CHIRAL PERTURBATION THEORY IN NEW SURROUNDINGS

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Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Overview

- Effective Field Theory
- Chiral Perturbation Theor(y)(ies)
- Three new applications: i.e. Lund the last two years

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- Chiral Perturbation Theor(y)(ies)
- Three new applications: i.e. Lund the last two years
  - Hard Pion Chiral Perturbation Theory
    JB+ Alejandro Celis, arXiv:0906.0302 and JB + Ilaria Jemos, arXiv:1006.1197, arXiv:1011.6531
  - Leading Logarithms to five loop order and large N
    JB + Lisa Carloni, arXiv:0909.5086,arXiv:1008.3499
  - Chiral Extrapolation Formulas for Technicolor and QCDlike theories
    JB + Jie LU, arXiv:0910.5424 and coming

Hadrons

- Hadron: $\alpha\delta\rho o\varsigma$ (hadros: stout, thick)
- Lepton: $\lambda\epsilon\pi\tau o\varsigma$ (leptos: small, thin, delicate) ($\varsigma = \sigma \neq \zeta$)

In those days we had n, p, π, ρ, K, Δ and e, μ.

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- Leptons: those that don't

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- Leptons: those that don't

But they are fundamentally different in other ways too:

- Leptons are known point particles up to about $10^{-19}m \sim \hbar c/(1\ \mathrm{TeV})$
- Hadrons have a typical size of $10^{-15}m$, proton charge radius is 0.875 fm

Hadrons

- Hadrons come in two types:
  - Fermions or half-integer spin: baryons ($\beta\alpha\rho\upsilon\varsigma$ barys, heavy)
  - Bosons or integer spin: mesons ($\mu\epsilon\varsigma o\varsigma$ mesos, intermediate)

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  - Mesons: quark and anti-quark
- Comments:
  - Quarks are as pointlike as leptons
  - Hadrons with different main constituents: glueballs (no quarks), hybrids (with a basic gluon) (probably) exist (mixing is the problem)

Hadron(ic) Physics

The study of the structure and interactions of hadrons

Flavour Physics

- There are six types (Flavours) of quarks in three generations or families
- up, down; strange, charm; bottom and top
- The only (known) interaction that changes quarks into each other (violates the separate quark numbers) is the weak interaction
- Violates also discrete symmetries: $\boldsymbol{C}$harge conjugation, $\boldsymbol{P}$arity and $\boldsymbol{T}$ime reversal.

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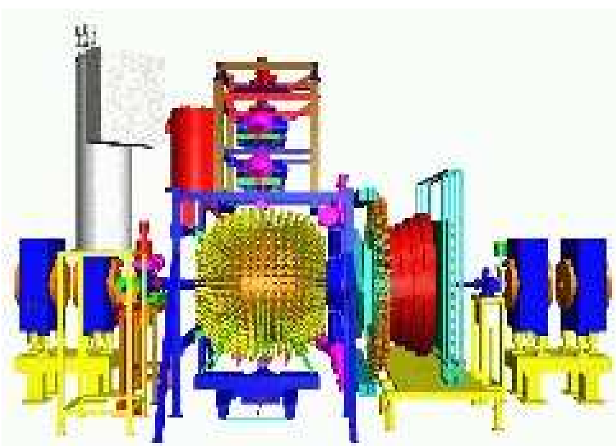
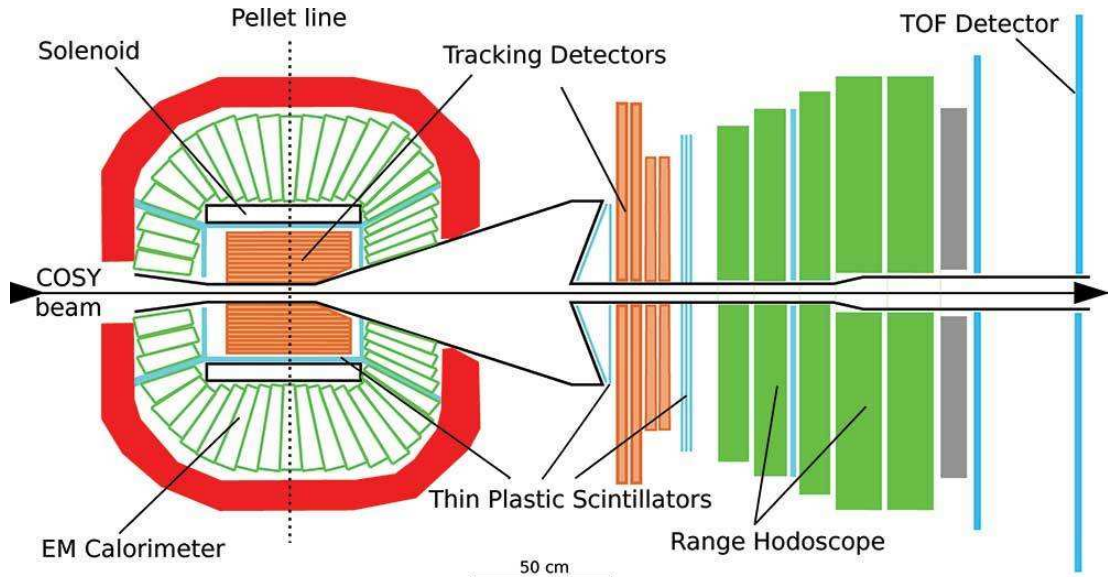
The study of quarks changing flavours (mainly) in decays

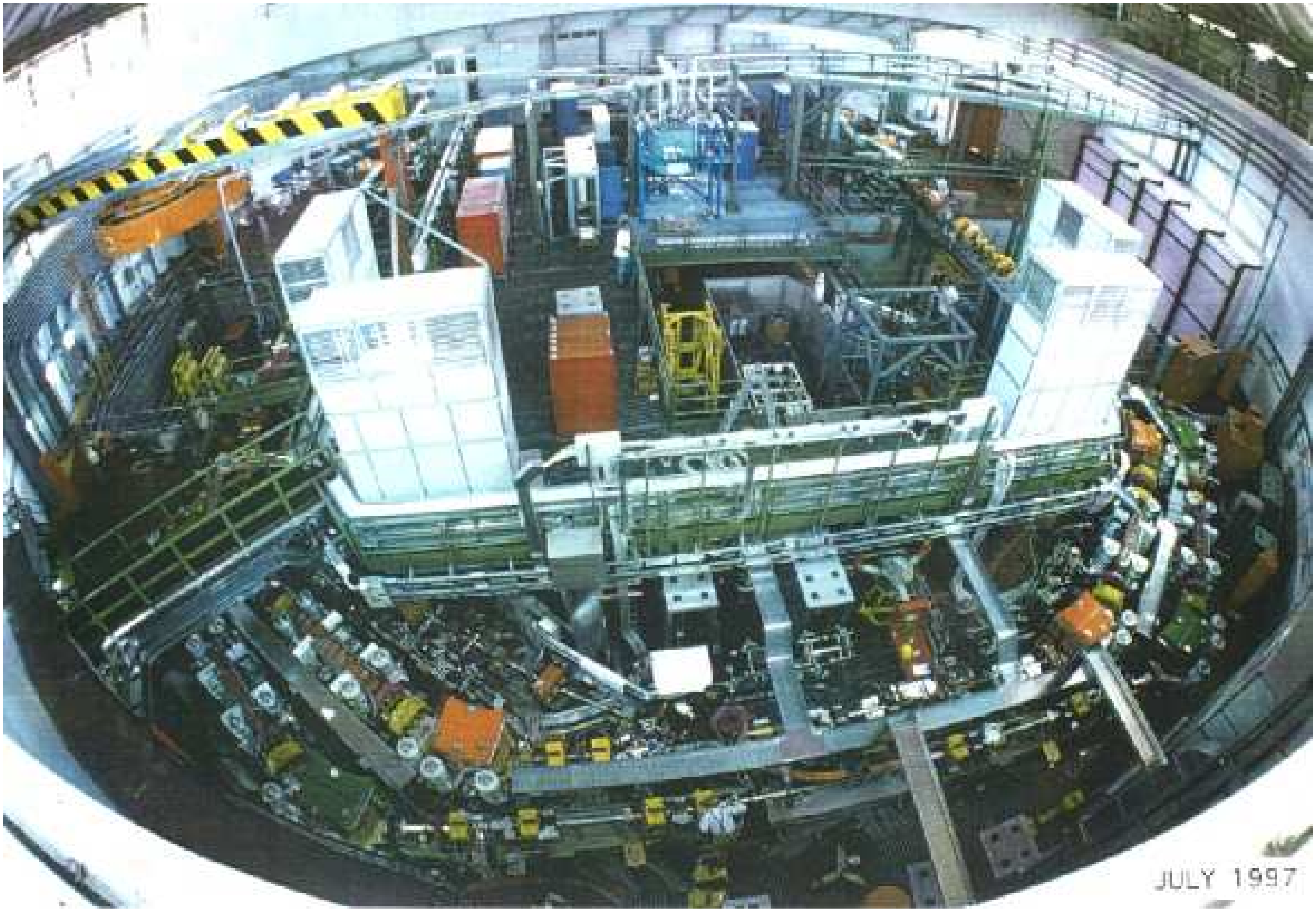
- Experimental research typically done at flavour/hadron factories

Hadron Physics: WASA@COSY



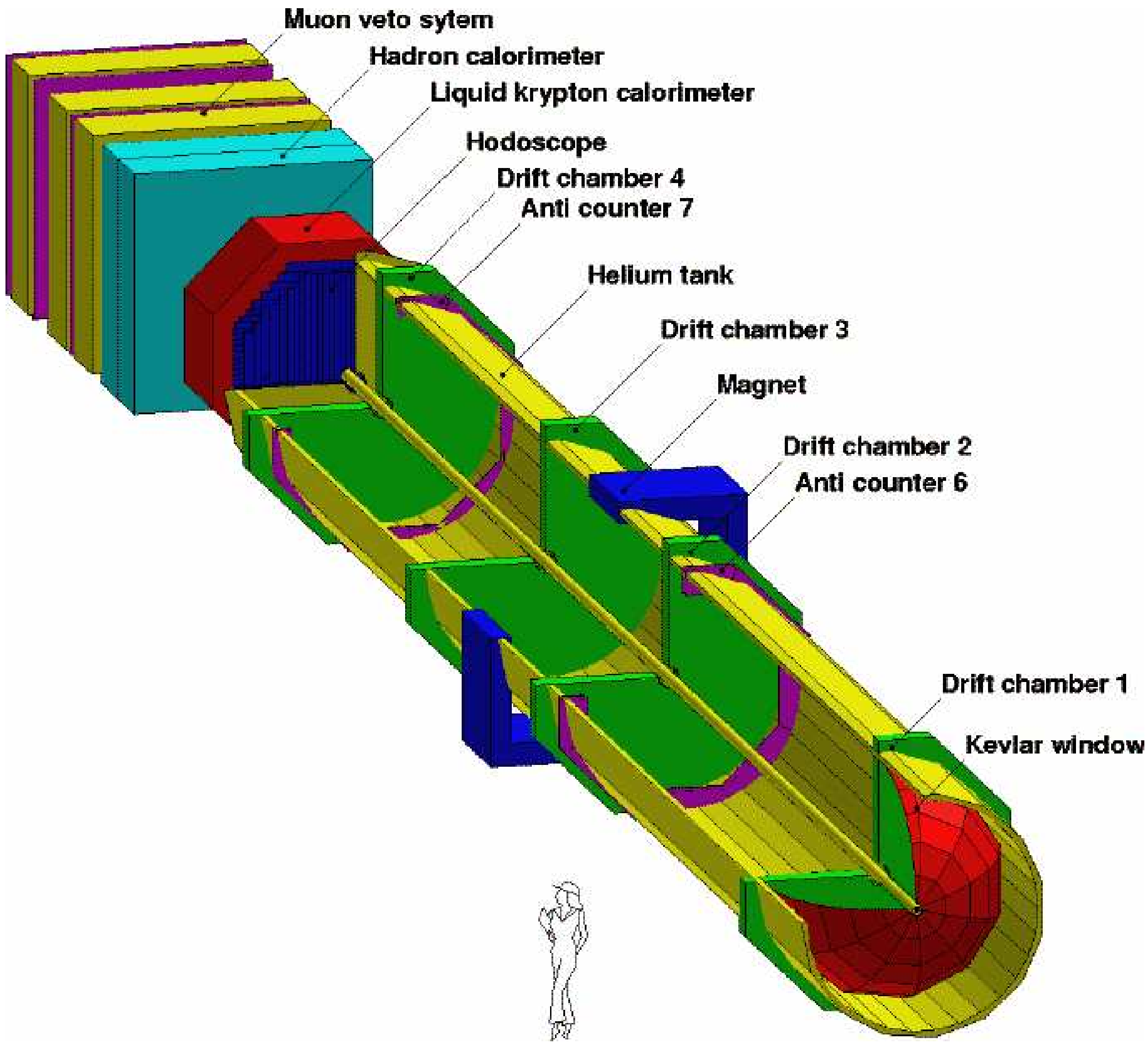


Flavour Physics: DAΦNE in Frascati



Flavour Physics: KEK B in Tsukuba

Flavour Physics: NA48/62 at CERN



Flavour Physics

- The Standard Model Lagrangian has four parts:

$$\underbrace{\mathcal{L}_H(\phi)}_{\text{Higgs}} + \underbrace{\mathcal{L}_G(W,Z,G)}_{\text{Gauge}}$$

$$\underbrace{\sum_{\psi=\text{fermions}} \bar\psi i \not{D}\, \psi}_{\text{gauge-fermion}} + \underbrace{\sum_{\psi,\psi'=\text{fermions}} g_{\psi\psi'}\bar\psi\phi\psi'}_{\text{Yukawa}}$$

- Last piece: weak interaction and mass eigenstates different
- Many extensions: much more complicated flavour changing sector

Flavour Physics

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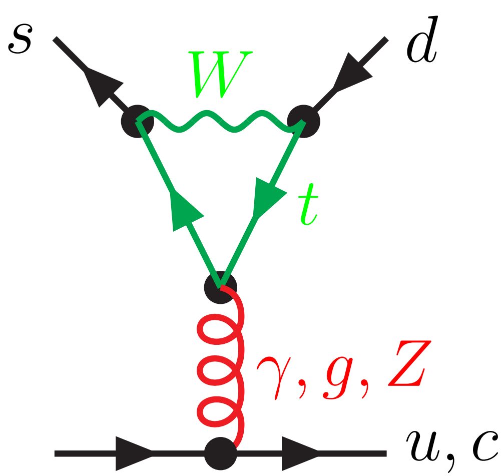

Heavy particles can contribute in loop

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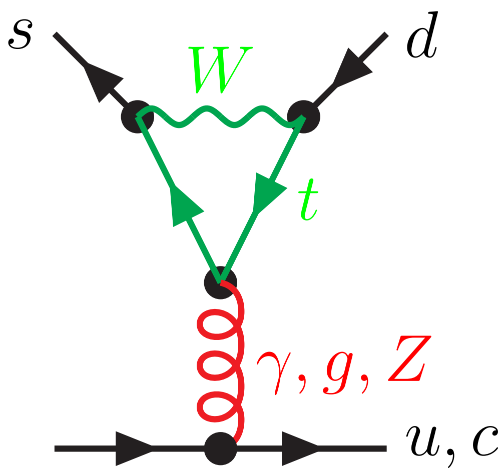


