

ISOSPIN BREAKING AT ORDER p^6 in $K_{\ell 3}$ DECAYS

Johan Bijnens

Lund University

bijnens@thep.lu.se

<http://www.thep.lu.se/~bijnens>

Various ChPT: <http://www.thep.lu.se/~bijnens/chpt.html>

Overview

- Why $K_{\ell 3}$ (and similar things)?
- List of papers or earlier results
- Formfactor relations
- Some aspects of the calculation
- Numerical results
 - $f_+(0)$
 - $f_+(t), r_{0-}(t)$
 - $f_0(t), r_{0-}^0(t)$
 - Δ_{CT}

$K_{\ell 3}$

Why look at $K_{\ell 3}$?

$$\Gamma (K^i \rightarrow \pi^j \ell^+ \nu_\ell) = C_{ij}^2 \frac{G_F^2 S_{EW} m_K^5}{192\pi^3} \left| V_{us} f_+^{K^i \pi^j}(0) \right|^2 \mathcal{I}_\ell^{ij} \left(1 + 2\Delta_{EM}^{ij} \right) .$$

\mathcal{I}_ℓ^{ij} : depends on measured formfactors
In principle well known

Similar formulas exist for $K \rightarrow \pi \nu \bar{\nu}$

$K_{\ell 3}$: Definitions

$$\langle \pi^0(p') | \bar{s} \gamma_\mu u(0) | K^+(p) \rangle = \frac{1}{\sqrt{2}} \left[(p' + p)_\mu f_+^{K^+ \pi^0}(t) + (p - p')_\mu f_-^{K^+ \pi^0}(t) \right]$$

$$\langle \pi^-(p') | \bar{s} \gamma_\mu u(0) | K^0(p) \rangle = \left[(p' + p)_\mu f_+^{K^0 \pi^-}(t) + (p - p')_\mu f_-^{K^0 \pi^-}(t) \right]$$

$$\langle \pi^+(p') | \bar{s} \gamma_\mu d(0) | K^+(p) \rangle = \left[(p' + p)_\mu f_+^{K^+ \pi^+}(t) + (p - p')_\mu f_-^{K^+ \pi^+}(t) \right]$$

$$\langle \pi^0(p') | \bar{s} \gamma_\mu d(0) | K^0(p) \rangle = \frac{-1}{\sqrt{2}} \left[(p' + p)_\mu f_+^{K^0 \pi^0}(t) + (p - p')_\mu f_-^{K^0 \pi^0}(t) \right]$$

8 form-factors, $t = (p' - p)^2$

$f_+^{K^i \pi^j}(0) = 1$ in the $SU(3)$ limit $m_u = m_d = m_s$

Isospin limit: $f_\pm = f_\pm^{K^\pm} = f_\pm^{K^+ \pi^0} = f_\pm^{K^0 \pi^-} = f_\pm^{K^+ \pi^+} = f_\pm^{K^0 \pi^0}$

$K_{\ell 3}$ Definitions

Scalar formfactor:
$$f_0(t) = f_+(t) + \frac{t}{m_{K^i}^2 - m_{\pi^j}^2} f_-(t)$$

$$f_l^{K^i \pi^j} = LO + NLO + NNLO + \dots$$

$\delta = m_u - m_d$, e^2 and p^2 expansion

$$f_{l=+,0} = 1 + \delta + \delta p^2 + e^2 p^2 + \delta p^4 + \dots$$

$K_{\ell 3}$ Definitions

Scalar formfactor:
$$f_0(t) = f_+(t) + \frac{t}{m_{K^i}^2 - m_{\pi^j}^2} f_-(t)$$

