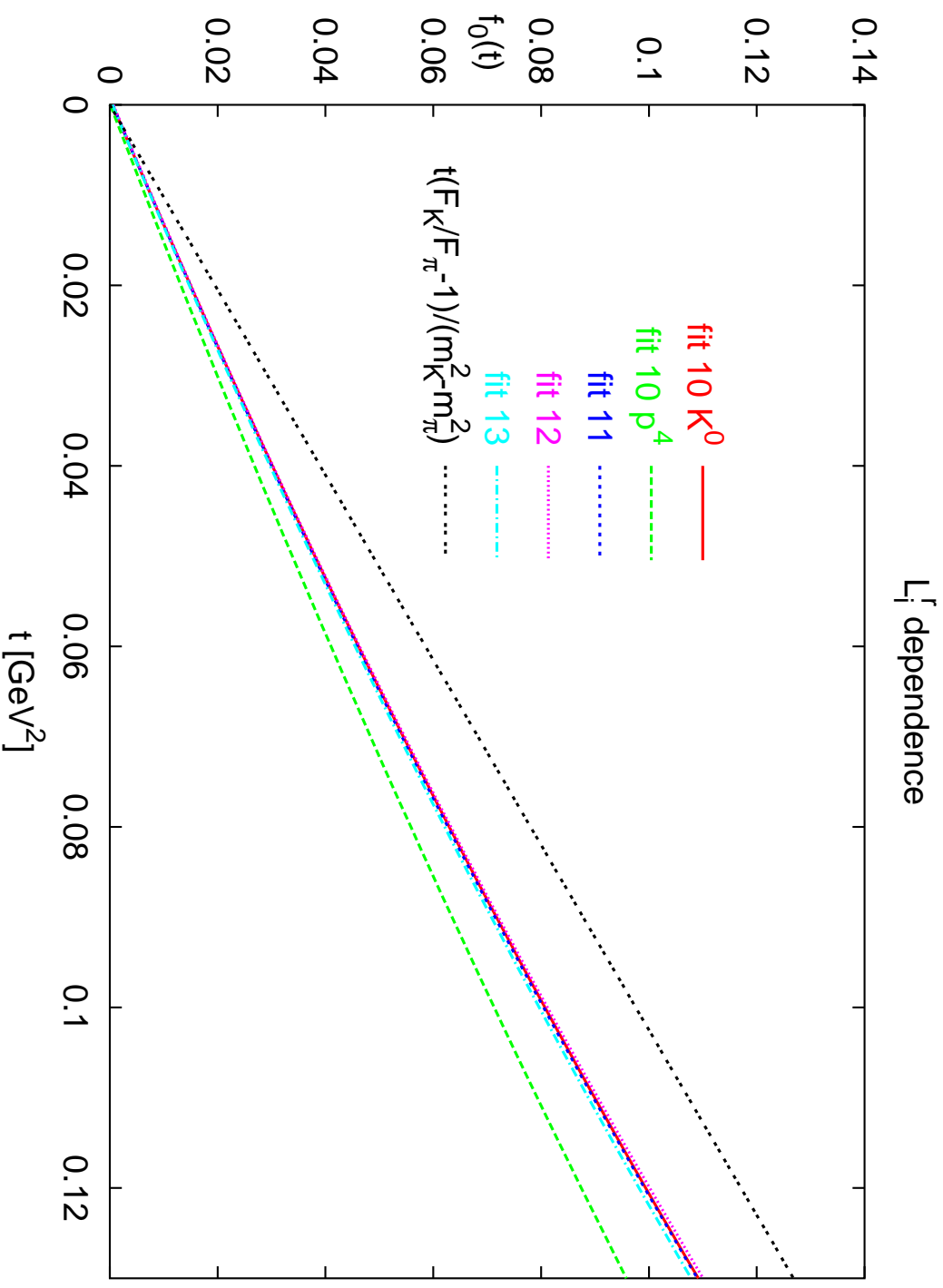
$$f_0(t) - f_0(0) \text{ for } C_i^r = 0$$

