



CHIRAL PERTURBATION THEORY AND MESONS

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Various ChPT: <http://www.thep.lu.se/~bijnens/chpt.html>

Overview

- 50, 40, 35, 30, 25, 20 and 15 years ago
- Chiral Perturbation Theory (ChPT, CHPT, χ PT)
- Expand in which quantities
- Two-flavour ChPT at NNLO: **one mass**
 - Calculations
 - LECs and Quark-mass dependence of m_π^2 , F_π
- Three-flavour ChPT at NNLO: **3-5 masses**
 - Calculations
 - Fits to data; some quark mass dependences
 - What about p^6 LECs; $\eta \rightarrow 3\pi$
- Even more flavours at NNLO (Partially Quenched)
- Renormalization group

Jubileum Papers: 50 years

The start:

- **M. Goldberger and S. Treiman**, *Decay of the pi meson*. Phys. Rev. 110:1178-1184, 1958. (330 citations)
- **Y. Nambu**, *Axial Vector Current Conservation in Weak Interactions*, Phys. Rev. Lett. 4 (1960) 380 (530 citations)
- **M. Gell-Mann and M. Lévy**, *The axial vector current in beta decay*. Nuovo Cim. 16 (1960) 705 (1229 citations)

Jubileum Papers: 40 and 35 years

Tree level and beginning of loops:

- **M. Gell-Mann, R.J. Oakes and B. Renner**, *Behavior of current divergences under $SU(3) \times SU(3)$* , Phys. Rev. 175 (1968) 2195 (1264 citations)
- **S. Coleman, J. Wess and B. Zumino**, *Structure of phenomenological Lagrangians. 1.*, Phys. Rev. 177 (1969) 2239 (1091 citations)
- **C. Callan, S. Coleman, J. Wess and B. Zumino**, *Structure of phenomenological Lagrangians. 2.*, Phys. Rev. 177 (1969) 2247 (932 citations)
- **P. Langacker and H. Pagels**, *Applications of Chiral Perturbation Theory: Mass Formulas and the Decay $\eta \rightarrow 3\pi$* , Phys. Rev. D10:2904,1974.
- **G. Ecker and J. Honerkamp**, *Application of invariant renormalization to the nonlinear chiral invariant pion lagrangian in the one-loop approximation*, Nucl.Phys.B35(1971)481

Jubileum Papers: 30 and 25 years

The restart:

- **Steven Weinberg**, *Phenomenological Lagrangians*, Physica A96 (1979) 327 (1884 citations)
- **J. Gasser and A. Zepeda**, *Approaching The Chiral Limit In QCD*, Nucl. Phys. B174 (1980) 445 (preprint in 1979)
- **Juerg Gasser and Heiri Leutwyler**, *Chiral Perturbation Theory to One Loop*, Annals Phys. 158 (1984) 142 (2407 citations)
- **Juerg Gasser and Heiri Leutwyler**, *Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark* Nucl. Phys. B250 (1985) 465 (2431 citations)
- **J. Bijnens, H. Sonoda and M. Wise**, *On the Validity of Chiral Perturbation Theory for $K^0-\overline{K}^0$ Mixing*, Phys. Rev. Lett. 53 (1984) 2367 [Here is where I started](#)

Jubileum Papers: 20 and 15 years

LECs from elsewhere and first full two-loop:

- G. Ecker, J. Gasser, A. Pich and E. de Rafael, *The Role of Resonances in Chiral Perturbation Theory*, Nucl. Phys. B321 (1989) 311 (826 citations)
- G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, *Chiral Lagrangians for Massive Spin 1 Fields*, Phys. Lett. B223 (1989) 425 (462 citations)
- S. Bellucci, J. Gasser and M.E. Sainio, *Low-energy photon-photon collisions to two loop order*, Nucl. Phys. B423 (1994) 80
- H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory*, Ann. Phys. 235 (1994) 165

Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

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Derivation from QCD:

H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory*,
Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

For lectures, review articles: see

<http://www.thep.lu.se/~bijmens/chpt.html>

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting in momenta/masses

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But
$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Chiral Perturbation Theory

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$

$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken \implies 8 massless degrees of freedom
and interaction vanishes at zero momentum

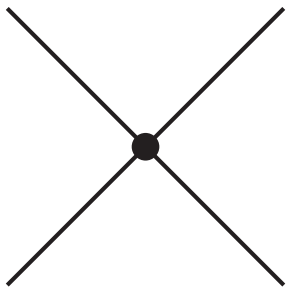
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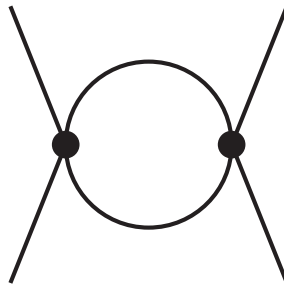
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Power counting in momenta: **Meson loops**



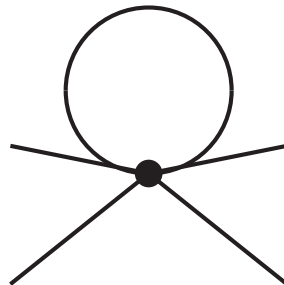
$$p^2$$



$$(p^2)^2 (1/p^2)^2 p^4 = p^4$$



$$1/p^2$$



$$(p^2) (1/p^2) p^4 = p^4$$

$$\int d^4 p$$

$$p^4$$

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
- ...
- \implies shortage of letters for the constants in the Lagrangians (LECs)

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
 - Two or Three (or even more) Flavours
 - Strong interaction and couplings to external currents/densities
 - Including (internal) electromagnetism
 - Including weak nonleptonic interactions
 - Treating kaon as heavy

Lagrangians

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}.$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4} \{ \langle D_\mu U^\dagger D^\mu U \rangle + \langle \chi^\dagger U + \chi U^\dagger \rangle \},$

$$D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu,$$

left and right external currents: $r(l)_\mu = v_\mu + (-)a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$

quark masses via scalar density: $s = \mathcal{M} + \dots$

$$\langle A \rangle = Tr_F (A)$$

Lagrangians

$$\begin{aligned}\mathcal{L}_4 = & L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle \\ & + L_3 \langle D^\mu U^\dagger D_\mu U D^\nu U^\dagger D_\nu U \rangle + L_4 \langle D^\mu U^\dagger D_\mu U \rangle \langle \chi^\dagger U + \chi U^\dagger \rangle \\ & + L_5 \langle D^\mu U^\dagger D_\mu U (\chi^\dagger U + U^\dagger \chi) \rangle + L_6 \langle \chi^\dagger U + \chi U^\dagger \rangle^2 \\ & + L_7 \langle \chi^\dagger U - \chi U^\dagger \rangle^2 + L_8 \langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger \rangle \\ & - i L_9 \langle F_{\mu\nu}^R D^\mu U D^\nu U^\dagger + F_{\mu\nu}^L D^\mu U^\dagger D^\nu U \rangle \\ & + L_{10} \langle U^\dagger F_{\mu\nu}^R U F^{L\mu\nu} \rangle + H_1 \langle F_{\mu\nu}^R F^{R\mu\nu} + F_{\mu\nu}^L F^{L\mu\nu} \rangle + H_2 \langle \chi^\dagger \chi \rangle\end{aligned}$$

L_i : Low-energy-constants (LECs)

H_i : Values depend on definition of currents/densities

These absorb the divergences of loop diagrams: $L_i \rightarrow L_i^r$

Renormalization: order by order in the powercounting

Lagrangians

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	\hat{L}_i^r, \hat{H}_i^r	11+2
p^6	c_i^r	52+4	C_i^r	90+4	K_i^r	112+3

p^2 : Weinberg 1966

p^4 : Gasser, Leutwyler 84,85

p^6 : JB, Colangelo, Ecker 99,00

- ▶ replica method \implies PQ obtained from N_F flavour
- ▶ All infinities known
- ▶ 3 flavour special case of 3+3 PQ: $\hat{L}_i^r, K_i^r \rightarrow L_i^r, C_i^r$
- ▶ 53 \rightarrow 52 [arXiv:0705.0576](https://arxiv.org/abs/0705.0576) [hep-ph]

Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars
- Chiral logarithms
- includes Isospin and the eightfold way ($SU(3)_V$)

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2} \log \frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu) \right] + \dots$$

$$M^2 = 2B\hat{m}$$

$B \neq B_0, F \neq F_0$ (two versus three-flavour)

LECs and μ

$$l_3^r(\mu)$$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i} l_i^r(\mu) - \log \frac{M_\pi^2}{\mu^2}.$$

Independent of the scale μ .

For 3 and more flavours, some of the $\gamma_i = 0$: $L_i^r(\mu)$

μ :

- m_π, m_K : chiral logs vanish
- pick larger scale
- 1 GeV then $L_5^r(\mu) \approx 0$ large N_c arguments????
- compromise: $\mu = m_\rho = 0.77$ GeV

Expand in what quantities?

- Expansion is in momenta and masses
- But is not unique: relations between masses (Gell-Mann–Okubo) exists
- Express orders in terms of physical masses and quantities (F_π , F_K)?
- Express orders in terms of lowest order masses?
- E.g. $s + t + u = 2m_\pi^2 + 2m_K^2$ in πK scattering

I prefer physical masses

- Thresholds correct
- Chiral logs are from physical particles propagating

An example

$$m_\pi = \frac{m_0}{1 + a \frac{m_0}{f_0}}$$

$$f_\pi = \frac{f_0}{1 + b \frac{m_0}{f_0}}$$

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$$m_\pi = \frac{m_0}{1 + a \frac{m_0}{f_0}} \quad f_\pi = \frac{f_0}{1 + b \frac{m_0}{f_0}}$$

$$m_\pi = m_0 - a \frac{m_0^2}{f_0} + a^2 \frac{m_0^3}{f_0^2} + \dots$$

$$f_\pi = f_0 \left(1 - b \frac{m_0}{f_0} + b^2 \frac{m_0^2}{f_0^2} + \dots \right)$$

