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# **HARD PION CHIRAL PERTURBATION THEORY**

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**Various ChPT:** `http://www.thep.lu.se/~bijmens/chpt.html`

# Overview

- Effective Field Theory
- Chiral Perturbation Theor(y)(ies)
- Hard Pion Chiral Perturbation Theory

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- Effective Field Theory
- Chiral Perturbation Theor(y)(ies)
- Hard Pion Chiral Perturbation Theory
  - $K_{\ell 3}$  Flynn-Sachrajda, arXiv:0809.1229
  - $K \rightarrow \pi\pi$  JB+ Alejandro Celis, arXiv:0906.0302
  - $F_{\pi}^S$  and  $F_{\pi}^V$  JB + Ilaria Jemos, arXiv:1011.6531 a two-loop check
  - $B, D \rightarrow \pi$  JB + Ilaria Jemos, arXiv:1006.1197
  - $B, D \rightarrow \pi, K, \eta$  JB + Ilaria Jemos, arXiv:1011.6531
  - $\chi_c(J = 0, 2) \rightarrow \pi\pi, KK, \eta\eta$  JB+Ilaria Jemos, arxiv:1109.5033
  - Some examples which do not have a chiral log prediction

# Wikipedia

`http://en.wikipedia.org/wiki/  
Effective\_field\_theory`

*In physics, an effective field theory is an approximate theory (usually a quantum field theory) that contains the appropriate degrees of freedom to describe physical phenomena occurring at a chosen length scale, but ignores the substructure and the degrees of freedom at shorter distances (or, equivalently, higher energies).*

# Effective Field Theory (EFT)

## Main Ideas:

- Use right degrees of freedom : essence of (most) physics
- If mass-gap in the excitation spectrum: neglect degrees of freedom above the gap.

Examples:

**Solid state physics:** conductors: neglect the empty bands above the partially filled one

**Atomic physics:** Blue sky: neglect atomic structure

# EFT: Power Counting

- ▣ gap in the spectrum  $\implies$  separation of scales
- ▣ with the lower degrees of freedom, build the most general effective Lagrangian

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Higher orders suppressed by powers of  $1/\Lambda$



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  - ▣ Where did my predictivity go ?
- $\implies$  Need some ordering principle: power counting
  - Higher orders suppressed by powers of  $1/\Lambda$
- ▣ Taylor series expansion does not work (convergence radius is zero when massless modes are present)
- ▣ Continuum of excitation states need to be taken into account

# References

- A. Manohar, Effective Field Theories (Schladming lectures), hep-ph/9606222
- I. Rothstein, Lectures on Effective Field Theories (TASI lectures), hep-ph/0308266
- G. Ecker, Effective field theories, Encyclopedia of Mathematical Physics, hep-ph/0507056
- D.B. Kaplan, Five lectures on effective field theory, nucl-th/0510023
- A. Pich, Les Houches Lectures, hep-ph/9806303
- S. Scherer, Introduction to chiral perturbation theory, hep-ph/0210398
- J. Donoghue, Introduction to the Effective Field Theory Description of Gravity, gr-qc/9512024

# Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

# Chiral Perturbation Theory

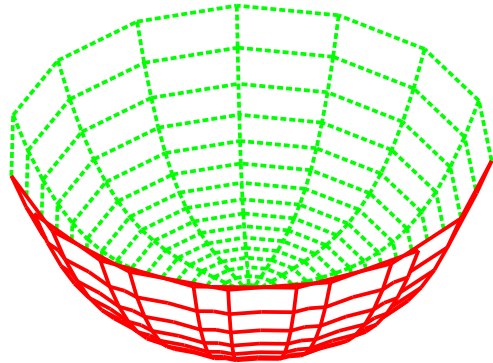
Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

Derivation from QCD:

H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory*,  
Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

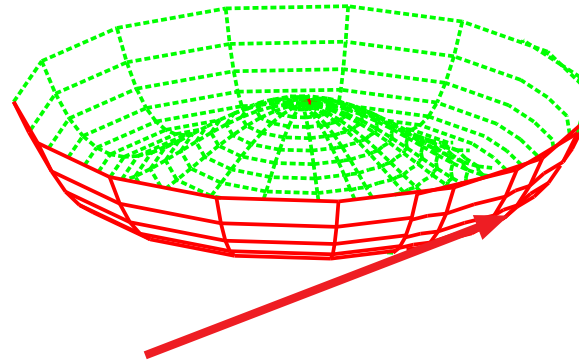
# The mass gap: Goldstone Modes

UNBROKEN:  $V(\phi)$



Only massive modes  
around lowest energy  
state (=vacuum)

BROKEN:  $V(\phi)$



Need to pick a vacuum  
 $\langle \phi \rangle \neq 0$ : Breaks symmetry  
No parity doublets  
Massless mode along bottom

For more complicated symmetries: need to describe the  
bottom mathematically:  $G \rightarrow H \implies G/H$

# The power counting

**Very important:**

**Low energy theorems: Goldstone bosons do not interact at zero momentum**

Heuristic proof:

- Which vacuum does not matter, choices related by symmetry
- $\phi(x) \rightarrow \phi(x) + \alpha$  should not matter
- Each term in  $\mathcal{L}$  must contain at least one  $\partial_\mu \phi$

# Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral  
Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so  $M_\rho$  or higher  
depending on the channel

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## Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange:  $SU(3)_V$

$$\text{But } \mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$$

So if  $m_q = 0$  then  $SU(3)_L \times SU(3)_R$ .



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So if  $m_q = 0$  then  $SU(3)_L \times SU(3)_R$ .