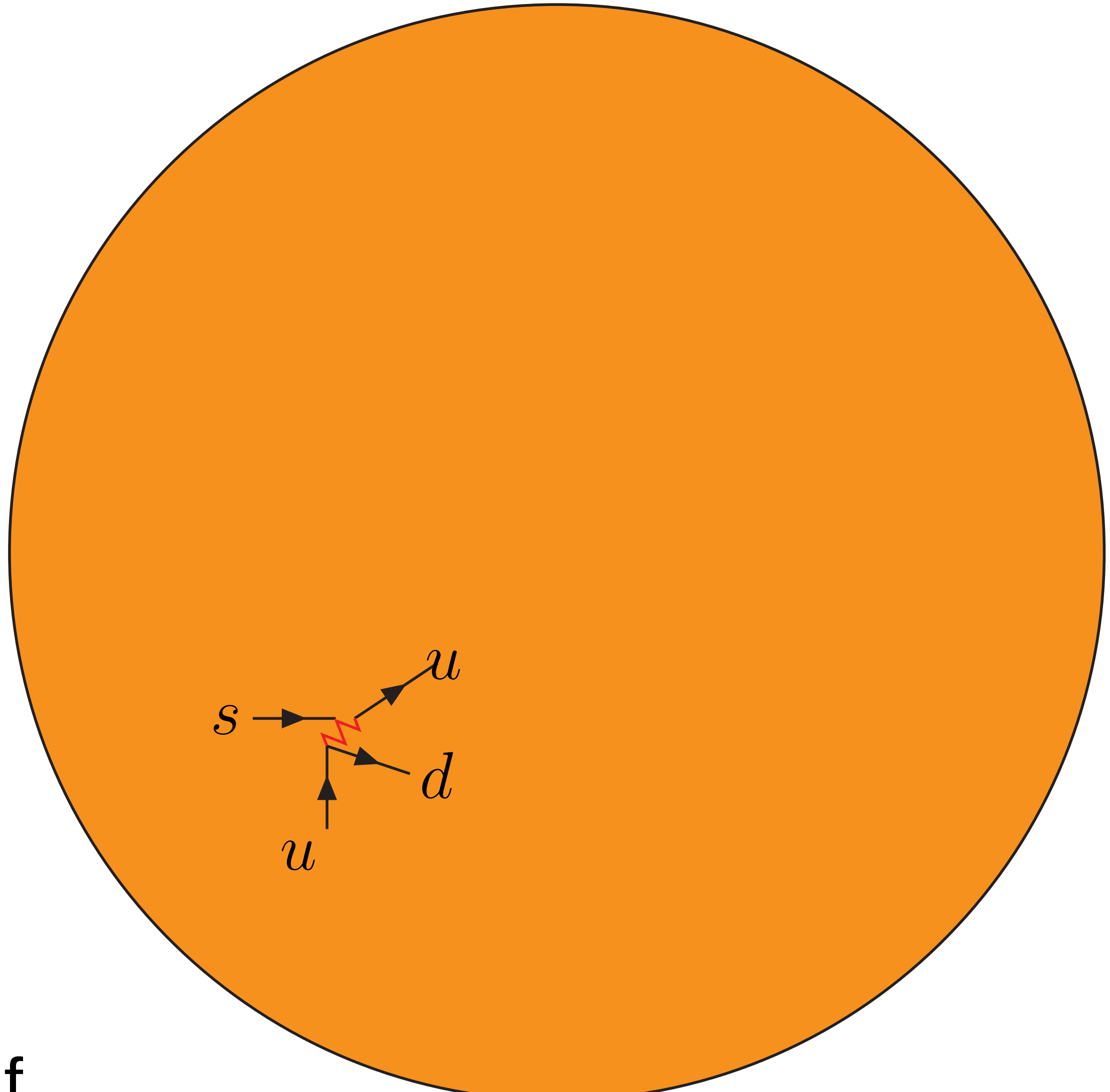
Heavy particles can contribute in loop

- Sometimes need a precise prediction for the standard model effect

Flavour Physics

A weak decay:

Hadron: 1 fm
W-boson: 10^{-3} fm



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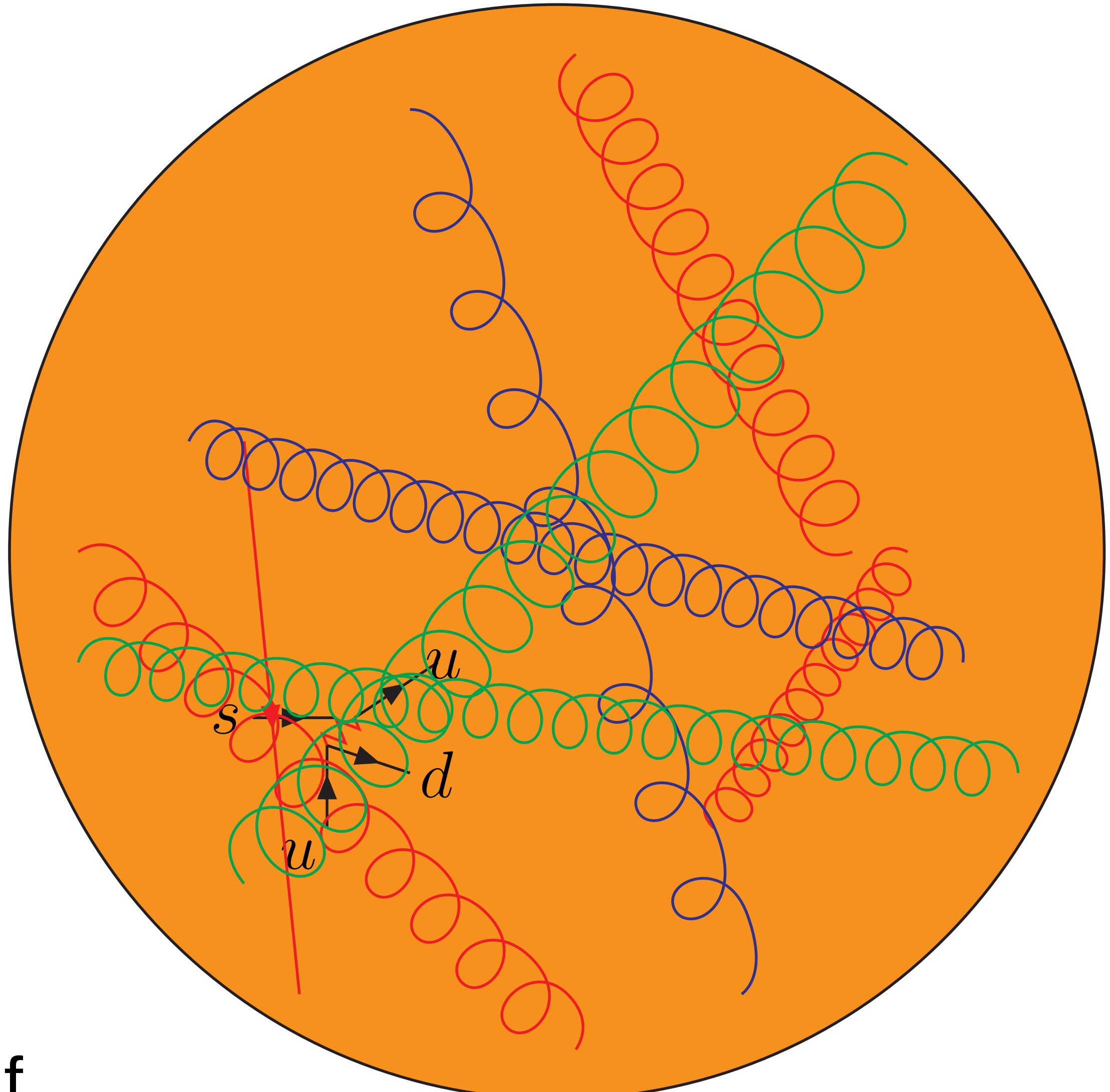

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- QCD: $\mathcal{L} = \overline{q}\gamma_\mu \left(\partial^\mu - i\frac{g}{2}G^\mu\right) q - \frac{1}{8}\mathrm{tr}\left(G_{\mu\nu}G^{\mu\nu}\right)$
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- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- $G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - ig\left(G_\mu G_\nu - G_\nu G_\mu\right)$
- gluons interact with themselves
- $e(\mu)$ smaller for smaller μ, $g(\mu)$ larger for smaller μ
- QCD: low scales no perturbation theory possible

Comments

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  - Lattice Gauge Theory:
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    - Do the resulting (very high dimensional) integral numerically
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  - Be less ambitious: try to solve some parts only: EFT

Wikipedia

`http://en.wikipedia.org/wiki/Effective_field_theory`

In physics, an effective field theory is an approximate theory (usually a quantum field theory) that contains the appropriate degrees of freedom to describe physical phenomena occurring at a chosen length scale, but ignores the substructure and the degrees of freedom at shorter distances (or, equivalently, higher energies).

Effective Field Theory (EFT)

Main Ideas:

- Use right degrees of freedom : essence of (most) physics
- If mass-gap in the excitation spectrum: neglect degrees of freedom above the gap.

Examples: { Solid state physics: conductors: neglect the empty bands above the partially filled one
Atomic physics: Blue sky: neglect atomic structure

EFT: Power Counting

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Higher orders suppressed by powers of $1/\Lambda$

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- ➠ $\infty \#$ parameters
- ➠ Where did my predictivity go ?

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Higher orders suppressed by powers of $1/\Lambda$

➠ Taylor series expansion does not work (convergence radius is zero when massless modes are present)

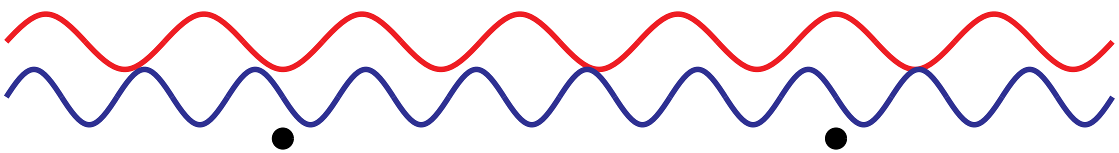
➠ Continuum of excitation states need to be taken into account

Example: Why is the sky blue ?

System: Photons of visible light and neutral atoms

Length scales: a few 1000 Å versus 1 Å

Atomic excitations suppressed by $\approx 10^{-3}$



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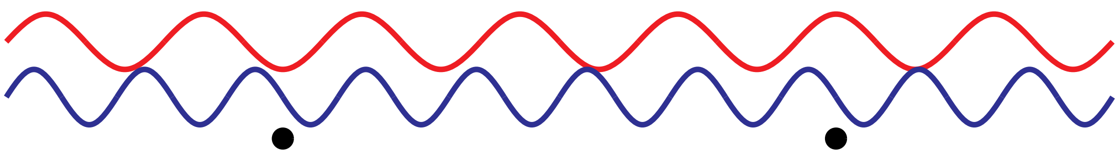

$$\mathcal{L}_A = \Phi_v^\dagger \partial_t \Phi_v + \dots \qquad \mathcal{L}_{\gamma A} = G F_{\mu\nu}^2 \Phi_v^\dagger \Phi_v + \dots$$

Units with $\hbar = c = 1$: G energy dimension -3:

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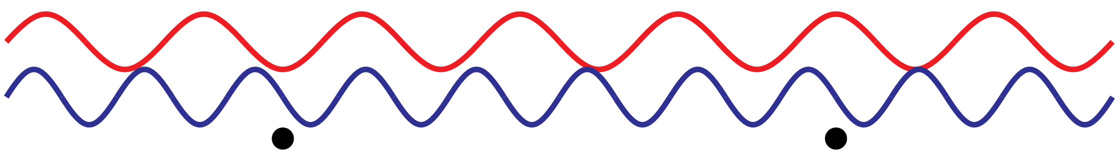

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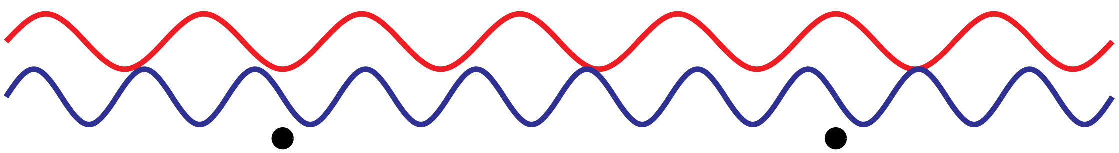

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blue light scatters a lot more than red $\begin{cases} \Longrightarrow \text{red sunsets} \\ \Longrightarrow \text{blue sky} \end{cases}$

Higher orders suppressed by 1 Å$/\lambda_\gamma$.

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Advantages

- Calculations are (relatively) simple
- It is general: model-independent
- Theory $\Longrightarrow$ errors can be estimated
- Systematic: ALL effects at a given order can be included
- Even if no convergence: classification of models often useful

Examples of EFT

- Fermi theory of the weak interaction
- Chiral Perturbation Theory: hadronic physics
- NRQCD
- SCET
- General relativity as an EFT
- 2,3,4 nucleon systems from EFT point of view
- Magnons and spin waves

references

- A. Manohar, Effective Field Theories (Schladming lectures), hep-ph/9606222
- I. Rothstein, Lectures on Effective Field Theories (TASI lectures), hep-ph/0308266
- G. Ecker, Effective field theories, Encyclopedia of Mathematical Physics, hep-ph/0507056
- D.B. Kaplan, Five lectures on effective field theory, nucl-th/0510023
- A. Pich, Les Houches Lectures, hep-ph/9806303
- S. Scherer, Introduction to chiral perturbation theory, hep-ph/0210398
- J. Donoghue, Introduction to the Effective Field Theory Description of Gravity, gr-qc/9512024

Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

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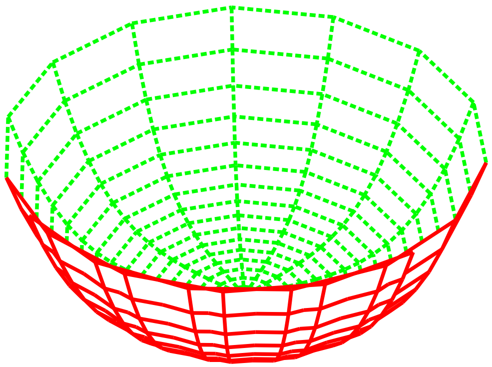
Derivation from QCD:

H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory,* Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

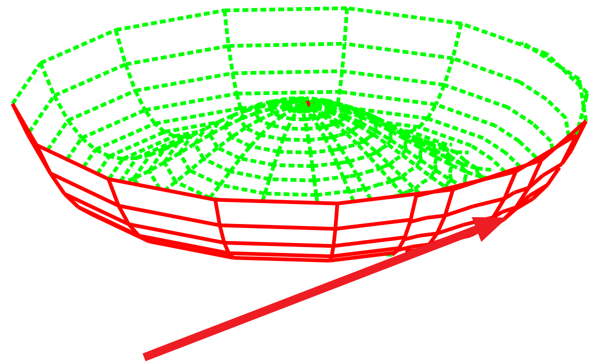
The mass gap: Goldstone Modes

UNBROKEN: $V(\phi)$



Only massive modes around lowest energy state (=vacuum)

BROKEN: $V(\phi)$



Need to pick a vacuum
$\langle \phi \rangle \neq 0$: Breaks symmetry
No parity doublets
Massless mode along bottom

For more complicated symmetries: need to describe the bottom mathematically: $G \to H \Longrightarrow G/H$

The two symmetry modes compared

Wigner-Eckart mode	Nambu-Goldstone mode
Symmetry group G	G spontaneously broken to subgroup H
Vacuum state unique	Vacuum state degenerate
Massive Excitations	Existence of a massless mode
States fall in multiplets of G	States fall in multiplets of H
Wigner Eckart theorem for G	Wigner Eckart theorem for H
	Broken part leads to low-energy theorems
Symmetry linearly realized	Full Symmetry, G, nonlinearly realized
	unbroken part, H, linearly realized

Some clarifications

- $\phi(x)$: orientation of vacuum in every space-time point
- Examples: spin waves, phonons
- Nonlinear: acting by a broken symmetry operator changes the vacuum, $\phi(x) \to \phi(x) + \alpha$
- The precise form of ϕ is *not* important but it must describe the space of vacua (field transformations possible)
- In gauge theories: the *local* symmetry allows the vacua to be different in every point, hence the Goldstone Boson might not be observable as a massless degree of freedom.