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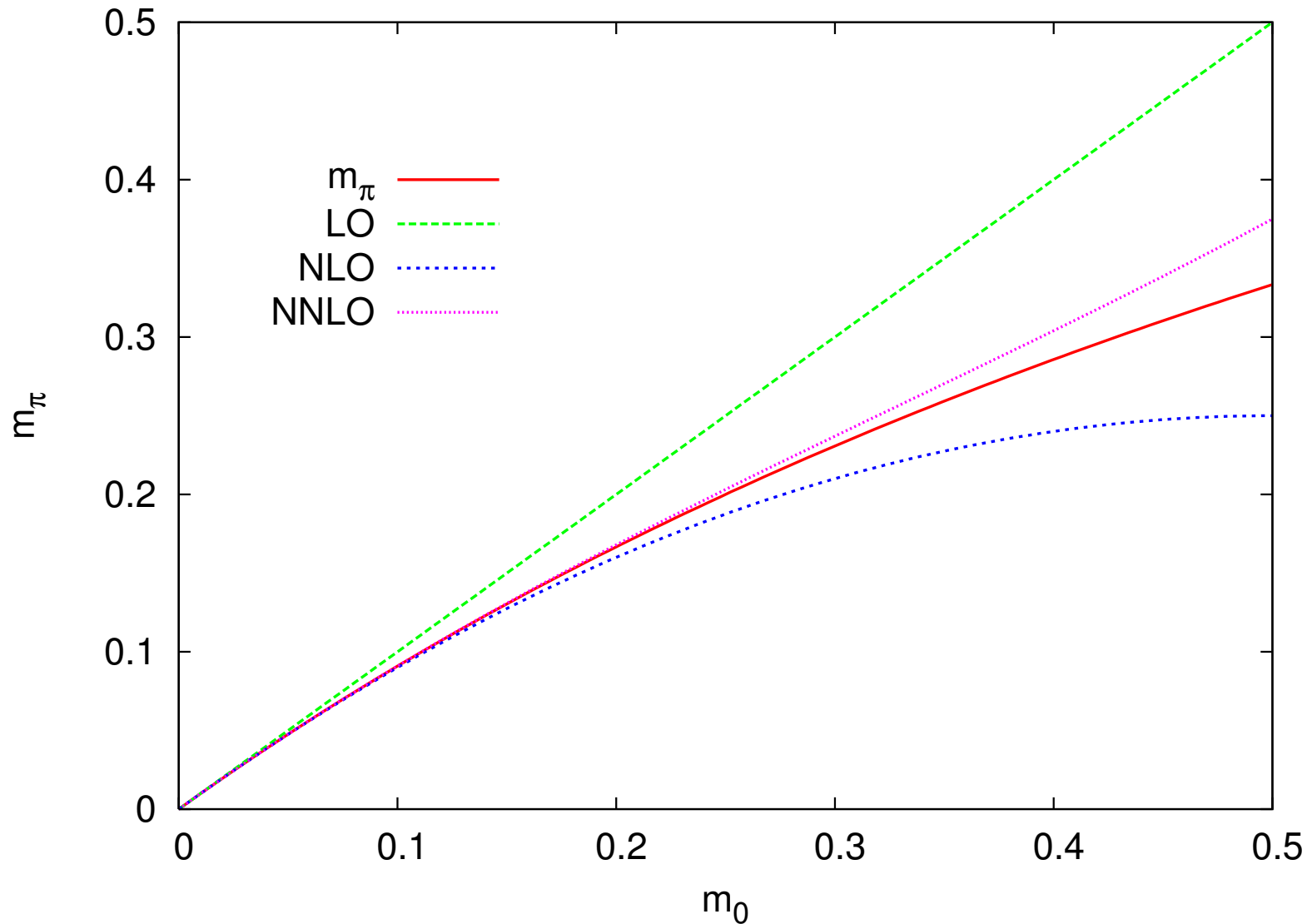
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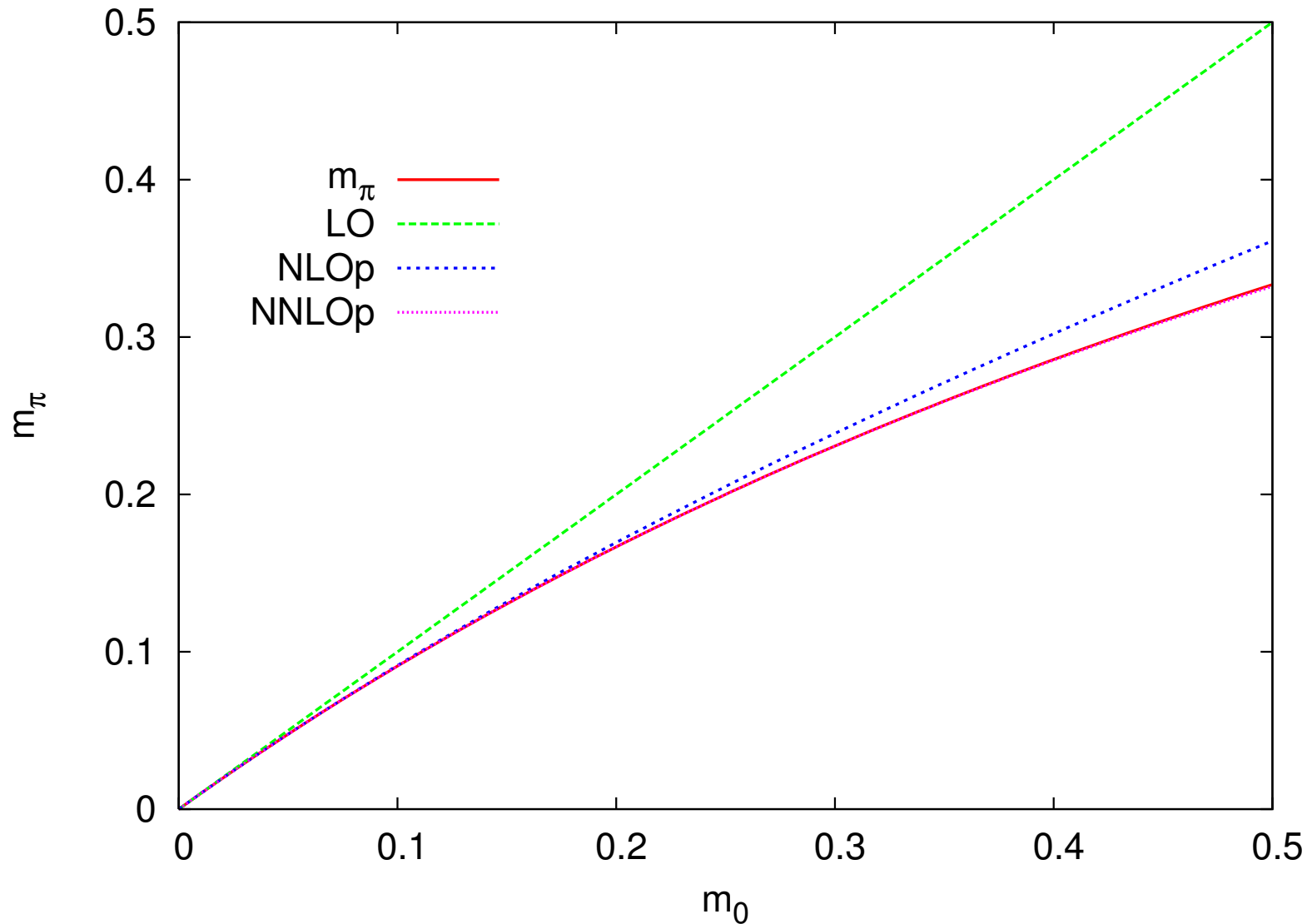
$$f_\pi = f_0 \left(1 - b \frac{m_\pi}{f_\pi} + b(2b - a) \frac{m_\pi^2}{f_\pi^2} + \dots \right)$$

$$a = 1 \quad b = 0.5 \quad f_0 = 1$$

An example: m_0/f_0



An example: m_π / f_π



Two-loop Two-flavour

Review paper on Two-Loops: JB, hep-ph/0604043 Prog. Part.
Nucl. Phys. 58 (2007) 521

Dispersive Calculation of the nonpolynomial part in q^2, s, t, u

- Gasser-Meißner: F_V, F_S : 1991 numerical
- Knecht-Moussallam-Stern-Fuchs: $\pi\pi$: 1995 analytical
- Colangelo-Finkemeier-Urech: F_V, F_S : 1996 analytical

Two-Loop Two-flavour

- Bellucci-Gasser-Sainio: $\gamma\gamma \rightarrow \pi^0\pi^0$: 1994
- Bürgi: $\gamma\gamma \rightarrow \pi^+\pi^-$, F_π , m_π : 1996
- JB-Colangelo-Ecker-Gasser-Sainio: $\pi\pi$, F_π , m_π : 1996-97
- JB-Colangelo-Talavera: $F_{V\pi}(t)$, $F_{S\pi}$: 1998
- JB-Talavera: $\pi \rightarrow \ell\nu\gamma$: 1997
- Gasser-Ivanov-Sainio: $\gamma\gamma \rightarrow \pi^0\pi^0$, $\gamma\gamma \rightarrow \pi^+\pi^-$: 2005-2006
- m_π , F_π , F_V , F_S , $\pi\pi$: simple analytical forms
- Colangelo-(Dürr-)Haefeli: Finite volume F_π , m_π : 2005-2006
- Kampf-Moussallam: $\pi^0 \rightarrow \gamma\gamma$ 2009

LECs

\bar{l}_1 to \bar{l}_4 : ChPT at order p^6 and the Roy equation analysis in $\pi\pi$ and F_S Colangelo, Gasser and Leutwyler, *Nucl. Phys. B* 603 (2001) 125 [hep-ph/0103088]

\bar{l}_5 and \bar{l}_6 : from F_V and $\pi \rightarrow \ell\nu\gamma$ JB,(Colangelo,)Talavera

$$\bar{l}_1 = -0.4 \pm 0.6 ,$$

$$\bar{l}_2 = 4.3 \pm 0.1 ,$$

$$\bar{l}_3 = 2.9 \pm 2.4 ,$$

$$\bar{l}_4 = 4.4 \pm 0.2 ,$$

$$\bar{l}_6 = 16.0 \pm 0.5 \pm 0.7 .$$

$$\bar{l}_6 - \bar{l}_5 = 3.0 \pm 0.3 \quad \pi \rightarrow e\nu\gamma ,$$

$$\bar{l}_5 = 12.24 \pm 0.21 \quad \tau \rightarrow \nu + \text{hadrons}$$

$l_7 \sim 5 \cdot 10^{-3}$ from π^0 - η mixing Gasser, Leutwyler 1984

LECs

Some combinations of order p^6 LECs are known as well: curvature of the scalar and vector formfactor, two more combinations from $\pi\pi$ scattering (implicit in b_5 and b_6),

Note: c_i^r for m_π , f_π , $\pi\pi$: small effect

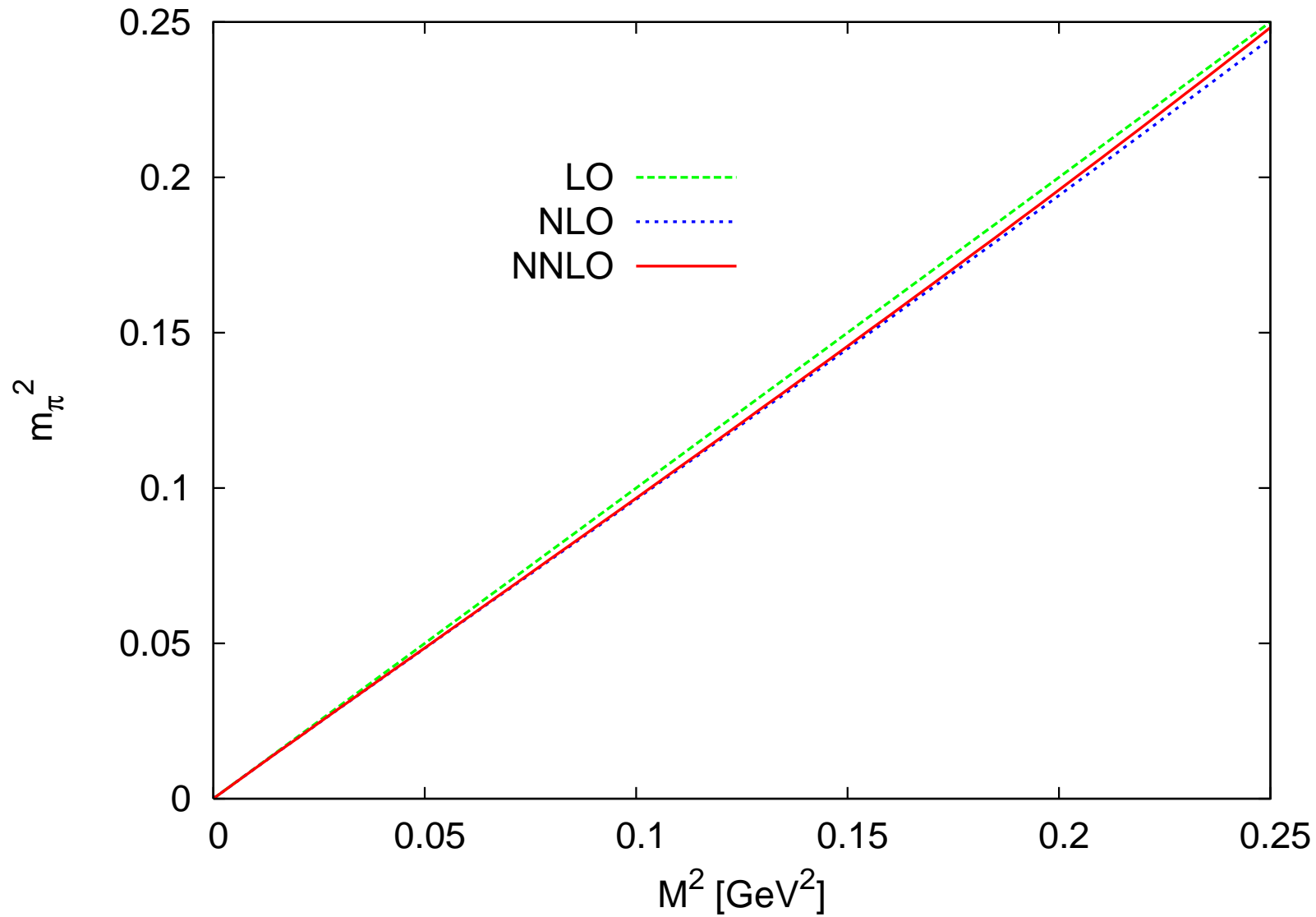
$c_i^r(770\text{MeV}) = 0$ for plots shown