Can also see that via



$$v < c, m_q \neq 0 \implies$$

$$v = c, m_q = 0 \not\implies$$



# Chiral Perturbation Theory

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$

$SU(3)_L \times SU(3)_R$  broken spontaneously to  $SU(3)_V$

8 generators broken  $\implies$  8 massless degrees of freedom  
**and** interaction vanishes at zero momentum

We have 8 candidates that are light compared to the other hadrons:  $\pi^0, \pi^+, \pi^-, K^+, K^-, K^0, \bar{K}^0, \eta$

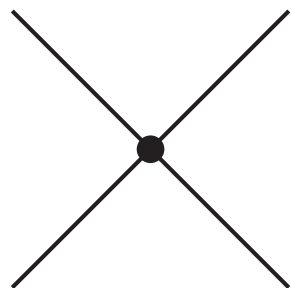
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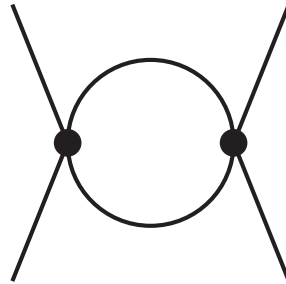
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Power counting in momenta (all lines soft):



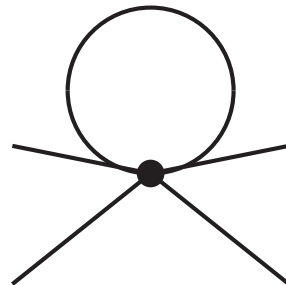
$$p^2$$



$$(p^2)^2 (1/p^2)^2 p^4 = p^4$$



$$1/p^2$$



$$(p^2) (1/p^2) p^4 = p^4$$

$$\int d^4p$$

$$p^4$$

# Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
  - Two or Three (or even more) Flavours
  - Strong interaction and couplings to external currents/densities
  - Including electromagnetism
  - Including weak nonleptonic interactions
  - Treating kaon as heavy

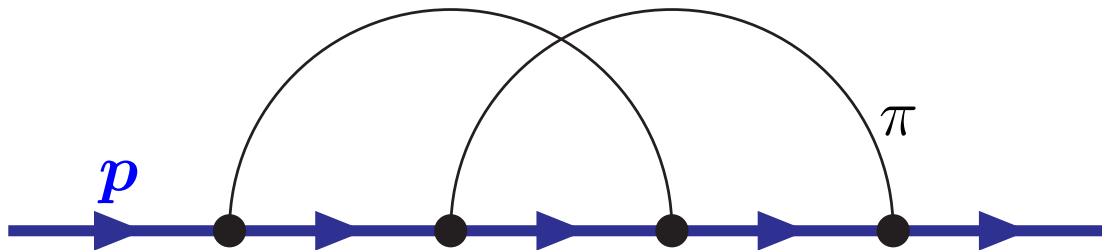
Many similarities with strongly interacting Higgs

# Hard pion ChPT?

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  - $p = M_B v + k$
  - Everything else soft
  - Works because baryon or  $b$  or  $c$  number conserved so the non soft line is continuous



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  - Decay constant works: takes away all heavy momentum
  - General idea:  $M_p$  dependence can always be reabsorbed in LECs, is analytic in the other parts  $k$ .

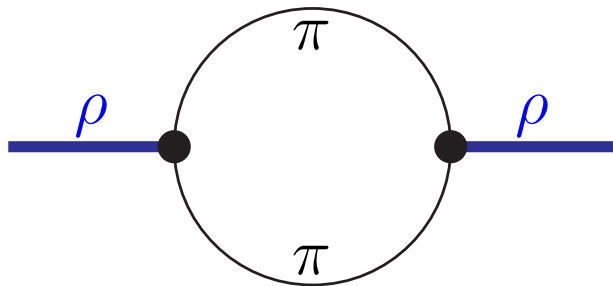
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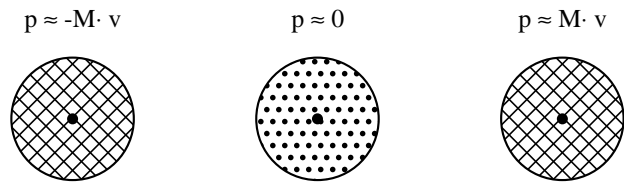


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  - (Vector) Meson:  $p = M_V v + k$
  - Everyone else soft or  $p = M_V v + k$
  - But (Heavy) (Vector) Meson ChPT decays strongly
    - First: keep diagrams where vectors always present
    - Applied to masses and decay constants
    - Decay constant works: takes away all heavy momentum
  - *It was argued that this could be done, the nonanalytic parts of diagrams with pions at large momenta are reproduced correctly* JB-Gosdzinsky-Talavera
  - Done both in relativistic and heavy meson formalism
  - **General idea:  $M_V$  dependence can always be reabsorbed in LECs, is analytic in the other parts  $k$ .**

# Toy model: one heavy one light scalar

JB-Gosdzinsky-Talavera 97



Momentum space

soft, particle on-shell,  
anti-particle on-shell

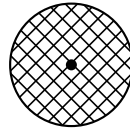
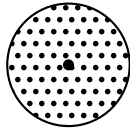
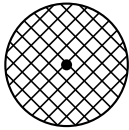
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$p \approx -M \cdot v$

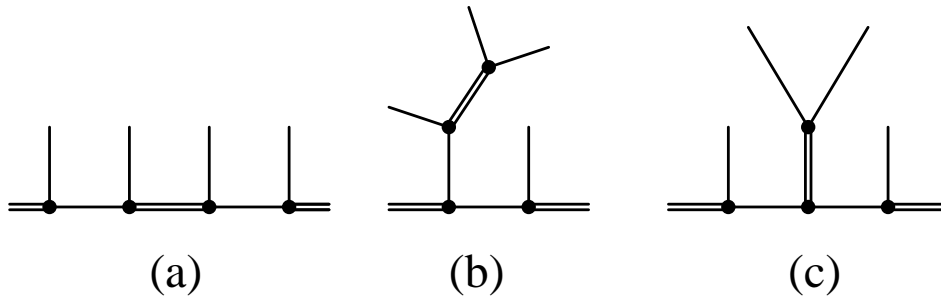
$p \approx 0$

$p \approx M \cdot v$

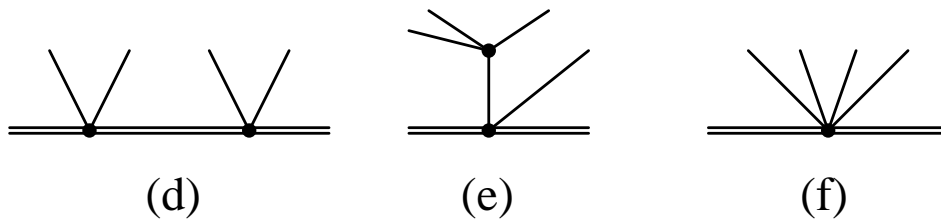


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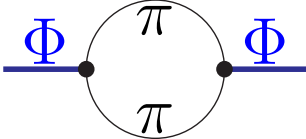
relativistic theory



heavy-meson theory

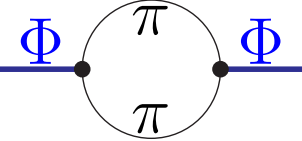
# Toy model: one heavy one light scalar

## Self-energy or mass corrections


$$\begin{aligned} \Pi_{\phi}^F &= 2\lambda^2 M^2 \frac{i}{16\pi^2} \left( \frac{1}{\epsilon} + \log 4\pi - \gamma_e - \log \frac{M^2}{\mu^2} \right) \\ &+ 4\lambda^2 m^2 \frac{i}{16\pi^2} \left( 1 - \log \frac{-m^2}{\mu^2} + \log \frac{M^2}{\mu^2} \right) + \mathcal{O}(1/M^2) \end{aligned}$$

