PHOTONS and PARTIAL QUENCHING

$\eta \to 3\pi$ AT TWO LOOPS: STATUS and PRELIMINARY RESULTS

Johan Bijnens

Lund University

bijnens@thep.lu.se

http://www.thep.lu.se/∼bijnens

Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Overview

- Chiral Perturbation Theory
- Photons and Partial Quenching
  - What is partially quenched?
  - PQChPT: what we did earlier
  - Electromagnetism in Lattice QCD
  - Electromagnetism in PQChPT
    - Lagrangians
    - Masses, Decay Constants
    - ΔM^2 and ΔF from *quenched photons*
- $\eta \to 3\pi$
  - What is known
  - Status of p^6 calculation
- Conclusions

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

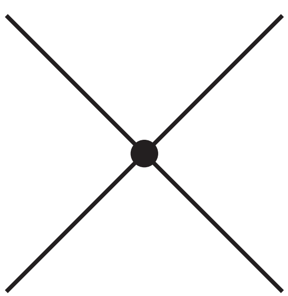
Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$

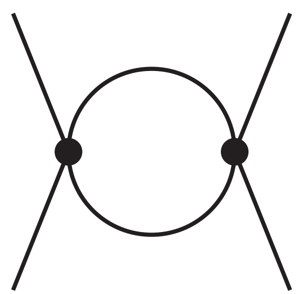
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

Power counting in momenta:



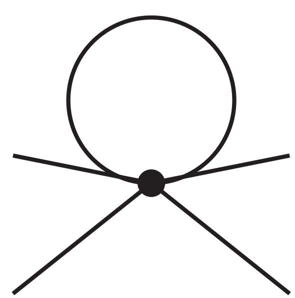
p^2



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

$1/p^2$

$\int d^4p$ $\qquad p^4$



$(p^2)\,(1/p^2)\,p^4 = p^4$

What is Partially Quenched?

In Lattice gauge theory one calculates

$$\langle 0|(\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)|\rangle$$

$$= \frac{\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}}}{\int [dq][d\overline{q}][dG]e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}}}$$

for Euclidean separations x
Integrals performed after rotation to Euclidean
(note that I use Minkowski notation throughout)
(note that $\not{D}$ includes also mass term)

What is Partially Quenched?

$$\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}(\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x)\,\det\,(\not{D}_G)_{\mathrm{QCD}}$$

What is Partially Quenched?

$$\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y \mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG] e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}} (\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x) \det (\not{D}_G)_{\mathrm{QCD}}$$

$\int [dG]$ done via importance sampling

What is Partially Quenched?

$$\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\text{QCD}}} \propto$$

$$\int [dG]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}(\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x)\,\det(\not{D}_G)_{\text{QCD}}$$

$\int [dG]$ done via importance sampling

- Quenched: get distribution from $e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}$ only
- Unquenched: include $\det(\not{D}_G)_{\text{QCD}}$ VERY expensive
- Partially quenched: $(\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x)$
  DIFFERENT Quarks then in $\det(\not{D}_G)_{\text{QCD}}$

What is Partially Quenched?

Why do this?

- Is not Quenched: Real QCD is continuous limit from Partially Quenched
- More handles to turn:
  - Allows more systematic studies by varying parameters
  - Sometimes allows to disentangle things from different observables
- $\det\left(\not{D}_G\right)_{\mathrm{QCD}}$: Sea quarks
- $(\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x)$: Valence Quarks

What is Partially Quenched?

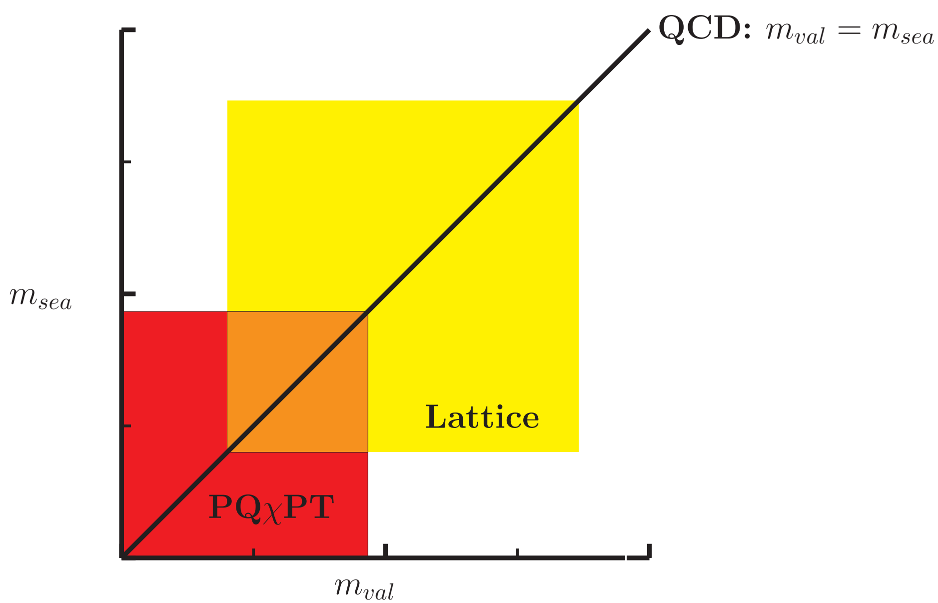
Why not do this?

- It is not QCD as soon as Valence$\neq$Sea
- not a Quantum Field Theory
- No unitarity
- No CPT theorem
- No spin statistics theorem

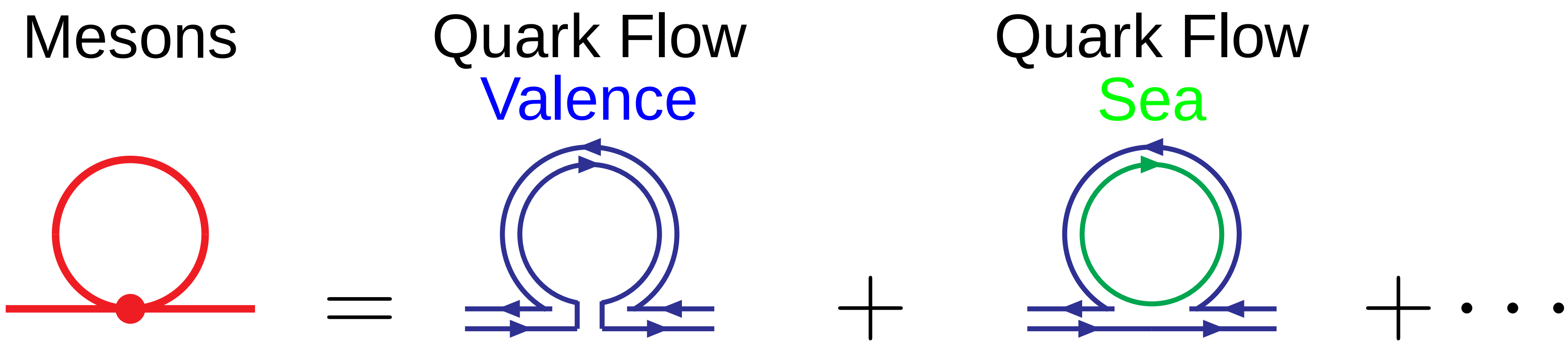
What is Partially Quenched?