expansion in m_π^2/F_π^2 shown

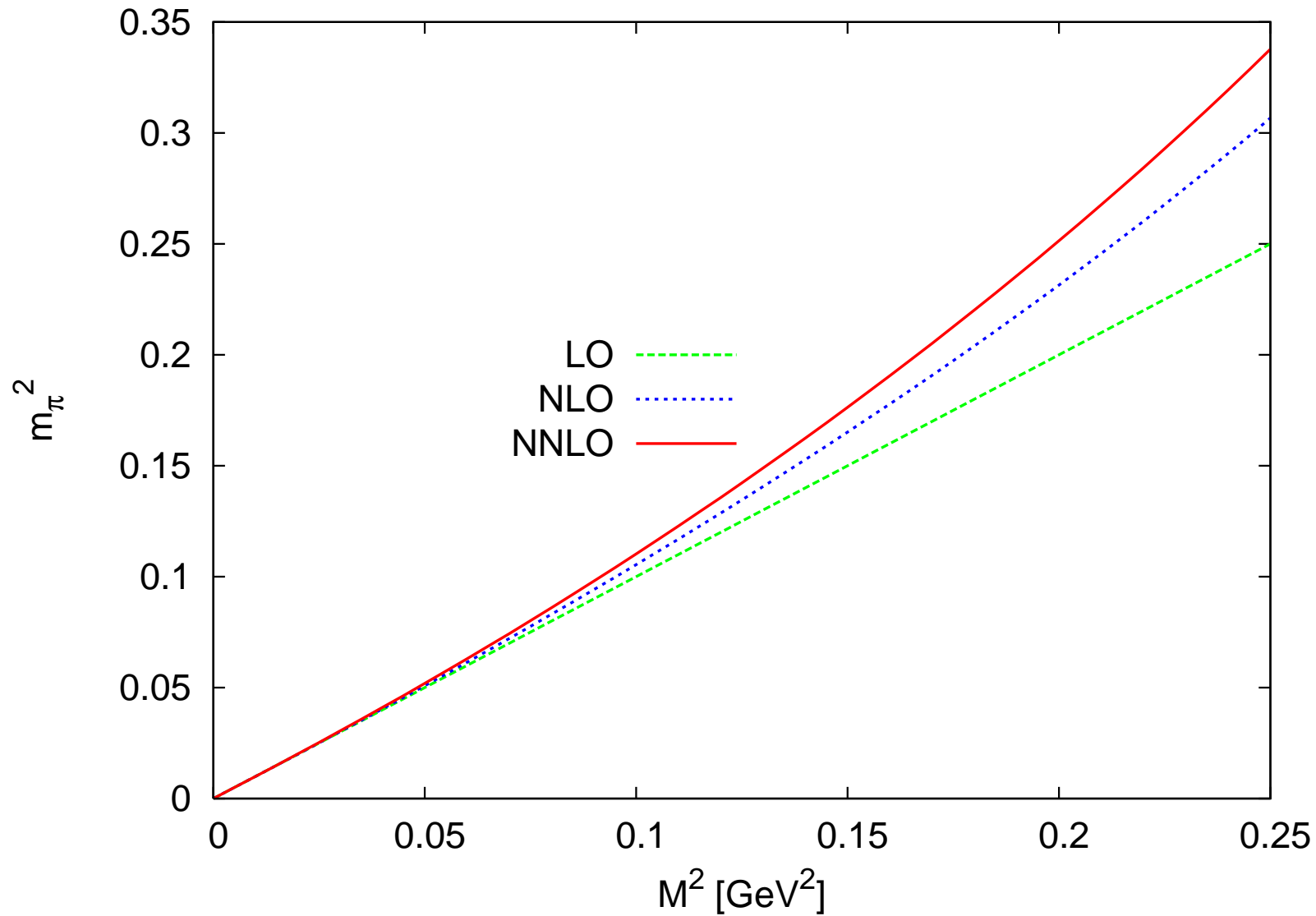
General observation:

- Obtainable from kinematical dependences: known
- Only via quark-mass dependence: poorly known

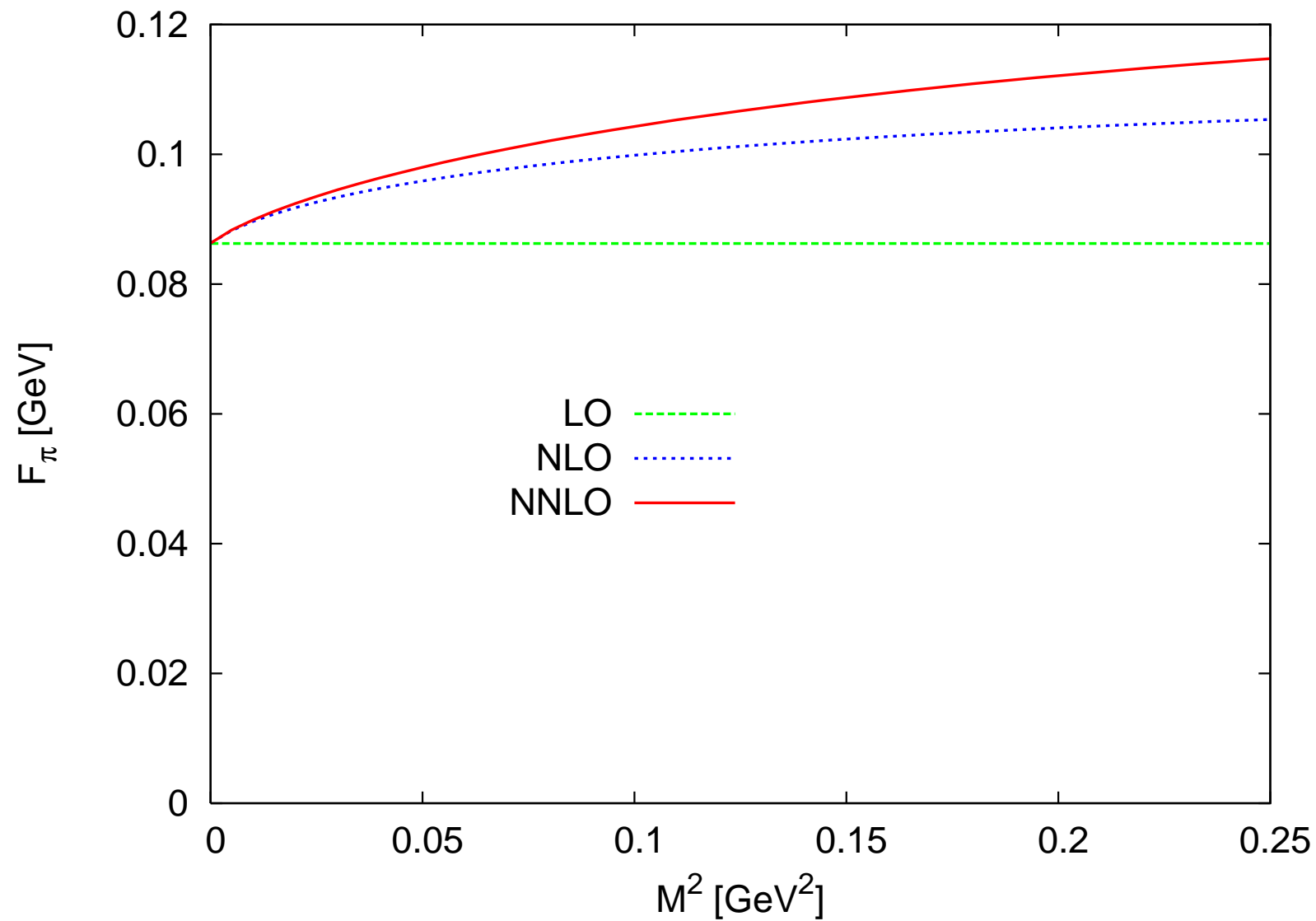
$$m_\pi^2$$



$$m_\pi^2 (\bar{l}_3 = 0)$$



F_π



Two-loop Three-flavour, ≤ 2001

- $\Pi_{VV\pi}, \Pi_{VV\eta}, \Pi_{VVK}$ Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera
- $\Pi_{VV\rho\omega}$ Maltman
- $\Pi_{AA\pi}, \Pi_{AA\eta}, F_\pi, F_\eta, m_\pi, m_\eta$ Kambor, Golowich; Amorós, JB, Talavera
- Π_{SS} Moussallam L_4^r, L_6^r
- $\Pi_{VVK}, \Pi_{AAK}, F_K, m_K$ Amorós, JB, Talavera
- $K_{\ell 4}, \langle \bar{q}q \rangle$ Amorós, JB, Talavera L_1^r, L_2^r, L_3^r
- $F_M, m_M, \langle \bar{q}q \rangle (m_u \neq m_d)$ Amorós, JB, Talavera $L_{5,7,8}^r, m_u/m_d$

Two-loop Three-flavour, ≥ 2001

- $F_{V\pi}, F_{VK^+}, F_{VK^0}$ Post, Schilcher; JB, Talavera L_9^r
- $K_{\ell 3}$ Post, Schilcher; JB, Talavera V_{us}
- $F_{S\pi}, F_{SK}$ (includes σ -terms) JB, Dhonte L_4^r, L_6^r
- $K, \pi \rightarrow \ell\nu\gamma$ Geng, Ho, Wu L_{10}^r
- $\pi\pi$ JB, Dhonte, Talavera
- πK JB, Dhonte, Talavera
- relation l_i^r and L_i^r, C_i^r Gasser, Haefeli, Ivanov, Schmid
- Finite volume $\langle \bar{q}q \rangle$ JB, Ghorbani
- $\eta \rightarrow 3\pi$: JB, Ghorbani
- $K_{\ell 3}$ isospin breaking JB, Ghorbani

Two-loop Three-flavour

Known to be in progress

- Finite Volume: sunsetintegrals

JB,Lähde

- relation c_i^r and L_i^r, C_i^r

Gasser,Haefeli,Ivanov,Schmid

- More analytical work on $K_{\ell 3}$

Greynat et al.

