HARD PION CHIRAL PERTURBATION THEORY

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Overview

- Effective Field Theory
- Chiral Perturbation Theor(y)(ies)
- Hard Pion Chiral Perturbation Theory

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- Hard Pion Chiral Perturbation Theory
  - $K_{\ell 3}$ Flynn-Sachrajda, arXiv:0809.1229
  - $K \to \pi\pi$ JB+ Alejandro Celis, arXiv:0906.0302
  - F_π^S and F_π^V JB + Ilaria Jemos, arXiv:1011.6531 a two-loop check
  - $B, D \to \pi$ JB + Ilaria Jemos, arXiv:1006.1197
  - $B, D \to \pi, K, \eta$ JB + Ilaria Jemos, arXiv:1011.6531
  - $\chi_c(J = 0, 2) \to \pi\pi, KK, \eta\eta$ JB+Ilaria Jemos, arxiv:1109.5033
  - Some examples which do not have a chiral log prediction

Wikipedia

`http://en.wikipedia.org/wiki/Effective_field_theory`

In physics, an effective field theory is an approximate theory (usually a quantum field theory) that contains the appropriate degrees of freedom to describe physical phenomena occurring at a chosen length scale, but ignores the substructure and the degrees of freedom at shorter distances (or, equivalently, higher energies).

Effective Field Theory (EFT)

Main Ideas:

- Use right degrees of freedom : essence of (most) physics
- If mass-gap in the excitation spectrum: neglect degrees of freedom above the gap.

Examples:
- Solid state physics: conductors: neglect the empty bands above the partially filled one
- Atomic physics: Blue sky: neglect atomic structure

EFT: Power Counting

- gap in the spectrum $\Longrightarrow$ separation of scales
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Higher orders suppressed by powers of $1/\Lambda$

EFT: Power Counting

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Higher orders suppressed by powers of $1/\Lambda$

- Taylor series expansion does not work (convergence radius is zero when massless modes are present)
- Continuum of excitation states need to be taken into account

References

- A. Manohar, Effective Field Theories (Schladming lectures), hep-ph/9606222
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- D.B. Kaplan, Five lectures on effective field theory, nucl-th/0510023
- A. Pich, Les Houches Lectures, hep-ph/9806303
- S. Scherer, Introduction to chiral perturbation theory, hep-ph/0210398
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Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

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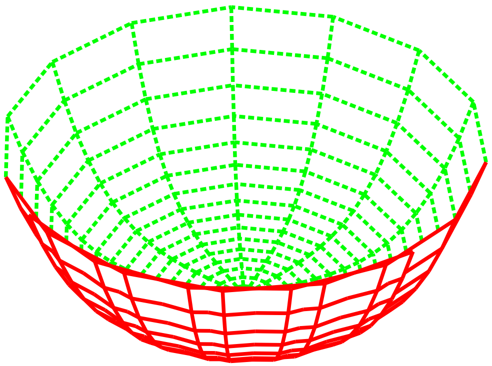
Derivation from QCD:

H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory,* Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

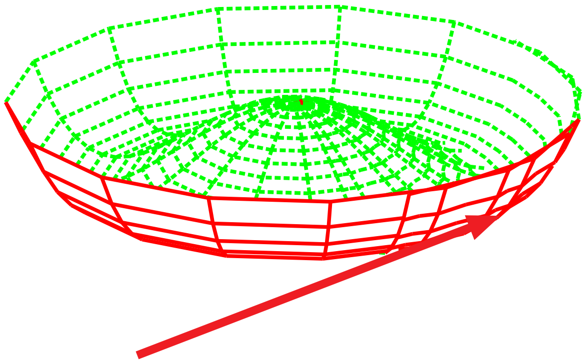
The mass gap: Goldstone Modes

UNBROKEN: $V(\phi)$



Only massive modes
around lowest energy
state (=vacuum)

BROKEN: $V(\phi)$



Need to pick a vacuum
$\langle \phi \rangle \neq 0$: Breaks symmetry
No parity doublets
Massless mode along bottom

For more complicated symmetries: need to describe the bottom mathematically: $G \to H \Longrightarrow G/H$

The power counting

Very important:

Low energy theorems: Goldstone bosons do not interact at zero momentum

Heuristic proof:

- Which vacuum does not matter, choices related by symmetry
- $\phi(x) \to \phi(x) + \alpha$ should not matter
- Each term in $\mathcal{L}$ must contain at least one $\partial_\mu \phi$

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

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Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

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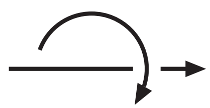
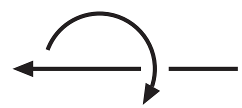
Can also see that via  $v < c,\ m_q \neq 0 \Longrightarrow$ 

$v = c,\ m_q = 0 \not\Longrightarrow$

Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

We have 8 candidates that are light compared to the other hadrons: $\pi^0, \pi^+, \pi^-, K^+, K^-, K^0, \overline{K^0}, \eta$

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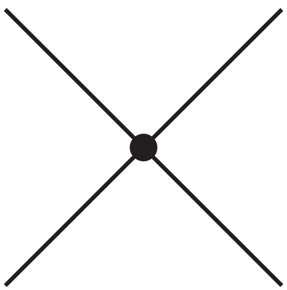
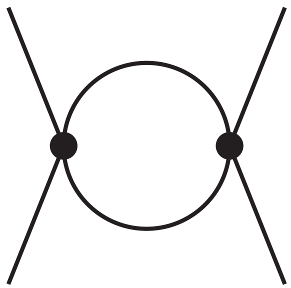
Power counting in momenta (all lines soft):

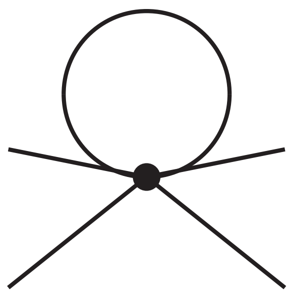


p^2



$(p^2)^2 \, (1/p^2)^2 \, p^4 = p^4$

$1/p^2$



$(p^2) \, (1/p^2) \, p^4 = p^4$

$\int d^4p$ $\quad p^4$

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
  - Two or Three (or even more) Flavours
  - Strong interaction and couplings to external currents/densities
  - Including electromagnetism
  - Including weak nonleptonic interactions
  - Treating kaon as heavy