The power counting

Very important:

Low energy theorems: Goldstone bosons do not interact at zero momentum

Heuristic proof:

- Which vacuum does not matter, choices related by symmetry
- $\phi(x) \rightarrow \phi(x) + \alpha$ should not matter
- Each term in $\mathcal{L}$ must contain at least one $\partial_\mu \phi$

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

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Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

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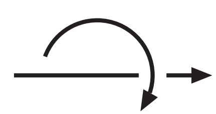
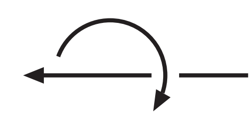
Can also see that via



$$v < c,\ m_q \neq 0 \Longrightarrow$$
$$v = c,\ m_q = 0 \not\Longrightarrow$$



Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom and interaction vanishes at zero momentum

We have 8 candidates that are light compared to hte other hadrons: $\pi^0, \pi^+, \pi^-, K^+, K^-, K^0, \overline{K^0}, \eta$

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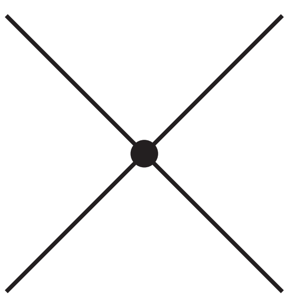
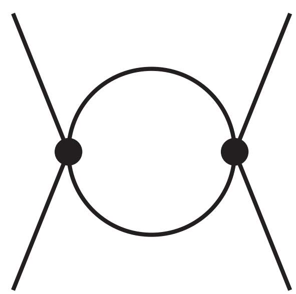
Power counting in momenta (all lines soft):



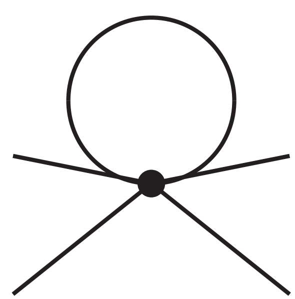
p^2



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

$1/p^2$

$\int d^4p$ $\quad p^4$



$(p^2)\,(1/p^2)\,p^4 = p^4$

Chiral Perturbation Theor**ies**

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
  - Two or Three (or even more) Flavours
  - Strong interaction and couplings to external currents/densities
  - Including electromagnetism
  - Including weak nonleptonic interactions
  - Treating kaon as heavy

Many similarities with strongly interacting Higgs

Hard pion ChPT?

- In Meson ChPT: the powercounting is from all lines in Feynman diagrams having soft momenta
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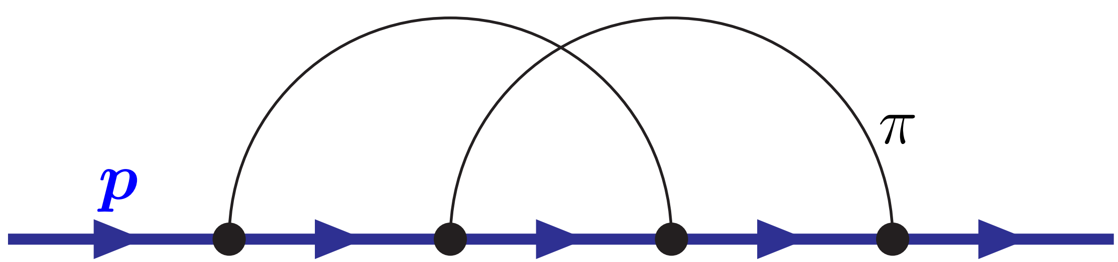
  - $p = M_B v + k$
  - Everything else soft
  - Works because baryon or b or c number conserved so the non soft line is continuous



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  - Decay constant works: takes away all heavy momentum
  - General idea: M_p dependence can always be reabsorbed in LECs, is analytic in the other parts k.

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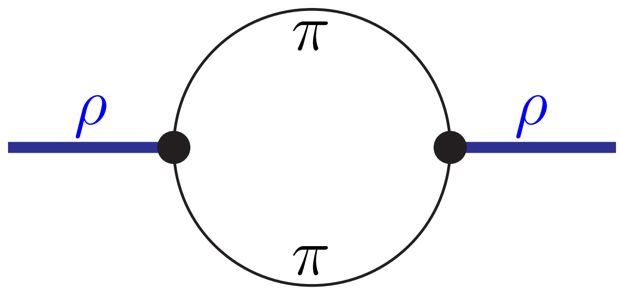
  - But (Heavy) (Vector) Meson ChPT decays strongly



Hard pion ChPT?

- (Heavy) (Vector or other) Meson ChPT:
  - (Vector) Meson: $p = M_V v + k$
  - Everyone else soft or $p = M_V v + k$
  - But (Heavy) (Vector) Meson ChPT decays strongly
    - First: keep diagrams where vectors always present
    - Applied to masses and decay constants
    - Decay constant works: takes away all heavy momentum
  - *It was argued that this could be done, the nonanalytic parts of diagrams with pions at large momenta are reproduced correctly* JB-Gosdzinsky-Talavera
  - Done both in relativistic and heavy meson formalism
  - General idea: M_V dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- Heavy Kaon ChPT:
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon goes through
  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,. . .
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
  - Works like all the previous *heavy* ChPT

Hard pion ChPT?

- Heavy Kaon ChPT:
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon goes through
  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,. . .
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
- Flynn-Sachrajda argued $K_{\ell 3}$ also for q^2 away from q^2_{max}.
- JB-Celis Argument generalizes to other processes with hard/fast pions and applied to $K \to \pi\pi$
- JB Jemos $B, D \to D, \pi, K, \eta$ vector formfactors and a two-loop check
- General idea: heavy/fast dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q\to 0}\langle\pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

Hard pion ChPT?

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$$\lim_{q\to 0}\langle \pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

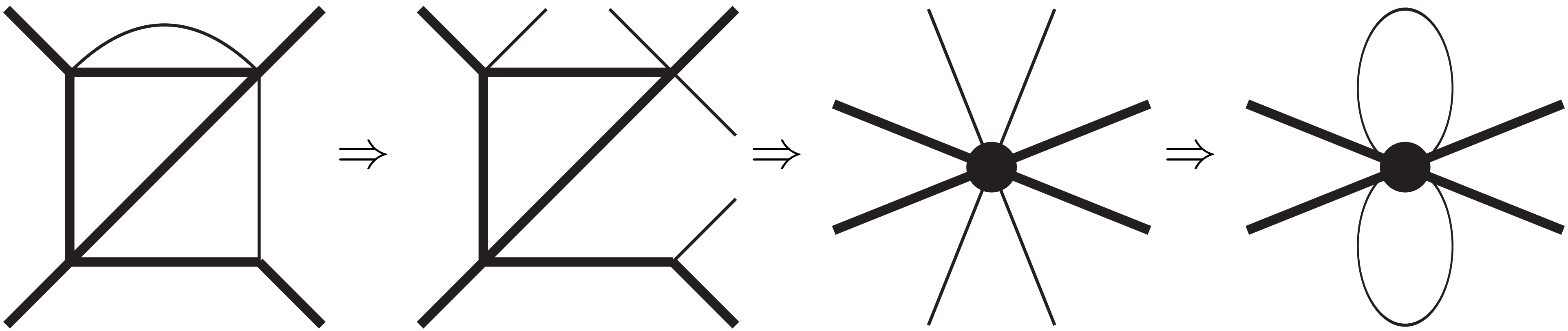
- Nothing prevents hard pions to be in the states α or β
- So by heavily using current algebra I should be able to get the light quark mass nonanalytic dependence

Hard pion ChPT?

Field Theory: a process at given external momenta

- Take a diagram with a particular internal momentum configuration
- Identify the soft lines and cut them
- The result part is analytic in the soft stuff
- So should be describably by an effective Lagrangian with coupling constants dependent on the external given momenta
- If symmetries present, Lagrangian should respect them
- Lagrangian should be complete in *neighbourhood*
- Loop diagrams with this effective Lagrangian *should* reproduce the nonanalyticities in the light masses
  Crucial part of the argument

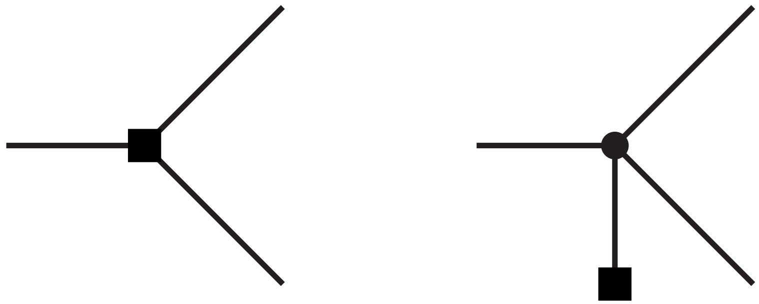
Hard pion ChPT?



This procedure works at one loop level, matching at tree level, nonanalytic dependence at one loop:

- Toy models and vector meson ChPT JB, Gosdzinsky, Talavera
- Recent work on relativistic meson ChPT Gegelia, Scherer et al.
- Extra terms kept in $K \to 2\pi$: a one-loop check
- Some preliminary two-loop checks

$K \to \pi\pi$: Tree level

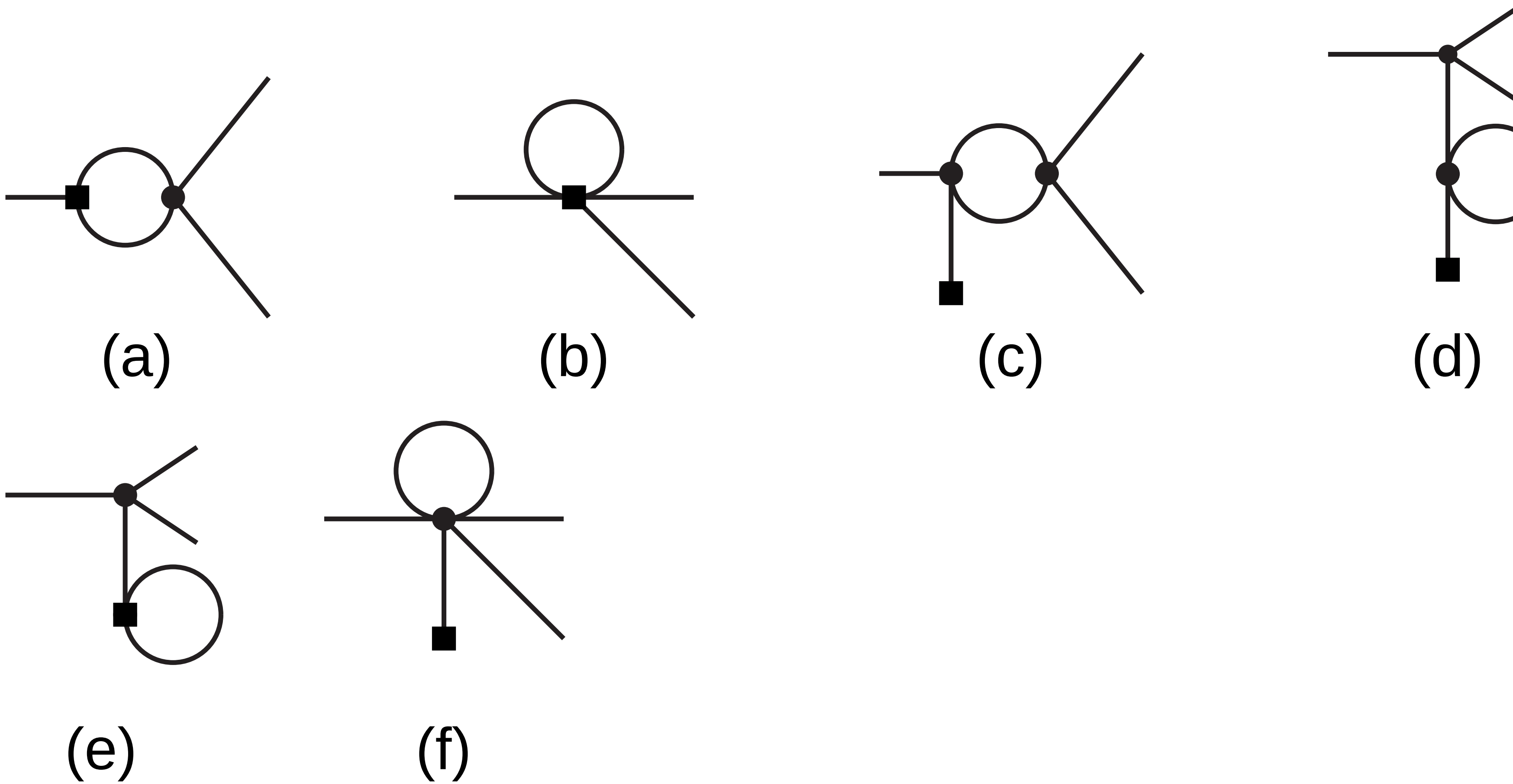


(a) (b)

$$A_0^{LO} = \frac{\sqrt{3}i}{2F^2}\left[-\frac{1}{2}E_1 + (E_2 - 4E_3)\,\overline{M}_K^2 + 2E_8\overline{M}_K^4 + A_1E_1\right]$$

$$A_2^{LO} = \sqrt{\frac{3}{2}}\frac{i}{F^2}\left[(-2D_1 + D_2)\,\overline{M}_K^2\right]$$

$K \to \pi\pi$: One loop



$K \to \pi\pi$: One loop

Diagram	A_0	A_2
Z	$-\frac{2F^2}{3}A_0^{LO}$	$-\frac{2F^2}{3}A_2^{LO}$
(a)	$\sqrt{3}i\left(-\frac{1}{3}E_1+\frac{2}{3}E_2\overline{M}_K^2\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{2}{3}D_2\overline{M}_K^2\right)$
(b)	$\sqrt{3}i\left(-\frac{5}{96}E_1-\left(\frac{7}{48}E_2+\frac{25}{12}E_3\right)\overline{M}_K^2+\frac{25}{24}E_8\overline{M}_K^4\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{61}{12}D_1+\frac{77}{24}D_2\right)\overline{M}_K^2$
(e)	$\sqrt{3}i\frac{3}{16}A_1E_1$	
(f)	$\sqrt{3}i\left(\frac{1}{8}E_1+\frac{1}{3}A_1E_1\right)$	

The coefficients of $\overline{A}(M^2)/F^4$ in the contributions to A_0 and A_2. Z denotes the part from wave-function renormalization.

