KAON PHYSICS: SOME TOPICS

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Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Kaon: What it isn't

www.kaon.org & www.kaon.com are also not "our" kaon

“Our” kaon

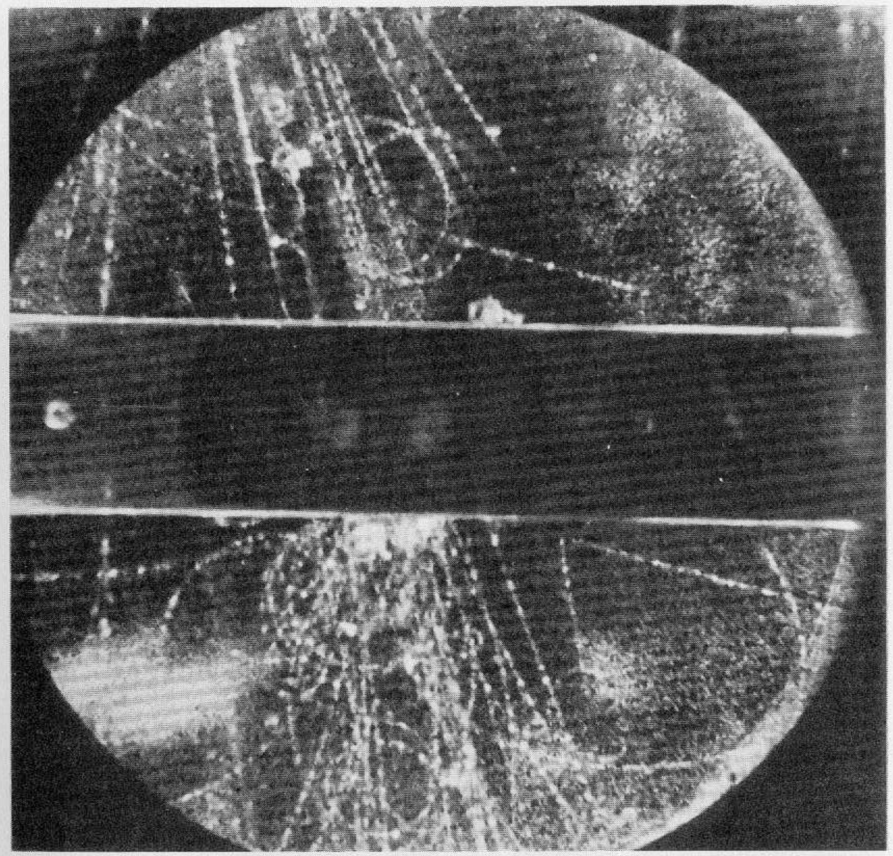

Figure 13-2 First picture showing a neutral particle decaying into 2 pions. Such a neutral particle is now called a K^0 particle. Originally the authors called them V particles. [From G. D. Rochester and C. C. Butler, *Nature*, **160**, 855 (1947).]

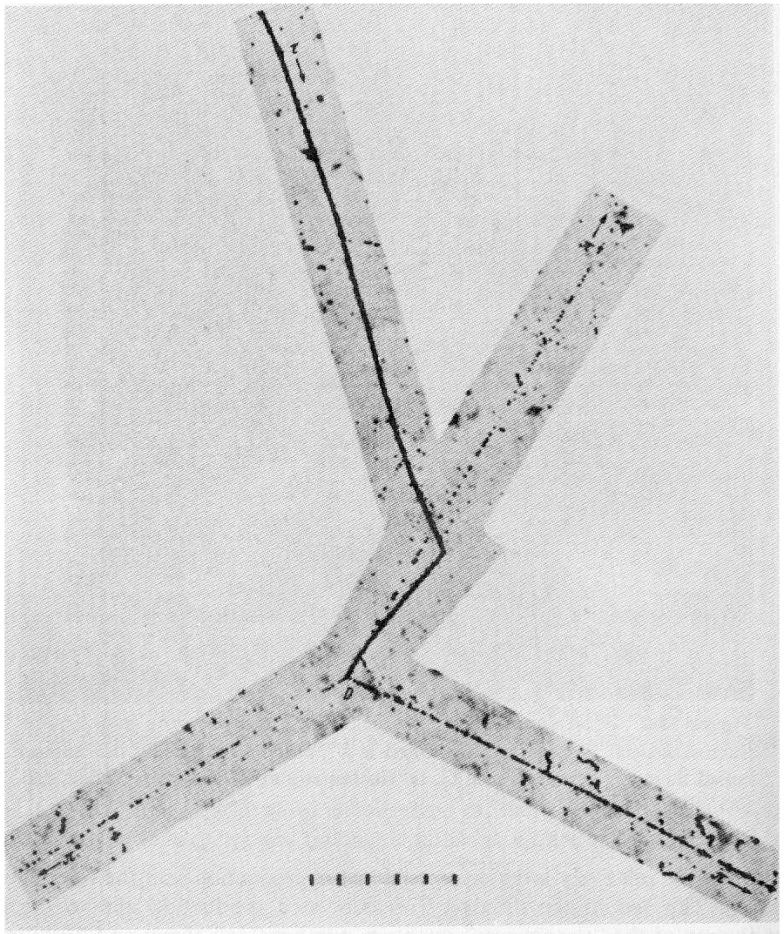

Figure 13-3 Decay of a τ meson (in modern notation $K^{\pm}$) into three pions. [Hodgson, 1951, from (PFP 59).]

Kaons from 62 and 58 years ago:

V-particles and a $K^+ \to \pi^+\pi^+\pi^-$ decay

Overview

- What do we want to learn about?
- Basic proporties: m_K and F_K
- Hadronic decays: $K \to \pi\pi$
    - $\delta_0 - \delta_2$
    - ε'/ε
    - In two-flavour ChPT
- Semi-leptonic decays
- Rare decays
- $K \to \pi$ vector form-factors

What do we want to learn about?

Standard Model Lagrangian has four parts:

$$\underbrace{\mathcal{L}_H(\phi)}_{\text{Higgs}} + \underbrace{\mathcal{L}_G(W,Z,G)}_{\text{Gauge}} \underbrace{\sum_{\psi=\text{fermions}} \bar\psi i \not{D}\,\psi}_{\text{gauge-fermion}} + \underbrace{\sum_{\psi,\psi'=\text{fermions}} g_{\psi\psi'}\bar\psi\phi\psi'}_{\text{Yukawa}}$$

What is tested ?:

gauge-fermion	Very well
Higgs	Real tests coming up
Gauge	Well tested in QCD, partly in electroweak
Yukawa	Rather well tested in B and K (Nobel 2008)

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Discrete symmetries: C, P, T
QCD and QED: C,P,T good; Field theory: CPT good
Weak: breaks P and C, Yukawa breaks CP

What do we want to learn about?