Many similarities with strongly interacting Higgs

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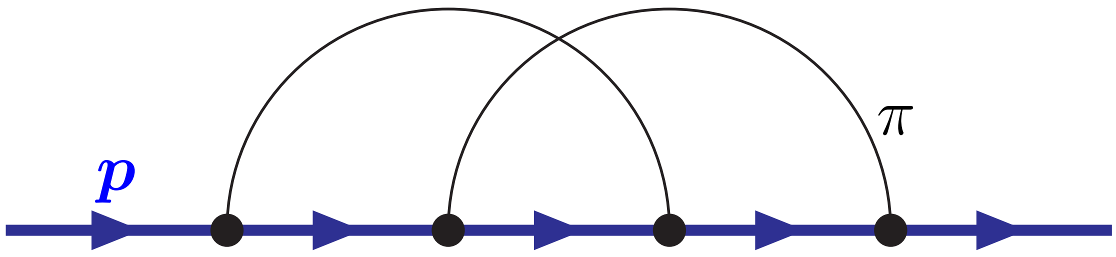
  - $p = M_B v + k$
  - Everything else soft
  - Works because baryon or b or c number conserved so the non soft line is continuous



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  - Decay constant works: takes away all heavy momentum
  - General idea: M_p dependence can always be reabsorbed in LECs, is analytic in the other parts k.

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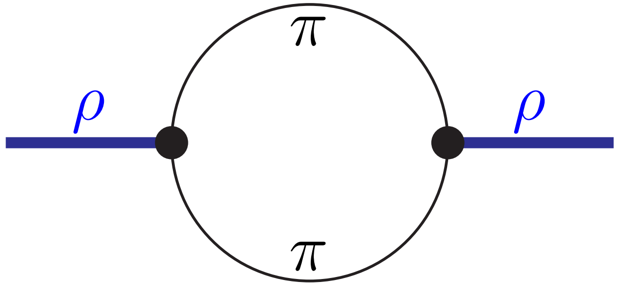

Hard pion ChPT?

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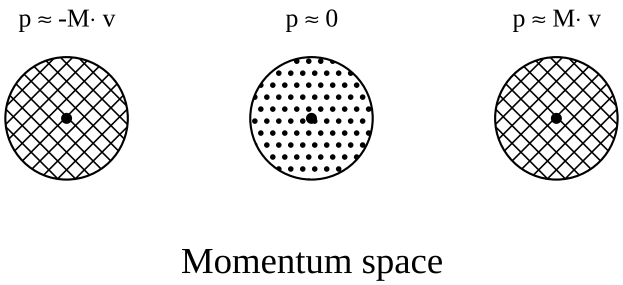
  - (Vector) Meson: $p = M_V v + k$
  - Everyone else soft or $p = M_V v + k$
  - But (Heavy) (Vector) Meson ChPT decays strongly
    - First: keep diagrams where vectors always present
    - Applied to masses and decay constants
    - Decay constant works: takes away all heavy momentum
  - *It was argued that this could be done, the nonanalytic parts of diagrams with pions at large momenta are reproduced correctly* JB-Gosdzinsky-Talavera
  - Done both in relativistic and heavy meson formalism
  - General idea: M_V dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Toy model: one heavy one light scalar

JB-Gosdzinsky-Talavera 97



soft, particle on-shell,
anti-particle on-shell

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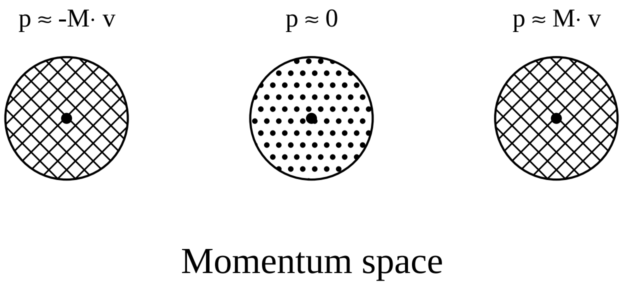

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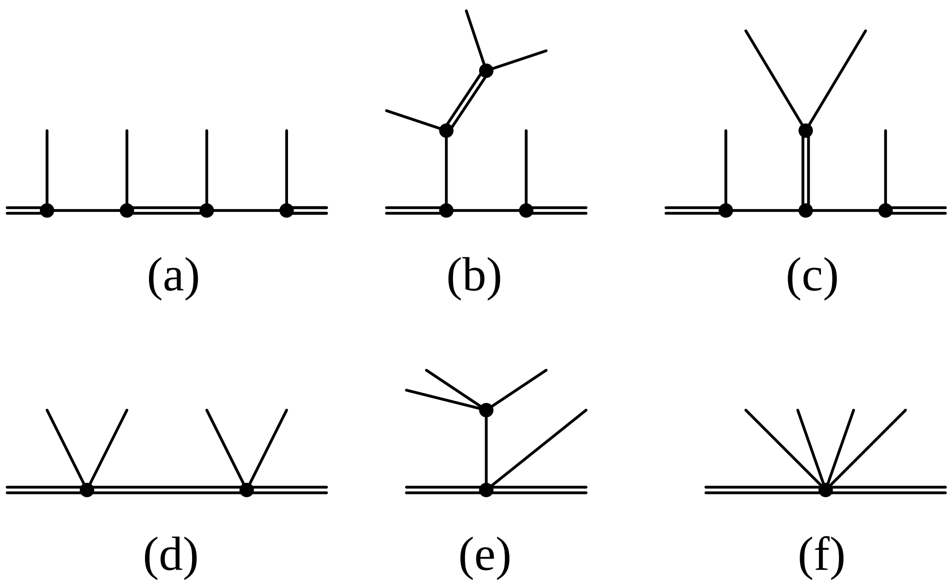
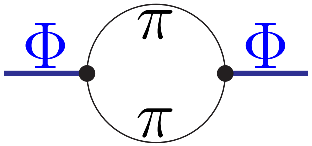


relativistic theory

heavy-meson theory

Toy model: one heavy one light scalar

Self-energy or mass corrections



$$\Pi_\phi^F = 2\lambda^2 M^2 \frac{i}{16\pi^2}\left(\frac{1}{\epsilon} + \log 4\pi - \gamma_e - \log\frac{M^2}{\mu^2}\right) + 4\lambda^2 m^2 \frac{i}{16\pi^2}\left(1 - \log\frac{-m^2}{\mu^2} + \log\frac{M^2}{\mu^2}\right) + \mathcal{O}(1/M^2)$$