- $$\overline{A}(M^2) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2}$$
- $K\pi$ intermediate state does not contribute, but did for Flynn-Sachrajda

$K \to \pi\pi$: One-loop

$$A_0^{NLO} = A_0^{LO}\left(1 + \frac{3}{8F^2}\overline{A}(M^2)\right) + \lambda_0 M^2 + \mathcal{O}(M^4)\,,$$

$$A_2^{NLO} = A_2^{LO}\left(1 + \frac{15}{8F^2}\overline{A}(M^2)\right) + \lambda_2 M^2 + \mathcal{O}(M^4)\,.$$

$K \to \pi\pi$: One-loop

$$A_0^{NLO} = A_0^{LO}\left(1+\frac{3}{8F^2}\overline{A}(M^2)\right)+\lambda_0 M^2+\mathcal{O}(M^4)\,,$$

$$A_2^{NLO} = A_2^{LO}\left(1+\frac{15}{8F^2}\overline{A}(M^2)\right)+\lambda_2 M^2+\mathcal{O}(M^4)\,.$$

Match with three flavour $SU(3)$ calculation Kambor, Missimer, Wyler; JB, Pallante, Prades

$$A_0^{(3)LO} = -\frac{i\sqrt{6}CF_0^4}{\overline{F}_K F^2}\left(G_8+\frac{1}{9}G_{27}\right)\overline{M}_K^2\,, \qquad A_2^{(3)LO} = -\frac{i10\sqrt{3}CF_0^4}{9\overline{F}_K F^2}G_{27}\overline{M}_K^2\,,$$

When using $F_\pi = F\left(1+\frac{1}{F^2}\overline{A}(M^2)+\frac{M^2}{F^2}l_4^r\right)\,,\ F_K = \overline{F}_K\left(1+\frac{3}{8F^2}\overline{A}(M^2)+\cdots\right)\,,$

logarithms at one-loop agree with above

Hard Pion ChPT: A two-loop check

- Similar arguments to JB-Celis, Flynn-Sachrajda work for the pion vector and scalar formfactor JB-Jemos
- Therefore at any t the chiral log correction must go like the one-loop calculation.
- But note the one-loop log chiral log is with $t >> m_\pi^2$
- Predicts

$$F_V(t, M^2) = F_V(t, 0)\left(1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

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- Note that $F_{V,S}(t,0)$ is now a coupling constant and can be complex
- Take the full two-loop ChPT calculation JB,Colangelo,Talavera and expand in $t >> m_\pi^2$.

A two-loop check

Full two-loop ChPT JB,Colangelo,Talavera, expand in $t >> m_\pi^2$:

$$F_V(t,M^2) = F_V(t,0)\left(1 - \frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$
$$F_S(t,M^2) = F_S(t,0)\left(1 - \frac{5}{2}\frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

with

$$F_V(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(\frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6}\ln\frac{t}{\mu^2}\right)$$
$$F_S(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(1 + 16\pi^2 l_4^r + i\pi - \ln\frac{t}{\mu^2}\right)$$

- The needed coupling constants are complex
- Both calculations have two-loop diagrams with overlapping divergences
- The chiral logs should be valid for any t where a pointlike interaction is a valid approximation

Electromagnetic formfactors

$$F_V^\pi(s) = F_V^{\pi\chi}(s)\left(1 + \frac{1}{F^2}\overline{A}(m_\pi^2) + \frac{1}{2F^2}\overline{A}(m_K^2) + \mathcal{O}(m_L^2)\right) ,$$

$$F_V^K(s) = F_V^{K\chi}(s)\left(1 + \frac{1}{2F^2}\overline{A}(m_\pi^2) + \frac{1}{F^2}\overline{A}(m_K^2) + \mathcal{O}(m_L^2)\right) .$$

$B, D \to \pi, K, \eta$

$$\left\langle P_f(p_f) \left| \overline{q}_i \gamma_\mu q_f \right| P_i(p_i) \right\rangle = (p_i + p_f)_\mu f_+(q^2) + (p_i - p_f)_\mu f_-(q^2)$$

$$\begin{aligned} f_{+B\to M}(t) &= f^\chi_{+B\to M}(t) F_{B\to M} \\ f_{-B\to M}(t) &= f^\chi_{-B\to M}(t) F_{B\to M} \end{aligned}$$

- $F_{B\to M}$ always same for f_+, f_- and f_0
- This is not heavy quark symmetry: not valid at endpoint and valid also for $K \to \pi$.
- Not like Low's theorem, not only dependence on external legs

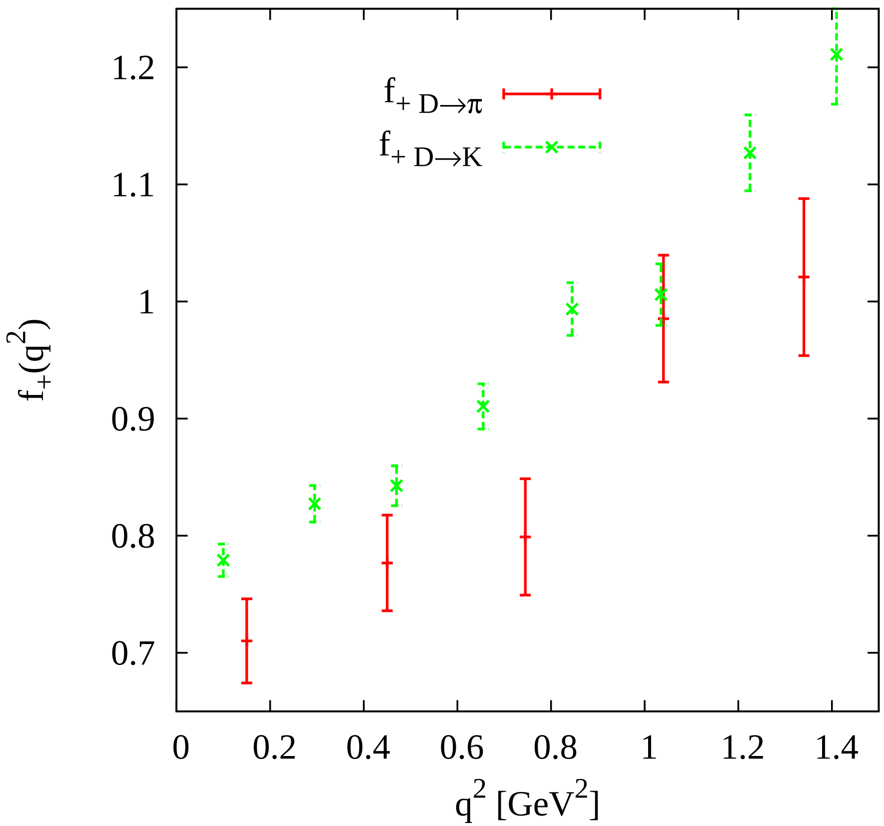
$B, D \to \pi, K, \eta$

$$
\begin{aligned}
F_{K\to\pi} &= 1 + \frac{3}{8F^2}\overline{A}(m_\pi^2) \qquad\qquad (2-\text{flavour})\\
F_{B\to\pi} &= 1 + \left(\frac{3}{8}+\frac{9}{8}g^2\right)\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4}+\frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24}+\frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},\\
F_{B\to K} &= 1 + \frac{9}{8}g^2\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{2}+\frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{6}+\frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},\\
F_{B\to\eta} &= 1 + \left(\frac{3}{8}+\frac{9}{8}g^2\right)\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4}+\frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24}+\frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},\\
F_{B_s\to K} &= 1 + \frac{3}{8}\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4}+\frac{3}{2}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24}+\frac{1}{2}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},\\
F_{B_s\to\eta} &= 1 + \left(\frac{1}{2}+\frac{3}{2}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{6}+\frac{1}{2}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2}.
\end{aligned}
$$

$F_{B_s\to\pi}$ vanishes due to the possible flavour quantum numbers.

Experimental check

CLEO data on $f_+(q^2)|V_{cq}|$ for $D \to \pi$ and $D \to K$ with $|V_{cd}| = 0.2253$, $|V_{cs}| = 0.9743$



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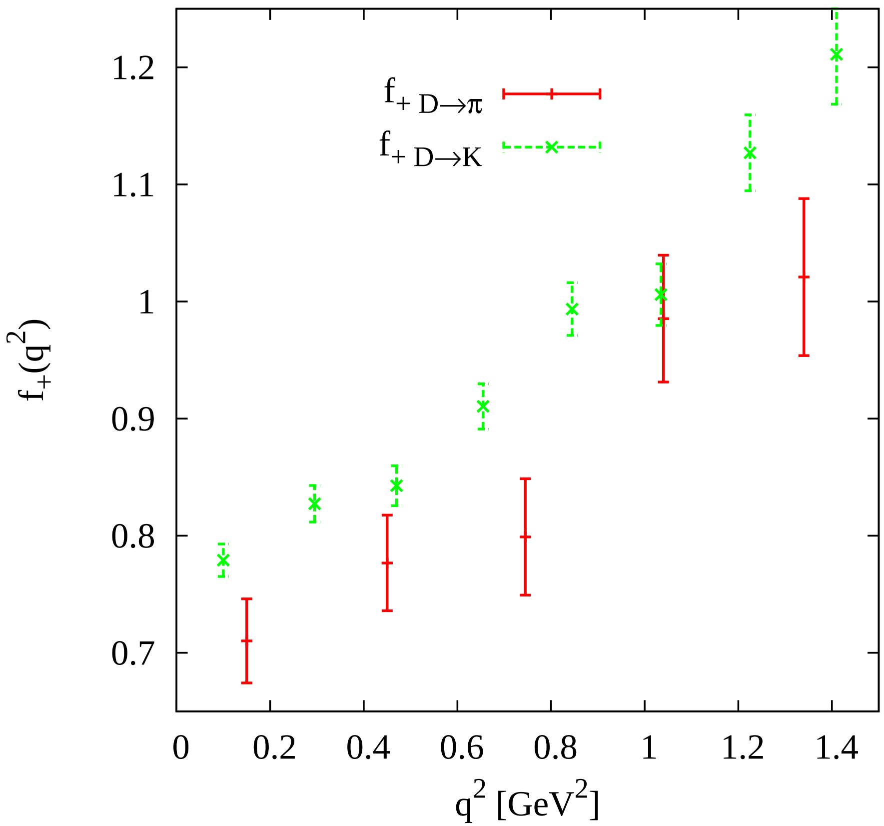

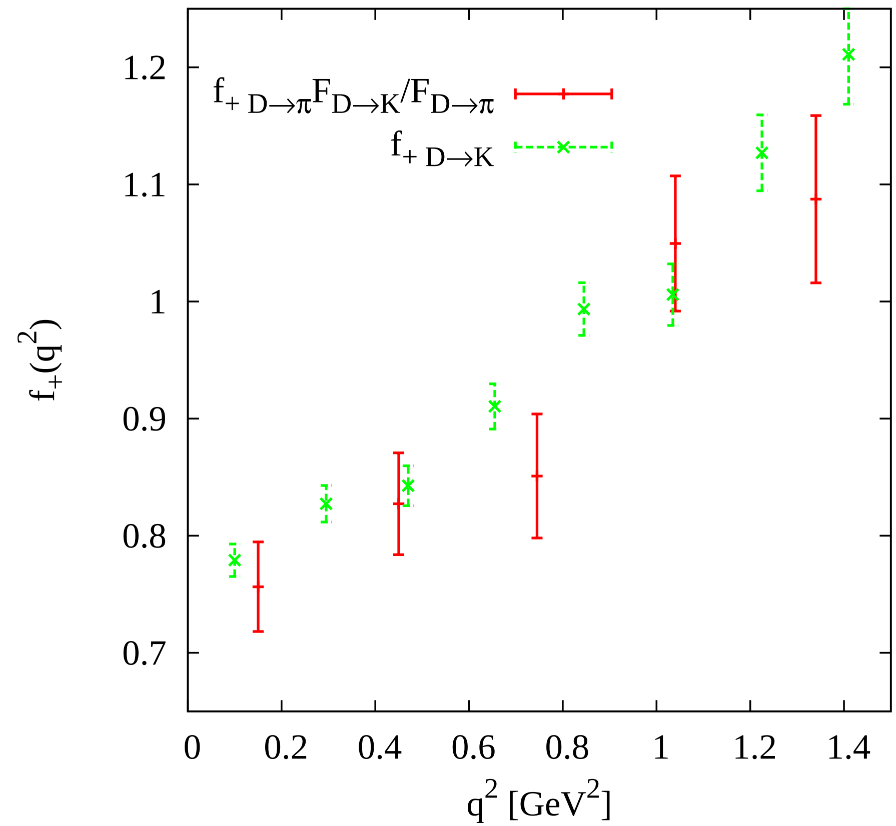

$$f_{+D\to\pi} = f_{+D\to K} F_{D\to\pi}/F_{D\to K}$$

Hard Pion ChPT: summary

Why is this useful:

- Lattice works actually around the strange quark mass
- need only extrapolate in m_u and m_d.
- Applicable in momentum regimes where usual ChPT might not work
- Three flavour case useful for B, D decays

Leading Logarithms

- Take a quantity with a single scale: $F(M)$
- The dependence on the scale in field theory is typically logarithmic
- $L = \log(\mu/M)$
- $F = F_0 + F_1^1 L + F_0^1 + F_2^2 L^2 + F_1^2 L + F_0^2 + F_3^3 L^3 + \cdots$
- Leading Logarithms: The terms $F_m^m L^m$

The F_m^m can be more easily calculated than the full result

- $\mu\,(dF/d\mu) \equiv 0$
- Ultraviolet divergences in Quantum Field Theory are always local

Renormalizable theories

- Loop expansion $\equiv \alpha$ expansion
- $F = \alpha + f_1^1 \alpha^2 L + f_0^1 \alpha^2 + f_2^2 \alpha^3 L^2 + f_1^2 \alpha^3 L + f_0^2 \alpha^3 + f_3^3 \alpha^4 L^3 + \cdots$
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- $\mu \frac{dF}{d\mu} \equiv F'$, $\mu \frac{d\alpha}{d\mu} \equiv \alpha'$, $\mu \frac{dL}{d\mu} = 1$
- $F' = \alpha' + f_1^1 \alpha^2 + f_1^1 2\alpha' \alpha L + f_2^2 \alpha^3 2L + f_2^2 3\alpha' \alpha^2 L^2$
  $+ f_1^2 \alpha^3 + f_1^2 3\alpha' \alpha^2 L + f_0^2 3\alpha' \alpha^2 + f_3^3 \alpha^3 3L^2 + f_3^3 4\alpha' \alpha^3 L^3 + \cdots$

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- $\boxed{\beta_0 = -f_1^1 = f_2^2 = -f_3^3 = \cdots}$

Renormalization Group

- Can be extended to other operators as well
- Underlying argument always $\mu \frac{dF}{d\mu} = 0$.
- Gell-Mann–Low, Callan–Symanzik, Weinberg–'t Hooft
- In great detail: J.C. Collins, *Renormalization*
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- In effective field theories: different Lagrangian at each order
- The recursive argument does not work

Weinberg's argument

- Weinberg, Physica A96 (1979) 327
- Two-loop leading logarithms can be calculated using only one-loop
- Weinberg consistency conditions
- $\pi\pi$ at 2-loop: Colangelo, hep-ph/9502285
- General at 2 loop: JB, Colangelo, Ecker, hep-ph/9808421
- Proof at all orders using β-functions
  Büchler, Colangelo, hep-ph/0309049
- Proof with diagrams: present work