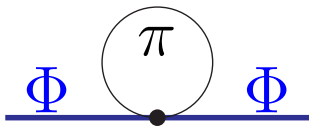
# Toy model: one heavy one light scalar

## Self-energy or mass corrections



A Feynman diagram for the self-energy correction  $\Pi_\phi^F$ . It consists of a horizontal blue line representing a  $\Phi$  particle. Two vertices on this line are connected by a circular loop. The top half of the loop is labeled with a  $\pi$  particle, and the bottom half is also labeled with a  $\pi$  particle.

$$\Pi_\phi^F = 2\lambda^2 M^2 \frac{i}{16\pi^2} \left( \frac{1}{\epsilon} + \log 4\pi - \gamma_e - \log \frac{M^2}{\mu^2} \right) + 4\lambda^2 m^2 \frac{i}{16\pi^2} \left( 1 - \log \frac{-m^2}{\mu^2} + \log \frac{M^2}{\mu^2} \right) + \mathcal{O}(1/M^2)$$

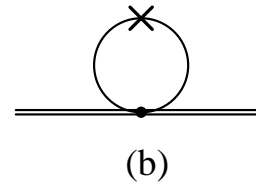
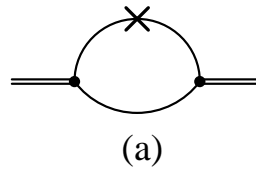


$$\Pi_\phi^E = -4i\lambda^2 \frac{m^2}{16\pi^2} \left( -\frac{1}{\epsilon} + \gamma_e - \log 4\pi - 1 + \log \frac{m^2}{\mu^2} \right) + \mathcal{O}(1/M^2)$$

$\log(m^2)$  terms **are the same**

# Toy model: one heavy one light scalar

Scalar formfactor:



(b):

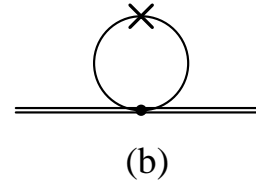
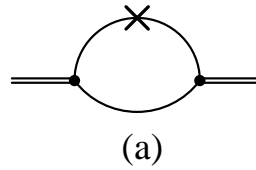
$$\frac{8i\lambda^2}{(4\pi)^2} \left[ \frac{1}{\epsilon} - \gamma_e + \log(4\pi) - \int_0^1 \log \left( \frac{m^2 - q^2 x(1-x) - i\epsilon}{\mu^2} \right) dx \right]$$

(a):

$$I = \frac{8i\lambda^2 M^2}{(4\pi)^2} \int_0^1 \frac{y dx dy}{(-m^2 + y(1-y)[Q^2 - q^2 x] + [xy - (xy)^2]q^2 + i\epsilon)}$$

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$1/M$ : away from  $y \approx 0$ ,  $y \approx 1$  expand in  $1/M^2$ ; in  $(1-y)$  and  $y$  near 1 and 0.

$$\begin{aligned} I &\approx \frac{8i\lambda^2 M^2}{(4\pi)^2} \int_0^1 dx \left\{ \int_{1-\delta}^1 \frac{dy}{(-m^2 + (1-y)Q^2 + x(1-x)q^2 + i\epsilon)} + \int_0^\alpha \frac{y dy}{-m^2 + yQ^2 + i\epsilon} \right\} \\ &= \frac{8i\lambda^2 M^2}{(4\pi)^2} \int_0^1 dx \left( -\frac{1}{M^2} \log \left| \frac{m^2 - x(1-x)q^2}{M^2} \right| - i\pi \int_0^\delta dz \delta(m^2 - x(1-x)q^2 - zQ^2) \right) \end{aligned}$$

full agreement in nonanalytic dependence on  $m^2$  and  $q^2$



# Hard pion ChPT?

- Heavy Kaon ChPT:
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon goes through
  - Applied to masses and  $\pi K$  scattering and decay constant [Roessl, Allton et al.,...](#)
  - Applied to  $K_{\ell 3}$  at  $q_{max}^2$  [Flynn-Sachrajda](#)
  - Works like all the previous *heavy* ChPT

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- [Flynn-Sachrajda](#) argued  $K_{\ell 3}$  also for  $q^2$  away from  $q_{max}^2$ .
- [JB-Celis](#) Argument generalizes to other processes with hard/fast pions and applied to  $K \rightarrow \pi\pi$
- [JB Jemos](#)  $B, D \rightarrow D, \pi, K, \eta$  vector formfactors, charmonium decays and a two-loop check
- **General idea: heavy/fast dependence can always be reabsorbed in LECs, is analytic in the other parts  $k$ .**

# Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q \rightarrow 0} \langle \pi^k(q) \alpha | O | \beta \rangle = -\frac{i}{F_\pi} \langle \alpha | [Q_5^k, O] | \beta \rangle,$$

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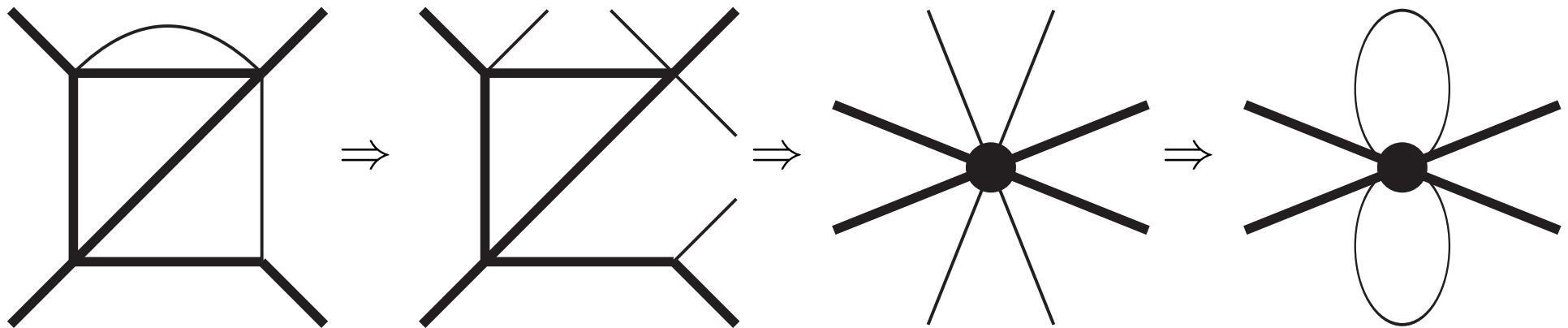
- Nothing prevents hard pions to be in the states  $\alpha$  or  $\beta$
- So by heavily using current algebra I should be able to get the light quark mass nonanalytic dependence