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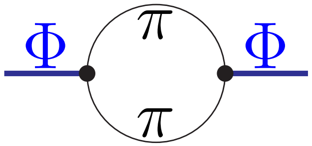

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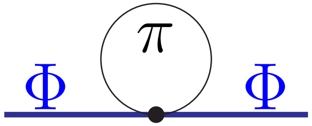
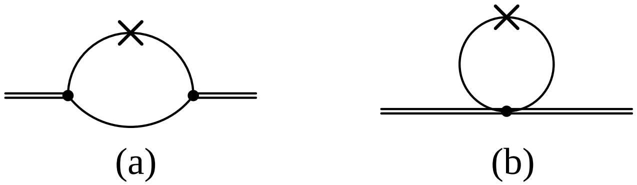


$$\Pi_\phi^E = -4i\lambda^2 \frac{m^2}{16\pi^2}\left(-\frac{1}{\epsilon} + \gamma_e - \log 4\pi - 1 + \log\frac{m^2}{\mu^2}\right) + \mathcal{O}(1/M^2)$$

$\log(m^2)$ terms are the same

Toy model: one heavy one light scalar

Scalar formfactor:



(b):

$$\frac{8i\lambda^2}{(4\pi)^2}\left[\frac{1}{\epsilon}-\gamma_e+\log(4\pi)-\int_0^1\log\left(\frac{m^2-q^2x(1-x)-i\varepsilon}{\mu^2}\right)dx\right]$$

(a):

$$I=\frac{8i\lambda^2M^2}{(4\pi)^2}\int_0^1\frac{ydxdy}{(-m^2+y(1-y)[Q^2-q^2x]+[xy-(xy)^2]q^2+i\varepsilon)}$$

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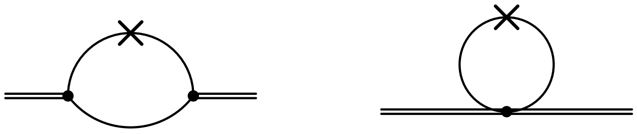

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$1/M$: away from $y\approx 0$, $y\approx 1$ expand in $1/M^2$; in $(1-y)$ and y near 1 and 0.

$$\begin{aligned}I &\approx \frac{8i\lambda^2M^2}{(4\pi)^2}\int_0^1 dx\left\{\int_{1-\delta}^1\frac{dy}{(-m^2+(1-y)Q^2+x(1-x)q^2+i\epsilon)}+\int_0^\alpha\frac{ydy}{-m^2+yQ^2+i\varepsilon}\right\}\\ &= \frac{8i\lambda^2M^2}{(4\pi)^2}\int_0^1 dx\left(-\frac{1}{M^2}\log\left|\frac{m^2-x(1-x)q^2}{M^2}\right|-i\,\pi\int_0^\delta dz\,\delta(m^2-x(1-x)q^2-zQ^2)\right)\end{aligned}$$

full agreement in nonanalytic dependence on m^2 and q^2

Hard pion ChPT?

- Heavy Kaon ChPT:
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon goes through
  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,. . .
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
  - Works like all the previous *heavy* ChPT

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  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,. . .
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
- Flynn-Sachrajda argued $K_{\ell 3}$ also for q^2 away from q^2_{max}.
- JB-Celis Argument generalizes to other processes with hard/fast pions and applied to $K \to \pi\pi$
- JB Jemos $B, D \to D, \pi, K, \eta$ vector formfactors, charmonium decays and a two-loop check
- General idea: heavy/fast dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q\to 0}\langle\pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

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$$\lim_{q\to 0}\langle \pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k,O\right]|\beta\rangle\,,$$

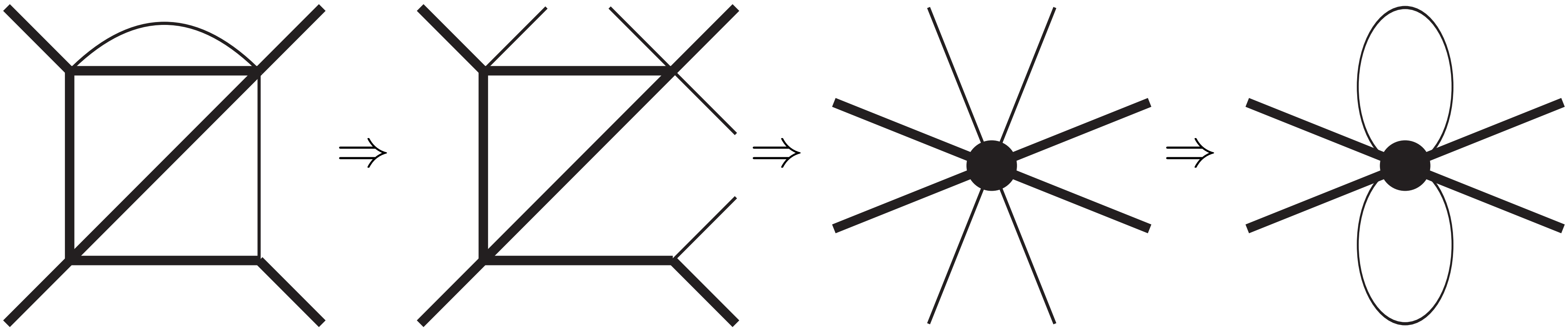
- Nothing prevents hard pions to be in the states α or β
- So by heavily using current algebra I should be able to get the light quark mass nonanalytic dependence

Hard pion ChPT?