- Yukawa sector
- QCD at low energies
- Possible beyond Standard Model effects

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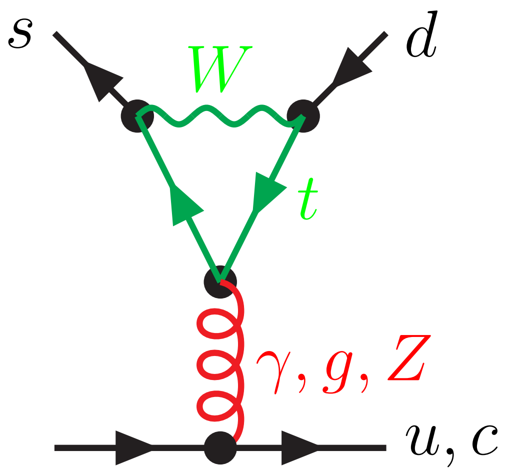


Heavy particles can contribute in loop

The problem

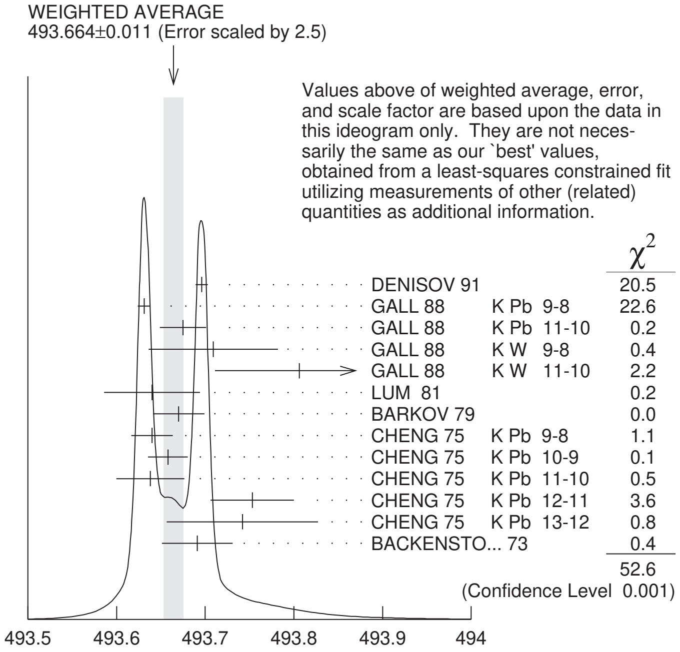
ENERGY SCALE	FIELDS	Effective Theory
M_W	$W, Z, \gamma, g;$ $\tau, \mu, e, \nu_\ell;$ t, b, c, s, u, d	Standard Model
	$\Downarrow$ using OPE	
$\lesssim m_c$	$\gamma, g; \mu, e, \nu_\ell;$ s, d, u	QCD,QED,$\mathcal{H}^{\lvert\Delta S\rvert=1,2}_{\text{eff}}$
	$\Downarrow$???	
M_K	$\gamma; \mu, e, \nu_\ell;$ π, K, η	CHPT

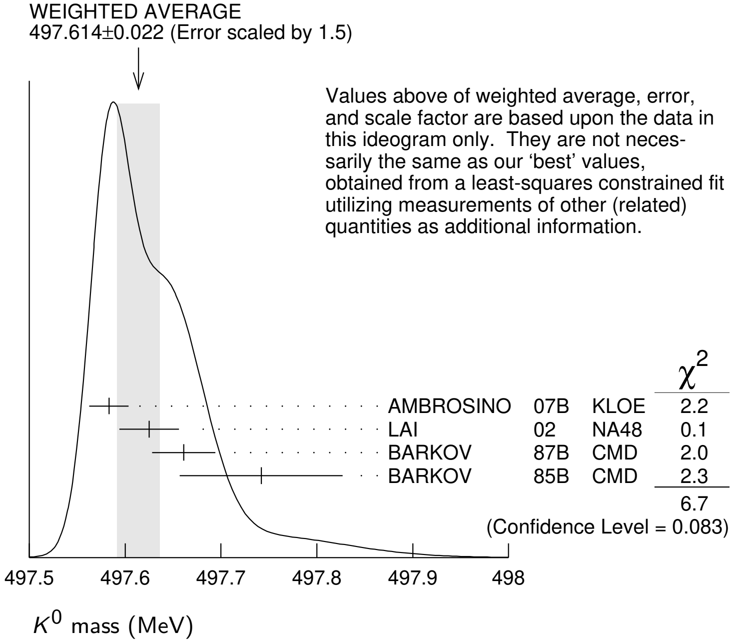
The problem

ENERGY SCALE	FIELDS	Effective Theory
M_W	W, Z, γ, g; τ, μ, e, ν_ℓ; t, b, c, s, u, d	Standard Model
	$\Downarrow$ using OPE	3-loop: Gorbahn et al.
$\lesssim m_c$	γ, g; μ, e, ν_ℓ; s, d, u	QCD,QED,$\mathcal{H}_{\text{eff}}^{\lvert\Delta S\rvert=1,2}$
	$\Downarrow$???	Lattice, Large N_c,...
M_K	γ; μ, e, ν_ℓ; π, K, η	CHPT

Basic properties: Mass

Mass:



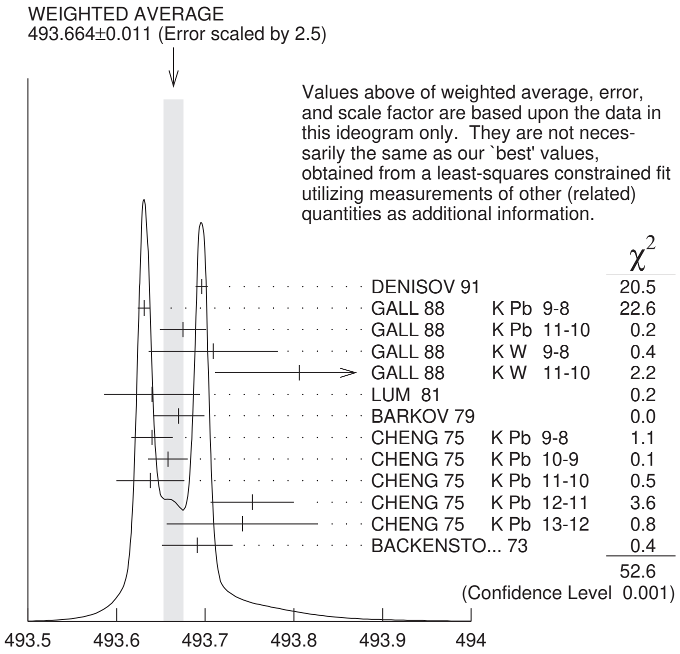


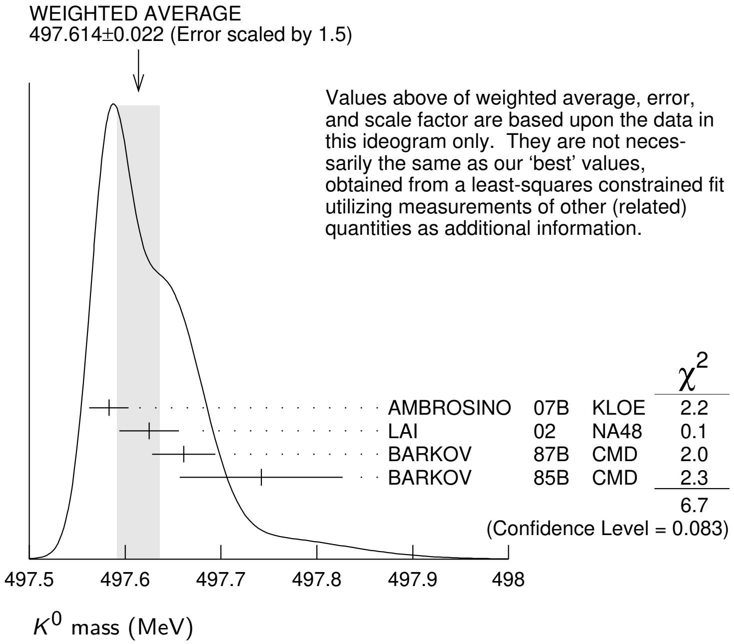
$$m_{K^+} = 493.677 \pm 0.016 \qquad m_{K^0} = 497.614 \pm 0.024$$

PDG2008, our fit

Basic properties: Mass

Mass:





$m_{K^+} = 493.677 \pm 0.016$ $\qquad$ $m_{K^0} = 497.614 \pm 0.024$

PDG2008, our fit

Question: Can KLOE and NA48/NA62 help with the K^+?

Basic properties: Decay constant

- $K_{\ell 2}$: V_{us} and F_K
- $K_{\ell 3}$: V_{us} and $f_+(0)$.

Two measurements, three quantities: need extra input.
See:

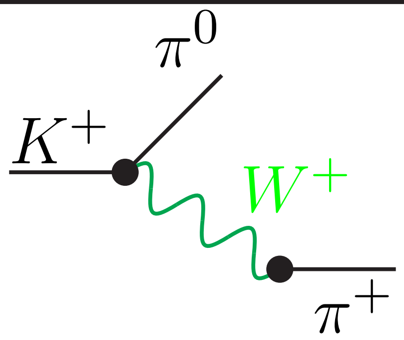
- Flavianet Kaon Working Group talk: B. Sciascia
- Flavianet Lattice Averaging Group: G. Colangelo
- Talks by A. Ramos, Z.h. Guo

$F_K/F_\pi = 0.193 \pm 0.002 \pm 0.006 \pm 0.001$ PDG2008
$F_K = 109.96 \pm 0.014 \pm 0.58 \pm 0.14$ MeV

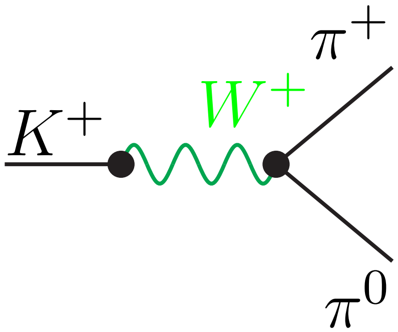
Uncertainty: decay rate, V_{us}, radiative corrections

$K \to \pi\pi$ and the $\Delta I = 1/2$ Rule

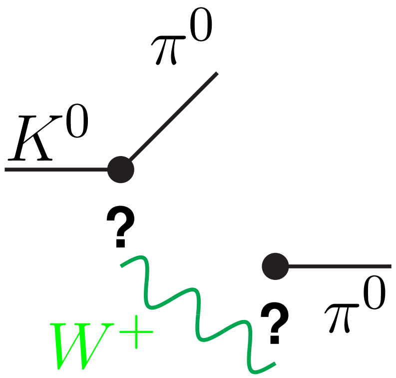
$K^+ \longrightarrow \pi^+\pi^0$:



$+$



$K^0 \longrightarrow \pi^0\pi^0$:



: IMPOSSIBLE

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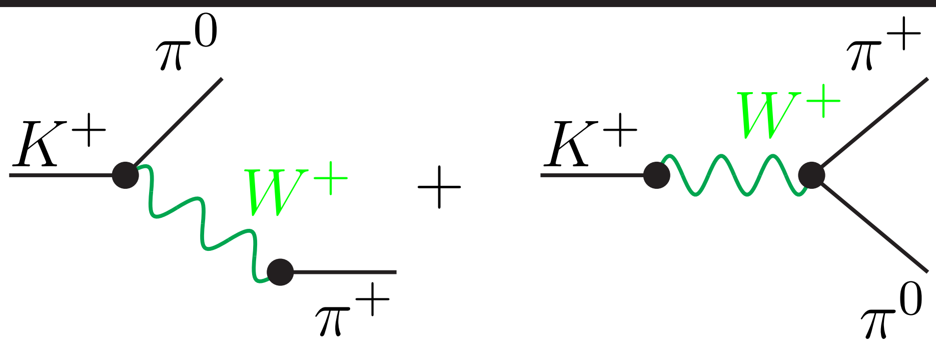

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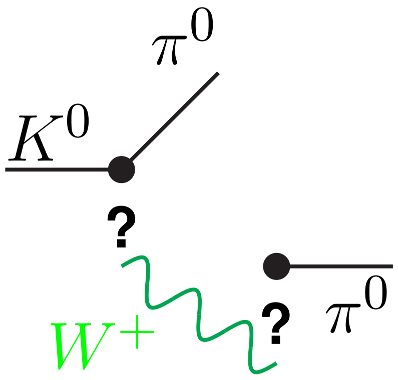

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Experimentally:

$$\Gamma(K^0 \longrightarrow \pi^0\pi^0) = \frac{1}{2}\Gamma(K_S \longrightarrow \pi^0\pi^0) = 2.3\ 10^{-12}\ \text{MeV}$$

$$\Gamma(K^+ \longrightarrow \pi^+\pi^0) = \qquad = 1.1\ 10^{-14}\ \text{MeV}$$

So the zero one is the largest !!!