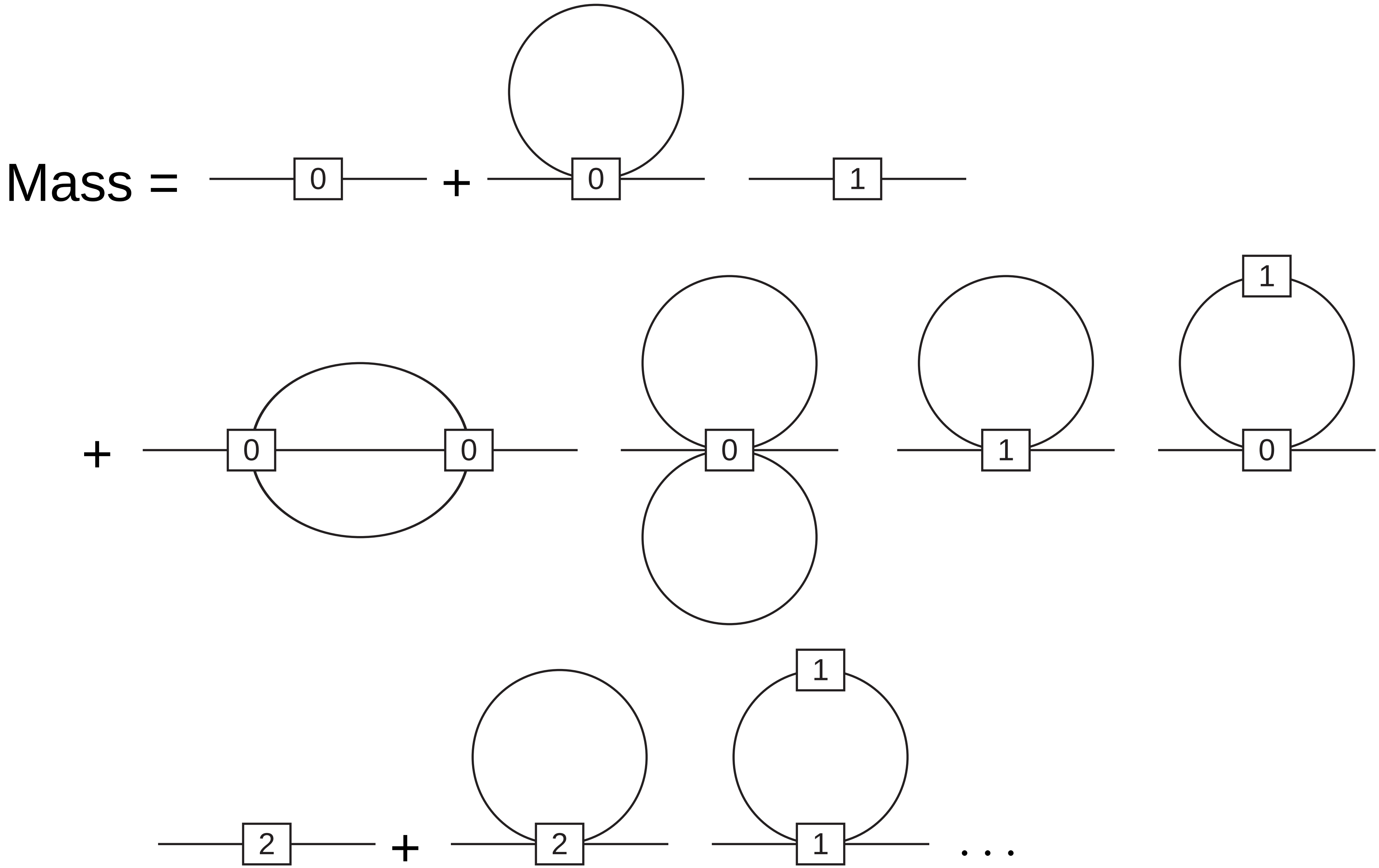
Weinberg's argument

- μ: dimensional regularization scale
- $d = 4 - w$
- loop-expansion $\equiv$ $\hbar$-expansion
- $\mathcal{L}^{\text{bare}} = \sum_{n\geq 0} \hbar^n \mu^{-nw} \mathcal{L}^{(n)}$
- $\mathcal{L}^{(n)} = \sum_i \left(\sum_{k=0,n} \frac{c_{ki}^{(n)}}{w^k} \right) \mathcal{O}_i^{(n)}$
- $c_{0i}^{(n)}$ have a direct μ-dependence
- $c_{ki}^{(n)}$ $k \geq 1$ only depend on μ through $c_{0i}^{(m<n)}$

Weinberg's argument

- L_l^n l-loop contribution at order $\hbar^n$
- Expand in divergences from the loops (not from the counterterms) $L_l^n = \sum_{k=0,l} \frac{1}{w^k} L_{kl}^n$
- Neglected positive powers: not relevant here, but should be kept in general
- $\{c\}_l^n$ all combinations $c_{k_1 j_1}^{(m_1)} c_{k_2 j_2}^{(m_2)} \ldots c_{k_r j_r}^{(m_r)}$ with $m_i \geq 1$, such that $\sum_{i=1,r} m_i = n$ and $\sum_{i=1,r} k_i = l$.
- $\{c_n^n\} \equiv \{c_{ni}^{(n)}\}$, $\{c\}_2^2 = \{c_{2i}^{(2)}, c_{1i}^{(1)} c_{1k}^{(1)}\}$
- $\mathcal{L}^{(n)} =$ $\boxed{\text{n}}$

Weinberg's argument



Weinberg's argument

- $\hbar^0$: L_0^0
- $\hbar^1$: $\frac{1}{w}\left(\mu^{-w}L_{00}^1(\{c\}_1^1) + L_{11}^1\right) + \mu^{-w}L_{00}^1(\{c\}_0^1) + L_{10}^1$

Weinberg's argument

- $\hbar^0$: L_0^0
- $\hbar^1$: $\frac{1}{w}\left(\mu^{-w}L^1_{00}(\{c\}^1_1) + L^1_{11}\right) + \mu^{-w}L^1_{00}(\{c\}^1_0) + L^1_{10}$
  - Expand $\mu^{-w} = 1 - w\log\mu + \frac{1}{2}w^2\log^2\mu + \cdots$
  - $1/w$ must cancel: $L^1_{00}(\{c\}^1_1) + L^1_{11} = 0$
    this determines the c^1_{1i}
  - Explicit $\log\mu$: $-\log\mu\, L^1_{00}(\{c\}^1_0) \equiv \log\mu\, L^1_{11}$

Weinberg's argument

- $\hbar^2$:
$$\frac{1}{w^2}\left(\mu^{-2w}L^2_{00}(\{c\}^2_2)+\mu^{-w}L^2_{11}(\{c\}^1_1)+L^2_{22}\right)$$
$$+\frac{1}{w}\left(\mu^{-2w}L^2_{00}(\{c\}^2_1)+\mu^{-w}L^2_{11}(\{c\}^1_0)+\mu^{-w}L^2_{10}(\{c\}^1_1)+L^2_{21}\right)$$
$$+\left(\mu^{-2w}L^2_{00}(\{c\}^2_0)+\mu^{-w}L^2_{10}(\{c\}^1_0)+L^2_{20}\right)$$
- $1/w^2$ and $\log\mu/w$ must cancel
$$L^2_{00}(\{c\}^2_2)+L^2_{11}(\{c\}^1_1)+L^2_{22}=0$$
$$2L^2_{00}(\{c\}^2_2)+L^2_{11}(\{c\}^1_1)=0$$

Weinberg's argument

- $\hbar^2$:
$$\frac{1}{w^2}\left(\mu^{-2w}L^2_{00}(\{c\}^2_2)+\mu^{-w}L^2_{11}(\{c\}^1_1)+L^2_{22}\right)$$
$$+\frac{1}{w}\left(\mu^{-2w}L^2_{00}(\{c\}^2_1)+\mu^{-w}L^2_{11}(\{c\}^1_0)+\mu^{-w}L^2_{10}(\{c\}^1_1)+L^2_{21}\right)$$
$$+\left(\mu^{-2w}L^2_{00}(\{c\}^2_0)+\mu^{-w}L^2_{10}(\{c\}^1_0)+L^2_{20}\right)$$
- $1/w^2$ and $\log\mu/w$ must cancel
$$L^2_{00}(\{c\}^2_2)+L^2_{11}(\{c\}^1_1)+L^2_{22}=0$$
$$2L^2_{00}(\{c\}^2_2)+L^2_{11}(\{c\}^1_1)=0$$
- Solution: $L^2_{00}(\{c\}^2_2)=-\frac{1}{2}L^2_{11}(\{c\}^1_1) \qquad L^2_{11}(\{c\}^1_1)=-2L^2_{22}$

Weinberg's argument

- $\hbar^2$:
$$\frac{1}{w^2}\left(\mu^{-2w}L^2_{00}(\{c\}^2_2)+\mu^{-w}L^2_{11}(\{c\}^1_1)+L^2_{22}\right)$$
$$+\frac{1}{w}\left(\mu^{-2w}L^2_{00}(\{c\}^2_1)+\mu^{-w}L^2_{11}(\{c\}^1_0)+\mu^{-w}L^2_{10}(\{c\}^1_1)+L^2_{21}\right)$$
$$+\left(\mu^{-2w}L^2_{00}(\{c\}^2_0)+\mu^{-w}L^2_{10}(\{c\}^1_0)+L^2_{20}\right)$$
- $1/w^2$ and $\log\mu/w$ must cancel
$$L^2_{00}(\{c\}^2_2)+L^2_{11}(\{c\}^1_1)+L^2_{22}=0$$
$$2L^2_{00}(\{c\}^2_2)+L^2_{11}(\{c\}^1_1)=0$$
- Solution: $L^2_{00}(\{c\}^2_2)=-\frac{1}{2}L^2_{11}(\{c\}^1_1)\qquad L^2_{11}(\{c\}^1_1)=-2L^2_{22}$
- Explicit $\log\mu$ dependence (one-loop is enough)
$$\frac{1}{2}\log^2\mu\left(4L^2_{00}(\{c\}^2_2)+L^2_{11}(\{c\}^1_1)\right)=-\frac{1}{2}L^2_{11}(\{c\}^1_1)\log^2\mu\,.$$

All orders

- $\hbar^n$:

$$\frac{1}{w^n}\left(\mu^{-nw}L^n_{00}(\{c\}^n_n) + \mu^{-(n-1)w}L^n_{11}(\{c\}^{n-1}_{n-1}) + \cdots\right.$$

$$\left. + \mu^{-w}L^n_{n-1\;n-1}(\{c\}^1_1) + L^n_{nn}\right) + \frac{1}{w^{n-1}}\cdots$$

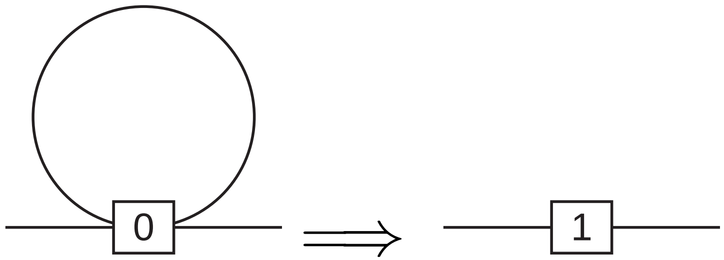
All orders

- $\hbar^n$:
$$\frac{1}{w^n}\Big(\mu^{-nw}L^n_{00}(\{c\}^n_n)+\mu^{-(n-1)w}L^n_{11}(\{c\}^{n-1}_{n-1})+\cdots$$
$$+\mu^{-w}L^n_{n-1\;n-1}(\{c\}^1_1)+L^n_{nn}\Big)+\frac{1}{w^{n-1}}\cdots$$
- $1/w^n, \log\mu/w^{n-1},\ldots,\log^{n-1}\mu/w$ cancel:
$$\sum_{i=0}^{n} i^j L^n_{n-i\;n-i}(\{c\}^i_i)=0 \qquad j=0,..,n-1.$$

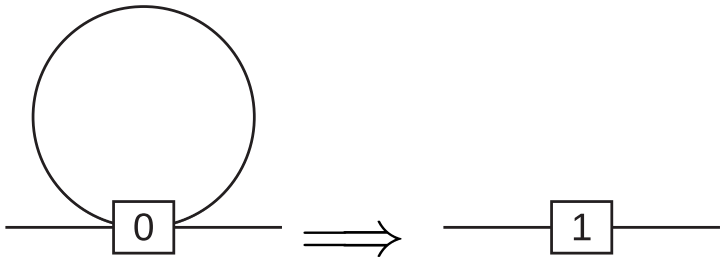
All orders

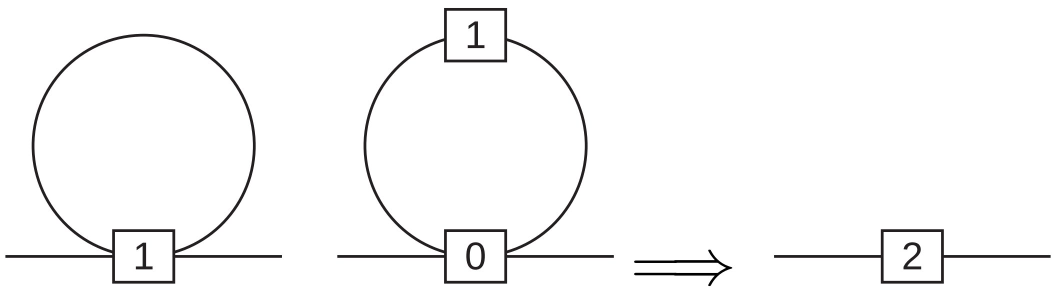
- $\hbar^n$:
$$\frac{1}{w^n}\Big(\mu^{-nw}L_{00}^n(\{c\}_n^n)+\mu^{-(n-1)w}L_{11}^n(\{c\}_{n-1}^{n-1})+\cdots$$
$$+\mu^{-w}L_{n-1\ n-1}^n(\{c\}_1^1)+L_{nn}^n\Big)+\frac{1}{w^{n-1}}\cdots$$
- $1/w^n, \log\mu/w^{n-1},\ldots,\log^{n-1}\mu/w$ cancel:
$$\sum_{i=0}^{n} i^j L_{n-i\ n-i}^n(\{c\}_i^i)=0 \qquad j=0,..,n-1.$$
- Solution: $L_{n-i\ n-i}^n(\{c\}_i^i)=(-1)^i\begin{pmatrix}n\\ i\end{pmatrix}L_{nn}^n$
- explicit leading $\log\mu$ dependence and divergence
$$\log^n\mu\frac{(-1)^{n-1}}{n}L_{11}^n(\{c\}_{n-1}^{n-1}) \qquad L_{00}^n(\{c\}_n^n)=-\frac{1}{n}L_{11}^n(\{c\}_{n-1}^{n-1})$$

Mass to $\hbar^2$

- $\hbar^1$: 

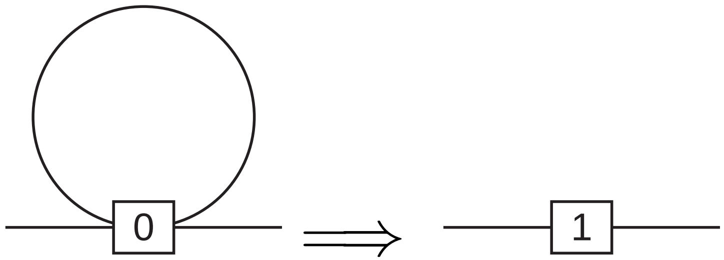
Mass to $\hbar^2$

- $\hbar^1$: 