# Hard pion ChPT?

Field Theory: a process at given external momenta

- Take a diagram with a particular internal momentum configuration
- Identify the soft lines and cut them
- The result part is analytic in the soft stuff
- So should be describable by an effective Lagrangian with coupling constants dependent on the external given momenta (Weinberg's folklore theorem)
- Envisage this effective Lagrangian as a Lagrangian in hadron fields but all possible orders of the momenta included.

# Hard pion ChPT?



This procedure works at one loop level, matching at tree level, nonanalytic dependence at one loop:

- Toy models and vector meson ChPT [JB, Gosdzinsky, Talavera](#)
- Recent work on relativistic baryon ChPT [Gegelia, Scherer et al.](#)
- Extra terms kept in many of our calculations: a one-loop check
- Some two-loop checks

# Hard pion ChPT?

- This effective Lagrangian as a Lagrangian in hadron fields but all possible orders of the momenta included: possibly an infinite number of terms
- If symmetries present, Lagrangian should respect them
- but my powercounting is gone

# Hard pion ChPT?

- This effective Lagrangian as a Lagrangian in hadron fields but all possible orders of the momenta included: **possibly an infinite number of terms**
- If symmetries present, Lagrangian should respect them
- In some cases we can prove that up to a certain order in the expansion in light masses, not momenta, matrix elements of higher order operators are reducible to those of lowest order.
- Lagrangian should be complete in *neighbourhood* of original process
- Loop diagrams with this effective Lagrangian *should* reproduce the nonanalyticities in the light masses  
**Crucial part of the argument**



# The main technical trick

- For getting soft singularities in an integral we need the meson close to on-shell
- This only happens in an area of order  $m^4$
- So typically  $\int d^4p \frac{1}{(p^2 - m^2)} \sim m^4/m^2$  but if  $\partial_\mu\phi$  on that propagator we get an extra factor of  $m$ .
- So extra derivatives are only at same order if they hit hard lines
- and then they are part of the hard part which can be expanded around

# $K \rightarrow 2\pi$ in $SU(2)$ ChPT

Add  $K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$  Roessl

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F^2}{4} (\langle u_\mu u^\mu \rangle + \langle \chi_+ \rangle),$$

$$\mathcal{L}_{\pi K}^{(1)} = \nabla_\mu K^\dagger \nabla^\mu K - \overline{M}_K^2 K^\dagger K,$$

$$\mathcal{L}_{\pi K}^{(2)} = A_1 \langle u_\mu u^\mu \rangle K^\dagger K + A_2 \langle u^\mu u^\nu \rangle \nabla_\mu K^\dagger \nabla_\nu K + A_3 K^\dagger \chi_+ K + \dots$$

Add a spurion for the weak interaction  $\Delta I = 1/2$ ,  $\Delta I = 3/2$

JB,Celis

$$t_k^{ij} \longrightarrow t_{k'}^{i'j'} = t_k^{ij} (g_L)_{k'}^k (g_L^\dagger)_{i'}^{i'} (g_L^\dagger)_j^{j'}$$

$$t_{1/2}^i \longrightarrow t_{1/2}^{i'} = t_{1/2}^i (g_L^\dagger)_{i'}^{i'}.$$

# $K \rightarrow 2\pi$ in $SU(2)$ ChPT

The  $\Delta I = 1/2$  terms:  $\tau_{1/2} = t_{1/2}u^\dagger$

$$\begin{aligned}\mathcal{L}_{1/2} = & iE_1 \tau_{1/2} K + E_2 \tau_{1/2} u^\mu \nabla_\mu K + iE_3 \langle u_\mu u^\mu \rangle \tau_{1/2} K \\ & + iE_4 \tau_{1/2} \chi_+ K + iE_5 \langle \chi_+ \rangle \tau_{1/2} K + E_6 \tau_{1/2} \chi_- K \\ & + E_7 \langle \chi_- \rangle \tau_{1/2} K + iE_8 \langle u_\mu u_\nu \rangle \tau_{1/2} \nabla^\mu \nabla^\nu K + \dots + h.c..\end{aligned}$$

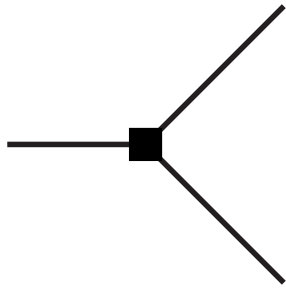
Note: higher order terms kept in both  $\mathcal{L}_{1/2}$  and  $\mathcal{L}_{\pi K}^{(2)}$  to check the arguments

Using partial integration, . . . :

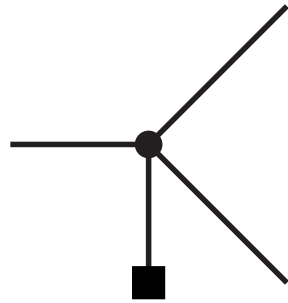
$$\begin{aligned}\langle \pi(p_1) \pi(p_2) | O | K(p_K) \rangle = \\ f(\overline{M}_K^2) \langle \pi(p_1) \pi(p_2) | \tau_{1/2} K | K(p_K) \rangle + \lambda M^2 + \mathcal{O}(M^4)\end{aligned}$$

$O$  any operator in  $\mathcal{L}_{1/2}$  or with more derivatives. Similar for  $\mathcal{L}_{3/2}$

# $K \rightarrow \pi\pi$ : Tree level



(a)