Field Theory: a process at given external momenta

- Take a diagram with a particular internal momentum configuration
- Identify the soft lines and cut them
- The result part is analytic in the soft stuff
- So should be describably by an effective Lagrangian with coupling constants dependent on the external given momenta (Weinberg's folklore theorem)
- Envisage this effective Lagrangian as a Lagrangian in hadron fields but all possible orders of the momenta included.

Hard pion ChPT?



This procedure works at one loop level, matching at tree level, nonanalytic dependence at one loop:

- Toy models and vector meson ChPT JB, Gosdzinsky, Talavera
- Recent work on relativistic baryon ChPT Gegelia, Scherer et al.
- Extra terms kept in many of our calculations: a one-loop check
- Some two-loop checks

Hard pion ChPT?

- This effective Lagrangian as a Lagrangian in hadron fields but all possible orders of the momenta included: possibly an infinite number of terms
- If symmetries present, Lagrangian should respect them
- but my powercounting is gone

Hard pion ChPT?

- This effective Lagrangian as a Lagrangian in hadron fields but all possible orders of the momenta included: possibly an infinite number of terms
- If symmetries present, Lagrangian should respect them
- In some cases we can prove that up to a certain order in the expansion in light masses, not momenta, matrix elements of higher order operators are reducible to those of lowest order.
- Lagrangian should be complete in *neighbourhood* of original process
- Loop diagrams with this effective Lagrangian *should* reproduce the nonanalyticities in the light masses
  Crucial part of the argument

The main technical trick

- For getting soft singularities in an integral we need the meson close to on-shell
- This only happens in an area of order m^4
- So typically $\int d^4p \, 1/(p^2 - m^2) \sim m^4/m^2$ but if $\partial_\mu \phi$ on that propagator we get an extra factor of m.
- So extra derivatives are only at same order if they hit hard lines
- and then they are part of the hard part which can be expanded around

$K \to 2\pi$ in $SU(2)$ ChPT

Add $K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$ Roessl

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{F^2}{4}\left(\langle u_\mu u^\mu\rangle + \langle \chi_+\rangle\right),$$
$$\mathcal{L}^{(1)}_{\pi K} = \nabla_\mu K^\dagger \nabla^\mu K - \overline{M}^2_K K^\dagger K\,,$$
$$\mathcal{L}^{(2)}_{\pi K} = A_1\langle u_\mu u^\mu\rangle K^\dagger K + A_2\langle u^\mu u^\nu\rangle \nabla_\mu K^\dagger \nabla_\nu K + A_3 K^\dagger \chi_+ K + \cdots$$

Add a spurion for the weak interaction $\Delta I = 1/2$, $\Delta I = 3/2$

JB,Celis

$$t^{ij}_k \longrightarrow t^{i'j'}_{k'} = t^{ij}_k (g_L)_{k'}{}^{k} (g_L^\dagger)_i{}^{i'} (g_L^\dagger)_j{}^{j'}$$
$$t^i_{1/2} \longrightarrow t^{i'}_{1/2} = t^i_{1/2} (g_L^\dagger)_i{}^{i'}.$$

$K \to 2\pi$ in $SU(2)$ ChPT

The $\Delta I = 1/2$ terms: $\tau_{1/2} = t_{1/2} u^\dagger$

$$\mathcal{L}_{1/2} = iE_1\, \tau_{1/2} K + E_2\, \tau_{1/2} u^\mu \nabla_\mu K + iE_3 \langle u_\mu u^\mu \rangle \tau_{1/2} K + iE_4 \tau_{1/2} \chi_+ K + iE_5 \langle \chi_+ \rangle \tau_{1/2} K + E_6 \tau_{1/2} \chi_- K + E_7 \langle \chi_- \rangle \tau_{1/2} K + iE_8 \langle u_\mu u_\nu \rangle \tau_{1/2} \nabla^\mu \nabla^\nu K + \cdots + h.c.\,.$$

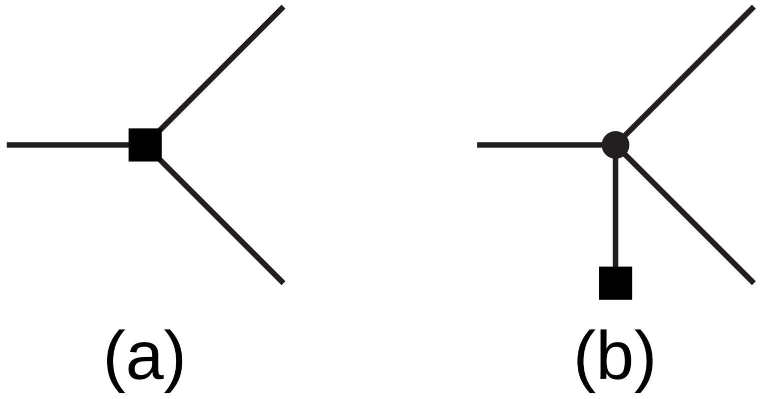
Note: higher order terms kept in both $\mathcal{L}_{1/2}$ and $\mathcal{L}^{(2)}_{\pi K}$ to check the arguments

Using partial integration,...:

$$\langle \pi(p_1)\pi(p_2)|O|K(p_K)\rangle = f(\overline{M}^2_K)\langle \pi(p_1)\pi(p_2)|\tau_{1/2} K|K(p_K)\rangle + \lambda M^2 + \mathcal{O}(M^4)$$

O any operator in $\mathcal{L}_{1/2}$ or with more derivatives. Similar for $\mathcal{L}_{3/2}$