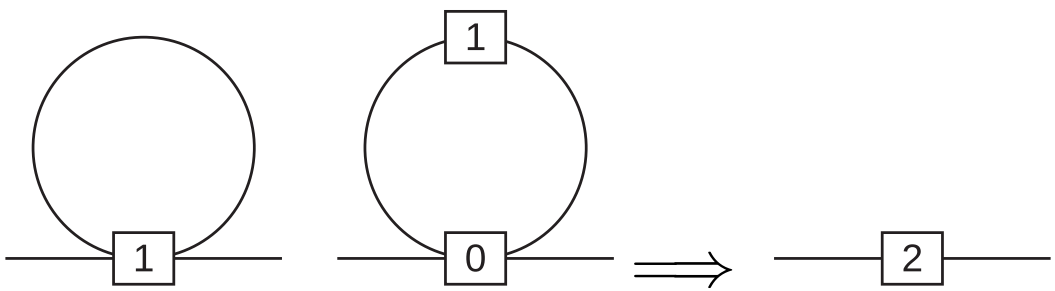
- $\hbar^2$: 

Mass to $\hbar^2$

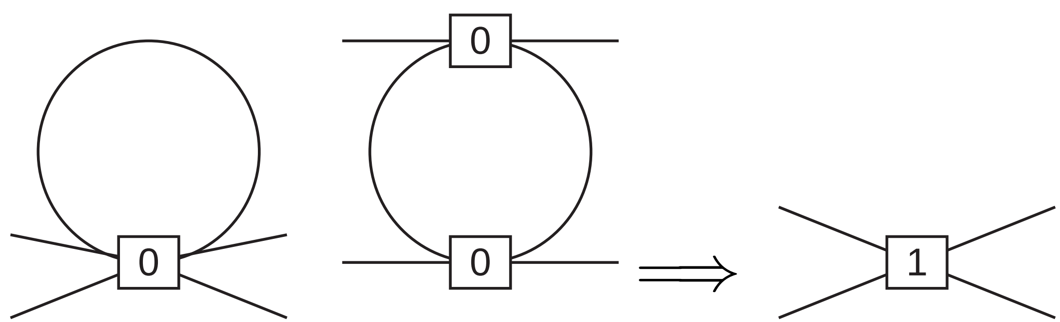
- $\hbar^1$:



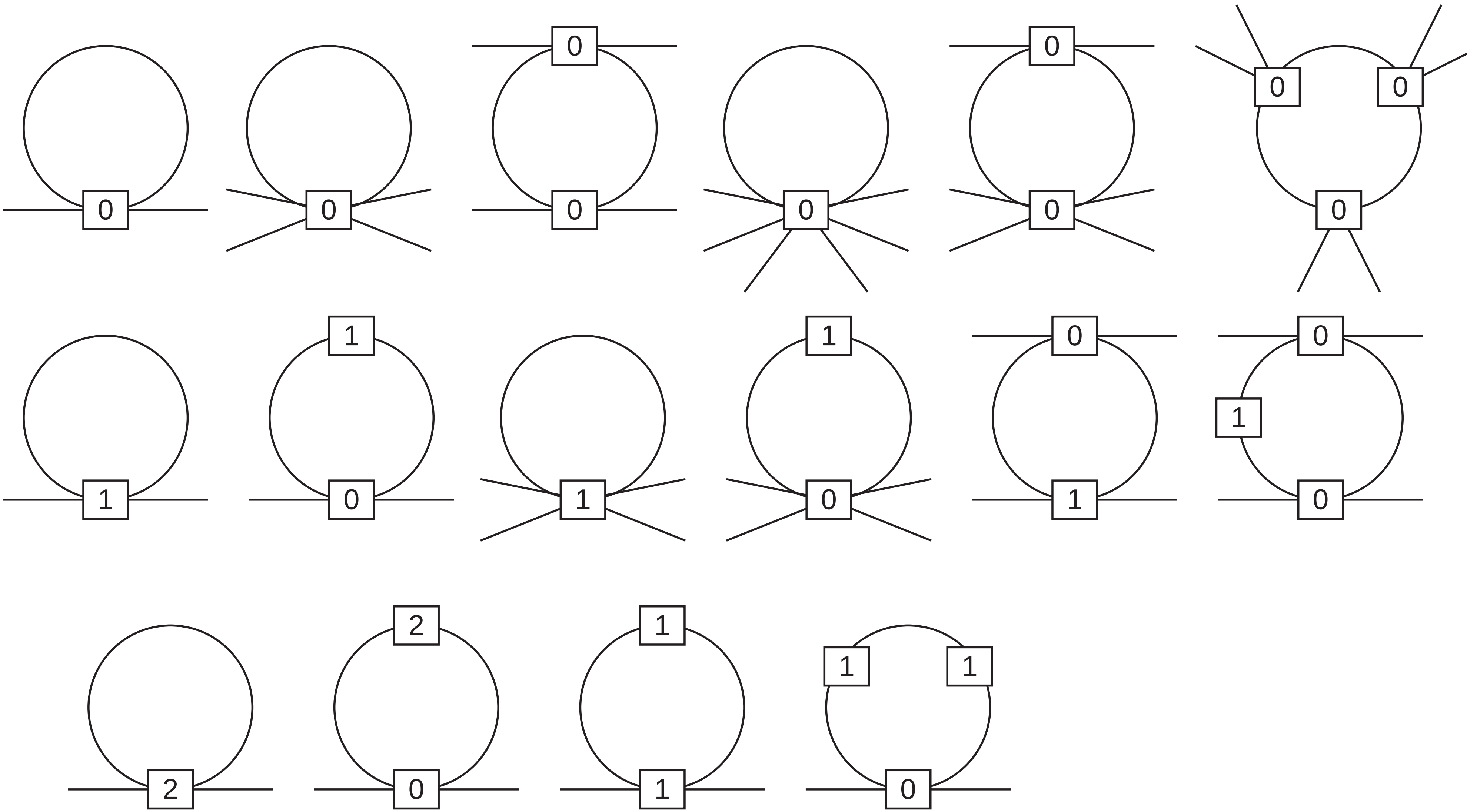
- $\hbar^2$:



- but also needs $\hbar^1$:



Mass to order $\hbar^3$



Mass+decay to $\hbar^5$

- $\hbar^1$: 18 + 27
- $\hbar^2$: 26 + 45
- $\hbar^3$: 33 + 51
- $\hbar^4$: 26 + 33
- $\hbar^5$: 13 + 13

- Calculate the divergence
- rewrite it in terms of a local Lagrangian
- Luckily: symmetry kept: we know result will be symmetrical, hence do not need to explicitly rewrite the Lagrangians in a nice form
- We keep all terms to have all 1PI (one particle irreducible) diagrams finite

Massive $O(N)$ sigma model

- $O(N+1)/O(N)$ nonlinear sigma model
- $\mathcal{L}_{n\sigma} = \frac{F^2}{2}\partial_\mu\Phi^T\partial^\mu\Phi + F^2\chi^T\Phi\,.$
- Φ is a real $N+1$ vector; $\Phi \to O\Phi$; $\Phi^T\Phi = 1$.
- Vacuum expectation value $\langle\Phi^T\rangle = (1\;0\ldots0)$
- Explicit symmetry breaking: $\chi^T = \left(M^2\;0\ldots0\right)$
- Both spontaneous and explicit symmetry breaking
- N-vector ϕ
- N (pseudo-)Nambu-Goldstone Bosons
- $N=3$ is two-flavour Chiral Perturbation Theory

Massive $O(N)$ sigma model: Φ vs ϕ

- $$\Phi_1 = \begin{pmatrix} \sqrt{1-\frac{\phi^T\phi}{F^2}} \\ \frac{\phi^1}{F} \\ \vdots \\ \frac{\phi^N}{F} \end{pmatrix} = \begin{pmatrix} \sqrt{1-\frac{\phi^T\phi}{F^2}} \\ \frac{\phi}{F} \end{pmatrix}$$ Gasser, Leutwyler

- $$\Phi_2 = \frac{1}{\sqrt{1+\frac{\phi^T\phi}{F^2}}}\begin{pmatrix} 1 \\ \frac{\phi}{F} \end{pmatrix}$$ similar to Weinberg

  $$\Phi_3 = \begin{pmatrix} 1-\frac{1}{2}\frac{\phi^T\phi}{F^2} \\ \sqrt{1-\frac{1}{4}\frac{\phi^T\phi}{F^2}}\,\frac{\phi}{F} \end{pmatrix}$$ only mass term

- $$\Phi_4 = \begin{pmatrix} \cos\sqrt{\frac{\phi^T\phi}{F^2}} \\ \sin\sqrt{\frac{\phi^T\phi}{F^2}}\,\frac{\phi}{\sqrt{\phi^T\phi}} \end{pmatrix}$$ CCWZ

Massive $O(N)$ sigma model: Checks

Need (many) checks:

- use the four different parametrizations
- compare with known results:

$$M^2_{phys} = M^2 \left(1 - \frac{1}{2} L_M + \frac{17}{8} L_M^2 + \cdots \right) ,$$

$$L_M = \frac{M^2}{16\pi^2 F^2} \log \frac{\mu^2}{\mathcal{M}^2}$$

Usual choice $\mathcal{M} = M$.

- large N (but known results only for massless case)
Coleman, Jackiw, Politzer 1974

Results

$$M^2_{\text{phys}} = M^2(1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \ldots)$$

$$L_M = \frac{M^2}{16\pi^2 F^2} \log \frac{\mu^2}{M^2}$$

i	a_i, $N = 3$	a_i for general N
1	$-\frac{1}{2}$	$1 - \frac{N}{2}$
2	$\frac{17}{8}$	$\frac{7}{4} - \frac{7N}{4} + \frac{5\ N^2}{8}$
3	$-\frac{103}{24}$	$\frac{37}{12} - \frac{113N}{24} + \frac{15\ N^2}{4} - N^3$
4	$\frac{24367}{1152}$	$\frac{839}{144} - \frac{1601\ N}{144} + \frac{695\ N^2}{48} - \frac{135\ N^3}{16} + \frac{231\ N^4}{128}$
5	$-\frac{8821}{144}$	$\frac{33661}{2400} - \frac{1151407\ N}{43200} + \frac{197587\ N^2}{4320} - \frac{12709\ N^3}{300} + \frac{6271\ N^4}{320} - \frac{7\ N^5}{2}$

Results

$$F_{\text{phys}} = F(1 + b_1 L_M + b_2 L_M^2 + b_3 L_M^3 + \ldots)$$

i	b_i for $N=3$	b_i for general N
1	1	$-\frac{1}{2} + \frac{N}{2}$
2	$-\frac{5}{4}$	$-\frac{1}{2} + \frac{7N}{8} - \frac{3N^2}{8}$
3	$\frac{83}{24}$	$-\frac{7}{24} + \frac{21N}{16} - \frac{73N^2}{48} + \frac{1N^3}{2}$
4	$-\frac{3013}{288}$	$\frac{47}{576} + \frac{1345N}{864} - \frac{14077N^2}{3456} + \frac{625N^3}{192} - \frac{105N^4}{128}$
5	$\frac{2060147}{51840}$	$-\frac{23087}{64800} + \frac{459413N}{172800} - \frac{189875N^2}{20736} + \frac{546941\ N^3}{43200} - \frac{1169\ N^4}{160} + \frac{3\ N^5}{2}$

Results

$$\langle \bar{q}_i q_i \rangle = -BF^2(1 + c_1 L_M + c_2 L_M^2 + c_3 L_M^3 + \ldots)$$

$M^2 = 2B\hat{m}$ $\qquad \chi^T = 2B(s\ 0\ \ldots 0)$

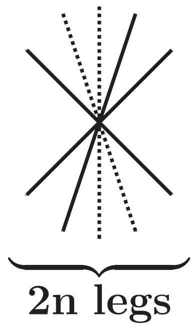
s corresponds to $\bar{u}u + \bar{d}d$ current

i	c_i for $N = 3$	c_i for general N
1	$\frac{3}{2}$	$\frac{N}{2}$
2	$-\frac{9}{8}$	$\frac{3N}{4} - \frac{3N^2}{8}$
3	$\frac{9}{2}$	$\frac{3N}{2} - \frac{3N^2}{2} + \frac{N^3}{2}$
4	$-\frac{1285}{128}$	$\frac{145N}{48} - \frac{55N^2}{12} + \frac{105N^3}{32} - \frac{105N^4}{128}$
5	46	$\frac{3007N}{480} - \frac{1471N^2}{120} + \frac{557\ N^3}{40} - \frac{1191\ N^4}{160} + \frac{3\ N^5}{2}$

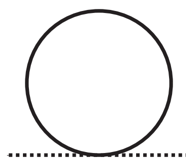
Anyone recognize any funny functions?

Large N

Power counting: pick $\mathcal{L}$ extensive in $N \Rightarrow F^2 \sim N$, $M^2 \sim 1$

- 

$\Leftrightarrow F^{2-2n} \sim \frac{1}{N^{n-1}}$

  
$\Leftrightarrow N$

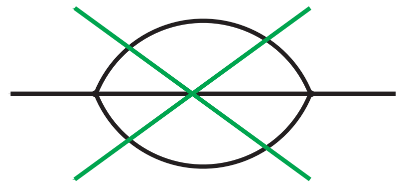
- 1PI diagrams:

$$\left.\begin{array}{l} N_L = N_I - \sum_n N_{2n} + 1 \\ 2N_I + N_E = \sum_n 2n N_{2n} \end{array}\right\} \Rightarrow N_L = \sum_n (n-1) N_{2n} - \tfrac{1}{2} N_E + 1$$

- diagram suppression factor: $\dfrac{N^{N_L}}{N^{N_E/2-1}}$

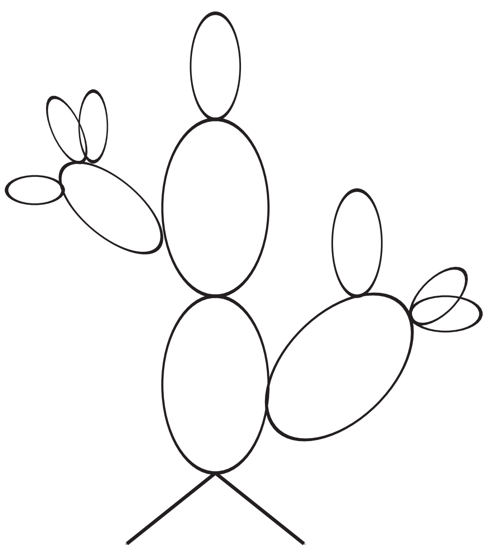
Large N

- diagrams with shared lines are suppressed



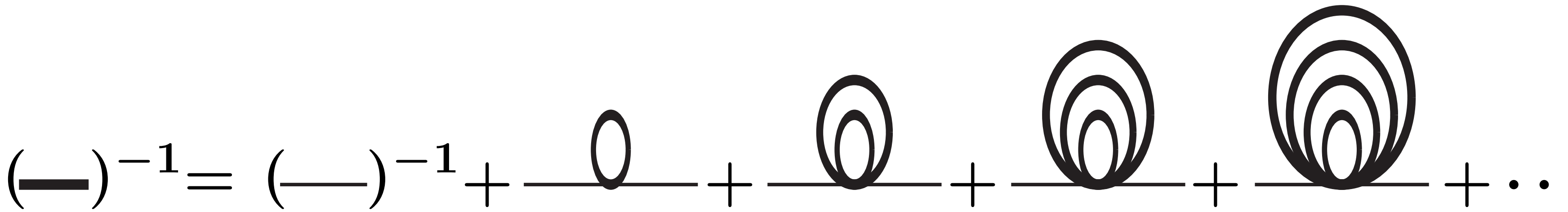
each new loop needs also a new flavour loop