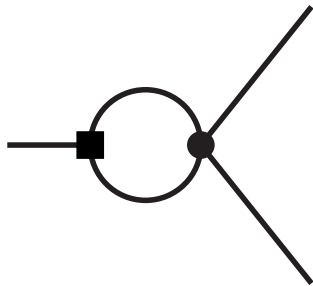


(b)

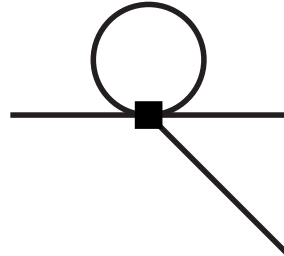
$$A_0^{LO} = \frac{\sqrt{3}i}{2F^2} \left[ -\frac{1}{2}E_1 + (E_2 - 4E_3) \overline{M}_K^2 + 2E_8 \overline{M}_K^4 + A_1 E_1 \right]$$

$$A_2^{LO} = \sqrt{\frac{3}{2}} \frac{i}{F^2} \left[ (-2D_1 + D_2) \overline{M}_K^2 \right]$$

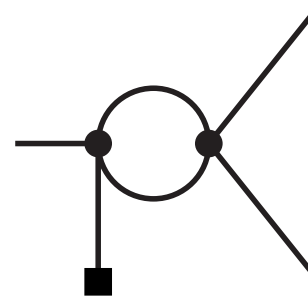
# $K \rightarrow \pi\pi$ : One loop



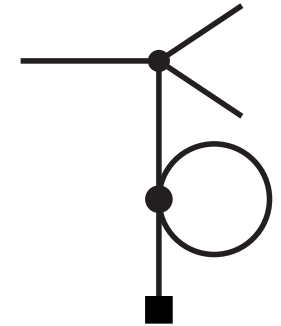
(a)



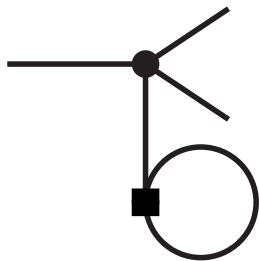
(b)



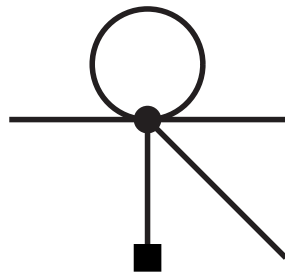
(c)



(d)



(e)



(f)

# $K \rightarrow \pi\pi$ : One loop

Diagram	$A_0$	$A_2$
$Z$	$-\frac{2F^2}{3} A_0^{LO}$	$-\frac{2F^2}{3} A_2^{LO}$
(a)	$\sqrt{3}i \left( -\frac{1}{3} E_1 + \frac{2}{3} E_2 \overline{M}_K^2 \right)$	$\sqrt{\frac{3}{2}}i \left( -\frac{2}{3} D_2 \overline{M}_K^2 \right)$
(b)	$\sqrt{3}i \left( -\frac{5}{96} E_1 - \left( \frac{7}{48} E_2 + \frac{25}{12} E_3 \right) \overline{M}_K^2 + \frac{25}{24} E_8 \overline{M}_K^4 \right)$	$\sqrt{\frac{3}{2}}i \left( -\frac{61}{12} D_1 + \frac{77}{24} D_2 \right) \overline{M}_K^2$
(e)	$\sqrt{3}i \frac{3}{16} A_1 E_1$	
(f)	$\sqrt{3}i \left( \frac{1}{8} E_1 + \frac{1}{3} A_1 E_1 \right)$	

The coefficients of  $\overline{A}(M^2)/F^4$  in the contributions to  $A_0$  and  $A_2$ .  $Z$  denotes the part from wave-function renormalization.

- $\overline{A}(M^2) = -\frac{M^2}{16\pi^2} \log \frac{M^2}{\mu^2}$
- $K\pi$  intermediate state does not contribute, but did for **Flynn-Sachrajda**

# $K \rightarrow \pi\pi$ : One-loop

$$A_0^{NLO} = A_0^{LO} \left( 1 + \frac{3}{8F^2} \bar{A}(M^2) \right) + \lambda_0 M^2 + \mathcal{O}(M^4),$$
$$A_2^{NLO} = A_2^{LO} \left( 1 + \frac{15}{8F^2} \bar{A}(M^2) \right) + \lambda_2 M^2 + \mathcal{O}(M^4).$$

# $K \rightarrow \pi\pi$ : One-loop

$$A_0^{NLO} = A_0^{LO} \left( 1 + \frac{3}{8F^2} \bar{A}(M^2) \right) + \lambda_0 M^2 + \mathcal{O}(M^4),$$

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Match with three flavour  $SU(3)$  calculation [Kambor, Missimer, Wyler; JB, Pallante, Prades](#)

$$A_0^{(3)LO} = -\frac{i\sqrt{6}CF_0^4}{\bar{F}_K F^2} \left( G_8 + \frac{1}{9}G_{27} \right) \bar{M}_K^2, \quad A_2^{(3)LO} = -\frac{i10\sqrt{3}CF_0^4}{9\bar{F}_K F^2} G_{27} \bar{M}_K^2,$$

When using  $F_\pi = F \left( 1 + \frac{1}{F^2} \bar{A}(M^2) + \frac{M^2}{F^2} l_4^r \right)$ ,  $F_K = \bar{F}_K \left( 1 + \frac{3}{8F^2} \bar{A}(M^2) + \dots \right)$ ,

**logarithms at one-loop agree with above**



# Hard Pion ChPT: A two-loop check

- Similar arguments to JB-Celis, Flynn-Sachrajda work for the pion vector and scalar formfactor JB-Jemos
- Therefore at any  $t$  the chiral log correction must go like the one-loop calculation.
- But note the one-loop log chiral log is with  $t \gg m_\pi^2$

- Predicts

$$F_V(t, M^2) = F_V(t, 0) \left( 1 - \frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(M^2) \right)$$
$$F_S(t, M^2) = F_S(t, 0) \left( 1 - \frac{5}{2} \frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(M^2) \right)$$