Isospin amplitudes

$$
\begin{aligned}
A[K^0 \to \pi^0\pi^0] &\equiv \sqrt{\frac{1}{3}}A_0 - \sqrt{\frac{2}{3}}A_2 \\
A[K^0 \to \pi^+\pi^-] &\equiv \sqrt{\frac{1}{3}}A_0 + \frac{1}{\sqrt{6}}A_2 \\
A[K^+ \to \pi^+\pi^0] &\equiv \frac{\sqrt{3}}{2}A_2
\end{aligned}
$$

$\left|\frac{A_0}{A_2}\right| = 22.4$ and the naive gives $\sqrt{2}$

$\left|\frac{A_0}{A_2}\right| = 18$ (p^2 part) fit to $K \to 2\pi, 3\pi$ up to order p^4 JB, Borg 2005

For later use: $A_I = -ia_I e^{i\delta_I}$

$\delta_0 - \delta_2$

New analysis a_0, a_2 and $\delta_0 - \delta_2$ Cirigilano, Ecker, Pich, 0907.1451

Older:

Cirigliano, Ecker, Neufeld, Pich, hep-ph/0310351
JB, Borg, hep-ph/0405025, hep-ph/0410333, hep-ph/0501163

$\delta_0 - \delta_2 = (60.8 \pm 2.2 \pm 3.1)^o = (57.9 \pm 1.5 \pm ?)^o$ (NLO fit for all)

Now fit to data but only use ChPT at order p^4 for the isospin breaking parts

$\delta_0 - \delta_2 = (52.84 \pm 0.83)^o$

Change 3^o from experiment and 5^o from theory.

$\delta_0 - \delta_2$

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$\delta_0 - \delta_2 = (52.84 \pm 0.83)^o$

$\delta_0 - \delta_2 = (47.7 \pm 1.5)^o$ $\pi\pi$ Colangelo, Gasser, Leutwyler

$$\left|\frac{A_0}{A_2}\right| = 21.63 \pm 0.04$$

CP-violation: ε and ε'

Define: $\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$ $\qquad \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}$

$$\varepsilon = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})}$$

$$\varepsilon' = \frac{1}{\sqrt{2}}\left(\frac{A(K_L \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})} - \varepsilon\frac{A(K_S \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})}\right)$$

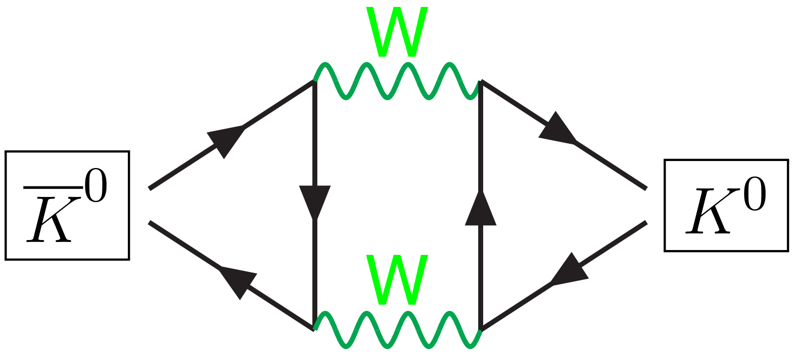
with approximations $|\mathrm{Re}a_0| >> |\mathrm{Re}a_2| >> |\mathrm{Im}a_0|, |\mathrm{Im}a_2|$, $|\varepsilon|, |\tilde{\varepsilon}| << 1$, $|\varepsilon'| << |\varepsilon|$, $\Delta m = m_L - m_S \approx \frac{\Delta\Gamma}{2}$, $\Gamma_L << \Gamma_S$ and Γ_{12} dominated by $\pi\pi$

$\Longrightarrow$

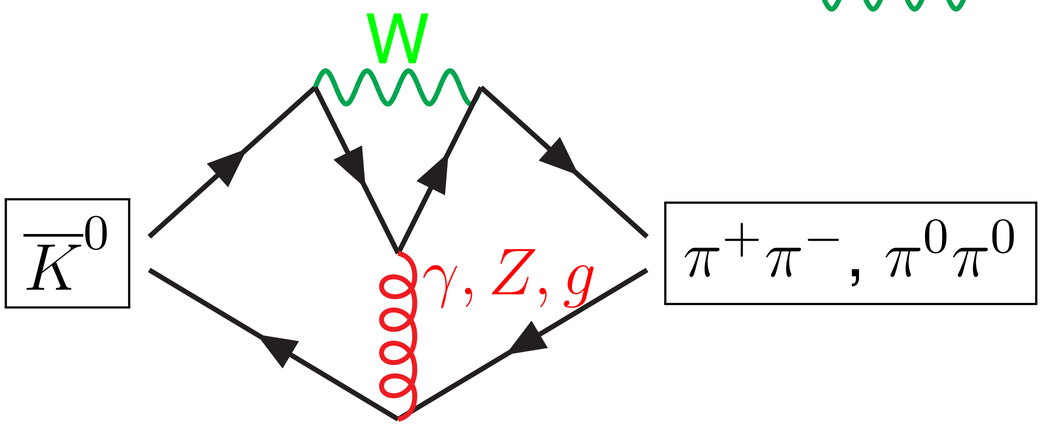
$$\begin{aligned} \eta_{+-} &= \varepsilon + \varepsilon' \\ \eta_{00} &= \varepsilon - 2\varepsilon' \end{aligned}$$

Experiment

$|\varepsilon| = 2.28 \cdot 10^{-3}$ from



ε' from



Experiment

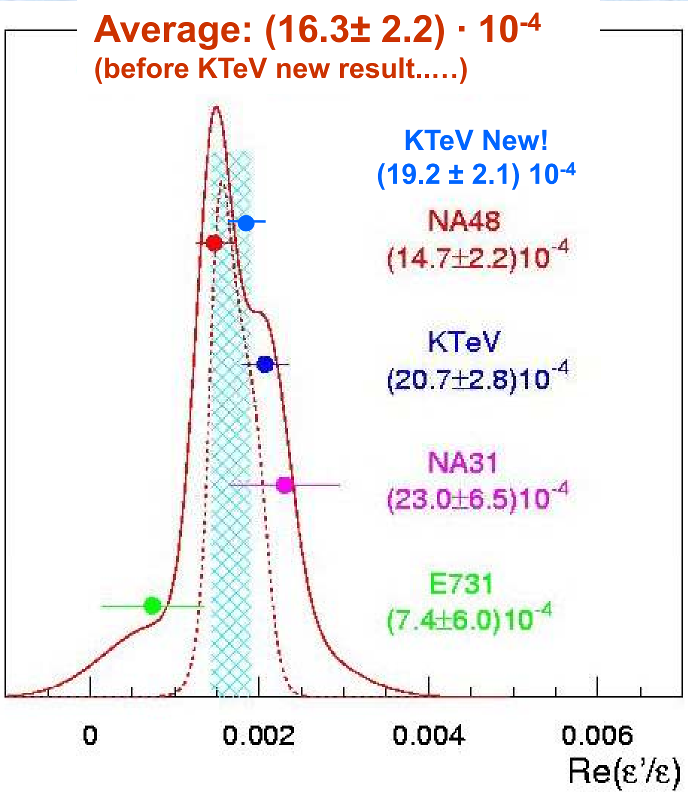
$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{1}{6}\left\{1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right\}$$