- in the large N limit only "*cactus*" diagrams survive:



large N: propagator

Generate recursively via a Gap equation



$\Rightarrow$ resum the series and look for the pole

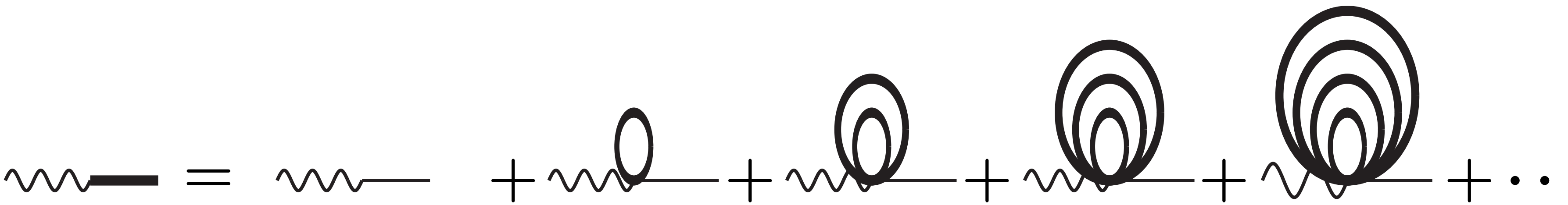
$$M^2 = M^2_{\text{phys}}\sqrt{1 + \frac{N}{F^2}\overline{A}(M^2_{\text{phys}})}$$

$$\overline{A}(m^2) = \frac{m^2}{16\pi^2}\log\frac{\mu^2}{m^2}\,.$$

Solve recursively, agrees with other result

Note: can be done for all parametrizations

large N: Decay constant



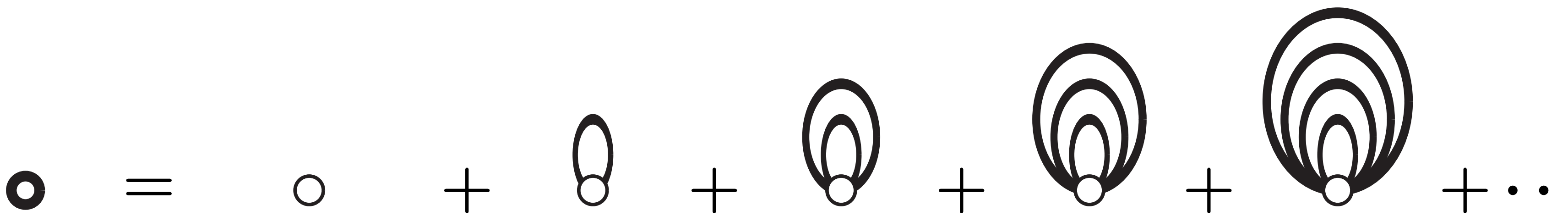
$\Rightarrow$ and include wave-function renormalization

$$F_{\text{phys}} = F\sqrt{1 + \frac{N}{F^2}\overline{A}(M^2_{\text{phys}})}$$

Solve recursively, agrees with other result

Note: can be done for all parametrizations

large N: Vacuum Expectation Value



$$\langle \bar{q}q \rangle_{\text{phys}} = \langle \bar{q}q \rangle_0 \sqrt{1 + \frac{N}{F^2}\overline{A}(M^2_{\text{phys}})}$$

Comments:

- These are the full* leading N results, not just leading log
- But depends on the choice of N-dependence of higher order coefficients
- Assumes higher LECs zero ($< N^{n+1}$ for $\hbar^n$)
- Large N as in $O(N)$ not large N_c

Large N: Checking expansions

$$M^2 = M^2_{\text{phys}}\sqrt{1+\frac{N}{F^2}\overline{A}(M^2_{\text{phys}})}$$

much smaller expansion coefficients than the table, try

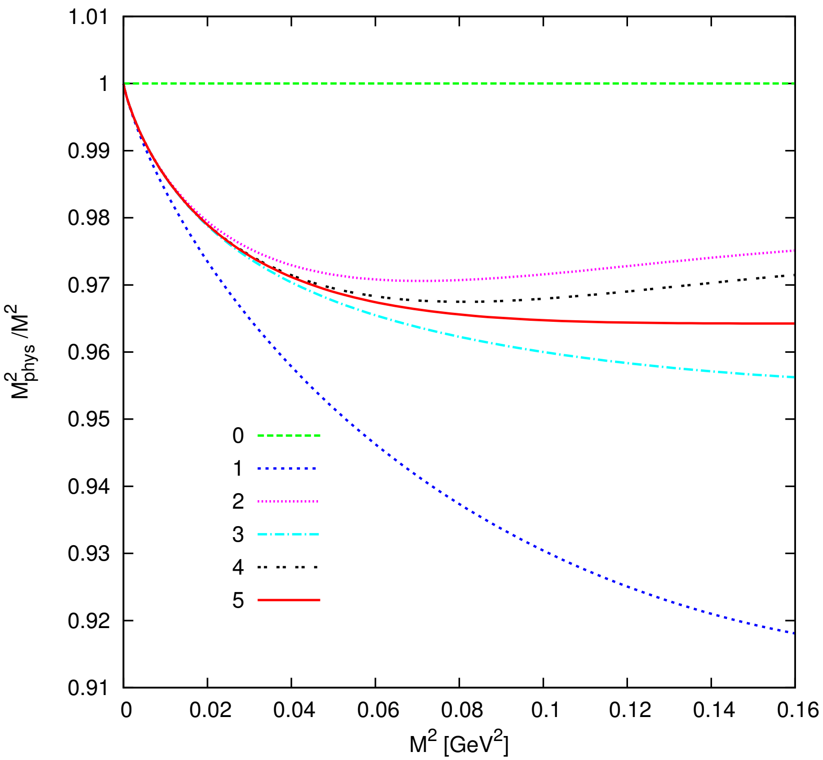
$$M^2 = M^2_{\text{phys}}(1 + d_1 L_{M_{\text{phys}}} + d_2 L^2_{M_{\text{phys}}} + d_3 L^3_{M_{\text{phys}}} + \ldots)$$

Large N: Checking expansions

i	d_i, $N=3$	d_i for general N
1	$\frac{1}{2}$	$-1+\frac{1}{2}\ N$
2	$-\frac{13}{8}$	$\frac{1}{4}-\frac{1}{4}\ N-\frac{1}{8}\ N^2$
3	$-\frac{19}{48}$	$\frac{2}{3}-\frac{11}{12}\ N+\frac{1}{16}\ N^3$
4	$-\frac{5773}{1152}$	$-\frac{8}{9}+\frac{107}{144}\ N-\frac{1}{6}\ N^2-\frac{1}{16}\ N^3-\frac{5}{128}\ N^4$
5	$-\frac{3343}{768}$	$-\frac{18383}{7200}+\frac{130807}{43200}\ N-\frac{2771}{2160}\ N^2-\frac{527}{1600}\ N^3+\frac{23}{640}\ N^4+\frac{7}{256}\ N^5$

i	a_i, $N=3$	a_i for general N
1	$-\frac{1}{2}$	$1-\frac{N}{2}$
2	$\frac{17}{8}$	$\frac{7}{4}-\frac{7N}{4}+\frac{5\ N^2}{8}$
3	$-\frac{103}{24}$	$\frac{37}{12}-\frac{113N}{24}+\frac{15\ N^2}{4}-N^3$
4	$\frac{24367}{1152}$	$\frac{839}{144}-\frac{1601\ N}{144}+\frac{695\ N^2}{48}-\frac{135\ N^3}{16}+\frac{231\ N^4}{128}$
5	$-\frac{8821}{144}$	$\frac{33661}{2400}-\frac{1151407\ N}{43200}+\frac{197587\ N^2}{4320}-\frac{12709\ N^3}{300}+\frac{6271\ N^4}{320}-\frac{7\ N^5}{2}$

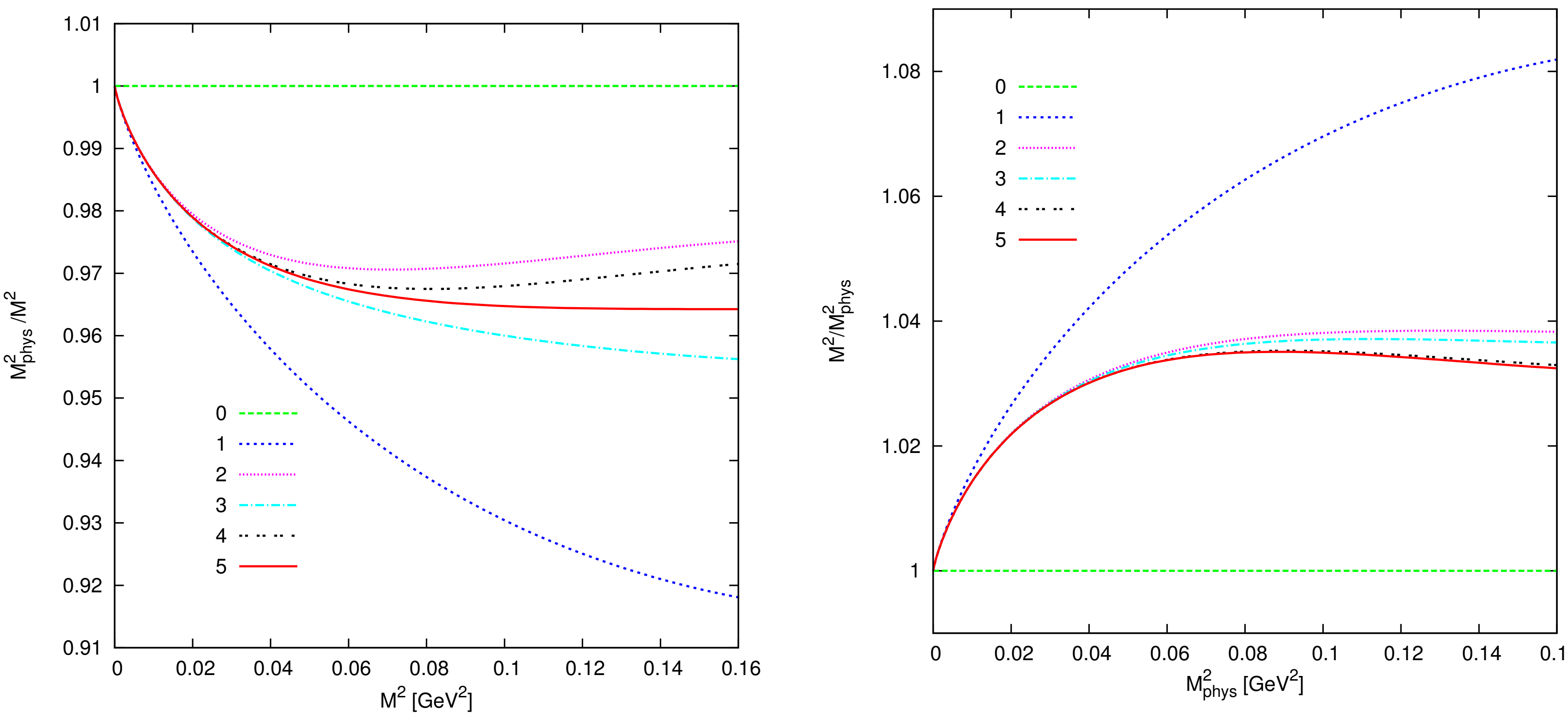
Numerical results



Left: $\frac{M^2_{\rm phys}}{M^2} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \cdots$

F = 90 MeV, $\mu = 0.77$ GeV

Numerical results



Left: $\frac{M^2_{\rm phys}}{M^2} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \cdots$

Right: $\frac{M^2}{M^2_{\rm phys}} = 1 + d_1 L_{M_{\rm phys}} + d_2 L^2_{M_{\rm phys}} + d_3 L^3_{M_{\rm phys}} + \cdots$

F = 90 MeV, $\mu = 0.77$ GeV

Other results

- Bissegger, Fuhrer, hep-ph/0612096 Dispersive methods, massless Π_S to five loops
- Kivel, Polyakov, Vladimirov, 0809.3236, 0904.3008, 1004.2197
  - In the massless case tadpoles vanish
  - hence the number of external legs needed does not grow
  - All 4-meson vertices via Legendre polynomials
  - can do divergence of all one-loop diagrams analytically
  - algebraic (but quadratic) recursion relations
  - massless $\pi\pi$, F_V and F_S to arbitrarily high order
  - large N agrees with Coleman, Wess, Zumino
  - large N is not a good approximation

Other results

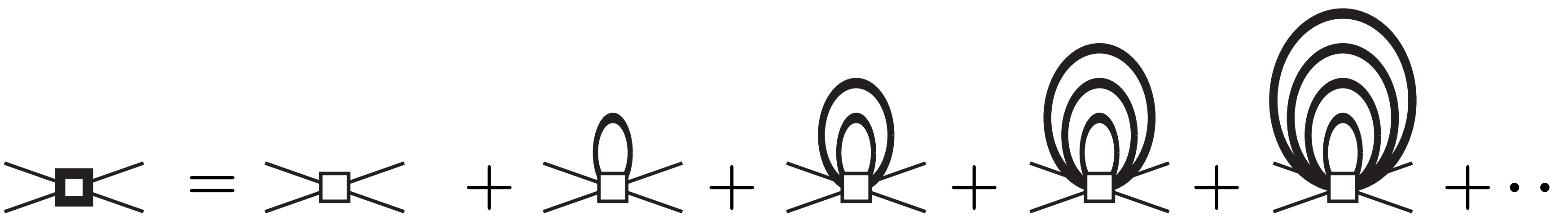
- JB,Carloni, arXiv:1008.3499
  - massive case: $\pi\pi$, F_V and F_S to 4-loop order
  - large N for these cases also for massive $O(N)$.
  - done using bubble resummations or recursion eqation which can be solved analytically (extension similar to gap equation)

Large N: $\pi\pi$-scattering

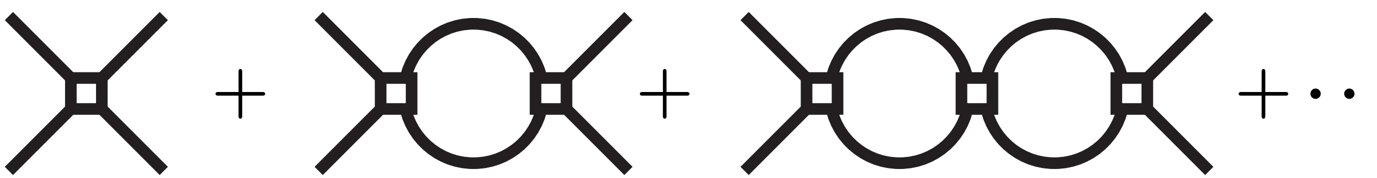
- Semiclassical methods Coleman, Jackiw, Politzer 1974
- Diagram resummation Dobado, Pelaez 1992
- $A(\phi^i\phi^j \to \phi^k\phi^l) =$
  $A(s,t,u)\delta^{ij}\delta^{kl} + A(t,u,s)\delta^{ik}\delta^{jl} + A(u,s,t)\delta^{il}\delta^{jk}$
- $A(s,t,u) = A(s,u,t)$
- Proof same as Weinberg's for $O(4)/O(3)$, group theory and crossing