- Note that  $F_{V,S}(t, 0)$  is now a coupling constant and can be complex

# Hard Pion ChPT: A two-loop check

- Take the full two-loop ChPT calculation  
JB, Colangelo, Talavera, valid for  $t, m_\pi^2 \ll \Lambda_\chi^2$
- Expand this for  $t \gg m_\pi^2$
- $t^2 \ln t, \dots$  terms go in  $F_{S,V}(t, 0)$
- But the one-loop for  $F_V(t, 0)$  is known and HPCChPT predicts how the chiral log  $m^2 \log m^2$  adds to this

$$F_V = 1 + x_2 \left[ \frac{1}{6}(s-4)\bar{J}(s) + s \left( -l_6^r - \frac{1}{6}L - \frac{1}{18N} \right) \right] + x_2^2 \left( P_V^{(2)} + U_V^{(2)} \right) + \mathcal{O}(x_2^3).$$

$$U_V^{(2)} = \bar{J}(s) \left[ \frac{1}{3}l_1^r(-s^2 + 4s) + \frac{1}{6}l_2^r(s^2 - 4s) + \frac{1}{3}l_4^r(s-4) + \frac{1}{6}l_6^r(-s^2 + 4s) - \frac{1}{36}L(s^2 + 8s - 48) \right. \\ \left. + \frac{1}{N} \left( \frac{7}{108}s^2 - \frac{97}{108}s + \frac{3}{4} \right) \right] + \frac{1}{9}K_1(s) + \frac{1}{9}K_2(s) \left( \frac{1}{8}s^2 - s + 4 \right) + \frac{1}{6}K_3(s) \left( s - \frac{1}{3} \right) - \frac{5}{3}K_4(s).$$

# A two-loop check

Full two-loop ChPT JB, Colangelo, Talavera, expand in  $t \gg m_\pi^2$ :

$$F_V(t, M^2) = F_V(t, 0) \left( 1 - \frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(M^2) \right)$$
$$F_S(t, M^2) = F_S(t, 0) \left( 1 - \frac{5}{2} \frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(M^2) \right)$$

with

$$F_V(t, 0) = 1 + \frac{t}{16\pi^2 F^2} \left( \frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6} \ln \frac{t}{\mu^2} \right)$$
$$F_S(t, 0) = 1 + \frac{t}{16\pi^2 F^2} \left( 1 + 16\pi^2 l_4^r + i\pi - \ln \frac{t}{\mu^2} \right)$$

- The needed coupling constants are complex
- Both calculations have two-loop diagrams with overlapping divergences
- The chiral logs should be valid for any  $t$  where a pointlike interaction is a valid approximation

# Electromagnetic formfactors

$$F_V^\pi(s) = F_V^{\pi\chi}(s) \left( 1 + \frac{1}{F^2} \bar{A}(m_\pi^2) + \frac{1}{2F^2} \bar{A}(m_K^2) + \mathcal{O}(m_L^2) \right),$$
$$F_V^K(s) = F_V^{K\chi}(s) \left( 1 + \frac{1}{2F^2} \bar{A}(m_\pi^2) + \frac{1}{F^2} \bar{A}(m_K^2) + \mathcal{O}(m_L^2) \right).$$

# $B, D \rightarrow \pi, K, \eta$

$$\langle P_f(p_f) | \bar{q}_i \gamma_\mu q_f | P_i(p_i) \rangle = (p_i + p_f)_\mu f_+(q^2) + (p_i - p_f)_\mu f_-(q^2)$$

$$f_{+B \rightarrow M}(t) = f_{+B \rightarrow M}^\chi(t) F_{B \rightarrow M}$$

$$f_{-B \rightarrow M}(t) = f_{-B \rightarrow M}^\chi(t) F_{B \rightarrow M}$$

- $F_{B \rightarrow M}$  is **always the same** for  $f_+$ ,  $f_-$  and  $f_0$
- This is not heavy quark symmetry: not valid at endpoint and valid also for  $K \rightarrow \pi$ .
- Not like Low's theorem, not only dependence on external legs
- Done in heavy meson and relativistic formalism

# $B, D \rightarrow \pi, K, \eta$

$$F_{K \rightarrow \pi} = 1 + \frac{3}{8F^2} \bar{A}(m_\pi^2) \quad (2 - \text{flavour})$$

$$F_{B \rightarrow \pi} = 1 + \left( \frac{3}{8} + \frac{9}{8}g^2 \right) \frac{\bar{A}(m_\pi^2)}{F^2} + \left( \frac{1}{4} + \frac{3}{4}g^2 \right) \frac{\bar{A}(m_K^2)}{F^2} + \left( \frac{1}{24} + \frac{1}{8}g^2 \right) \frac{\bar{A}(m_\eta^2)}{F^2},$$

$$F_{B \rightarrow K} = 1 + \frac{9}{8}g^2 \frac{\bar{A}(m_\pi^2)}{F^2} + \left( \frac{1}{2} + \frac{3}{4}g^2 \right) \frac{\bar{A}(m_K^2)}{F^2} + \left( \frac{1}{6} + \frac{1}{8}g^2 \right) \frac{\bar{A}(m_\eta^2)}{F^2},$$

$$F_{B \rightarrow \eta} = 1 + \left( \frac{3}{8} + \frac{9}{8}g^2 \right) \frac{\bar{A}(m_\pi^2)}{F^2} + \left( \frac{1}{4} + \frac{3}{4}g^2 \right) \frac{\bar{A}(m_K^2)}{F^2} + \left( \frac{1}{24} + \frac{1}{8}g^2 \right) \frac{\bar{A}(m_\eta^2)}{F^2},$$