Do anyway

- Cheap computationally
- Allows extra studies
- Hope it works near real QCD case
- Is a well defined Euclidean statistical model



ChPT and Lattice QCD

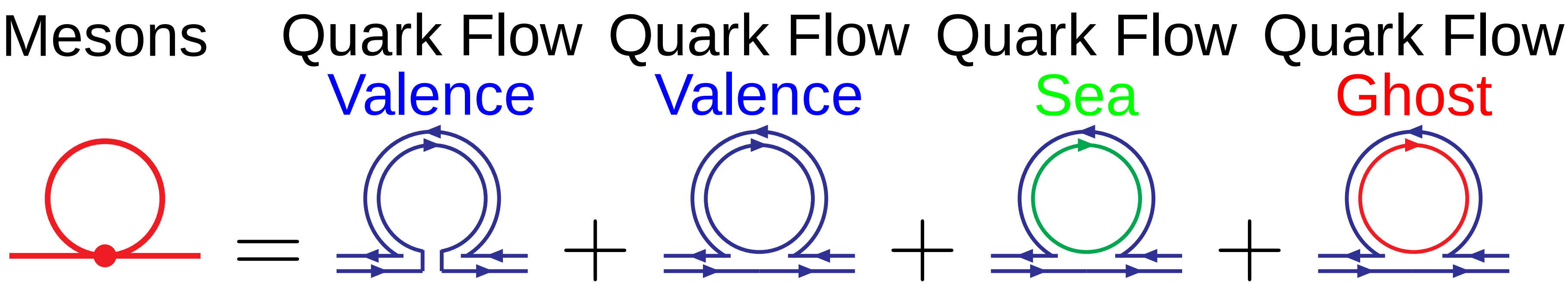


They need to be treated separately: i.e. different quark masses

Partially Quenched ChPT (PQChPT)

PQChPT at Two Loops: General

Add ghost quarks: remove the unwanted free valence loops



Possible problem: QCD $\Longrightarrow$ ChPT relies heavily on unitarity

Partially quenched: at least one dynamical sea quark
$\Longrightarrow$ Φ_0 is heavy: remove from PQChPT

Symmetry group becomes $SU(n_v + n_s|n_v) \times SU(n_v + n_s|n_v)$ (approximately)

PQChPT: General

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

Very important note: ChPT is a limit of PQChPT
$\Longrightarrow$ LECs from ChPT are linear combinations of LECs of PQChPT with the same number of sea quarks.

E.g. $L_1^r = L_0^{r(3pq)}/2 + L_1^{r(3pq)}$

PQChPT: What we did earlier

One-loop: Bernard, Golterman, Sharpe, Shoresh, Pallante,. . .

Two loops: Subject started

$m^2_{\pi^+}$ simplest mass case: JB,Danielsson,Lähde, hep-lat/0406017

Other mass combinations:

F_{π^+}: JB,Lähde, hep-lat/0501014

F_{π^+}, $m^2_{\pi^+}$ two sea quarks: JB,Lähde, hep-lat/0506004

$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0602003

Neutral masses: JB,Danielsson, hep-lat/0606017

Actual Calculations:
- heavy use of FORM Vermaseren
- use PQ without super Φ_0 in supersymmetric formalism
- Main problem: sheer size of the expressions

Iso breaking from lattice data: a and L extrapolations needed

Including electromagnetism

- High precision: need to include electromagnetism
- Large violations of Dashen's theorem expected

Including electromagnetism

- High precision: need to include electromagnetism
- Large violations of Dashen's theorem expected
- Dashen: $\left(m^2_{K^+} - m^2_{K^0}\right)_{\text{em}} = \left(m^2_{\pi^+} - m^2_{\pi^0}\right)_{\text{em}}$
- JB,Donoghue,Holstein,Wyler 93:
  $\left(m^2_{K^+} - m^2_{K^0}\right)_{\text{em}} \approx 2 \left(m^2_{\pi^+} - m^2_{\pi^0}\right)_{\text{em}}$

Including electromagnetism

- High precision: need to include electromagnetism
- Large violations of Dashen's theorem expected
- Dashen: $\left(m^2_{K^+} - m^2_{K^0}\right)_{\text{em}} = \left(m^2_{\pi^+} - m^2_{\pi^0}\right)_{\text{em}}$
- JB,Donoghue,Holstein,Wyler 93:
  $\left(m^2_{K^+} - m^2_{K^0}\right)_{\text{em}} \approx 2\left(m^2_{\pi^+} - m^2_{\pi^0}\right)_{\text{em}}$
- Needs checking
- Lattice: Duncan, Eichten,...
- Analytical: JB, Prades, Ananthanarayan, Moussallam,...

Why so little lattice?

$$\int [dq][d\overline{q}][dG][dA](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG][dA]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}-(1/4)F_{\mu\nu}F^{\mu\nu}} \times$$

$$(\not{D}^u_{G,A})^{-1}(x,0)(\not{D}^d_{G,A})^{-1}(0,x)\,\det\left(\not{D}_{G,A}\right)_{\mathrm{QCD}}$$

Why so little lattice?

$$\int [dq][d\overline{q}][dG][dA](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG][dA]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}-(1/4)F_{\mu\nu}F^{\mu\nu}} \times$$

$$(\not{D}^u_{G,A})^{-1}(x,0)(\not{D}^d_{G,A})^{-1}(0,x)\,\det\left(\not{D}_{G,A}\right)_{\mathrm{QCD}}$$

$\int [dG][dA]$ done via importance sampling

Why so little lattice?

$$\int [dq][d\overline{q}][dG][dA](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y \mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG][dA]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}-(1/4)F_{\mu\nu}F^{\mu\nu}} \times$$

$$(\not{D}^u_{G,A})^{-1}(x,0)(\not{D}^d_{G,A})^{-1}(0,x)\,\det\left(\not{D}_{G,A}\right)_{\mathrm{QCD}}$$

$\int [dG][dA]$ done via importance sampling

- Quenched: get distribution from $e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}-(1/4)F_{\mu\nu}F^{\mu\nu}}$ only
- Unquenched: include $\det\left(\not{D}_{G,A}\right)_{\mathrm{QCD}}$ VERY expensive

Other possibilities:

- Include $\det\left(\not{D}_G\right)_{\mathrm{QCD}}$, but not $\det\left(\not{D}_{G,A}\right)_{\mathrm{QCD}}$
- Allows to reuse pure QCD gluon configurations

Other possibilities:

- Include $\det\left(\not{D}_G\right)_{\mathrm{QCD}}$, but not $\det\left(\not{D}_{G,A}\right)_{\mathrm{QCD}}$
- Allows to reuse pure QCD gluon configurations
- Partially quenched: $(\not{D}^u_{G,A})^{-1}(x,0)(\not{D}^d_{G,A})^{-1}(0,x)$ DIFFERENT mass and charge Quarks then in $\det\left(\not{D}_{G,A}\right)_{\mathrm{QCD}}$
- Allows the reuse if we set $\{q_{sea}\} \equiv \{0\}$

Other possibilities:

- Include $\det(\not{D}_G)_{\mathrm{QCD}}$, but not $\det(\not{D}_{G,A})_{\mathrm{QCD}}$
- Allows to reuse pure QCD gluon configurations
- Partially quenched: $(\not{D}^u_{G,A})^{-1}(x,0)(\not{D}^d_{G,A})^{-1}(0,x)$ DIFFERENT mass and charge Quarks then in $\det(\not{D}_{G,A})_{\mathrm{QCD}}$
- Allows the reuse if we set $\{q_{sea}\} \equiv \{0\}$
- Done to $\mathcal{O}(e^2, e^2p^2)$: JB, Danielsson
  - Lagrangian
  - Off-diagonal mesons: masses and decay constants

EM PQChPT

- The whole trace to supertrace goes through again
- Need to include the spurions with valence, sea and ghost sector
- ghost: masses and charges equal to valence
- n_F flavour Lagrangian and infinities carry across
  - trace $\rightarrow$ supertrace
  - $n_F \longrightarrow n_{sea}$

EM PQChPT: Lagrangian

$$r_\mu = v_\mu + eQ_R A_\mu + a_\mu \qquad l_\mu = v_\mu + eQ_L A_\mu - a_\mu$$

$$Q_R = Q_L = Q = \text{diag}(q_1, \ldots, q_9) \quad \text{with} \quad \langle Q \rangle = q_4 + q_5 + q_6 = 0.$$

$$u = \sqrt{U}$$

$$\mathcal{Q}_\mathrm{L} = u Q_\mathrm{L} u^\dagger \qquad \hat{\nabla}_\mu \mathcal{Q}_\mathrm{L} = u D_\mu Q_\mathrm{L} u^\dagger$$

$$\mathcal{Q}_\mathrm{R} = u^\dagger Q_\mathrm{R} u \qquad \hat{\nabla}_\mu \mathcal{Q}_\mathrm{R} = u^\dagger D_\mu Q_\mathrm{R} u,$$

$$D^\mu Q_L = \partial^\mu Q_L - i\,[l^\mu, Q_L] \qquad D^\mu Q_R = \partial^\mu Q_R - i\,[r^\mu, Q_R]$$

EM PQChPT: Lagrangian

$$\mathcal{L}_2 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\lambda(\partial_\mu A^\mu)^2 + \frac{F_0^2}{4}\langle u^\mu u_\mu + \chi_+\rangle + e^2 Z_E F_0^4 \langle \mathcal{Q}_\mathrm{L} \mathcal{Q}_\mathrm{R}\rangle$$

$$\mathcal{L}_4 = \mathcal{L}_{S4} + \mathcal{L}_{S2E2}.$$

$$\mathcal{L}_{S2E2} = e^2 F_0^2 \left(\sum_{i=1}^{14} K_i^E Q_i^s + K_{18}^E Q_{18}^s + K_{19}^E Q_{19}^s \right).$$

$$Q_{18}^s = \langle \mathcal{Q}_\mathrm{L} u_\mu \mathcal{Q}_\mathrm{L} u^\mu + \mathcal{Q}_\mathrm{R} u_\mu \mathcal{Q}_\mathrm{R} u^\mu \rangle$$

$$Q_{19}^s = \langle \mathcal{Q}_\mathrm{L} u_\mu \mathcal{Q}_\mathrm{R} u^\mu \rangle$$

K_i^E $i = 1, \ldots, 14$ terms: same form as Urech, but supertrace,. . .

K_{18}^E and K_{19}^E 'new' ones.

EM PQChPT: Lagrangian

$$Q_1^s = \frac{1}{2}\left\langle \mathcal{Q}_{\rm L}^2 + \mathcal{Q}_{\rm R}^2 \right\rangle \left\langle u_\mu u^\mu \right\rangle \qquad Q_2^s = \left\langle \mathcal{Q}_{\rm L}\mathcal{Q}_{\rm R} \right\rangle \left\langle u_\mu u^\mu \right\rangle$$

$$Q_3^s = -\left\langle \mathcal{Q}_{\rm L} u_\mu \right\rangle \left\langle \mathcal{Q}_{\rm L} u^\mu \right\rangle - \left\langle \mathcal{Q}_{\rm R} u_\mu \right\rangle \left\langle \mathcal{Q}_{\rm R} u^\mu \right\rangle$$

$$Q_4^s = \left\langle \mathcal{Q}_{\rm L} u_\mu \right\rangle \left\langle \mathcal{Q}_{\rm R} u^\mu \right\rangle \qquad Q_5^s = \left\langle \left(\mathcal{Q}_{\rm L}^2 + \mathcal{Q}_{\rm R}^2 \right) u_\mu u^\mu \right\rangle$$

$$Q_6^s = \left\langle \left(\mathcal{Q}_{\rm L}\mathcal{Q}_{\rm R} + \mathcal{Q}_{\rm R}\mathcal{Q}_{\rm L} \right) u_\mu u^\mu \right\rangle \qquad Q_7^s = \frac{1}{2}\left\langle \mathcal{Q}_{\rm L}^2 + \mathcal{Q}_{\rm R}^2 \right\rangle \left\langle \chi_+ \right\rangle$$

$$Q_8^s = \left\langle \mathcal{Q}_{\rm L}\mathcal{Q}_{\rm R} \right\rangle \left\langle \chi_+ \right\rangle \qquad Q_9^s = \left\langle (\mathcal{Q}_{\rm L}^2 + \mathcal{Q}_{\rm R}^2)\chi_+ \right\rangle$$

$$Q_{10}^s = \left\langle (\mathcal{Q}_{\rm L}\mathcal{Q}_{\rm R} + \mathcal{Q}_{\rm R}\mathcal{Q}_{\rm L})\chi_+ \right\rangle$$

$$Q_{11}^s = \left\langle (\mathcal{Q}_{\rm R}\mathcal{Q}_{\rm L} - \mathcal{Q}_{\rm L}\mathcal{Q}_{\rm R})\chi_- \right\rangle$$

$$Q_{12}^s = i\left\langle \left[\hat{\nabla}_\mu \mathcal{Q}_{\rm R}, \mathcal{Q}_{\rm R} \right] u^\mu - \left[\hat{\nabla}_\mu \mathcal{Q}_{\rm L}, \mathcal{Q}_{\rm L} \right] u^\mu \right\rangle$$

$$Q_{13}^s = \left\langle \hat{\nabla}_\mu \mathcal{Q}_{\rm L} \hat{\nabla}^\mu \mathcal{Q}_{\rm R} \right\rangle$$

$$Q_{14}^s = \left\langle \hat{\nabla}_\mu \mathcal{Q}_{\rm L} \hat{\nabla}^\mu \mathcal{Q}_{\rm L} + \hat{\nabla}_\mu \mathcal{Q}_{\rm R} \hat{\nabla}^\mu \mathcal{Q}_{\rm R} \right\rangle$$

EM PQChPT: Lagrangian

Relation with unquenched:

$$K_1 = K_1^E + K_{18}^E\,, \qquad K_2 = K_2^E + \frac{1}{2}K_{19}^E\,,$$
$$K_3 = K_1^E - K_{18}^E\,, \qquad K_4 = K_4^E + K_{19}^E\,,$$
$$K_5 = K_5^E - 2K_{18}^E\,, \qquad K_6 = K_6^E - K_{19}^E\,,$$
$$K_i = K_i^E; \quad i = 7,\dots,14$$