Inputs

Fit: Amoros, JB Talavera 2001

$$K_{\ell 4}: F(0), G(0), \lambda$$

E865 BNL

$$m_{\pi^0}^2, m_{\eta}^2, m_{K^+}^2, m_{K^0}^2$$

em with Dashen violation

$$F_{\pi^+}$$

$$F_{K^+} / F_{\pi^+}$$

$$m_s / \hat{m}$$

$$24 \text{ (26)}$$

$$\hat{m} = (m_u + m_d) / 2$$

$$L_4^r, L_6^r$$

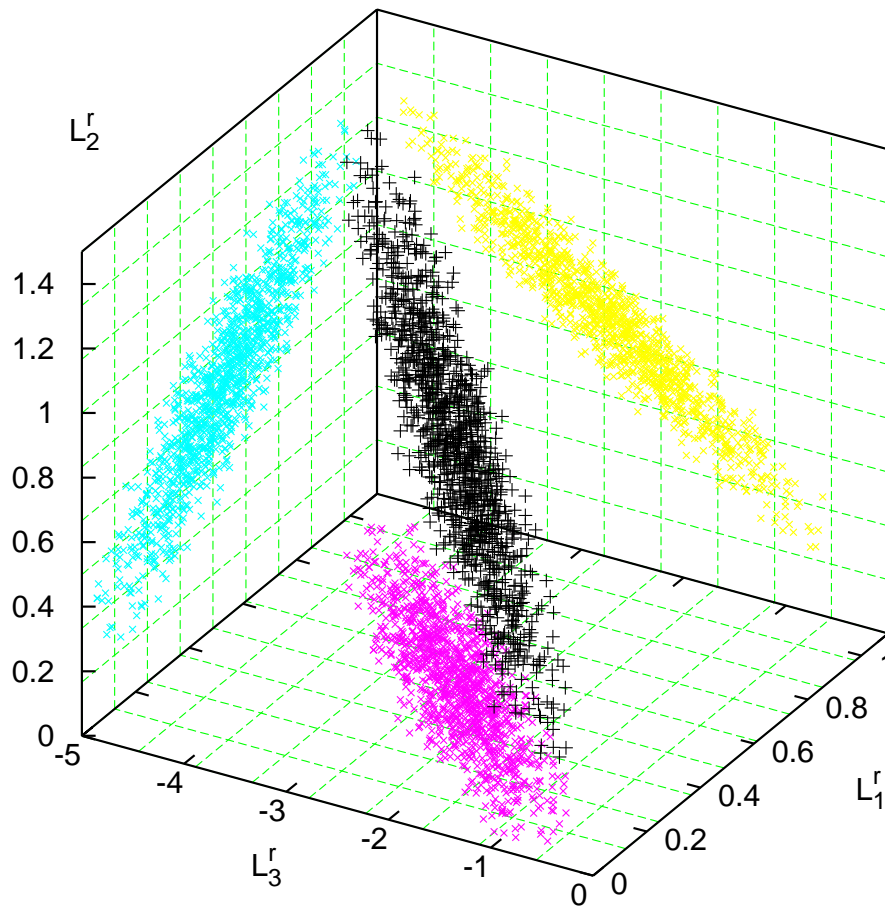
Comprehensive new fit in progress

Outputs: I

	fit 10	same p^4	fit B	fit D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.35 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54

- ▣ errors are very correlated
- ▣ $\mu = 770$ MeV; 550 or 1000 within errors
- ▣ varying C_i^r factor 2 about errors
- ▣ $L_4^r, L_6^r \approx -0.3, \dots, 0.6 \cdot 10^{-3}$ OK
- ▣ fit B: small corrections to pion “sigma” term, fit scalar radius
- ▣ fit D: fit $\pi\pi$ and πK thresholds

Correlations



(older fit)

$$10^3 L_1^r = 0.52 \pm 0.23$$

$$10^3 L_2^r = 0.72 \pm 0.24$$

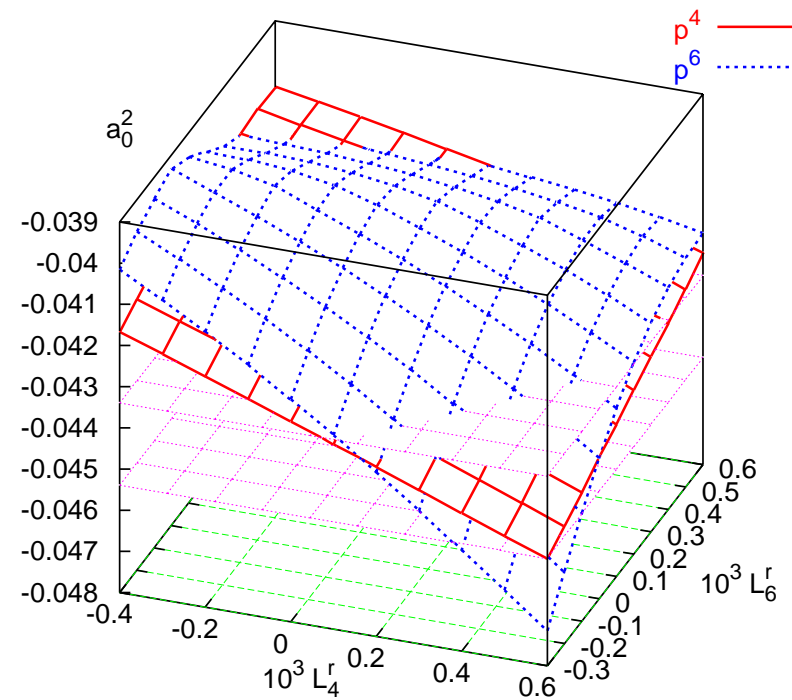
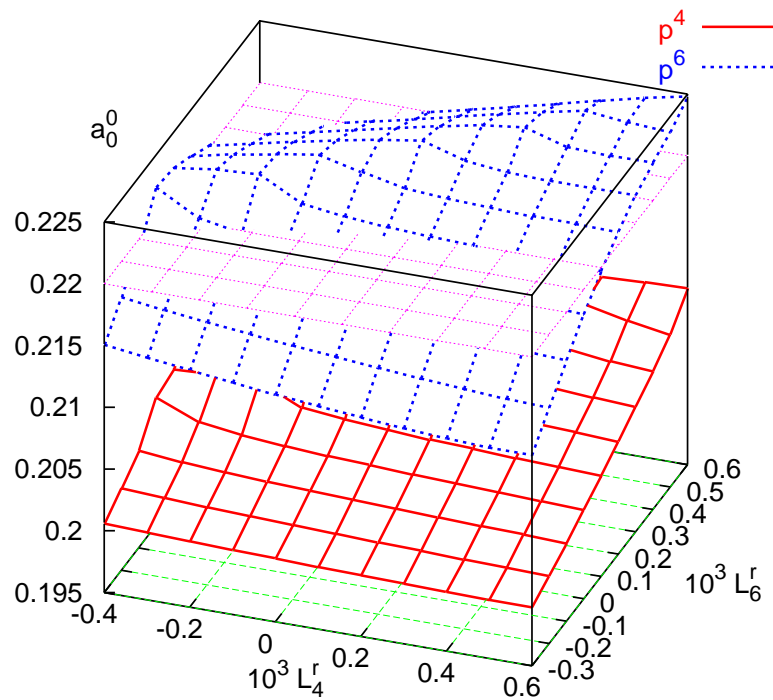
$$10^3 L_3^r = -2.70 \pm 0.99$$

Outputs: II

	fit 10	same p^4	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
$m_\pi^2: p^4, p^6$	0.006,0.258	0.009, $\equiv 0$	-0.138,0.009	-0.091,0.133
$m_K^2: p^4, p^6$	0.007,0.306	0.075, $\equiv 0$	-0.149,0.094	-0.096,0.201
$m_\eta^2: p^4, p^6$	-0.052,0.318	0.013, $\equiv 0$	-0.197,0.073	-0.151,0.197
m_u/m_d	0.45 ± 0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}: p^4, p^6$	0.169,0.051	0.22, $\equiv 0$	0.153,0.067	0.159,0.061