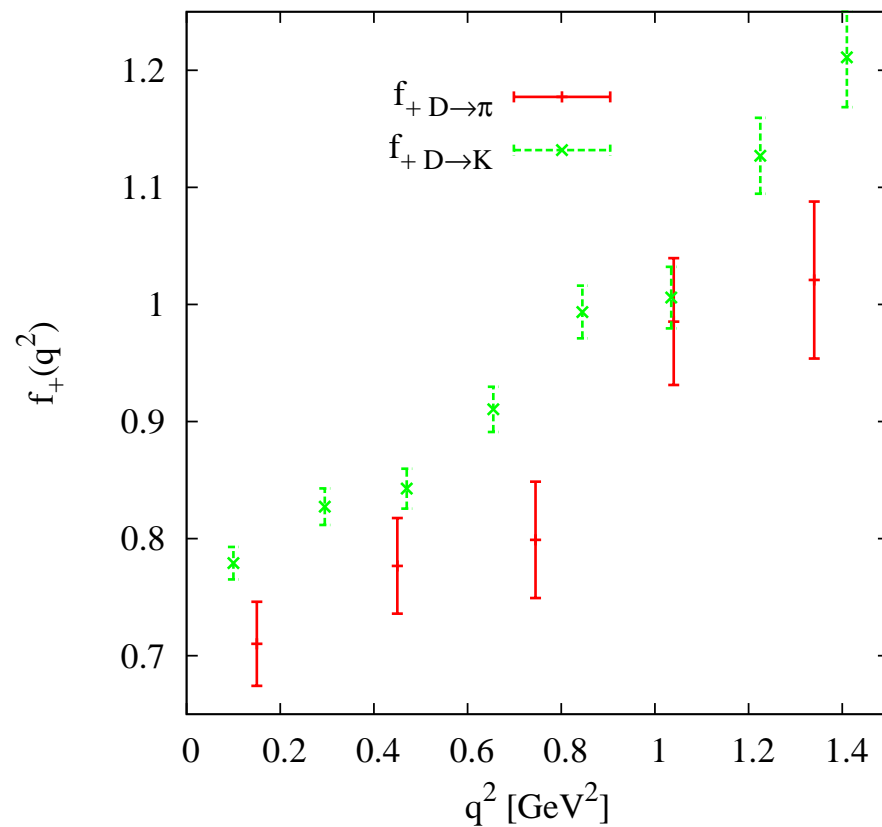
$$F_{B_s \rightarrow K} = 1 + \frac{3}{8} \frac{\bar{A}(m_\pi^2)}{F^2} + \left( \frac{1}{4} + \frac{3}{2}g^2 \right) \frac{\bar{A}(m_K^2)}{F^2} + \left( \frac{1}{24} + \frac{1}{2}g^2 \right) \frac{\bar{A}(m_\eta^2)}{F^2},$$

$$F_{B_s \rightarrow \eta} = 1 + \left( \frac{1}{2} + \frac{3}{2}g^2 \right) \frac{\bar{A}(m_K^2)}{F^2} + \left( \frac{1}{6} + \frac{1}{2}g^2 \right) \frac{\bar{A}(m_\eta^2)}{F^2}.$$

$F_{B_s \rightarrow \pi}$  vanishes due to the possible flavour quantum numbers.

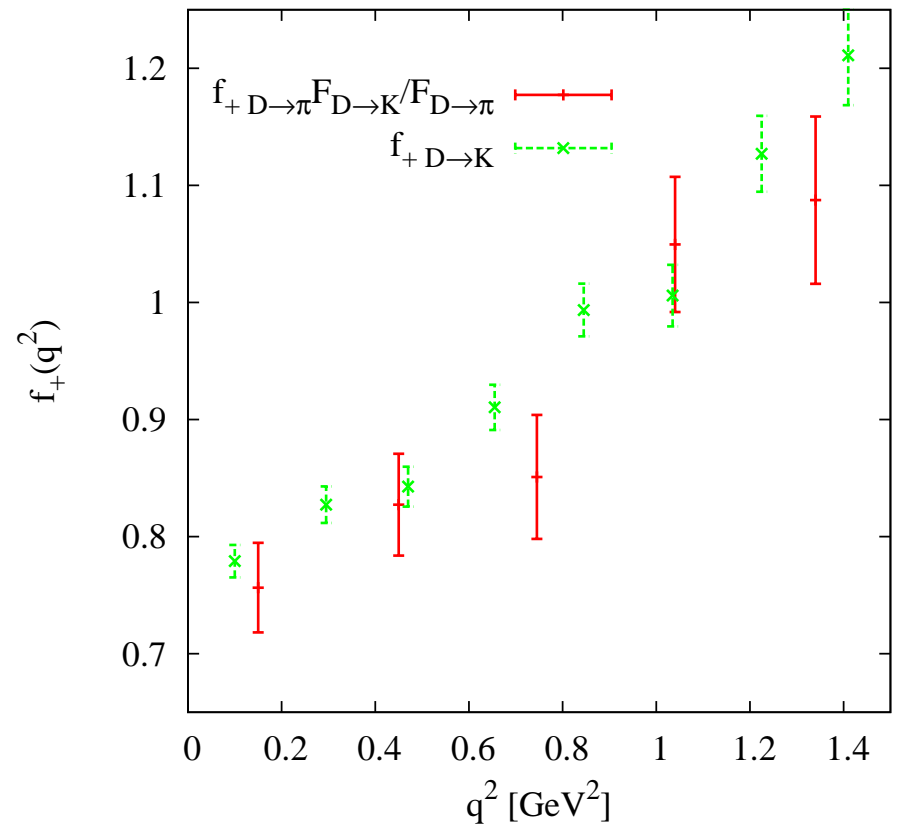
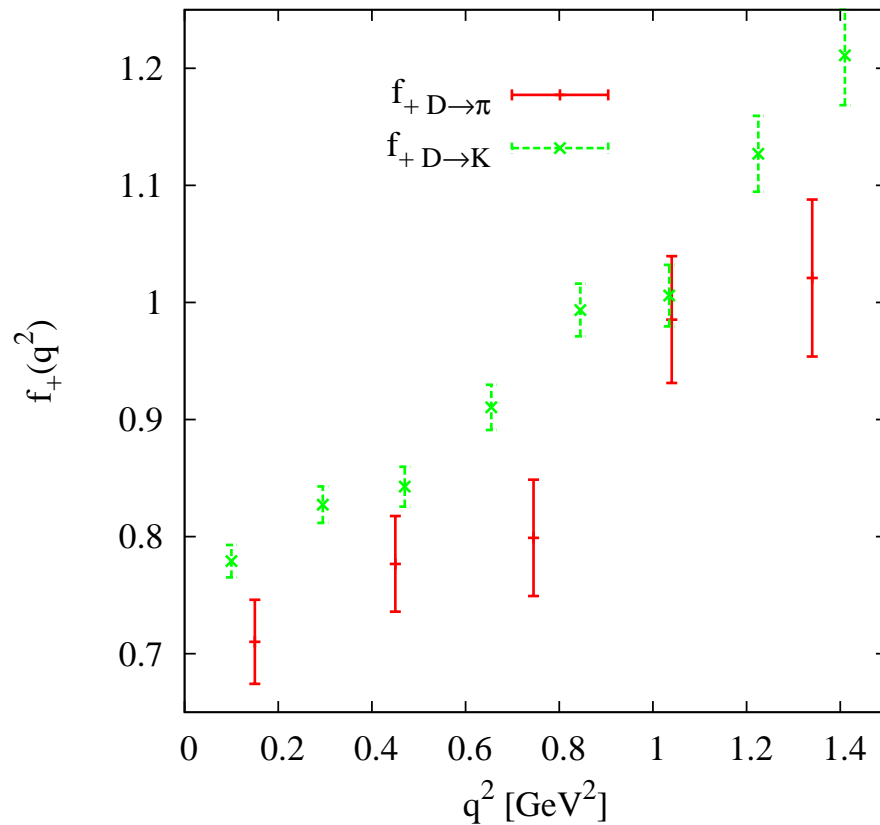
# Experimental check