NA31	$(23.0 \pm 6.5) \times 10^{-4}$
E731	$(7.4 \pm 5.9) \times 10^{-4}$
KTeV 99	$(28.0 \pm 4.1) \times 10^{-4}$
KTeV 03	$(20.7 \pm 2.8) \times 10^{-4}$
KTeV 09	$(19.2 \pm 2.1) \times 10^{-4}$
NA48 97	$(18.5 \pm 7.3) \times 10^{-4}$
NA48 98	$(12.2 \pm 4.9) \times 10^{-4}$
NA48 03	$(14.7 \pm 2.2) \times 10^{-4}$
ALL	$(16.8 \pm 1.4) \times 10^{-4}$

Overview

ε'/ε Result

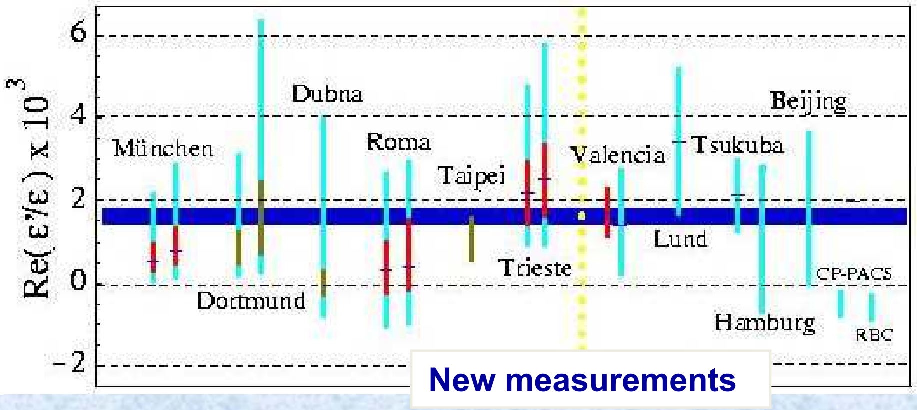
Re(ε'/ε) = (14.7 ± 2.2) x 10^{-4}



Observation of Direct CP Violation in $K^0 \rightarrow 2\pi$ decays

Tiny particle - antiparticle asimmetry

Theoretical predictions (SM)



KAON09, June 9-12, 2009 Monica Pepe - INFN Perugia 9

Theory for ε'/ε

Analytic: Flavianet: Lund/Granada, Valencia, Marseille

all some years old

Time for some new attempts?

Theory for ε'/ε

Analytic: Flavianet: Lund/Granada, Valencia, Marseille

all some years old

Time for some new attempts?

Lattice: Mainly concentrated on A_2 part

reason: loose fermion loops or eye graphs

ChPT: needed for extrapolations

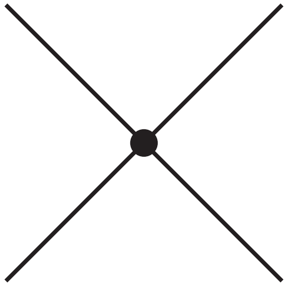
Three-flavour ChPT: large corrections Two-flavour possible?

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting in momenta/masses

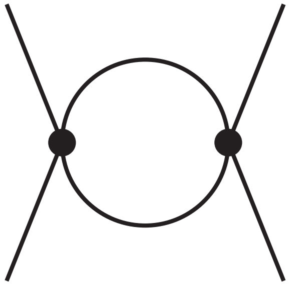
Power counting in momenta: Meson loops



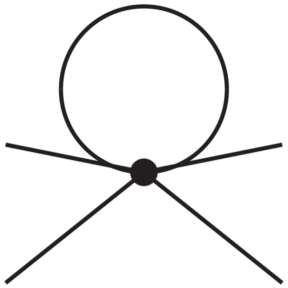
p^2

$1/p^2$

$\int d^4p$ $\quad p^4$



$$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$$



$$(p^2)\,(1/p^2)\,p^4 = p^4$$

Hard pion ChPT?

- Baryon and Heavy Meson ChPT: $p, n, \ldots\ B, B^*$ or D, D^*
  - $p = M_B v + k$
  - Everything else soft
  - Works because baryon or b or c number conserved.
  - Decay constant works: takes away all heavy momentum
  - General idea: M_B dependence can always be reabsorbed in LECs, is analytic in the other parts k.

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  - Decay constant works: takes away all heavy momentum
  - General idea: M_B dependence can always be reabsorbed in LECs, is analytic in the other parts k.
- (Heavy) (Vector or other) Meson ChPT:
  - (Vector) Meson: $p = M_V v + k$
  - Everyone else soft or $p = M_V + k$
  - General idea: M_V dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- (Heavy) (Vector) Meson ChPT:
  - $p = M_V v + k$
  - First: only keep diagrams where vectors always present
  - Applied to masses and decay constants
  - Decay constant works: takes away all heavy momentum
  - *It was argued that this could be done, the nonanalytic parts of diagrams with pions at large momenta are reproduced correctly*
  - Done both in relativistic and heavy meson type of formalism

Hard pion ChPT?

- Heavy Kaon ChPT:
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon always goes through
  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,...
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
  - Works like all the previous *heavy* ChPT

Hard pion ChPT?

- Heavy Kaon ChPT:
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  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,...
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
- Flynn-Sachrajda also argued that $K_{\ell 3}$ could be done for q^2 away from q^2_{max}.
- JB-Celis Argument generalizes to other processes with hard/fast pions and applied to $K \to \pi\pi$
- General idea: heavy/fast dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q\to 0}\langle\pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k,O\right]|\beta\rangle\,,$$

Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q\to 0}\langle\pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

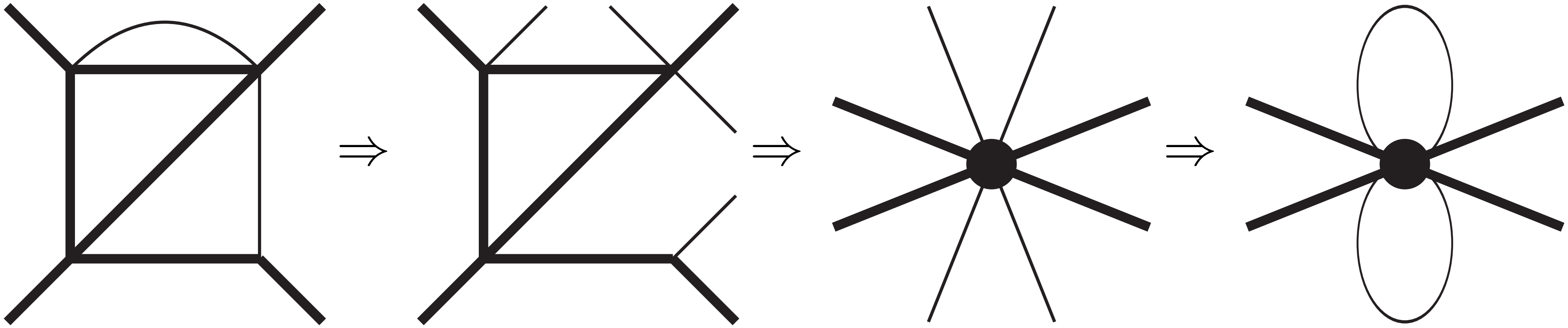
- Nothing prevents hard pions to be in the states α or β
- So by heavily using current algebra I should be able to get the light quark mass nonanalytic dependence

Hard pion ChPT?