▣ $m_u = 0$ always very far from the fits

▣ F_0 : pion decay constant in the chiral limit

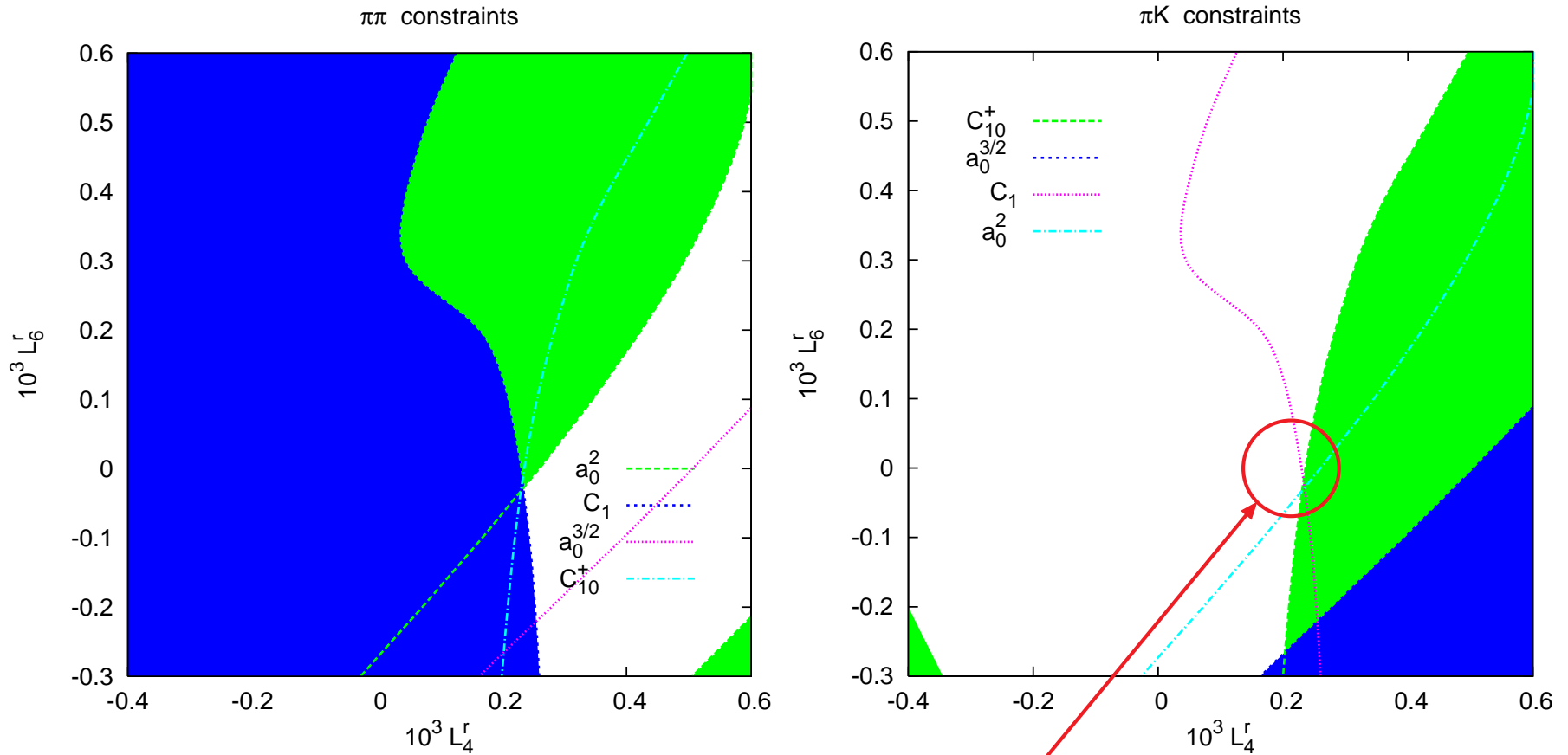


$$a_0^0 = 0.220 \pm 0.005, \quad a_0^2 = -0.0444 \pm 0.0010$$

Colangelo, Gasser, Leutwyler

$$a_0^0 = 0.159 \quad a_0^2 = -0.0454 \quad \text{at order } p^2$$

$\pi\pi$ and πK



preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

General fitting needs more work and systematic studies

Quark mass dependences

Updates of plots in

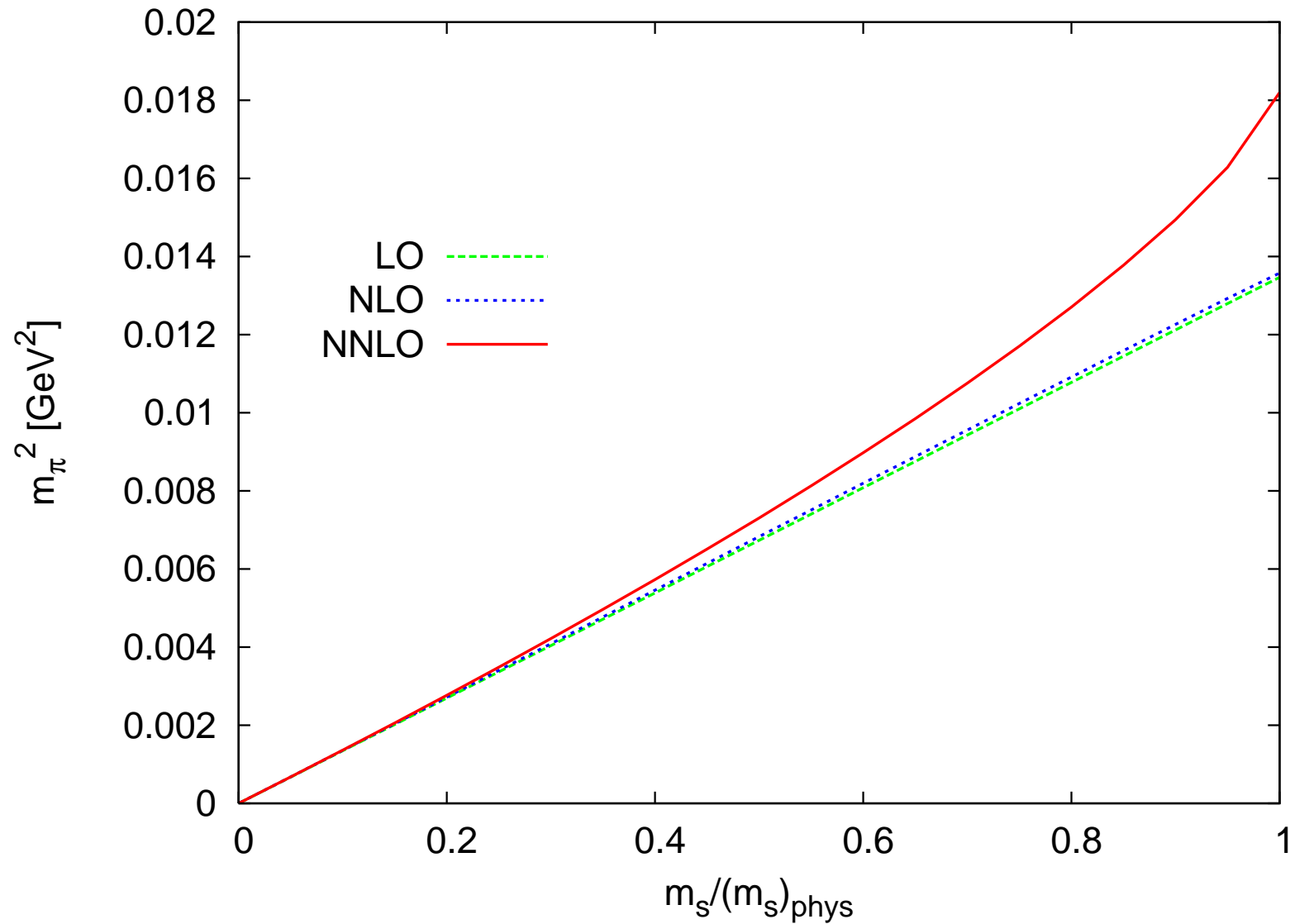
Amorós, JB and Talavera, hep-ph/0003258, Nucl. Phys. B585 (2000) 293

Some new ones

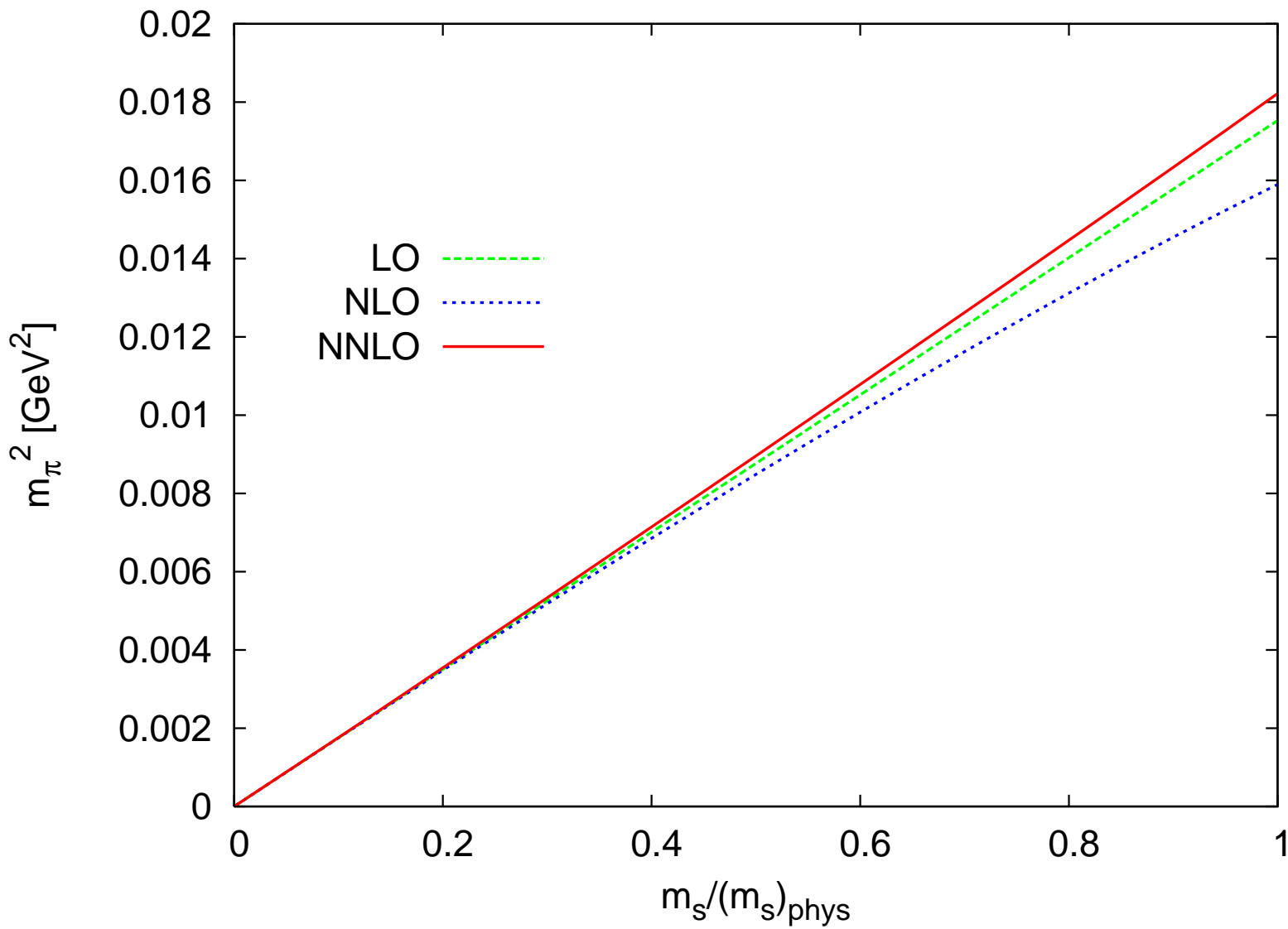
Procedure: calculate a consistent set of $m_\pi, m_K, m_\eta, f_\pi$ with the given input values (done iteratively)

- vary $m_s / (m_s)_{phys}$, keep $m_s / \hat{m} = 24$
 $m_\pi^2, F_K / F_\pi$

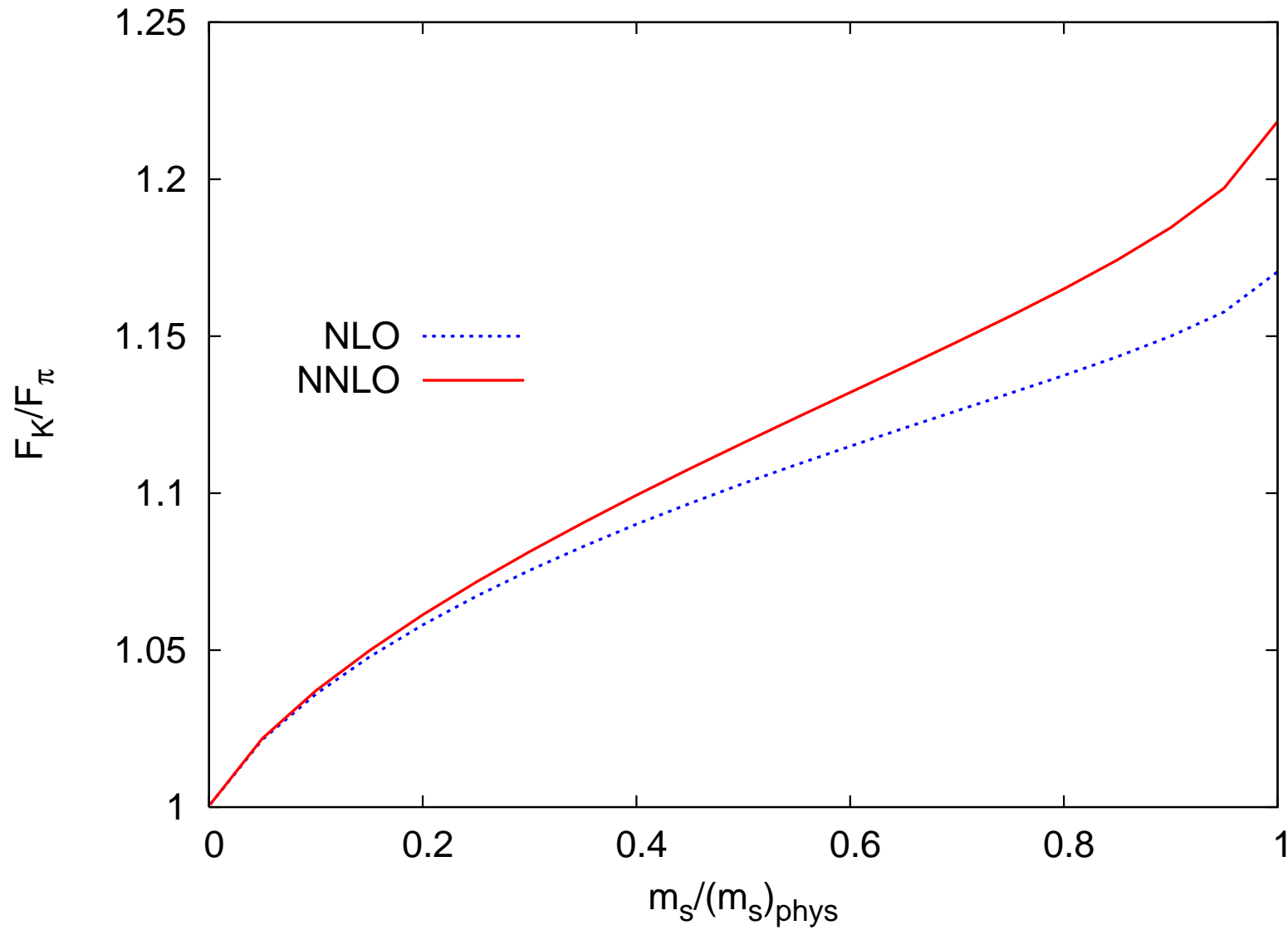
m_π^2 fit 10



m_π^2 fit D

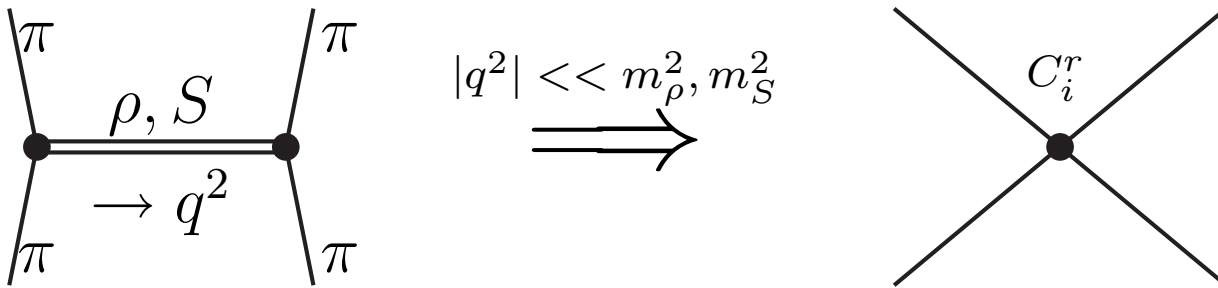


F_K/F_π fit 10



Most analysis use:

C_i^r from (single) resonance approximation



Motivated by large N_c : large effort goes in this

Ananthanarayan, JB, Cirigliano, Donoghue, Ecker, Gamiz, Golterman, Kaiser, Kampf, Knecht, Moussallam, Peris, Pich, Prades, Portoles, de Rafael,...

$$\begin{aligned}
 \mathcal{L}_V &= -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \frac{1}{2}m_V^2\langle V_\mu V^\mu\rangle - \frac{f_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle \\
 &\quad - \frac{ig_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + f_\chi\langle V_\mu[u^\mu, \chi_-]\rangle \\
 \mathcal{L}_A &= -\frac{1}{4}\langle A_{\mu\nu}A^{\mu\nu}\rangle + \frac{1}{2}m_A^2\langle A_\mu A^\mu\rangle - \frac{f_A}{2\sqrt{2}}\langle A_{\mu\nu}f_-^{\mu\nu}\rangle \\
 \mathcal{L}_S &= \frac{1}{2}\langle \nabla^\mu S \nabla_\mu S - M_S^2 S^2 \rangle + c_d\langle S u^\mu u_\mu \rangle + c_m\langle S \chi_+ \rangle \\
 \mathcal{L}_{\eta'} &= \frac{1}{2}\partial_\mu P_1 \partial^\mu P_1 - \frac{1}{2}M_{\eta'}^2 P_1^2 + i\tilde{d}_m P_1 \langle \chi_- \rangle.
 \end{aligned}$$

$$f_V = 0.20, \quad f_\chi = -0.025, \quad g_V = 0.09, \quad c_m = 42 \text{ MeV}, \quad c_d = 32 \text{ MeV}, \quad \tilde{d}_m = 20 \text{ MeV},$$

$$m_V = m_\rho = 0.77 \text{ GeV}, \quad m_A = m_{a_1} = 1.23 \text{ GeV}, \quad m_S = 0.98 \text{ GeV}, \quad m_{P_1} = 0.958 \text{ GeV}$$

f_V, g_V, f_χ, f_A : experiment

c_m and c_d from resonance saturation at $\mathcal{O}(p^4)$

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

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- Weakest point in the numerics
- However not all results presented depend on this
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What we did about it:

- Vary resonance estimate by factor of two
- Vary the scale μ at which it applies: 600-900 MeV
- Check the estimates for the measured ones
- Again: kinematic can be had, quark-mass dependence difficult

Comparisons of C_i^r

Kampf-Moussallam 2006 using our $\pi\pi$ and πK results

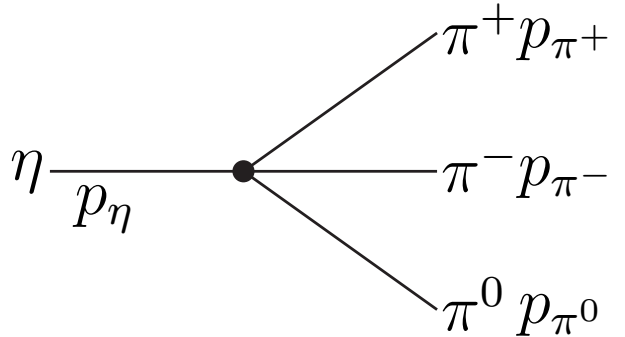
input	$C_1^r + 4C_3^r$	C_2^r	$C_4^r + 3C_3^r$	$C_1^r + 4C_3^r + 2C_2^r$
$\pi K : C_{30}^+, C_{11}^+, C_{20}^-$	20.7 ± 4.9	-9.2 ± 4.9	9.9 ± 2.5	2.3 ± 10.8
$\pi K : C_{30}^+, C_{11}^+, C_{01}^-$	28.1 ± 4.9	-7.4 ± 4.9	21.0 ± 2.5	13.4 ± 10.8
$\pi\pi$			23.5 ± 2.3	18.8 ± 7.2
Resonance model	7.2	-0.5	10.0	6.2

Correlated analysis in progress (slowly)

$\eta \rightarrow 3\pi$

Reviews: JB, Gasser, Phys.Scripta T99(2002)34 [hep-ph/0202242]

JB, Acta Phys. Slov. 56(2005)305 [hep-ph/0511076]


$$\begin{aligned} s &= (p_{\pi^+} + p_{\pi^-})^2 = (p_\eta - p_{\pi^0})^2 \\ t &= (p_{\pi^-} + p_{\pi^0})^2 = (p_\eta - p_{\pi^+})^2 \\ u &= (p_{\pi^+} + p_{\pi^0})^2 = (p_\eta - p_{\pi^-})^2 \end{aligned}$$

$$s + t + u = m_\eta^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2 \equiv 3s_0.$$

$$\langle \pi^0 \pi^+ \pi^- \mathbf{out} | \eta \rangle = i (2\pi)^4 \delta^4 (p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}) A(s, t, u).$$

$$\langle \pi^0 \pi^0 \pi^0 \mathbf{out} | \eta \rangle = i (2\pi)^4 \delta^4 (p_\eta - p_1 - p_2 - p_3) \bar{A}(s_1, s_2, s_3)$$

$$\bar{A}(s_1, s_2, s_3) = A(s_1, s_2, s_3) + A(s_2, s_3, s_1) + A(s_3, s_1, s_2),$$

$\eta \rightarrow 3\pi$: Lowest order (LO)

Pions are in $I = 1$ state $\implies A \sim (m_u - m_d)$ or α_{em}

- α_{em} effect is small (but large via $m_{\pi^+} - m_{\pi^0}$)
- $\eta \rightarrow \pi^+ \pi^- \pi^0 \gamma$ needs to be included directly

$\eta \rightarrow 3\pi$: Lowest order (LO)

Pions are in $I = 1$ state $\implies A \sim (m_u - m_d)$ or α_{em}

$$\text{ChPT:Cronin 67: } A(s, t, u) = \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2} \left\{ 1 + \frac{3(s - s_0)}{m_\eta^2 - m_\pi^2} \right\}$$

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$$\text{with } Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \text{ or } R \equiv \frac{m_s - \hat{m}}{m_d - m_u} \quad \hat{m} = \frac{1}{2}(m_u + m_d)$$

$$A(s, t, u) = \frac{1}{Q^2} \frac{m_K^2}{m_\pi^2} (m_\pi^2 - m_K^2) \frac{\mathcal{M}(s, t, u)}{3\sqrt{3}F_\pi^2},$$

$$A(s, t, u) = \frac{\sqrt{3}}{4R} M(s, t, u)$$

$$\text{LO: } \mathcal{M}(s, t, u) = \frac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2} \quad M(s, t, u) = \frac{1}{F_\pi^2} \left(\frac{4}{3}m_\pi^2 - s \right)$$

$\eta \rightarrow 3\pi$ beyond p^4 : p^2 and p^4

$\Gamma(\eta \rightarrow 3\pi) \propto |A|^2 \propto Q^{-4}$ allows a PRECISE measurement

$Q \approx 24$ gives lowest order $\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) \approx 66 \text{ eV}$.

Other Source from $m_{K^+}^2 - m_{K^0}^2 \sim Q^{-2} \implies Q = 20.0 \pm 1.5$

Lowest order prediction $\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) \approx 140 \text{ eV}$.

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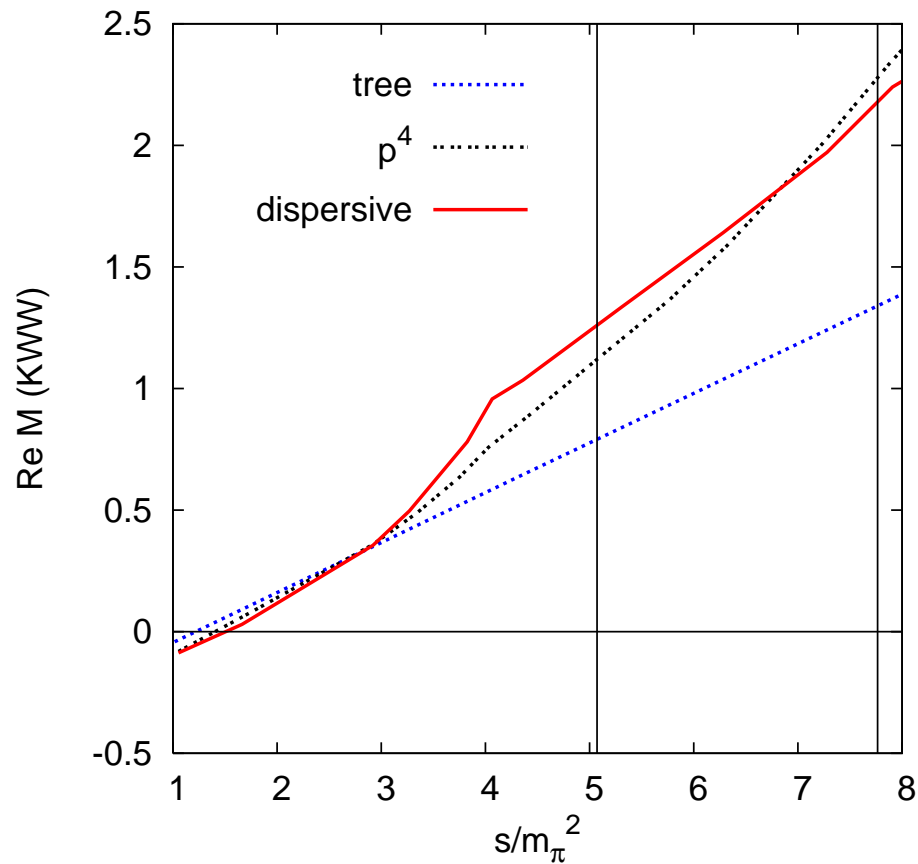
At order p^4 Gasser-Leutwyler 1985:
$$\frac{\int dLIPS |A_2 + A_4|^2}{\int dLIPS |A_2|^2} = 2.4,$$

(LIPS=Lorentz invariant phase-space)

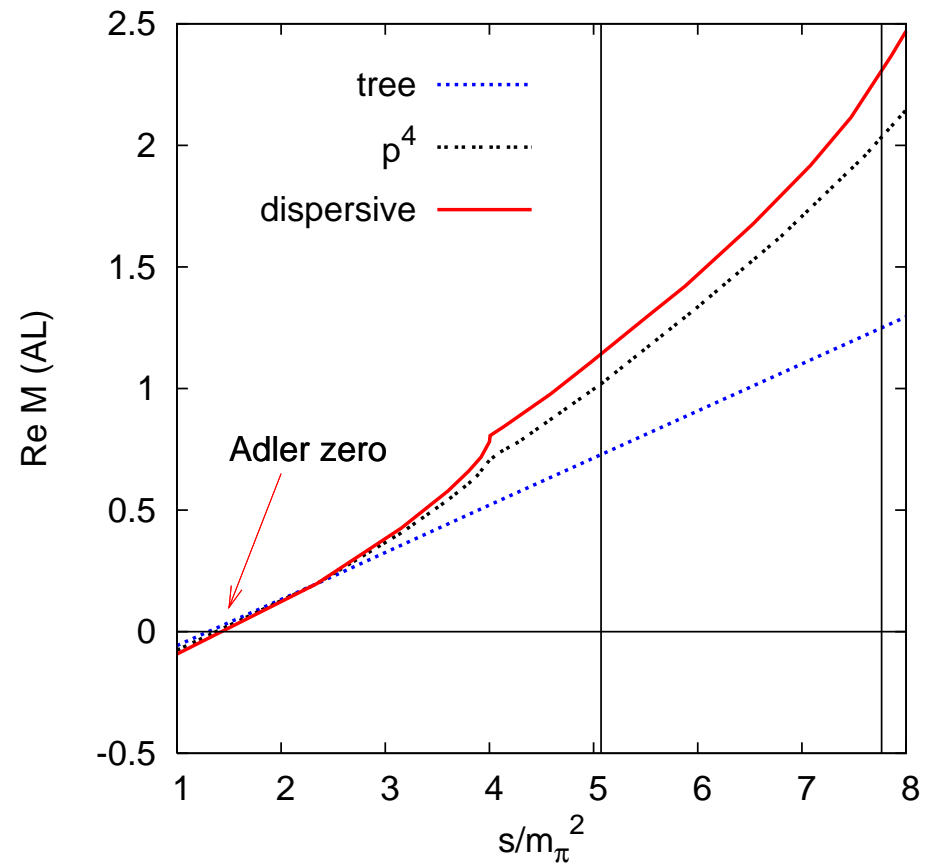
Major source: large S -wave final state rescattering