Large N: $\pi\pi$-scattering

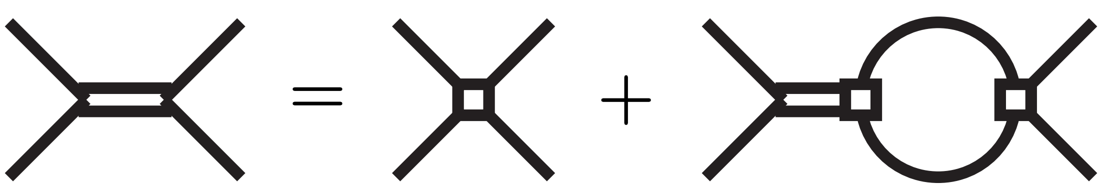
- Cactus diagrams for $A(s,t,u)$
- Branch with no momentum: resummed by —
- Branch starting at vertex: resum by



- The full result is then



- Can be summarized by a recursive equation



Large N: $\pi\pi$ scattering

$$y = \frac{N}{F^2}\overline{A}(M^2_{\text{phys}})$$

$$A(s,t,u) = \frac{\frac{s}{F^2(1+y)} - \frac{M^2}{F^2(1+y)^{3/2}}}{1 - \frac{N}{2}\left(\frac{s}{F^2(1+y)} - \frac{M^2}{F^2(1+y)^{3/2}}\right)\overline{B}(M^2_{\text{phys}}, M^2_{\text{phys}}, s)}$$

or

$$A(s,t,u) = \frac{\frac{s-M^2_{\text{phys}}}{F_{\text{phys}}}}{1 - \frac{N}{2}\frac{s-M^2_{\text{phys}}}{F^2_{\text{phys}}}\overline{B}(M^2_{\text{phys}}, M^2_{\text{phys}}, s)}$$

- $M^2 \to 0$ agrees with the known results
- Agrees with our 4-loop results

Conclusions Leading Logs

- Several quantities in massive $O(N)$ LL known to high loop order
- Large N in massive $O(N)$ model solved
- Had hoped: recognize the series also for general N
- Limited essentially by CPU time and size of intermediate files
- Some first studies on convergence etc.
- $\pi\pi$, F_V and F_S to four-loop order
- The technique can be generalized to other models/theories
  - $SU(N) \times SU(N)/SU(N)$
  - One nucleon sector

QCDlike and/or technicolor theories

A typical gaugegroup and N_F fermions:

- QCD or complex: $q^T = (q_1\ q_2 \ldots\ q_{N_F})$
  - Global $G = SU(N_F)_L \times SU(N_F)_R$
    $q_L \to g_L q_L$ and $g_R \to g_R q_R$
  - Vacuum condensate $\Sigma_{ij} = \langle \overline{q}_j q_i \rangle \propto \delta_{ij}$
  - Conserved $H = SU(N_F)$ $g_L = g_R$ $\Sigma_{ij} \to \Sigma_{ij}$
  - q in complex prepresentation of gauge group

QCDlike and/or technicolor theories

A typical gaugegroup and N_F fermions:

- Real (e.g. adjoint):
  - $\tilde{q}_{Ri} \equiv C\overline{q}_{Li}^T$ is in the same gauge group representation as q_{Ri}
  - $\hat{q}^T = (q_{R1} \ \ldots \ q_{RN_F} \ \tilde{q}_{R1} \ \ldots \ \tilde{q}_{RN_F})$
  - Global $G = SU(2N_F)$ and $\hat{q} \to g\hat{q}$
  - Vacuum condensate $\langle \overline{q}_j q_i \rangle$ is really $\langle (\hat{q}_j)^T C \hat{q}_i \rangle \propto J_{Sij}$

    $$J_S = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$
  - Conserved symmetry part has $gJ_Sg^T = J_S$
  - $H = SO(2N_F)$

QCDlike and/or technicolor theories

A typical gaugegroup and N_F fermions:

- Pseudoeal (e.g. two-colours):
  - $\tilde{q}_{R\alpha i} \equiv \epsilon_{\alpha\beta} C \bar{q}^T_{L\beta i}$ is in the same gauge group representation as $q_{R\alpha i}$
  - $\hat{q}^T = (q_{R1} \; \ldots \; q_{RN_F} \; \tilde{q}_{R1} \; \ldots \; \tilde{q}_{RN_F})$
  - Global $G = SU(2N_F)$ and $\hat{q} \to g\hat{q}$
  - Vacuum condensate $\langle \overline{q}_j q_i \rangle$ is really

    $$\epsilon_{\alpha\beta} \langle (\hat{q}_{\alpha j})^T C \hat{q}_{\beta i} \rangle \propto J_{Aij} \; J_A = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$
  - Conserved symmetry part has $g J_A g^T = J_A$
  - $H = Sp(2N_F)$

Lagrangians

In arXiv:0910.5424 we showed that there is a very similar way of phrasing the two theories using $u = \exp\left(\frac{i}{\sqrt{2}F}\phi^a X^a\right)$

But the matrices X^a are:

- Complex or $SU(N) \times SU(N)/SU(N)$: all $SU(N)$ generators
- Real or $SU(2N)/SO(2N)$: $SU(2N)$ generator with $X^a J_S = J_S X^{aT}$
- Pseudoeal or $SU(2N)/Sp(2N)$: $SU(2N)$ generator with $X^a J_A = J_A X^{aT}$
- Note that the latter are not the usual ways of parametrizing $SO(2N)$ and $Sp(2N)$ matrices

Lagrangians

- $u \to h u g_L^\dagger \equiv g_R u h^\dagger$ for complex
- $u \to h u g^\dagger$ for real, pseudoreal
- h is in the conserved part of the group for all cases
- $u_\mu = i\left(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger\right) \to h u_\mu h^\dagger$
- external fields can also be included.
- a generalized mass term $\chi_\pm \to h \chi_\pm h^\dagger$ can be defined with $\chi_\pm = u^\dagger \tilde{\chi} u^\dagger \pm u \tilde{\chi} u$
- $\mathcal{L}_{LO} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+ \rangle$
- $$\begin{aligned} \mathcal{L}_4 \quad = \quad & L_0 \langle u^\mu u^\nu u_\mu u_\nu \rangle + L_1 \langle u^\mu u_\mu \rangle \langle u^\nu u_\nu \rangle + L_2 \langle u^\mu u^\nu \rangle \langle u_\mu u_\nu \rangle + L_3 \langle u^\mu u_\mu u^\nu u_\nu \rangle \\ & + L_4 \langle u^\mu u_\mu \rangle \langle \chi_+ \rangle + L_5 \langle u^\mu u_\mu \chi_+ \rangle + L_6 \langle \chi_+ \rangle^2 + L_7 \langle \chi_- \rangle^2 + \frac{1}{2} L_8 \langle \chi_+^2 + \chi_-^2 \rangle \\ & - i L_9 \langle f_{+\mu\nu} u^\mu u^\nu \rangle + \frac{1}{4} L_{10} \langle f_+^2 - f_-^2 \rangle + H_1 \langle l_{\mu\nu} l^{\mu\nu} + r_{\mu\nu} r^{\mu\nu} \rangle + H_2 \langle \chi \chi^\dagger \rangle \,. \end{aligned}$$

Divergences etc

Calculating for equal mass case goes though using:

$$
\begin{aligned}
\text{QCD}: \quad & \langle X^a A X^a B\rangle = \langle A\rangle\langle B\rangle - \frac{1}{N_F}\langle AB\rangle\,, \\
& \langle X^a A\rangle\langle X^a B\rangle = \langle AB\rangle - \frac{1}{N_F}\langle A\rangle\langle B\rangle\,. \\
\text{Adjoint}: \quad & \langle X^a A X^a B\rangle = \frac{1}{2}\langle A\rangle\langle B\rangle + \frac{1}{2}\left\langle A J_S B^T J_S\right\rangle - \frac{1}{2N_F}\langle AB\rangle\,, \\
& \langle X^a A\rangle\langle X^a B\rangle = \frac{1}{2}\langle AB\rangle + \frac{1}{2}\left\langle A J_S B^T J_S\right\rangle - \frac{1}{2N_F}\langle A\rangle\langle B\rangle\,. \\
2-\text{colour}: \quad & \langle X^a A X^a B\rangle = \frac{1}{2}\langle A\rangle\langle B\rangle + \frac{1}{2}\left\langle A J_A B^T J_A\right\rangle - \frac{1}{2N_F}\langle AB\rangle\,, \\
& \langle X^a A\rangle\langle X^a B\rangle = \frac{1}{2}\langle AB\rangle - \frac{1}{2}\left\langle A J_A B^T J_A\right\rangle - \frac{1}{2N_F}\langle A\rangle\langle B\rangle
\end{aligned}
$$

So for the subtraction $L_i = (c\mu)^{d-4}\left[\frac{\Gamma_i}{16\pi^2(d-4)} + L_i^r\right]$

with $\ln c = -(1/2)(\ln 4\pi + \Gamma'(1) + 1)$

Divergences etc

i	QCD	Adjoint	2-colour
0	$N_F/48$	$(N_F+4)/48$	$(N_F-4)/48$
1	$1/16$	$1/32$	$1/32$
2	$1/8$	$1/16$	$1/16$
3	$N_F/24$	$(N_F-2)/24$	$(N_F+2)/24$
4	$1/8$	$1/16$	$1/16$
5	$N_F/8$	$N_F/8$	$N_F/8$
6	$(N_F^2+2)/(16N_F^2)$	$(N_F^2+1)/(32N_F^2)$	$(N_F^2+1)/(32N_F^2)$
7	0	0	0
8	$(N_F^2-4)/(16N_F)$	$(N_F^2+N_F-2)/(16N_F)$	$(N_F^2-N_F-2)/(16N_F)$
9	$N_F/12$	$(N_F+1)/2$	$(N_F-1)/2$
10	$-N_F/12$	$-(N_F+1)/2$	$-(N_F-1)/2$
1’	$-N_F/24$	$-(N_F+1)/4$	$-(N_F+1)/4$
2’	$(N_F^2-4)/(8N_F)$	$(N_F^2+N_F-2)/(8N_F)$	$(N_F^2-N_F-2)/(8N_F)$

Vacuum expectation value

All cases: $\langle \overline{q}q\rangle_{\rm LO} \equiv \sum_{i=1,N_F}\langle \overline{q}_{Ri}q_{Li} + \overline{q}_{Li}q_{Ri}\rangle_{\rm LO} = -N_F B_0 F^2$

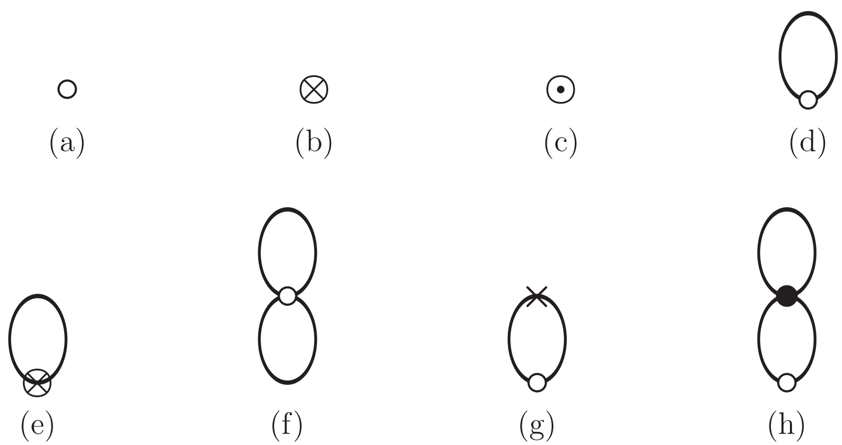
$M^2 = 2B_0\hat{m}$ and $\overline{A}(M^2) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2}$.

$\langle \overline{q}q\rangle = \langle \overline{q}q\rangle_{\rm LO} + \langle \overline{q}q\rangle_{\rm NLO} + \langle \overline{q}q\rangle_{\rm NNLO}$.

$$\langle \overline{q}q\rangle_{\rm NLO} = \langle \overline{q}q\rangle_{\rm LO}\left(a_V\frac{\overline{A}(M^2)}{F^2} + b_V\frac{M^2}{F^2}\right),$$

$$\langle \overline{q}q\rangle_{\rm NNLO} = \langle \overline{q}q\rangle_{\rm LO}\left(c_V\frac{\overline{A}(M^2)^2}{F^4} + \frac{M^2\overline{A}(M^2)}{F^4}\left(d_V + \frac{e_V}{16\pi^2}\right) + \frac{M^4}{F^4}\left(f_V + \frac{g_V}{16\pi^2}\right)\right).$$

Diagrams:



Vacuum expectation value

	QCD	
a_V	$n-\frac{1}{n}$	
b_V	$16nL_6^r+8L_8^r+4H_2^r$	
c_V	$\frac{3}{2}\left(-1+\frac{1}{n^2}\right)$	
d_V	$-24\left(n^2-1\right)\left(L_A+\frac{1}{n}L_B\right)$	$L_A=L_4^r-2L_6^r$
e_V	$1-\frac{1}{n^2}$	$L_B=L_5^r-2L_8^r$
f_V	$48\left(K_{25}^r+nK_{26}^r+n^2K_{27}^r\right)$	
g_V	$8\left(n^2-1\right)\left(L_A+\frac{1}{n}L_B\right)$	
	Adjoint	2-colour
a_V	$n+\frac{1}{2}-\frac{1}{2n}$	$n-\frac{1}{2}-\frac{1}{2n}$
b_V	$32nL_6^r+8L_8^r+4H_2^r$	$32nL_6^r+8L_8^r+4H_2^r$
c_V	$\frac{3}{8}\left(-1+\frac{1}{n^2}-\frac{2}{n}+2n\right)$	$\frac{3}{8}\left(-1+\frac{1}{n^2}+\frac{2}{n}-2n\right)$
d_V	$-12\left(2n^2+n-1\right)\left(2L_A+\frac{1}{n}L_B\right)$	$-12\left(2n^2-n-1\right)\left(2L_A+\frac{1}{n}L_B\right)$
e_V	$\frac{1}{4}\left(1-\frac{1}{n^2}+\frac{2}{n}-2n\right)$	$\frac{1}{4}\left(1-\frac{1}{n^2}-\frac{2}{n}+2n\right)$
f_V	r_{VA}^r	r_{VT}^r
g_V	$4\left(2n^2+n-1\right)\left(2L_A+\frac{1}{n}L_B\right)$	$4\left(2n^2-n-1\right)\left(2L_A+\frac{1}{n}L_B\right)$

Other results

- M^2_{phys}
- F_{phys}
- Meson-meson scattering: being written up
- Equal mass case: allows to get fully analytical result

QCDlike: conclusions

- Different symmetry patterns can appear for different gaugegroups and fermion representations
- Nonperturbartive: lattice needs extrapolation formulae
- Masses, decay constant and VEV: done to NNLO
- Meson-meson scattering: being written up
- Two-pointfunctions and fromfactors for precision observables: planned

Conclusions

- A general introduction to Effective Field Theory
- Three applications:
  - Hard Pion ChPT: a new application domain for EFT and a first result
  - Leading Logarithms and large N: some progress in getting results at high loop orders, but hoped for patterns not seen (except large N calculated)
  - Two-loop results for the equal mass case for different symmetry patterns. $SU(N) \times SU(N)/SU(N)$, $SU(2N)/SO(2N)$, $SU(2N)/Sp(2N)$