Infinities: taken over from n_F-flavour calculation of Knecht-Urech (after putting in minimal basis and correcting misprint)

Masses

$$M^2_{\rm phys} = \chi_{e,ij} + \delta^{(4){\rm vs}}/F_0^2 \qquad \chi_{e,ij} = \chi_{ij} + 2Z_E e^2 F_0^2 \left(q_i - q_j\right)^2$$

$$\begin{aligned}
\delta^{(4)23} = & \left[48L_6^r - 24L_4^r\right]\bar\chi_1\chi_{13} + \left[16L_8^r - 8L_5^r\right]\chi_{13}^2 \\
& -48e^2Z_EF_0^2L_4^rq_{13}^2\bar\chi_1 - 16e^2Z_EF_0^2L_5^rq_{13}^2\chi_{13} \\
& -e^2F_0^2\left[12K_1^{Er}+12K_2^{Er}-12K_7^{Er}-12K_8^{Er}\right]\overline{Q}_2\chi_{13} \\
& -e^2F_0^2\left[4K_5^{Er}+4K_6^{Er}\right]q_p^2\chi_{13} + e^2F_0^2\left[4K_9^{Er}+4K_{10}^{Er}\right]q_p^2\chi_p \\
& +12e^2F_0^2K_8^{Er}q_{13}^2\bar\chi_1 + 8e^2F_0^2\left[K_{10}^{Er}+K_{11}^{Er}\right]q_{13}^2\chi_{13} \\
& -e^2F_0^2\left[8K_{18}^{Er}+4K_{19}^{Er}\right]q_1q_3\chi_{13} - 1/3\bar A(\chi_m)R^m_{n13}\chi_{13} \\
& -1/3\bar A(\chi_p)R^p_{q\pi\eta}\chi_{13} + e^2F_0^2\bar A(\chi_{13})q_{13}^2 \\
& +2e^2Z_EF_0^2\bar A(\chi_{1s})q_{1s}q_{13} - 2e^2Z_EF_0^2\bar A(\chi_{3s})q_{3s}q_{13} \\
& +4e^2F_0^2\bar B(\chi_\gamma,\chi_{13},\chi_{13})q_{13}^2\chi_{13} - 4e^2F_0^2\bar B_1(\chi_\gamma,\chi_{13},\chi_{13})q_{13}^2\chi_{13}.
\end{aligned}$$

$$\overline{Q}_2 = (q_4^2+q_5^2+q_6^2)/3$$

Dashen's theorem again

$$\Delta M^2 = M^2(\chi_1,\chi_3,q_1,q_3) - M^2(\chi_1,\chi_3,q_3,q_3) \\ -M^2(\chi_1,\chi_1,q_1,q_3) + M^2(\chi_1,\chi_1,q_3,q_3)$$

very close to

$$\Delta M_D^2 = (m_{K^+}^2 - m_{K^0}^2) - (m_{\pi^+}^2 - m_{\pi^0}^2)\,.$$

$\{q_{sea}\}$ dependence predicted:

can be had from quenched photons

Decay Constants

$$F_{\rm phys} = F_0 \left[1 + \frac{f^{(4)\rm vs}}{F_0^2} + \mathcal{O}(p^6, e^2p^4)\right], \qquad F_{LO} = F_0$$

$$\begin{aligned}
f^{(4)23} \;=\; & +12L_4^r\bar{\chi}_1 + 4L_5^r\chi_{13} + 6e^2F_0^2\left[K_1^{Er} + K_2^{Er}\right]\overline{Q}_2 \\
& +2e^2F_0^2\left[K_5^{Er} + K_6^{Er}\right]q_p^2 + 2e^2F_0^2K_{12}^{Er}q_{13}^2 \\
& +e^2F_0^2\left[4K_{18}^{Er} + 2K_{19}^{Er}\right]q_1q_3 - 1/12\bar{A}(\chi_m)R_{mn13}^v \\
& +\bar{A}(\chi_p)\left[1/6R_{q\pi\eta}^p - 1/12R_p^c\right] \\
& +1/4\bar{A}(\chi_{e,ps}) - 1/12\bar{B}(\chi_p,\chi_p,0)R_p^d \\
& +2e^2F_0^2\bar{B}'(\chi_\gamma,\chi_{13},\chi_{13})q_{13}^2\chi_{13} \\
& -e^2F_0^2\bar{B}_1(\chi_\gamma,\chi_{13},\chi_{13})q_{13}^2 - 2e^2F_0^2\bar{B}_1'(\chi_\gamma,\chi_{13},\chi_{13})q_{13}^2\chi_{13}
\end{aligned}$$

Decay Constants

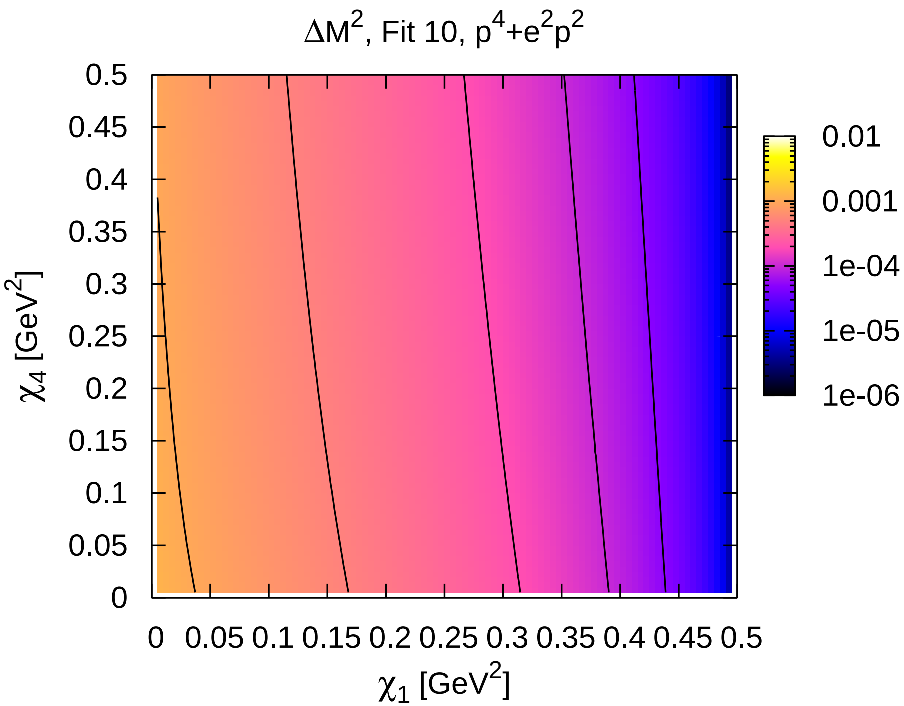
$$\Delta F = \Big[F(\chi_1,\chi_3,q_1,q_3,\{q_{sea}\}) - F(\chi_1,\chi_3,0,0,\{0\}) - F(\chi_1,\chi_1,q_1,q_3,\{q_{sea}\}) + F(\chi_1,\chi_1,0,0,\{0\})\Big]/F_0$$