Experiment: $295 \pm 17 \text{ eV}$ (PDG 2006)

$\eta \rightarrow 3\pi$ beyond p^4 : dispersive



Along $s = u$ KWW

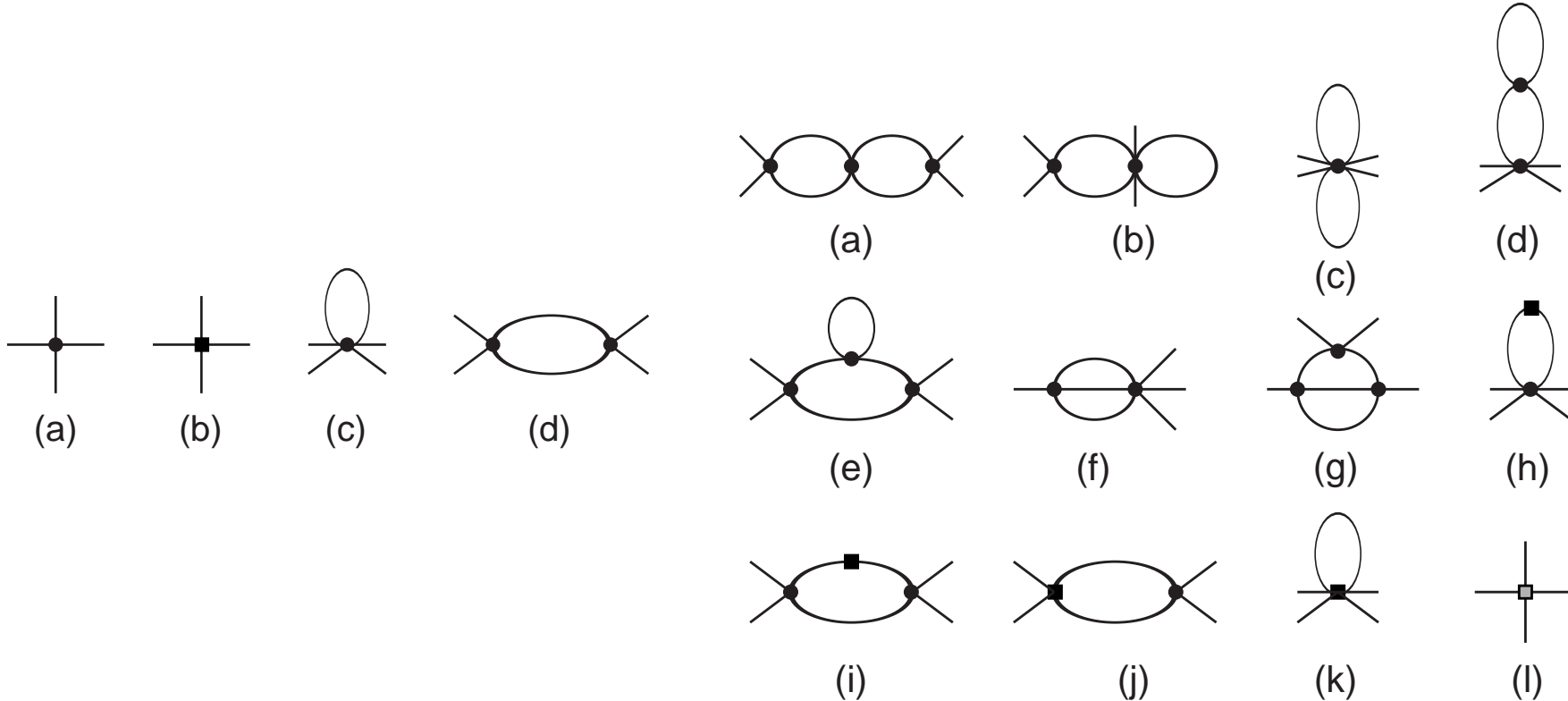


Along $s = u$ AL

Two Loop Calculation: why

- In $K_{\ell 4}$ dispersive gave about half of p^6 in amplitude
- Same order in ChPT as masses for consistency check on m_u/m_d
- Check size of 3 pion dispersive part
- At order p^4 unitarity about half of correction
- Technology exists:
 - Two-loops: Amorós, JB, Dhonte, Talavera, . . .
 - Dealing with the mixing π^0 - η :
Amorós, JB, Dhonte, Talavera 01
- JB, Ghorbani, [arXiv:0709.0230 \[hep-ph\]](https://arxiv.org/abs/0709.0230)
 - Dealing with the mixing π^0 - η : extended to $\eta \rightarrow 3\pi$

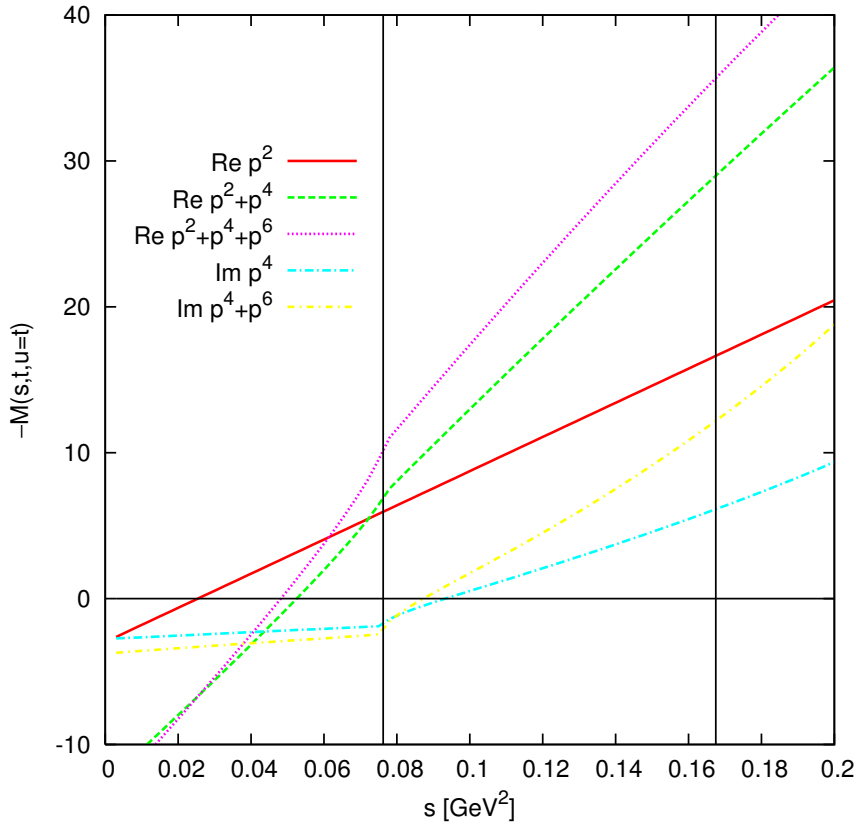
Diagrams



- Include mixing, renormalize, pull out factor $\frac{\sqrt{3}}{4R}, \dots$
- Two independent calculations (comparison major amount of work)

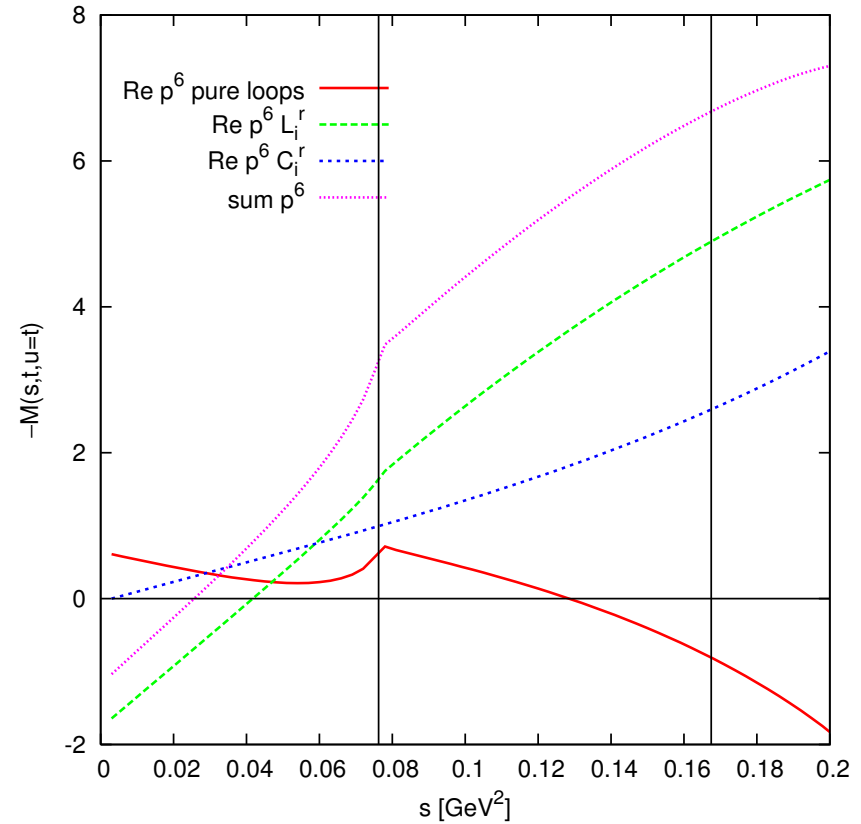
$\eta \rightarrow 3\pi: M(s, t = u)$

L_i fit 10 and C_i



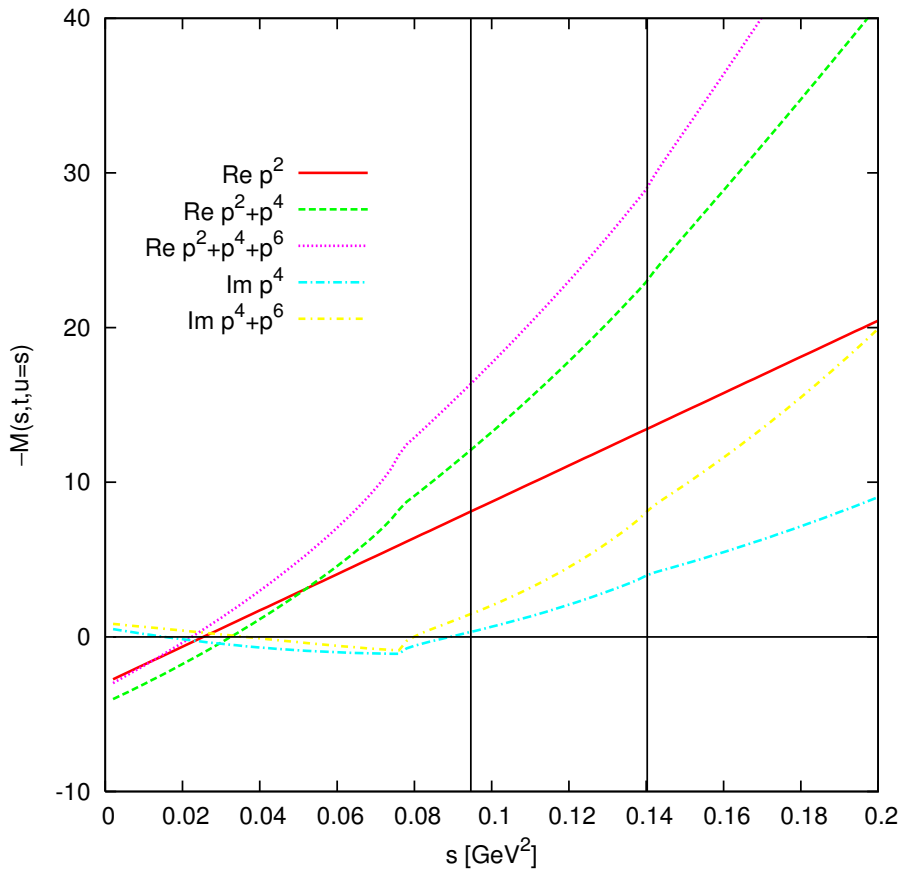
Along $t = u$

L_i fit 10 and C_i



Along $t = u$ parts

$$\eta \rightarrow 3\pi: M(s = u, t)$$



Along $s = u$

Shape agrees with AL

Correction larger:
20-30% in amplitude

Dalitz plot

$$x = \sqrt{3} \frac{T_+ - T_-}{Q_\eta} = \frac{\sqrt{3}}{2m_\eta Q_\eta} (u - t)$$