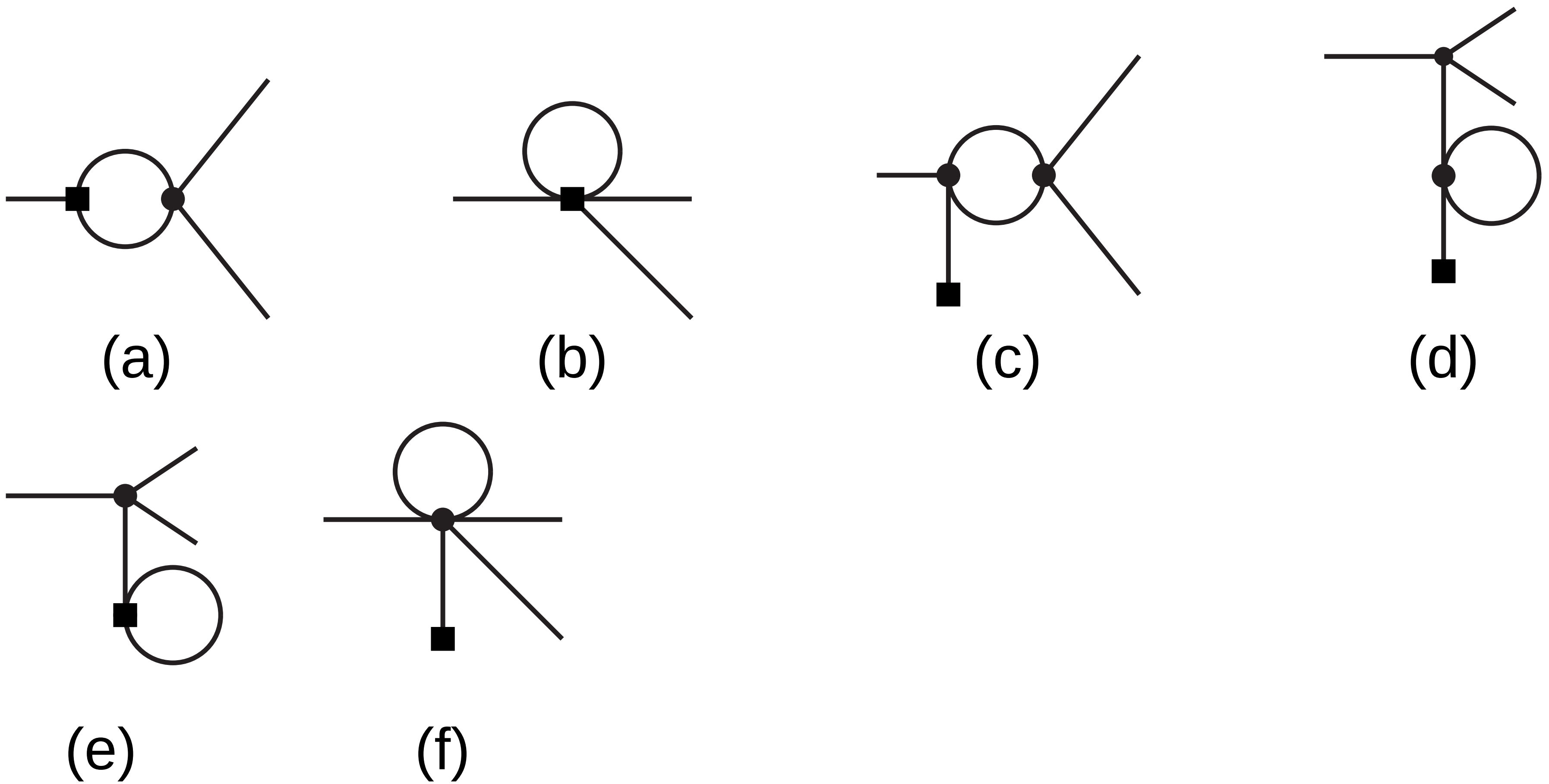
$K \to \pi\pi$: Tree level



$$A_0^{LO} = \frac{\sqrt{3}i}{2F^2}\left[-\frac{1}{2}E_1 + (E_2 - 4E_3)\,\overline{M}_K^2 + 2E_8\overline{M}_K^4 + A_1E_1\right]$$

$$A_2^{LO} = \sqrt{\frac{3}{2}}\frac{i}{F^2}\left[(-2D_1 + D_2)\,\overline{M}_K^2\right]$$

$K \to \pi\pi$: One loop



$K \to \pi\pi$: One loop

Diagram	A_0	A_2
Z	$-\frac{2F^2}{3} A_0^{LO}$	$-\frac{2F^2}{3} A_2^{LO}$
(a)	$\sqrt{3}i\left(-\frac{1}{3}E_1 + \frac{2}{3}E_2\overline{M}_K^2\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{2}{3}D_2\overline{M}_K^2\right)$
(b)	$\sqrt{3}i\left(-\frac{5}{96}E_1 - \left(\frac{7}{48}E_2 + \frac{25}{12}E_3\right)\overline{M}_K^2 + \frac{25}{24}E_8\overline{M}_K^4\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{61}{12}D_1 + \frac{77}{24}D_2\right)\overline{M}_K^2$
(e)	$\sqrt{3}i\frac{3}{16}A_1E_1$	
(f)	$\sqrt{3}i\left(\frac{1}{8}E_1 + \frac{1}{3}A_1E_1\right)$	

The coefficients of $\overline{A}(M^2)/F^4$ in the contributions to A_0 and A_2. Z denotes the part from wave-function renormalization.

- $\overline{A}(M^2) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2}$
- $K\pi$ intermediate state does not contribute, but did for Flynn-Sachrajda

$K \to \pi\pi$: One-loop

$$A_0^{NLO} = A_0^{LO}\left(1 + \frac{3}{8F^2}\overline{A}(M^2)\right) + \lambda_0 M^2 + \mathcal{O}(M^4)\,,$$

$$A_2^{NLO} = A_2^{LO}\left(1 + \frac{15}{8F^2}\overline{A}(M^2)\right) + \lambda_2 M^2 + \mathcal{O}(M^4)\,.$$

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$$A_2^{NLO} = A_2^{LO}\left(1+\frac{15}{8F^2}\overline{A}(M^2)\right)+\lambda_2 M^2+\mathcal{O}(M^4)\,.$$

Match with three flavour $SU(3)$ calculation Kambor, Missimer, Wyler; JB, Pallante, Prades

$$A_0^{(3)LO} = -\frac{i\sqrt{6}CF_0^4}{\overline{F}_K F^2}\left(G_8+\frac{1}{9}G_{27}\right)\overline{M}_K^2\,,\qquad A_2^{(3)LO} = -\frac{i10\sqrt{3}CF_0^4}{9\overline{F}_K F^2}G_{27}\overline{M}_K^2\,,$$