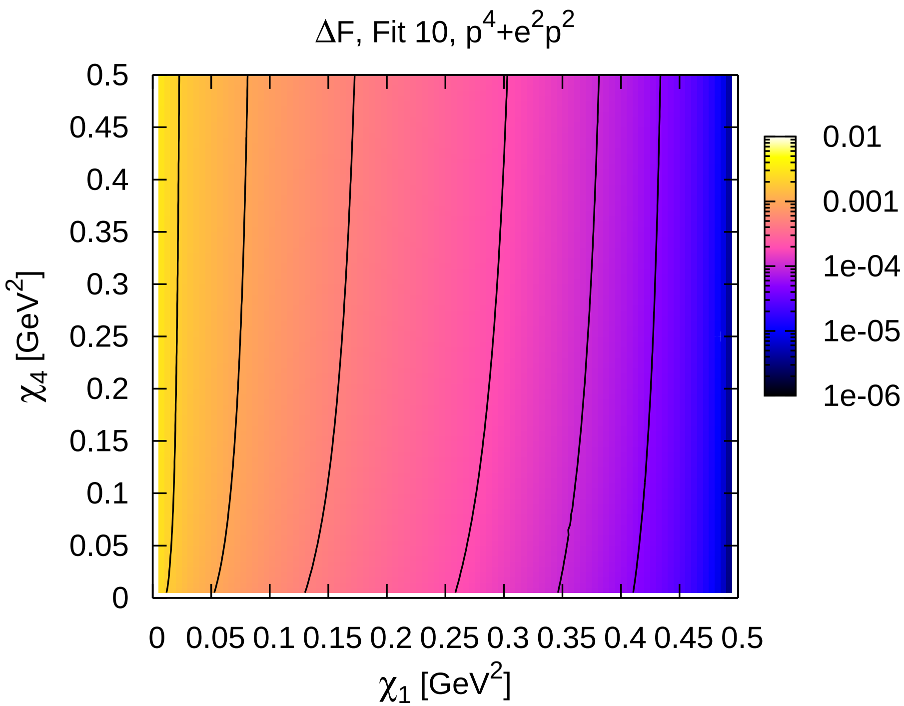
$\{q_{sea}\}$ dependence predicted:

Actually: no dependence on K_i^{Er}

can be had from quenched photons

Masses/Decay Constants





$\chi_3 = \chi_6 = 0.5$ GeV^2

K_i^E chosen to be same mass combinations as JB,Prades

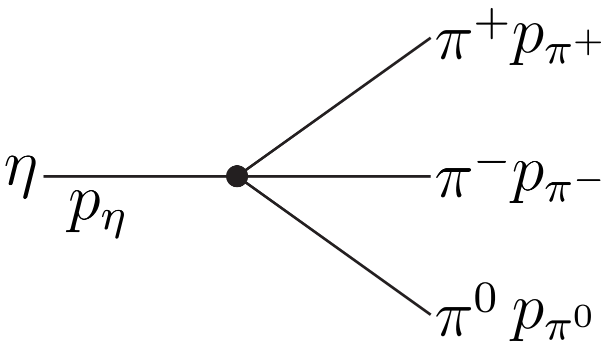
Remember: Eta Physics Handbook

Physica Scripta, Vol. T99, 2002

Contents

$\eta \to 3\pi$ beyond p^4: Basic

Review: JB, Gasser, Phys.Scripta T99(2002)34 [hep-ph/0202242]



$$s = (p_{\pi^+} + p_{\pi^-})^2 = (p_\eta - p_{\pi^0})^2$$
$$t = (p_{\pi^-} + p_{\pi^0})^2 = (p_\eta - p_{\pi^+})^2$$
$$u = (p_{\pi^+} + p_{\pi^0})^2 = (p_\eta - p_{\pi^-})^2$$

$$s + t + u = m_\eta^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2 \equiv 3s_0\,.$$

$$\langle \pi^0\pi^+\pi^- \text{out} | \eta \rangle = i\,(2\pi)^4\ \delta^4\,(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0})\ A(s,t,u)\,.$$

$$\langle \pi^0\pi^0\pi^0 \text{out} | \eta \rangle = i\,(2\pi)^4\ \delta^4\,(p_\eta - p_1 - p_2 - p_3)\ \overline{A}(s_1,s_2,s_3)$$

$$\overline{A}(s_1,s_2,s_3) = A(s_1,s_2,s_3) + A(s_2,s_3,s_1) + A(s_3,s_1,s_2)\,,$$

$\eta \to 3\pi$ beyond p^4: Lowest order

Pions are in $I = 1$ state $\Longrightarrow A \sim (m_u - m_d)$ or α_{em}

- α_{em} effect is small (but large via $m_{\pi^+} - m_{\pi^0}$)
- $\eta \to \pi^+\pi^-\pi^0\gamma$ needs to be included directly

$\eta \to 3\pi$ beyond p^4: Lowest order

Pions are in $I = 1$ state $\Longrightarrow A \sim (m_u - m_d)$ or α_{em}

- α_{em} effect is small (but large via $m_{\pi^+} - m_{\pi^0}$)
- $\eta \to \pi^+\pi^-\pi^0\gamma$ needs to be included directly

ChPT:Cronin 67: $A(s,t,u) = \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \frac{3(s-s_0)}{m_\eta^2 - m_\pi^2}\right\}$

$\eta \to 3\pi$ beyond p^4: Lowest order

Pions are in $I = 1$ state $\Longrightarrow A \sim (m_u - m_d)$ or α_{em}

- α_{em} effect is small (but large via $m_{\pi^+} - m_{\pi^0}$)
- $\eta \to \pi^+\pi^-\pi^0\gamma$ needs to be included directly

ChPT:Cronin 67: $A(s,t,u) = \dfrac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \dfrac{3(s-s_0)}{m_\eta^2 - m_\pi^2}\right\}$

or with $Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$, $\qquad \hat{m} = \frac{1}{2}(m_u + m_d)$

$$A(s,t,u) = \frac{1}{Q^2}\frac{m_K^2}{m_\pi^2}(m_\pi^2 - m_K^2)\frac{M(s,t,u)}{3\sqrt{3}F_\pi^2},$$

with at lowest order $M(s,t,u) = \dfrac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$.

$\eta \longrightarrow 3\pi$ beyond p^4: p^2 and p^4

$\Gamma\left(\eta \to 3\pi\right) \propto |A|^2 \propto Q^{-4}$ allows a PRECISE measurement

$\boxed{Q \approx 24}$ gives lowest order $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 66$ eV.

Other Source from $m^2_{K^+} - m^2_{K^0} \sim Q^{-2} \Longrightarrow \boxed{Q = 20.0 \pm 1.5}$

Lowest order prediction $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 140$ eV.

$\eta \to 3\pi$ beyond p^4: p^2 and p^4

$\Gamma\left(\eta \to 3\pi\right) \propto |A|^2 \propto Q^{-4}$ allows a PRECISE measurement

$\boxed{Q \approx 24}$ gives lowest order $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 66$ eV .

Other Source from $m^2_{K^+} - m^2_{K^0} \sim Q^{-2} \Longrightarrow \boxed{Q = 20.0 \pm 1.5}$
Lowest order prediction $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 140$ eV .