Field Theory: a process at given external momenta

- Take a diagram with a particular internal momentum configuration
- Identify the soft lines and cut them
- The result part is analytic in the soft stuff
- So should be describably by an effective Lagrangian with coupling constants dependent on the external given momenta
- If symmetries present, Lagrangian should respect them
- Lagrangian should be complete in *neighbourhood*
- Loop diagrams with this effective Lagrangian *should* reproduce the nonanalyticities in the light masses
  Crucial part of the argument

Hard pion ChPT?



This procedure works at one loop level, matching at tree level, nonanalytic dependence at one loop:

- Toy models and vector meson ChPT JB, Gosdzinsky, Talavera
- Recent work on relativistic meson ChPT Gegelia, Scherer et al.
- Extra terms kept in $K \to 2\pi$: a one-loop check
- Some preliminary two-loop checks

$K \to 2\pi$ in $SU(2)$ ChPT

Add $K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$ Roessl

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{F^2}{4}\left(\langle u_\mu u^\mu \rangle + \langle \chi_+ \rangle\right),$$
$$\mathcal{L}^{(1)}_{\pi K} = \nabla_\mu K^\dagger \nabla^\mu K - \overline{M}^2_K K^\dagger K\,,$$
$$\mathcal{L}^{(2)}_{\pi K} = A_1 \langle u_\mu u^\mu \rangle K^\dagger K + A_2 \langle u^\mu u^\nu \rangle \nabla_\mu K^\dagger \nabla_\nu K + A_3 K^\dagger \chi_+ K + \cdots$$

Add a spurion for the weak interaction $\Delta I = 1/2$, $\Delta I = 3/2$

JB,Celis

$$t^{ij}_k \longrightarrow t^{i'j'}_{k'} = t^{ij}_k (g_L)_{k'}{}^k (g_L^\dagger)_i{}^{i'} (g_L^\dagger)_j{}^{j'}$$
$$t^i_{1/2} \longrightarrow t^{i'}_{1/2} = t^i_{1/2} (g_L^\dagger)_i{}^{i'}.$$

$K \to 2\pi$ in $SU(2)$ ChPT

The $\Delta I = 1/2$ terms: $\tau_{1/2} = t_{1/2} u^\dagger$

$$\begin{aligned}\mathcal{L}_{1/2} &= iE_1\,\tau_{1/2}K + E_2\,\tau_{1/2}u^\mu\nabla_\mu K + iE_3\langle u_\mu u^\mu\rangle\tau_{1/2}K \\ &\quad +iE_4\tau_{1/2}\chi_+K + iE_5\langle\chi_+\rangle\tau_{1/2}K + E_6\tau_{1/2}\chi_-K \\ &\quad +E_7\langle\chi_-\rangle\tau_{1/2}K + iE_8\langle u_\mu u_\nu\rangle\tau_{1/2}\nabla^\mu\nabla^\nu K + \cdots + h.c.\,.\end{aligned}$$

Note: higher order terms kept in both $\mathcal{L}_{1/2}$ and $\mathcal{L}^{(2)}_{\pi K}$ to check the arguments

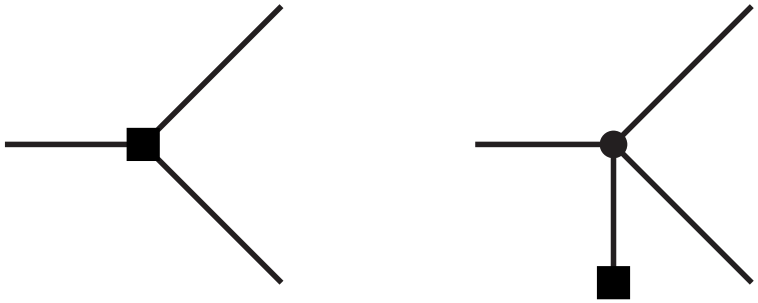
Using partial integration,. . . :

$\langle\pi(p_1)\pi(p_2)|O|K(p_K)\rangle =$

$f(\overline{M}^2_K)\langle\pi(p_1)\pi(p_2)|\tau_{1/2}K|K(p_K)\rangle + \lambda M^2 + \mathcal{O}(M^4)$

O any operator in $\mathcal{L}_{1/2}$ or with more derivatives. Similar for $\mathcal{L}_{3/2}$