	$K_{\ell 3}^0$	$K_{\ell 3}^+$
p^4	-0.02266	-0.02275
p^6 loops only	0.01130	0.01104
p^6-L_i fit 10	0.00332	0.00320
p^6-L_i fit 11	0.00375	0.00355
p^6-L_i fit 12	0.00216	0.00189
p^6-L_i fit 13	0.00539	0.00526
p^6-L_i p^4 fit	0.00891	0.00863

$$\Delta(0) = -0.0080$$

$$\pm 0.0057 \text{ [loops]}$$

$$\pm 0.0028 [L_i^r]$$



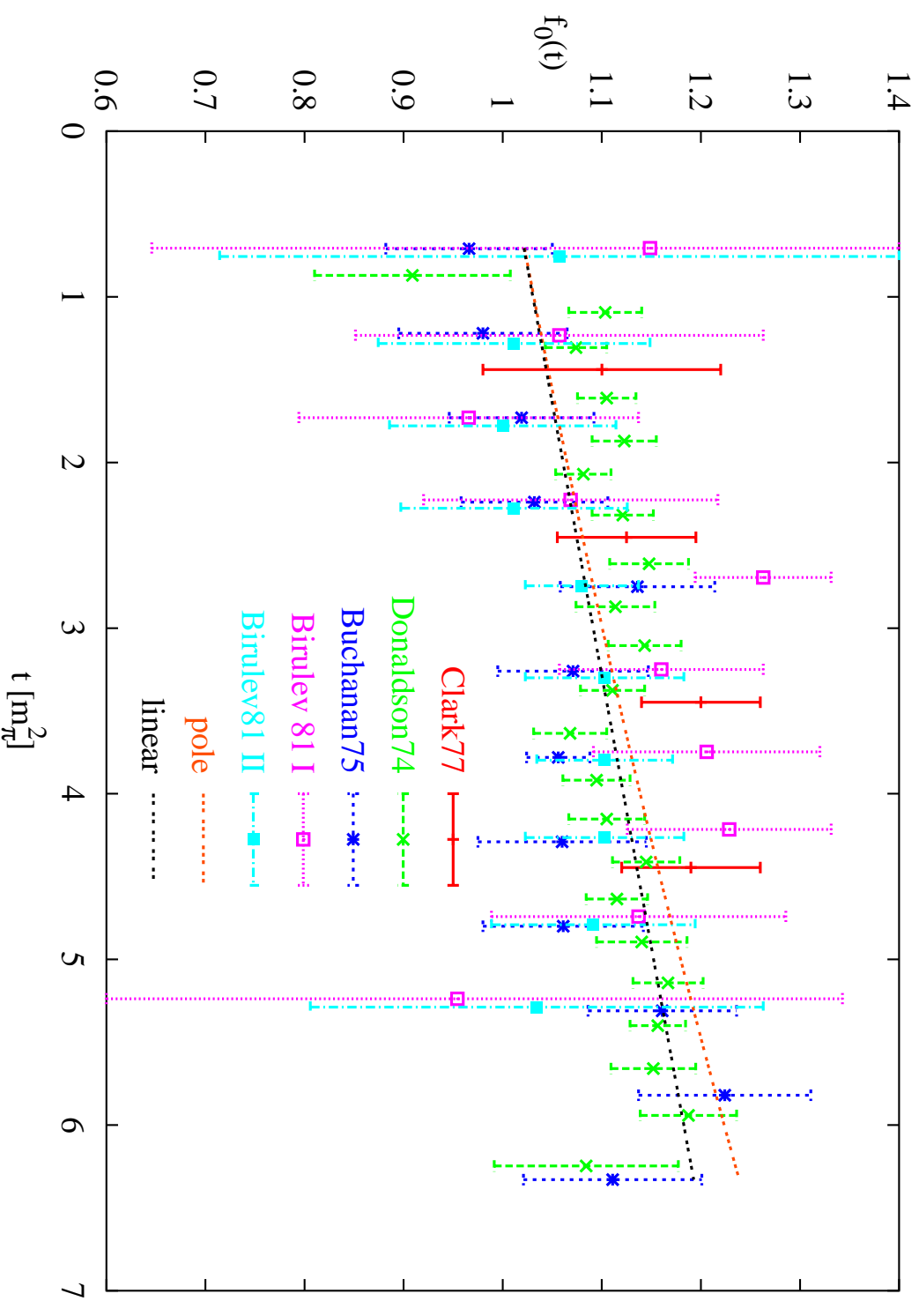
$f_0(t)$ curvature plus estimates

$$C_{12}^{rr}|_{SMD} = -\frac{F_\pi^4}{8m_S^2} \approx -1.0 \cdot 10^{-5}$$

$$\Lambda_0 = 0.020 \pm 0.010 \implies$$

$$2C_{12}^{rr} + C_{34}^{rr} = (1.0 \pm 1.7) \cdot 10^{-5}$$

$$f_+(0)|c_i^{rr} \approx 0.0 \pm 0.1$$



Conclusions and Work in Progress

- Small numeric disagreement with *Post-Schlicher*: under investigation
- C_{12}^{rr} from curvature in pion scalar formfactor: JB, Dhonte
- Isospin breaking contribution will be known to the same precision JB, Talavera
- Need precision measurements of $f_+(t)$ and $f_0(t)$
- Do not neglect curvature in the analysis
- ChPT can predict the needed curvatures