At order p^4: $\dfrac{\int dLIPS|A_2 + A_4|^2}{\int dLIPS|A_2|^2} = 2.4\,,$

($LIPS$=Lorentz invariant phase-space)

Major source: large S-wave final state rescattering

Experiment: $\Gamma(\eta \to 3\pi) = 295 \pm 17$ eV.

$\eta \to 3\pi$ beyond p^4: Dispersive

Try to resum the S-wave rescattering:

Anisovich-Leutwyler (AL), Kambor,Wiesendanger,Wyler (KWW)

Different method but similar approximations

Here: simplified version of AL

Up to p^8: No absorptive parts from $\ell \geq 2$

$\Longrightarrow M(s,t,u) =$

$$M_0(s) + (s-u)M_1(t) + (s-t)M_1(t) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

M_I: “roughly” contributions with isospin 0,1,2

$\eta \to 3\pi$ beyond p^4: Dispersive

3 body dispersive: difficult: keep only 2 body cuts

start from $\pi\eta \to \pi\pi$ ($m_\eta^2 < 3m_\pi^2$) standard dispersive analysis

analytically continue to physical m_η^2.

$$M_I(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \, \frac{\mathrm{Im}M_I(s')}{s'-s-i\varepsilon}$$

$$\mathrm{Im}M_I(s') \longrightarrow \mathrm{disc}M_I(s) = \frac{1}{2i}\left(M_I(s+i\varepsilon) - M_I(s-i\varepsilon)\right)$$

$$\begin{aligned}
M_0(s) &= a_0 + b_0 s + c_0 s^2 + \frac{s^3}{\pi}\int \frac{ds'}{s'^3}\,\frac{\mathrm{disc}M_0(s')}{s'-s-i\varepsilon}\,,\\
M_1(s) &= a_1 + b_1 s + \frac{s^2}{\pi}\int \frac{ds'}{s'^2}\,\frac{\mathrm{disc}M_1(s')}{s'-s-i\varepsilon}\,,\\
M_2(s) &= a_2 + b_2 s + c_2 s^2 + \frac{s^3}{\pi}\int \frac{ds'}{s'^3}\,\frac{\mathrm{disc}M_2(s')}{s'-s-i\varepsilon}\,.
\end{aligned}$$

$\eta \longrightarrow 3\pi$ **beyond** p^4

AL: Lowest order is $M(s,t,u) = \dfrac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$

zero at $s_4/3\,m_\pi^2$: remains in the neighbourhood:

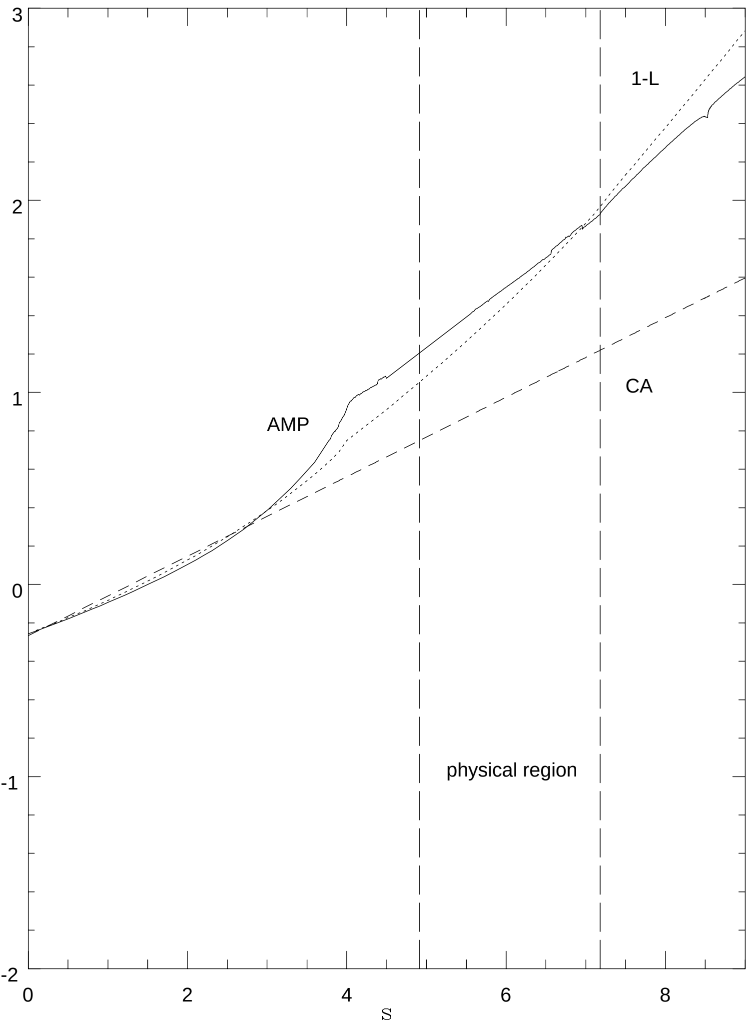
match position of s_A and slope of Adler zero.

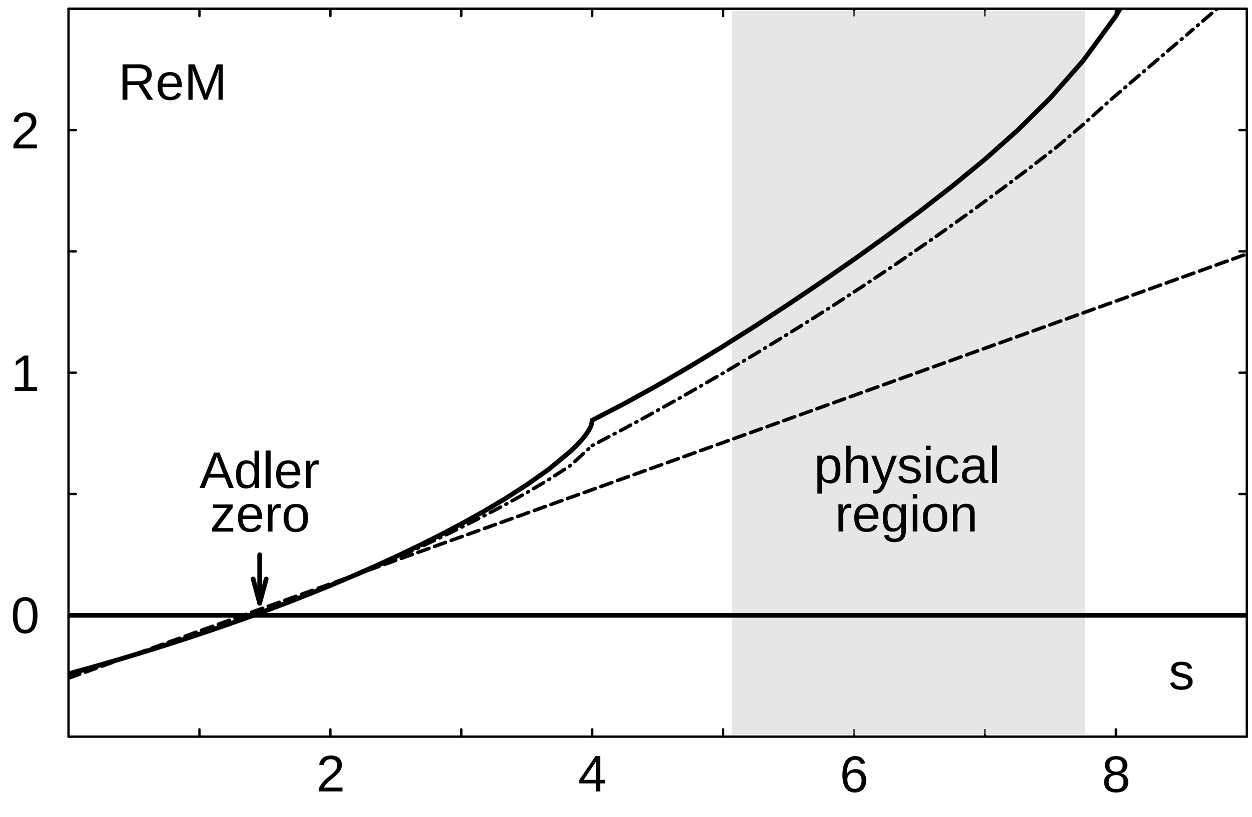
KWW: fix amplitude at some place(s) in s,t,u plane to be equal to p^4