When using $F_\pi = F\left(1+\frac{1}{F^2}\overline{A}(M^2)+\frac{M^2}{F^2}l_4^r\right)\,,\ F_K = \overline{F}_K\left(1+\frac{3}{8F^2}\overline{A}(M^2)+\cdots\right)\,,$

logarithms at one-loop agree with above

Hard Pion ChPT: A two-loop check

- Similar arguments to JB-Celis, Flynn-Sachrajda work for the pion vector and scalar formfactor JB-Jemos
- Therefore at any t the chiral log correction must go like the one-loop calculation.
- But note the one-loop log chiral log is with $t >> m_\pi^2$
- Predicts

$$F_V(t, M^2) = F_V(t, 0)\left(1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

$$F_S(t, M^2) = F_S(t, 0)\left(1 - \frac{5}{2}\frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

- Note that $F_{V,S}(t, 0)$ is now a coupling constant and can be complex

Hard Pion ChPT: A two-loop check

- Take the full two-loop ChPT calculation JB,Colangelo,Talavera, valid for $t, m_\pi^2 \ll \Lambda_\chi^2$
- Expand this for $t \gg m_\pi^2$
- $t^2 \ln t, \ldots\ldots$ terms go in $F_{S,V}(t,0)$
- But the one-loop for $F_V(t,0)$ is known and HPChPT predicts how the chiral log $m^2 \log m^2$ adds to this

$$F_V = 1 + x_2 \left[\frac{1}{6}(s-4)\bar{J}(s) + s\left(-l_6^r - \frac{1}{6}L - \frac{1}{18N}\right)\right] + x_2^2\left(P_V^{(2)} + U_V^{(2)}\right) + \mathcal{O}(x_2^3)\,.$$

$$U_V^{(2)} = \bar{J}(s)\left[\frac{1}{3}l_1^r(-s^2+4s) + \frac{1}{6}l_2^r(s^2-4s) + \frac{1}{3}l_4^r(s-4) + \frac{1}{6}l_6^r(-s^2+4s) - \frac{1}{36}L(s^2+8s-48)\right.$$
$$\left. + \frac{1}{N}\left(\frac{7}{108}s^2 - \frac{97}{108}s + \frac{3}{4}\right)\right] + \frac{1}{9}K_1(s) + \frac{1}{9}K_2(s)\left(\frac{1}{8}s^2 - s + 4\right) + \frac{1}{6}K_3(s)\left(s - \frac{1}{3}\right) - \frac{5}{3}K_4(s)\,.$$

A two-loop check

Full two-loop ChPT JB,Colangelo,Talavera, expand in $t >> m_\pi^2$:

$$F_V(t,M^2) = F_V(t,0)\left(1 - \frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$
$$F_S(t,M^2) = F_S(t,0)\left(1 - \frac{5}{2}\frac{M^2}{16\pi^2F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

with

$$F_V(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(\frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6}\ln\frac{t}{\mu^2}\right)$$
$$F_S(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(1 + 16\pi^2 l_4^r + i\pi - \ln\frac{t}{\mu^2}\right)$$

- The needed coupling constants are complex
- Both calculations have two-loop diagrams with overlapping divergences
- The chiral logs should be valid for any t where a pointlike interaction is a valid approximation

Electromagnetic formfactors

$$
\begin{aligned}
F_V^{\pi}(s) &= F_V^{\pi\chi}(s)\left(1+\frac{1}{F^2}\overline{A}(m_\pi^2)+\frac{1}{2F^2}\overline{A}(m_K^2)+\mathcal{O}(m_L^2)\right)\,,\\
F_V^{K}(s) &= F_V^{K\chi}(s)\left(1+\frac{1}{2F^2}\overline{A}(m_\pi^2)+\frac{1}{F^2}\overline{A}(m_K^2)+\mathcal{O}(m_L^2)\right)\,.
\end{aligned}
$$

$B, D \to \pi, K, \eta$

$$\left\langle P_f(p_f) \left| \overline{q}_i \gamma_\mu q_f \right| P_i(p_i) \right\rangle = (p_i + p_f)_\mu f_+(q^2) + (p_i - p_f)_\mu f_-(q^2)$$

$$f_{+B\to M}(t) = f^{\chi}_{+B\to M}(t) F_{B\to M}$$
$$f_{-B\to M}(t) = f^{\chi}_{-B\to M}(t) F_{B\to M}$$

- $F_{B\to M}$ is always the same for f_+, f_- and f_0
- This is not heavy quark symmetry: not valid at endpoint and valid also for $K \to \pi$.
- Not like Low's theorem, not only dependence on external legs
- Done in heavy meson and relativistic formalism

$B, D \to \pi, K, \eta$

$$F_{K\to\pi} = 1 + \frac{3}{8F^2}\overline{A}(m_\pi^2) \qquad (2 - \text{flavour})$$

$$F_{B\to\pi} = 1 + \left(\frac{3}{8} + \frac{9}{8}g^2\right)\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4} + \frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24} + \frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},$$

$$F_{B\to K} = 1 + \frac{9}{8}g^2\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{2} + \frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{6} + \frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},$$

$$F_{B\to\eta} = 1 + \left(\frac{3}{8} + \frac{9}{8}g^2\right)\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4} + \frac{3}{4}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24} + \frac{1}{8}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},$$