CLEO data on  $f_+(q^2)|V_{cq}|$  for  $D \rightarrow \pi$  and  $D \rightarrow K$  with  $|V_{cd}| = 0.2253$ ,  $|V_{cs}| = 0.9743$



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$$f_{+D \rightarrow \pi} = f_{+D \rightarrow K} F_{D \rightarrow \pi} / F_{D \rightarrow K}$$



# Applications to charmonium

- We look at decays  $\chi_{c0}, \chi_{c2} \rightarrow \pi\pi, KK, \eta\eta$
- $J/\psi, \psi(nS), \chi_{c1}$  decays to the same final state break isospin or  $U$ -spin or  $V$ -spin, they thus proceed via electromagnetism or quark mass differences: more difficult.
- So construct a Lagrangian with a chiral singlet scalar and tensor field.
- $$\mathcal{L}_{\chi_c} = E_1 F_0^2 \chi_0 \langle u^\mu u_\mu \rangle + E_2 F_0^2 \chi_2^{\mu\nu} \langle u_\mu u_\nu \rangle .$$

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- $\mathcal{L}_{\chi_c} = E_1 F_0^2 \chi_0 \langle u^\mu u_\mu \rangle + E_2 F_0^2 \chi_2^{\mu\nu} \langle u_\mu u_\nu \rangle .$
- **No chiral logarithm corrections**
- Expanding the energy-momentum tensor result **Donoghue-Leutwyler** at large  $q^2$  agrees.
- These decays should have small  $SU(3)_V$  breaking

# Charmonium

- Phase space correction:  $|\vec{p}_1| = \sqrt{m_\chi^2 - 4m_P^2}/2$ .
- $\chi_{c0}$ :
  - $A \propto p_1 \cdot p_2 = (m_\chi^2 - 2m_P^2)/2$ .
  - $\implies G_0 = \sqrt{BR/|\vec{p}_1|/(p_1 \cdot p_2)}$ .
- $\chi_{c2}$ :
  - $A \propto T_\chi^{\mu\nu} p_{1\mu} p_{2\nu}$ . (polarization tensor)
  - $|A|^2 \propto \frac{1}{5} \sum_{pol} T_\chi^{\mu\nu} p_{1\mu} p_{2\nu} T_\chi^{*\alpha\beta} p_{1\alpha} p_{2\beta} = \frac{1}{30} (m_\chi^2 - 4m_P^2)^2 \propto |\vec{p}_1|^4$ .
  - $\implies G_2 = \sqrt{BR/|\vec{p}_1|/|\vec{p}_1|^2}$ .
- $\times 2$  for  $K_S^0 K_S^0$  to  $K^0 \bar{K}^0$ ,  $\times 2/3$  for  $\pi\pi$  to  $\pi^+ \pi^-$ .

# Charmonium

	$\chi_{c0}$		$\chi_{c2}$	
Mass	3414.75 $\pm$ 0.31 MeV		3556.20 $\pm$ 0.09 MeV	
Width	10.4 $\pm$ 0.6 MeV		1.97 $\pm$ 0.11 MeV	
Final state	$10^3$ BR	$10^{10} G_0 [\text{MeV}^{-5/2}]$	$10^3$ BR	$10^{10} G_2 [\text{MeV}^{-5/2}]$
$\pi\pi$	8.5 $\pm$ 0.4	3.15 $\pm$ 0.07	2.42 $\pm$ 0.13	3.04 $\pm$ 0.08
$K^+ K^-$	6.06 $\pm$ 0.35	3.45 $\pm$ 0.10	1.09 $\pm$ 0.08	2.74 $\pm$ 0.10
$K_S^0 K_S^0$	3.15 $\pm$ 0.18	3.52 $\pm$ 0.10	0.58 $\pm$ 0.05	2.83 $\pm$ 0.12
$\eta\eta$	3.03 $\pm$ 0.21	2.48 $\pm$ 0.09	0.59 $\pm$ 0.05	2.06 $\pm$ 0.09
$\eta'\eta'$	2.02 $\pm$ 0.22	2.43 $\pm$ 0.13	< 0.11	< 1.2

Experimental results for  $\chi_{c0}, \chi_{c2} \rightarrow PP$  and the factors corrected for the known  $m^2$  effects.

- $\pi\pi$  and  $KK$  are good to 10% (Note: 20% for  $F_K/F_\pi$ )
- $\eta\eta$  OK

# Caveat utilitor: let the user beware

- This is not a simple straightforward process
- Especially the proof that it all reduces to a single type of lowest order term can be tricky.
- Some examples where it does not work easily:
  - $VV$  two-point function has two types of lowest order terms:  $\langle LR \rangle$  and  $\langle LL \rangle + \langle RR \rangle$  (no derivative structure indicated)
  - Scalar form factors in three flavour ChPT, again two types of lowest order terms  $\langle \chi_+ \rangle$  and  $\langle \chi_+ \rangle \langle u_\mu u^\mu \rangle$

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  - Scalar form factors in three flavour ChPT, again two types of lowest order terms  $\langle \chi_+ \rangle$  and  $\langle \chi_+ \rangle \langle u_\mu u^\mu \rangle$ 
    - In  $SU(2)$  these two types are the same hence our check still worked for the scalar form-factor
    - For the vector formfactor the second type vanishes or gives for the  $SU(2)$  case no contribution because of  $G$ -parity.

# Summary

Why is this useful:

- Lattice works actually around the strange quark mass
- need only extrapolate in  $m_u$  and  $m_d$ .
- Applicable in momentum regimes where usual ChPT might not work
- Three flavour case useful for  $B, D, \chi_c$  decays

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Can we use it for  $\eta, \eta'$ ?

- Some examples of  $\eta$  in final state included
- $\eta \rightarrow 3\pi$ : soft pions so nothing new
- $\eta' \rightarrow 3\pi$ : In principle yes but nothing to compare to, e.g.  
 $\eta' \rightarrow KK\pi$  is not an observable