$$y = \frac{3T_0}{Q_\eta} - 1 = \frac{3((m_\eta - m_{\pi^0})^2 - s)}{2m_\eta Q_\eta} - 1 \stackrel{\text{iso}}{=} \frac{3}{2m_\eta Q_\eta} (s_0 - s)$$

$$Q_\eta = m_\eta - 2m_{\pi^+} - m_{\pi^0}$$

T^i is the kinetic energy of pion π^i

$$z = \frac{2}{3} \sum_{i=1,3} \left(\frac{3E_i - m_\eta}{m_\eta - 3m_\pi^0} \right)^2 \quad E_i \text{ is the energy of pion } \pi^i$$

$$|M|^2 = A_0^2 (1 + ay + by^2 + dx^2 + fy^3 + gx^2y + \dots)$$

$$|\overline{M}|^2 = \overline{A}_0^2 (1 + 2\alpha z + \dots)$$

Experiment: charged

Exp.	a	b	d
KLOE	$-1.090 \pm 0.005^{+0.008}_{-0.019}$	$0.124 \pm 0.006 \pm 0.010$	$0.057 \pm 0.006^{+0.007}_{-0.016}$
Crystal Barrel	-1.22 ± 0.07	0.22 ± 0.11	0.06 ± 0.04 (input)
Layter et al.	-1.08 ± 0.014	0.034 ± 0.027	0.046 ± 0.031
Gormley et al.	-1.17 ± 0.02	0.21 ± 0.03	0.06 ± 0.04

KLOE has: $f = 0.14 \pm 0.01 \pm 0.02$.

Crystal Barrel: d input, but a and b insensitive to d

Theory: charged

	A_0^2	a	b	d	f
LO	120	-1.039	0.270	0.000	0.000
NLO	314	-1.371	0.452	0.053	0.027
NLO ($L_i^r = 0$)	235	-1.263	0.407	0.050	0.015
NNLO	538	-1.271	0.394	0.055	0.025
NNLOp (y from T^0)	574	-1.229	0.366	0.052	0.023
NNLOq (incl $(x, y)^4$)	535	-1.257	0.397	0.076	0.004
NNLO ($\mu = 0.6$ GeV)	543	-1.300	0.415	0.055	0.024
NNLO ($\mu = 0.9$ GeV)	548	-1.241	0.374	0.054	0.025
NNLO ($C_i^r = 0$)	465	-1.297	0.404	0.058	0.032
NNLO ($L_i^r = C_i^r = 0$)	251	-1.241	0.424	0.050	0.007
dispersive (KWW)	—	-1.33	0.26	0.10	—
tree dispersive	—	-1.10	0.33	0.001	—
absolute dispersive	—	-1.21	0.33	0.04	—
error	18	0.075	0.102	0.057	0.160

NLO to
NNLO:
Little
change

Error on
 $|M(s, t, u)|^2$:

$|M^{(6)} M(s, t, u)|$

Experiment: neutral

Exp.	α		\overline{A}_0^2	α
Crystal Ball (MAMI C)	-0.032 ± 0.003	LO	1090	0.000
Crystal Ball (MAMI B)	$-0.032 \pm 0.002 \pm 0.002$	NLO	2810	0.013
WASA/COSY	$-0.027 \pm 0.008 \pm 0.005$	NLO ($L_i^r = 0$)	2100	0.016
KLOE 2007	$-0.027 \pm 0.004^{+0.004}_{-0.006}$	NNLO	4790	0.013
KLOE (prel)	$-0.014 \pm 0.005 \pm 0.004$	NNLOq	4790	0.014
Crystal Ball (BNL)	-0.031 ± 0.004	NNLO ($C_i^r = 0$)	4140	0.011
WASA/CELSIUS	$-0.026 \pm 0.010 \pm 0.010$	NNLO ($L_i^r, C_i^r = 0$)	2220	0.016
Crystal Barrel	$-0.052 \pm 0.017 \pm 0.010$	dispersive (KWW)	—	—(0.007—0.014)
GAMS2000	-0.022 ± 0.023	tree dispersive	—	-0.0065
SND	$-0.010 \pm 0.021 \pm 0.010$	absolute dispersive	—	-0.007
		Borasoy	—	-0.031
		error	160	0.032

Note: NNLO ChPT gets a_0^0 in $\pi\pi$ correct

α is difficult

Expand amplitudes and isospin:

$$M(s, t, u) = A \left(1 + \tilde{a}(s - s_0) + \tilde{b}(s - s_0)^2 + \tilde{d}(u - t)^2 + \dots \right)$$

$$\overline{M}(s, t, u) = A \left(3 + (\tilde{b} + 3\tilde{d}) \left((s - s_0)^2 + (t - s_0)^2 + (u - s_0)^2 \right) + \dots \right)$$

Gives relations ($R_\eta = (2m_\eta Q_\eta)/3$)

$$a = -2R_\eta \operatorname{Re}(\tilde{a}), \quad b = R_\eta^2 \left(|\tilde{a}|^2 + 2\operatorname{Re}(\tilde{b}) \right), \quad d = 6R_\eta^2 \operatorname{Re}(\tilde{d}).$$

$$\alpha = \frac{1}{2} R_\eta^2 \operatorname{Re}(\tilde{b} + 3\tilde{d}) = \frac{1}{4} (d + b - R_\eta^2 |\tilde{a}|^2) \leq \frac{1}{4} \left(d + b - \frac{1}{4} a^2 \right)$$

equality if $\operatorname{Im}(\tilde{a}) = 0$

Large cancellation in α , overestimate of b likely the problem

r and decay rates

$$r \equiv \frac{\Gamma(\eta \rightarrow \pi^0 \pi^0 \pi^0)}{\Gamma(\eta \rightarrow \pi^+ \pi^- \pi^0)}$$

$$r_{\text{LO}} = 1.54$$

$$r_{\text{NLO}} = 1.46$$

$$r_{\text{NNLO}} = 1.47$$

$$r_{\text{NNLO } C_i^r=0} = 1.46$$

PDG 2006

$$r = 1.49 \pm 0.06 \quad \text{our average .}$$

$$r = 1.43 \pm 0.04 \quad \text{our fit ,}$$

Good agreement

R and Q

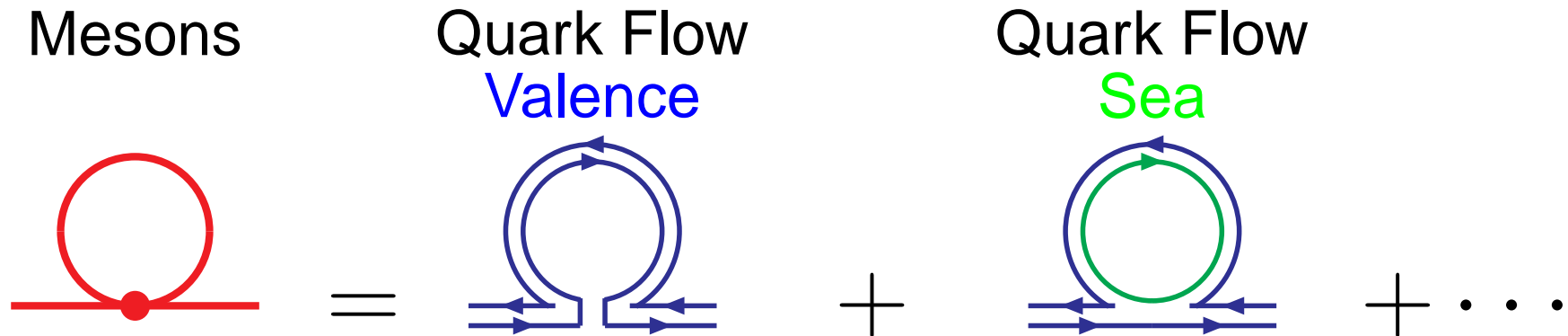
	LO	NLO	NNLO	NNLO ($C_i^r = 0$)
$R(\eta)$	19.1	31.8	42.2	38.7
R (Dashen)	44	44	37	—
R (Dashen-violation)	36	37	32	—
$Q(\eta)$	15.6	20.1	23.2	22.2
Q (Dashen)	24	24	22	—
Q (Dashen-violation)	22	22	20	—

$$\text{LO from } R = \frac{m_{K^0}^2 + m_{K^+}^2 - 2m_{\pi^0}^2}{2(m_{K^0}^2 - m_{K^+}^2)} \quad (\text{QCD part only})$$

NLO and NNLO from masses: Amorós, JB, Talavera 2001

$$Q^2 = \frac{m_s + \hat{m}}{2\hat{m}} R = 12.7R \quad (m_s/\hat{m} = 24.4)$$

Partial Quenching and ChPT



Lattice QCD: **Valence** is **easy** to deal with, **Sea** **very difficult**

They can be treated separately: i.e. different quark masses
Partially Quenched QCD and ChPT (PQChPT)

\geq 3-flavour: PQChPT

PQChPT: treat closed quark-loops differently from external quarks,

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

Very important note: ChPT is a limit of PQChPT
 \implies LECs from ChPT are linear combinations of LECs of PQChPT with the **same** number of sea quarks.

$$\text{E.g. } L_1^r = L_0^{r(3pq)} / 2 + L_1^{r(3pq)}$$

PQChPT

One-loop: Bernard, Golterman, Sharpe, Shoresh, Pallante,...

with electromagnetism: JB, Danielsson, hep-lat/0610127

Two loops:

$m_{\pi^+}^2$ **simplest mass case:** JB, Danielsson, Lähde, hep-lat/0406017

F_{π^+} : JB, Lähde, hep-lat/0501014

$F_{\pi^+}, m_{\pi^+}^2$ **two sea quarks:** JB, Lähde, hep-lat/0506004

$m_{\pi^+}^2$: JB, Danielsson, Lähde, hep-lat/0602003

Neutral masses: JB, Danielsson, hep-lat/0606017

Lattice data: a and L extrapolations needed

Programs available from me (Fortran)

Formulas: <http://www.thep.lu.se/~bijmens/chpt.html>

Renormalization group

Weinberg 79: nonlocal divergences must cancel \implies consistency conditions between graphs with different numbers of loops (but same order in the power counting)

This allows to calculate the leading logarithms to any order from one-loop diagrams [Buchler Colangelo 2003](#)

- double logs in $\pi\pi$ [Colangelo 95](#)
- all double logs [JB, Ecker, Colangelo 1998](#)
- leading logs to five loops for (massless) Scalar two-point function [Bissegger Fuhrer 2007](#)
- three loops for generalized GPD [Kivel Polyakov 2007](#)
- Recursion relations in the massless $O(N+1)/O(N)$ sigma model for many quantities [Kivel, Polyakov, Vladimirov 2008](#)

Renormalization group

Underlying **practical** problem: the number of needed terms increases fast with order \implies need a good way to handle this.

KPV: write the 4-meson vertex using Legendre polynomials
could perform all loopintegrals
 \implies algebraic recursion relations

It works in the chiral limit since tadpoles vanish:
simplification: the number of external legs in the vertices needed does not go up.

Conclusions

- Modern ChPT is doing fine:
- Two flavour ChPT is in good shape: precision science in many ways
- Three flavour ChPT: corrections are larger there seem to be some problems, but many parameters (scalar sector) rather uncertain, errors very quantity dependent
- Partially quenched: useful for the lattice
- New application areas continue to be found