Both find moderate increases over p^4: ~ 15% in amplitude

Dalitzplot distributions provide a check

$\eta \to 3\pi$ beyond p^4





Two Loop Calculation: why

- In $K_{\ell 4}$ dispersive gave about half of p^6 in amplitude
- Same order in ChPT as masses for consistency check on m_u/m_d
- Check size of 3 pion dispersive part
- Technology exists:
  - Two-loops: Amorós,JB,Dhonte,Talavera,...
  - Dealing with the mixing π^0-η: Amorós,JB,Dhonte,Talavera 01

Status

$A(s,t,u) = \sin\epsilon\; M(s,t,u)$ $\qquad\qquad \epsilon \approx \frac{\sqrt{3}}{4}\frac{m_d-m_u}{m_s-\hat{m}}$

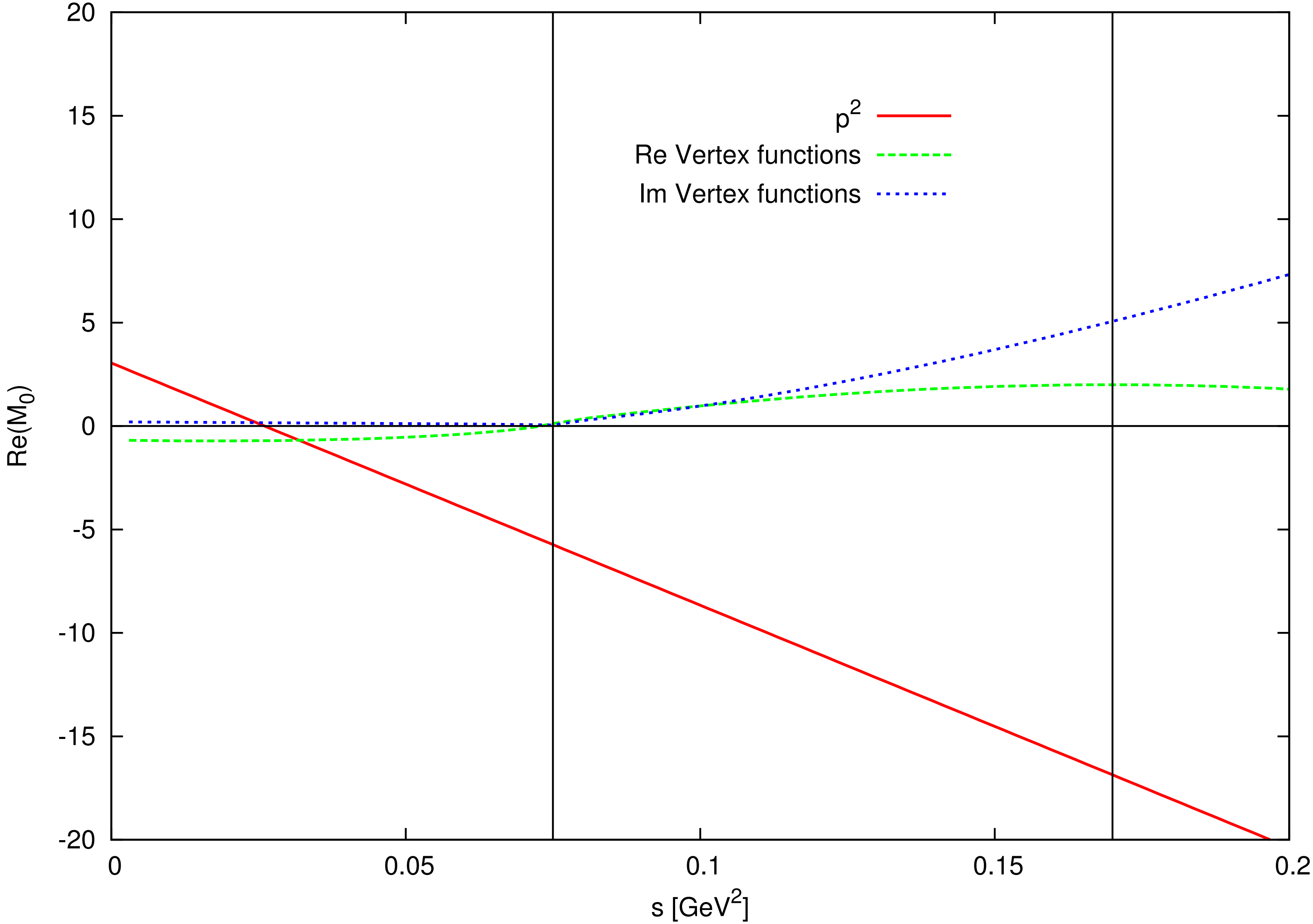
$M(s,t,u) =$

$M_0(s) + (s-u)M_1(t) + (s-t)M_1(t) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$