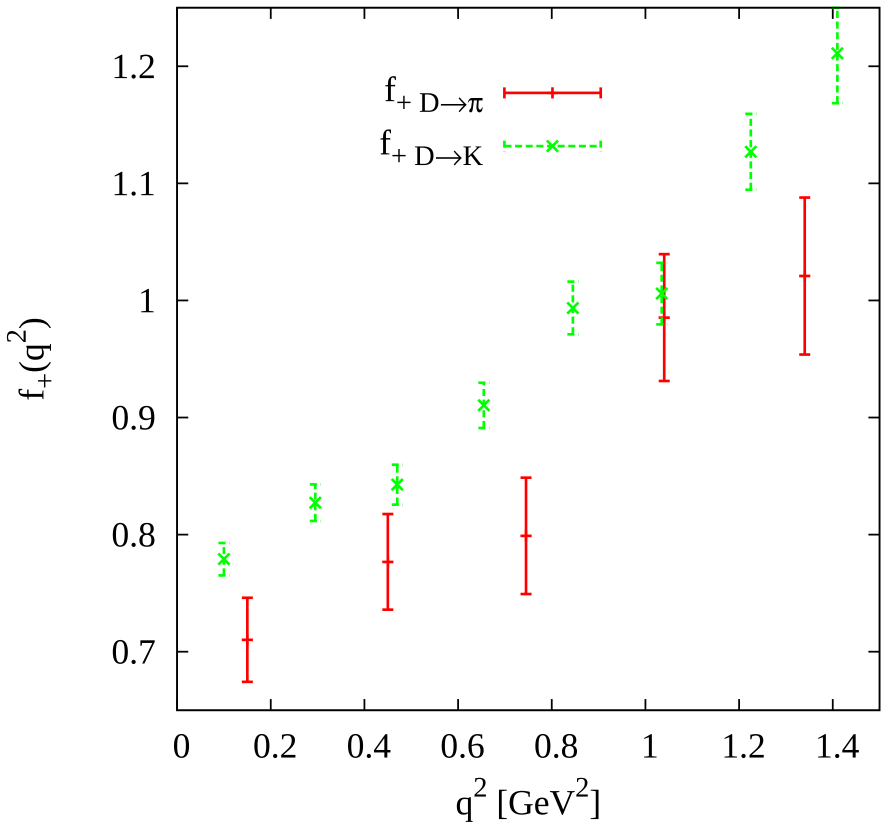
$$F_{B_s\to K} = 1 + \frac{3}{8}\frac{\overline{A}(m_\pi^2)}{F^2} + \left(\frac{1}{4} + \frac{3}{2}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{24} + \frac{1}{2}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2},$$

$$F_{B_s\to\eta} = 1 + \left(\frac{1}{2} + \frac{3}{2}g^2\right)\frac{\overline{A}(m_K^2)}{F^2} + \left(\frac{1}{6} + \frac{1}{2}g^2\right)\frac{\overline{A}(m_\eta^2)}{F^2}.$$

$F_{B_s\to\pi}$ vanishes due to the possible flavour quantum numbers.

Experimental check

CLEO data on $f_+(q^2)|V_{cq}|$ for $D \to \pi$ and $D \to K$ with $|V_{cd}| = 0.2253$, $|V_{cs}| = 0.9743$



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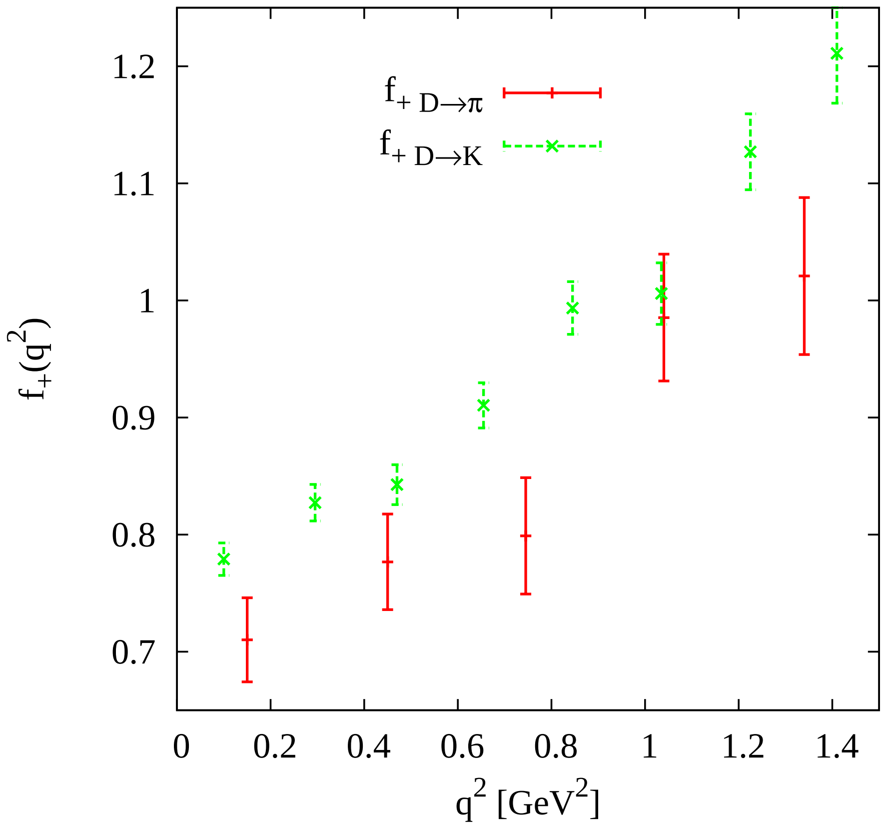