Tree level

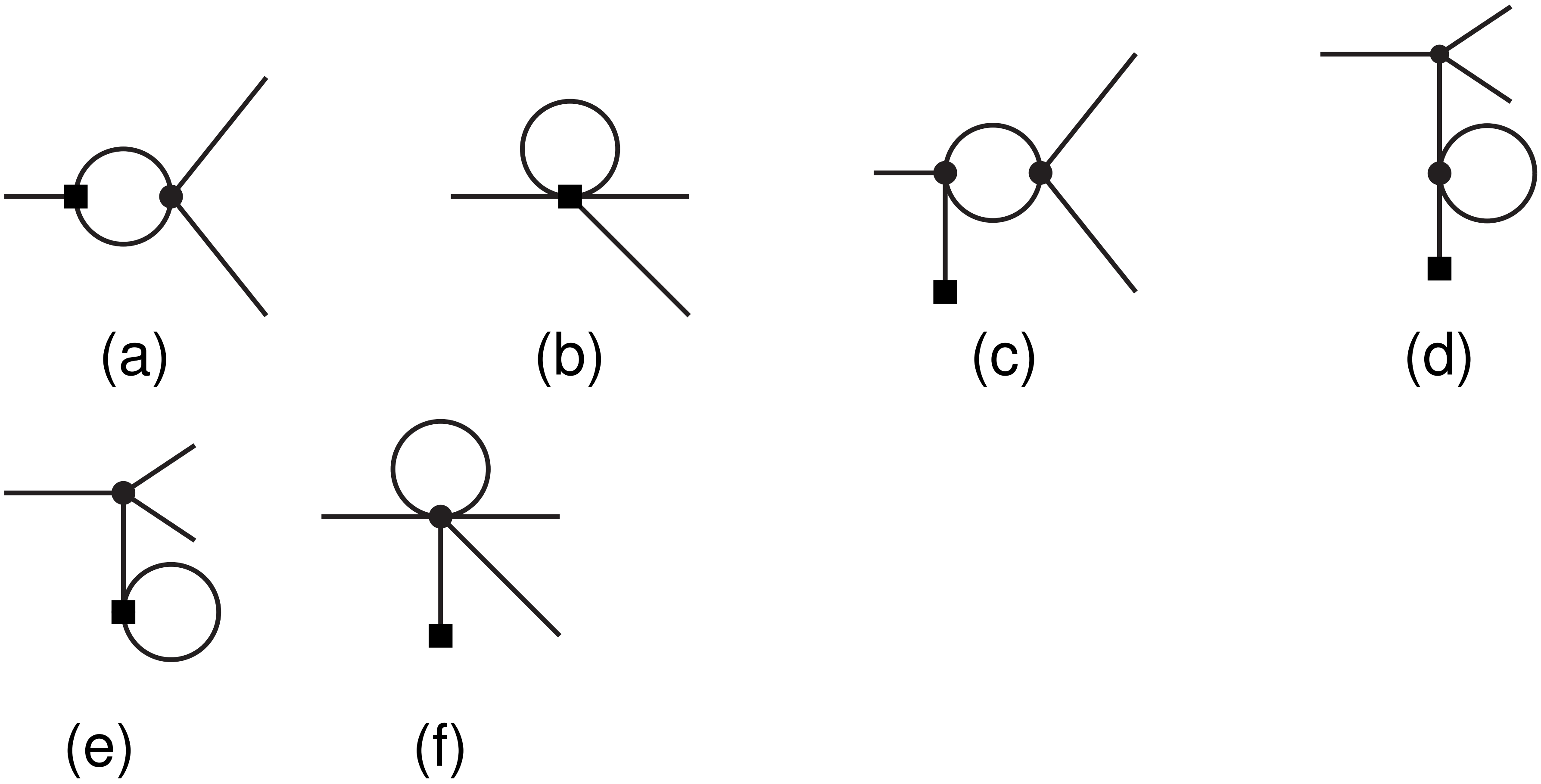


(a) (b)

$$A_0^{LO} = \frac{\sqrt{3}i}{2F^2}\left[-\frac{1}{2}E_1 + (E_2 - 4E_3)\,\overline{M}_K^2 + 2E_8\overline{M}_K^4 + A_1E_1\right]$$

$$A_2^{LO} = \sqrt{\frac{3}{2}}\frac{i}{F^2}\left[(-2D_1 + D_2)\,\overline{M}_K^2\right]$$

One loop



One loop

Diagram	A_0	A_2
Z	$-\frac{2F^2}{3}A_0^{LO}$	$-\frac{2F^2}{3}A_2^{LO}$
(a)	$\sqrt{3}i\left(-\frac{1}{3}E_1+\frac{2}{3}E_2\overline{M}_K^2\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{2}{3}D_2\overline{M}_K^2\right)$
(b)	$\sqrt{3}i\left(-\frac{5}{96}E_1-\left(\frac{7}{48}E_2+\frac{25}{12}E_3\right)\overline{M}_K^2+\frac{25}{24}E_8\overline{M}_K^4\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{61}{12}D_1+\frac{77}{24}D_2\right)\overline{M}_K^2$
(e)	$\sqrt{3}i\frac{3}{16}A_1E_1$	
(f)	$\sqrt{3}i\left(\frac{1}{8}E_1+\frac{1}{3}A_1E_1\right)$	

The coefficients of $\overline{A}(M^2)/F^4$ in the contributions to A_0 and A_2. Z denotes the part from wave-function renormalization.

- $\overline{A}(M^2) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2}$
- $K\pi$ intermediate state does not contribute, but did for Flynn-Sachrajda

One-loop

$$
\begin{aligned}
A_0^{NLO} &= A_0^{LO}\left(1+\frac{3}{8F^2}\overline{A}(M^2)\right)+\lambda_0 M^2+\mathcal{O}(M^4)\,,\\
A_2^{NLO} &= A_2^{LO}\left(1+\frac{15}{8F^2}\overline{A}(M^2)\right)+\lambda_2 M^2+\mathcal{O}(M^4)\,.
\end{aligned}
$$

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Match with three flavour $SU(3)$ calculation Kambor, Missimer, Wyler; JB, Pallante, Prades

$$A_0^{(3)LO} = -\frac{i\sqrt{6}CF_0^4}{\overline{F}_K F^2}\left(G_8+\frac{1}{9}G_{27}\right)\overline{M}_K^2\,,\qquad A_2^{(3)LO} = -\frac{i10\sqrt{3}CF_0^4}{9\overline{F}_K F^2}G_{27}\overline{M}_K^2\,,$$

When using $F_\pi = F\left(1+\frac{1}{F^2}\overline{A}(M^2)+\frac{M^2}{F^2}l_4^r\right)\,,\ F_K = \overline{F}_K\left(1+\frac{3}{8F^2}\overline{A}(M^2)+\cdots\right)\,,$

logarithms at one-loop agree with above

A two-loop check

- Similar arguments to JB-Celis, Flynn-Sachrajda work for the pion vector and scalar formfactor
- Therefore at any t the chiral log correction must go like the one-loop calculation.
- But note the one-loop log chiral log is with $t >> m_\pi^2$
- Predicts

$$F_V(t, M^2) = F_V(t, 0)\left(1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$
$$F_S(t, M^2) = F_S(t, 0)\left(1 - \frac{5}{2}\frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

- Note that $F_{V,S}(t, 0)$ is now a coupling constant and can be complex

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- Take the full two-loop ChPT calculation JB,Colangelo,Talavera and expand in $t >> m_\pi^2$.

A two-loop check

Full two-loop ChPT JB,Colangelo,Talavera, expand in $t >> m_\pi^2$:

$$F_V(t,M^2) = F_V(t,0)\left(1 - \frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$
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with

$$F_V(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(\frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6}\ln\frac{t}{\mu^2}\right)$$
$$F_S(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(1 - 16\pi^2 l_4^r + i\pi - \ln\frac{t}{\mu^2}\right)$$

- The needed coupling constants are complex
- Both calculations have two-loop diagrams with overlapping divergences
- The chiral logs should be valid for any t where a pointlike interaction is a valid approximation

Semileptonic Decays

- $K \to \ell\nu$: known to order p^6 including isospin breaking and electromagnetic corrections. Also important for lepton-universality tests with $\pi_{e2}/\pi_{\mu2}$ and $K_{e2}/K_{\mu2}$

  Cirigliano-Rosell NA62

- $K \to \pi\ell\nu$: known to order p^6, isospin breaking included, electromagnetic corrections also studied in detail