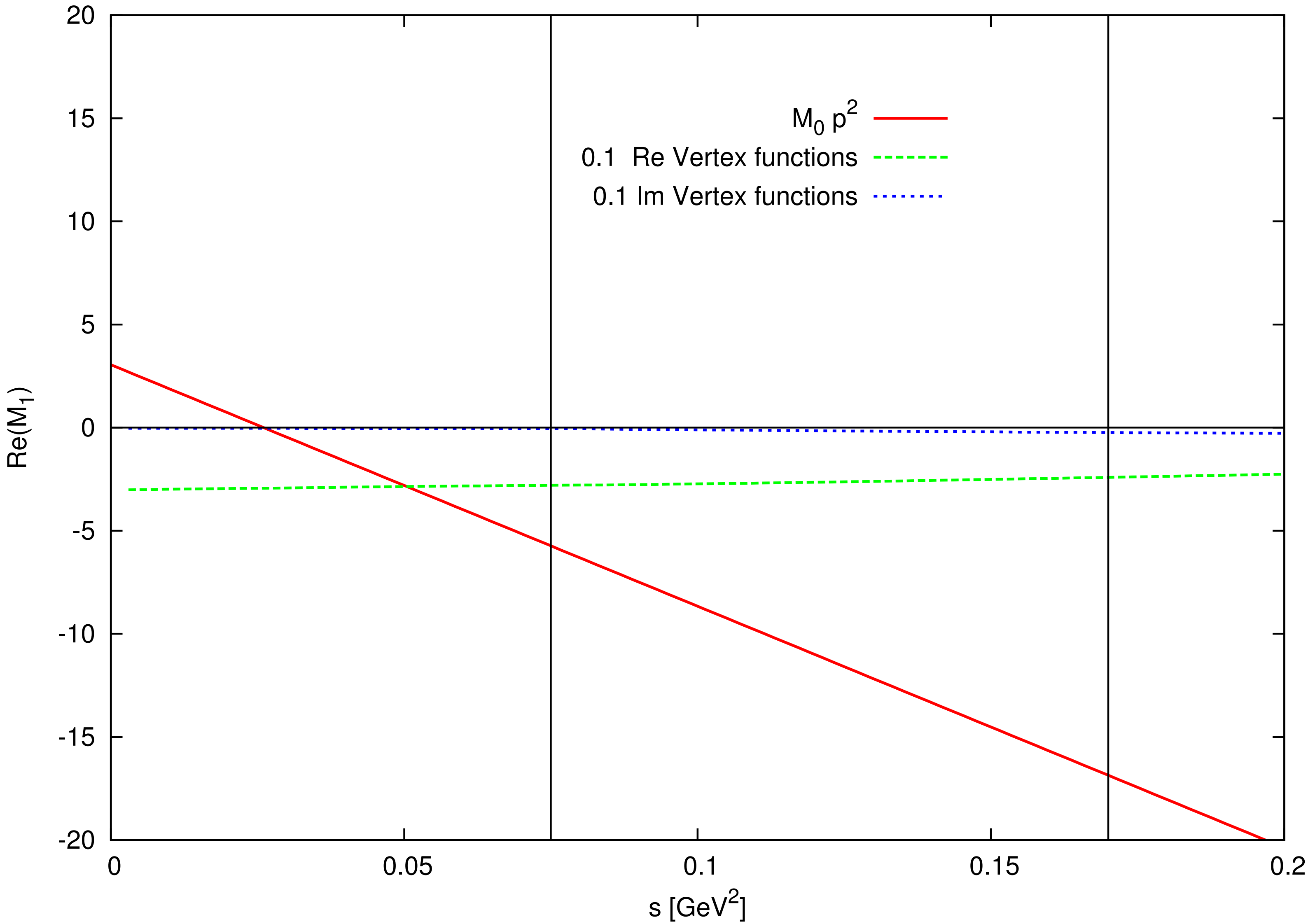
$M^{(2)} = \frac{1}{F_\pi^2}\left(\frac{4}{3}m_\pi^2 - s\right)$

	JB	Ghorbani	Talavera
p^4	All agree and with GL		
p^6 ∞	All cancel		
p^6 Vertex	Checked and agree		in progress
p^6 Rest	Comparison in progress		
p^6 Numerics	Very preliminary		Very preliminary

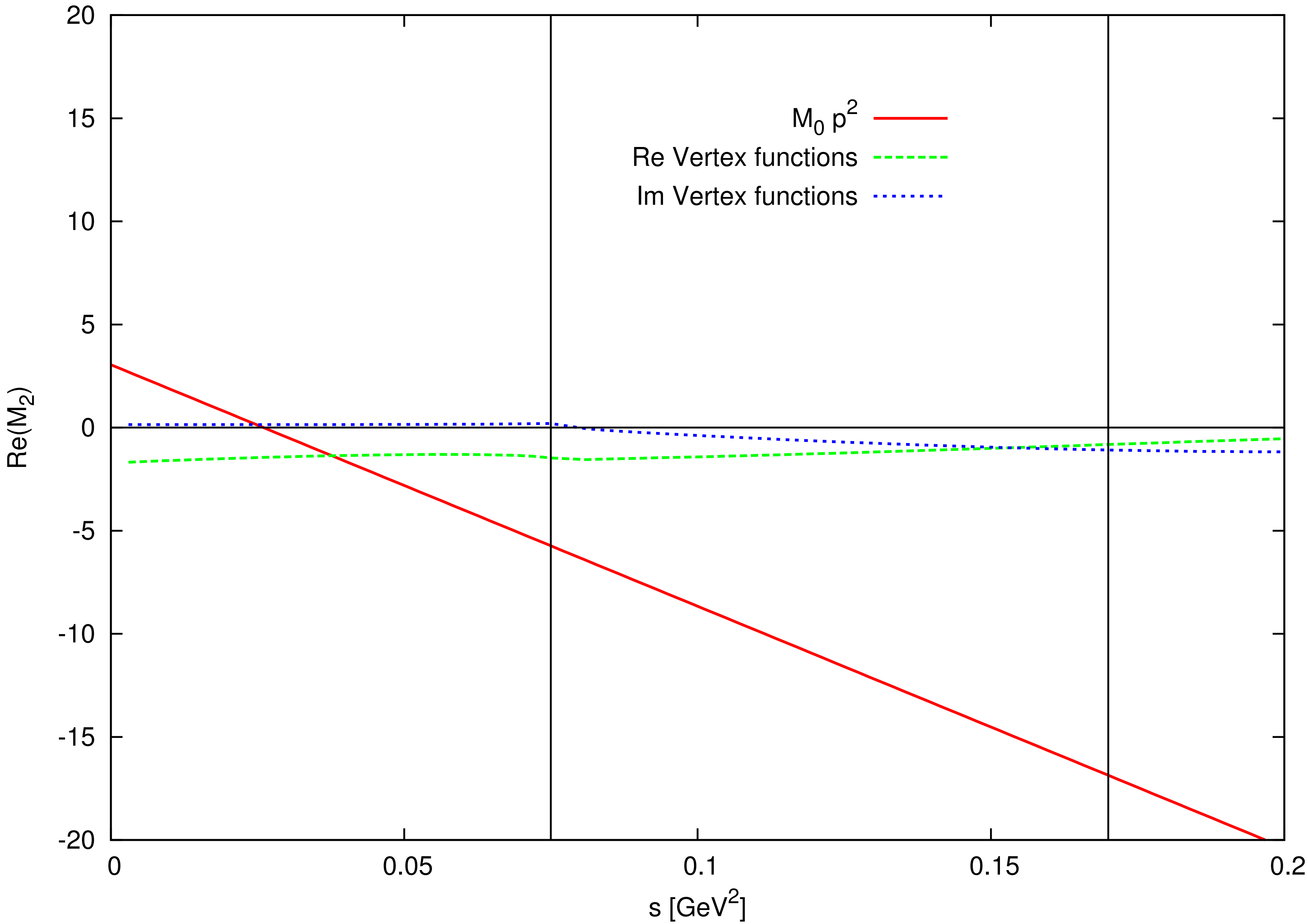
M_0: Vertex function contribution



M_1: Vertex function contribution



M_2: Vertex function contribution



Conclusions

- PQChPT
  - PQChPT at two loops: basic calculations done
  - For formulas: `http://www.thep.lu.se/~bijnens/chpt.html`
  - Numerical programs: ask me (or a collaborator)
  - Photons now also included (but NLO)
  - ΔM^2 and ΔF can be done with quenched photons
  - Lattice: Please use (and cite ☺) our work
- $\eta \to 3\pi$: All three have expressions: do we agree?