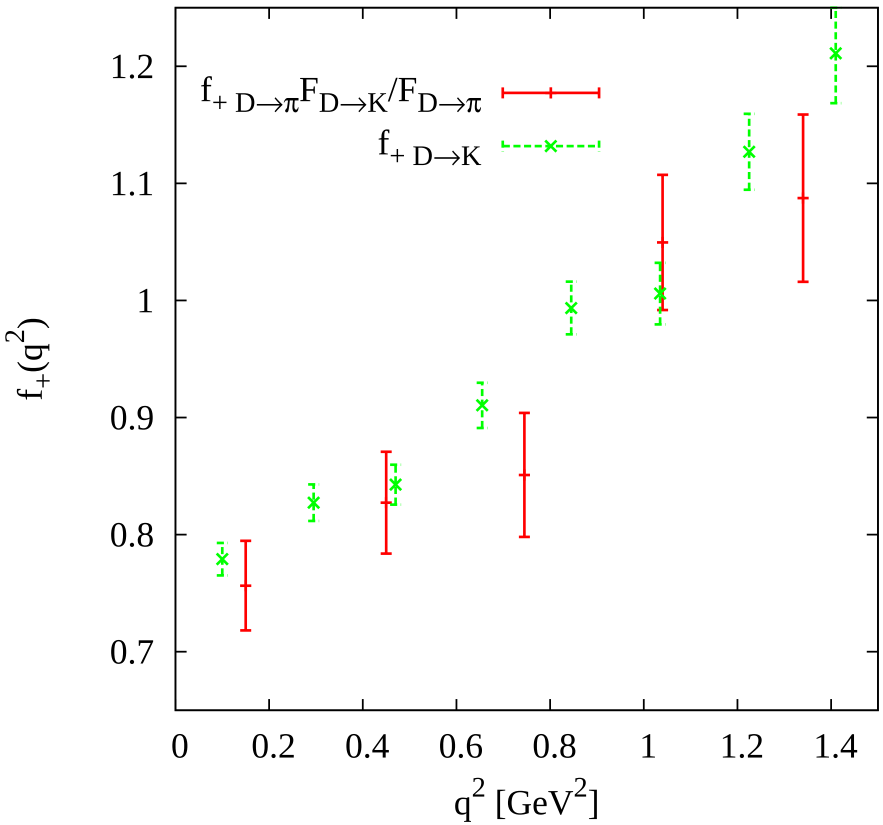

$$f_{+D\to\pi} = f_{+D\to K} F_{D\to\pi}/F_{D\to K}$$

Applications to charmonium

- We look at decays $\chi_{c0}, \chi_{c2} \to \pi\pi, KK, \eta\eta$
- $J/\psi, \psi(nS), \chi_{c1}$ decays to the same final state break isospin or U-spin or V-spin, they thus proceed via electromagnetism or quark mass differences: more difficult.
- So construct a Lagrangian with a chiral singlet scalar and tensor field.
- $\mathcal{L}_{\chi_c} = E_1 F_0^2\, \chi_0 \left\langle u^\mu u_\mu \right\rangle + E_2 F_0^2\, \chi_2^{\mu\nu} \left\langle u_\mu u_\nu \right\rangle .$

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- No chiral logarithm corrections
- Expanding the energy-momentum tensor result Donoghue-Leutwyler at large q^2 agrees.
- These decays should have small $SU(3)_V$ breaking

Charmonium

- Phase space correction: $|\vec{p}_1| = \sqrt{m_\chi^2 - 4m_P^2}/2$.
- χ_{c0}:
  - $A \propto p_1 \cdot p_2 = (m_\chi^2 - 2m_P^2)/2$.
  - $\Longrightarrow$ $\boxed{G_0 = \sqrt{BR/|\vec{p}_1|}/(p_1.p_2)}$.
- χ_{c2}:
  - $A \propto T_\chi^{\mu\nu} p_{1\mu} p_{2\nu}$. (polarization tensor)
  - $|A|^2 \propto \frac{1}{5}\sum_{pol} T_\chi^{\mu\nu} p_{1\mu} p_{2\nu} T_\chi^{*\alpha\beta} p_{1\alpha} p_{2\beta} = \frac{1}{30}\left(m_\chi^2 - 4m_P^2\right)^2 \propto |\vec{p}_1|^4$.
  - $\Longrightarrow$ $\boxed{G_2 = \sqrt{BR/|\vec{p}_1|}/|\vec{p}_1|^2}$.
- $\times 2$ for $K_S^0 K_S^0$ to $K^0\overline{K^0}$, $\times 2/3$ for $\pi\pi$ to $\pi^+\pi^-$.

Charmonium

	χ_{c0}		χ_{c2}	
Mass	3414.75 ± 0.31 MeV		3556.20 ± 0.09 MeV	
Width	10.4 ± 0.6 MeV		1.97 ± 0.11 MeV	
Final state	10^3 BR	$10^{10}\,G_0[\mathrm{MeV}^{-5/2}]$	10^3 BR	$10^{10}G_2[\mathrm{MeV}^{-5/2}]$
$\pi\pi$	8.5 ± 0.4	3.15 ± 0.07	2.42 ± 0.13	3.04 ± 0.08
K^+K^-	6.06 ± 0.35	3.45 ± 0.10	1.09 ± 0.08	2.74 ± 0.10
$K^0_S K^0_S$	3.15 ± 0.18	3.52 ± 0.10	0.58 ± 0.05	2.83 ± 0.12
$\eta\eta$	3.03 ± 0.21	2.48 ± 0.09	0.59 ± 0.05	2.06 ± 0.09
$\eta'\eta'$	2.02 ± 0.22	2.43 ± 0.13	< 0.11	< 1.2

Experimental results for $\chi_{c0}, \chi_{c2} \to PP$ and the factors corrected for the known m^2 effects.

- $\pi\pi$ and KK are good to 10% (Note: 20% for F_K/F_π)
- $\eta\eta$ OK

Caveat utilitor: let the user beware

- This is not a simple straightforward process
- Especially the proof that it all reduces to a single type of lowest order term can be tricky.
- Some examples where it does not work easily:
  - VV two-point function has two types of lowest order terms: $\langle LR\rangle$ and $\langle LL\rangle + \langle RR\rangle$ (no derivative structure indicated)
  - Scalar form factors in three flavour ChPT, again two types of lowest order terms $\langle\chi_+\rangle$ and $\langle\chi_+\rangle\langle u_\mu u^\mu\rangle$

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  - Scalar form factors in three flavour ChPT, again two types of lowest order terms $\langle \chi_+ \rangle$ and $\langle \chi_+ \rangle \langle u_\mu u^\mu \rangle$
    - In $SU(2)$ these two types are the same hence our check still worked for the scalar form-factor
    - For the vector formfactor the second type vanishes or gives for the $SU(2)$ case no contribution because of G-parity.

Summary

Why is this useful:

- Lattice works actually around the strange quark mass
- need only extrapolate in m_u and m_d.
- Applicable in momentum regimes where usual ChPT might not work
- Three flavour case useful for B, D, χ_c decays

Summary

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Can we use it for η, η'?

- Some examples of η in final state included
- $\eta \to 3\pi$: soft pions so nothing new
- $\eta' \to 3\pi$: In principle yes but nothing to compare to, e.g. $\eta' \to KK\pi$ is not an observable