  JB-Ghorbani, Kastner-Neufeld, Cirigilano-Gianonotti-Neufeld NA48, KLOE, KTeV

  $\Longrightarrow V_{us}$

- $K \to \pi\pi\ell\nu$: F, G and H known to p^6, R only to p^4, isospin breaking studied at one-loop and in nonrelativistic EFT

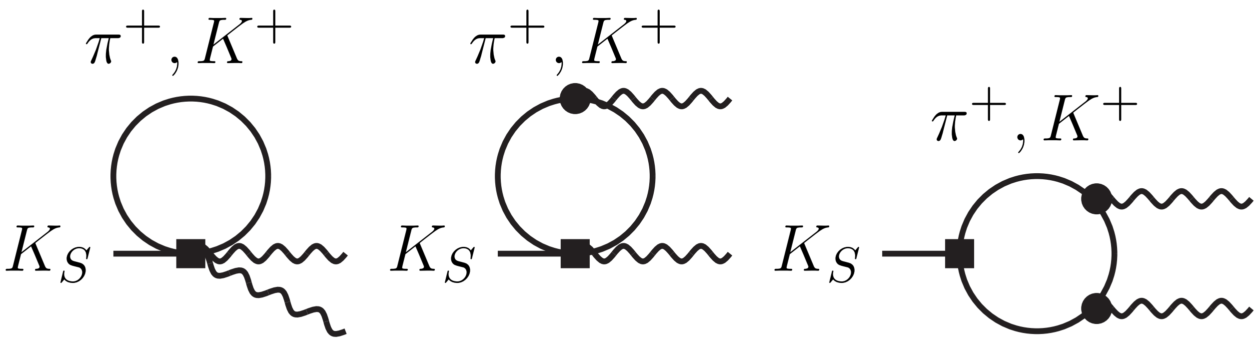
- $K \to \pi\pi\pi\ell\nu$: known to p^2

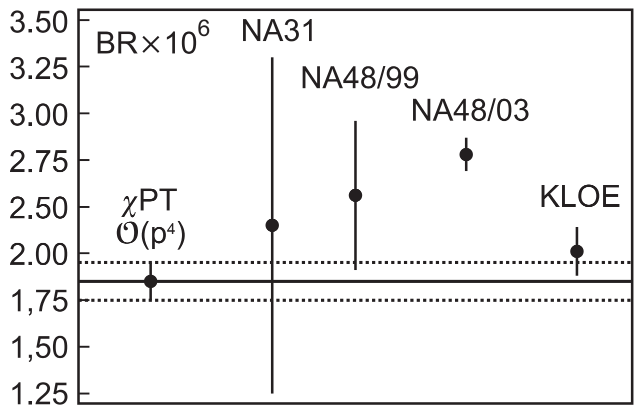
Nonleptonic weak interaction

- Mainly done to one-loop with estimates of higher order corrections
- Big success: prediction of $K_S \to \gamma\gamma$ D'Ambrosio, Espriu, Goity
- Extended to $K \to \pi\ell^+\ell^-$ and $K \to \pi\gamma\gamma$ Ecker, Pich, de Rafael
- Put generally together: Kambor, Missimer, Wyler
- K^0-$\overline{K}^0$, $K \to 2\pi$, $K \to 3\pi$: all done, also including isospin breaking and electromagnetic corrections Kambor, Missimer, Wyler, JB, Pallante, Prades, Dhonte, Borg, Ciriglino, Pich, Ecker
- Already very many parameters at NLO Kambor, Missimer, Wyler, Ecker, Esposito-Farese
- Cusps in $K \to 3\pi$ used for $\pi\pi$ scattering determination Cabbibo, Isidori,...
- Recent review: D'Ambrosio in EFT09

$K_S \to \gamma\gamma$

Well predicted by CHPT at order p^4 from Goity, D'Ambrosio, Espriu





Prediction was: BR = $2.1 \cdot 10^{-6}$

NA48: $2.78(6)(4) \cdot 10^{-6}$ (PLB 551 2003)

KLOE: $2.26(12)(6) \cdot 10^{-6}$ (JHEP 05 (2008) 051)

No full p^6 calculation exists, FSI effects estimated

$K \to \pi$ vector form-factors

The electromagnetic corrections are known to order e^2p^2 Mescia, Smith and the $\delta = m_u - m_d$ to order p^4 MS and p^6 JB,Ghorbani

MS noticed a curious relation to order p^4

$$r(t) \equiv \frac{f_\ell^{K^+\pi^0}(t)\, f_\ell^{K^0\pi^0}(t)}{f_\ell^{K^0\pi^-}(t)\, f_\ell^{K^+\pi^+}(t)} = 1 + \mathcal{O}(\delta^2)$$

BG: this follows from isospin and the Wigner-Eckart theorem $\Longrightarrow$ valid to all orders in ChPT.

$$\begin{aligned}
f_\ell^{K^+\pi^0}(t) &= f_\ell^A(t) + \delta f_\ell^B(t) + \mathcal{O}(\delta^2)\,,\\
f_\ell^{K^0\pi^-}(t) &= f_\ell^A(t) - \delta f_\ell^D(t) + \mathcal{O}(\delta^2)\,,\\
f_\ell^{K^+\pi^+}(t) &= f_\ell^A(t) + \delta f_\ell^D(t) + \mathcal{O}(\delta^2)\,,\\
f_\ell^{K^0\pi^0}(t) &= f_\ell^A(t) - \delta f_\ell^B(t) + \mathcal{O}(\delta^2)\,,
\end{aligned}$$

$K \to \pi$ vector form-factors

	$f_+^{K^+\pi^0}$	$f_+^{K^0\pi^-}$	$f_+^{K^+\pi^+}$	$f_+^{K^0\pi^0}$
order p^2	1.02465	1.00000	1.00000	0.97514
order p^4	−0.01775	−0.02292	−0.02197	−0.02838
order p^6	0.00809	0.01470	0.01391	0.02095
p^6 2-loop	0.00159	0.01145	0.01081	0.02092
p^6 L_i^r-dependent	0.00650	0.00325	0.00309	0.00004
sum of p^2, p^4 and p^6	1.01499	0.99177	0.99194	0.96772

The different contributions to $f_+^{K^i\pi^j}(0)$ using the amplitudes *including isospin breaking*. The part depending on the C_i^r is *not* included. We used here $m_u/m_d = 0.45$.

The long distance part of $K \to \pi\bar{\nu}\nu$ is thus under good control.

If have V_{us}: measure from $K_{\ell 3}^+$ and $K_{\ell 3}^0$ f_ℓ^A and f_ℓ^B.

Conclusions

- Not a full review of kaon physics
- Pointed out some places where more work could be useful
  - m_{K^+}
  - ε'/ε New ideas anyone?
- $K \to \pi\pi$ $SU(2)$ chiral logarithm known
- Vector form-factors for rare decays: can be had fully experimentally