$$f_l^{K^i \pi^j} = LO + NLO + NNLO + \dots$$

$\delta = m_u - m_d$, e^2 and p^2 expansion

$$f_{l=+,0} = 1 + \delta + \delta p^2 + e^2 p^2 + \delta p^4 + \dots$$

LO is also tree diagrams from p^2 -Lagrangian: p^2

NLO is also tree diagrams from p^4 -Lagrangian: p^4

NNLO is also tree diagrams from p^6 -Lagrangian: p^6

$K_{\ell 3}$: (subset of) papers

- H. Leutwyler and M. Roos,
Z.Phys.C25:91,1984. $K_{\ell 3} f_+(0) \delta p^2$
- J. Gasser and H. Leutwyler,
Nucl.Phys.B250:517-538,1985. $K_{\ell 3} f_+(t) \delta p^2, f_0(t) p^2$
- V. Cirigliano et al., hep-ph/0110153,
Eur.Phys.J.C23:121-133,2002 $K_{\ell 3} f_+(t), f_0(t) \delta p^2, e^2 p^2$
- P. Post and K.Schilcher, hep-ph/0112352
Eur.Phys.J.C25,427 2002 $K_{\ell 3} f_+(t), f_0(t) p^4$ numerics??
- J. Bijnens and P. Talavera, hep-ph/0303103,
Nucl. Phys. B669 (2003) 341-362 $K_{\ell 3} f_+(t), f_0(t) p^4$
- V. Cirigliano et al., hep-ph/0503108,
JHEP 0504 (2005) 006 $C_i p^4$

$K_{\ell 3}$: (subset of) papers

- M. Jamin, J. Oller and A. Pich, JHEP 0402 (2004) 047 $C_i p^4$, dispersive
- F. Mescia and C. Smith, arXiv:0705.2025 $f_+(t) \delta p^2, e^2 p^2$
- J. Bijnens and K. Ghorbani, arXiv:0711.0148 $f_{+,0}(t) \delta p^4$
- V. Bernard, M. Oertel, E. Passemar and J. Stern, hep-ph/0603202, arXiv:0707.4194[hep-ph]
dispersive, CT
- A. Kastner and H. Neufeld, arXiv:0805.2222[hep-ph]
putting a lot together, see previous talk

Formfactor relation

MS (and us) noticed:

$$f_{\ell}^{K^+\pi^0}(t) - f_{\ell}^{K^0\pi^-}(t) - f_{\ell}^{K^+\pi^+}(t) + f_{\ell}^{K^0\pi^0}(t) = 0 + \mathcal{O}(\delta^2),$$

$$r(t) \equiv \frac{f_{\ell}^{K^+\pi^0}(t) f_{\ell}^{K^0\pi^0}(t)}{f_{\ell}^{K^0\pi^-}(t) f_{\ell}^{K^+\pi^+}(t)} = 1 + \mathcal{O}(\delta^2)$$

GL noticed:

$$r_{0-}(t) = \frac{f_+^{K^+\pi^0}(t)}{f_+^{K^0\pi^-}(t)} = \text{constant}$$

Formfactor relations

$r(t) = 1 + \delta^2$ is **always** true, also for $r^-(t)$ and $r^0(t)$

Vector operator is $I=1/2$

$(m_u - m_d)(\bar{u}u - \bar{d}d)$ is $I=1$

$$f_\ell^{K^+\pi^0}(t) = f_\ell^A(t) + \delta f_\ell^B(t) + \mathcal{O}(\delta^2),$$

$$f_\ell^{K^0\pi^-}(t) = f_\ell^A(t) - \delta f_\ell^D(t) + \mathcal{O}(\delta^2),$$

$$f_\ell^{K^+\pi^+}(t) = f_\ell^A(t) + \delta f_\ell^D(t) + \mathcal{O}(\delta^2),$$

$$f_\ell^{K^0\pi^0}(t) = f_\ell^A(t) - \delta f_\ell^B(t) + \mathcal{O}(\delta^2),$$

Photon exchange has $I = 0, 1, 2$, $I = 2$ part breaks relation

Relation also true for scalar currents $\bar{s}u, \bar{s}d$

Formfactor relations

$r_{0-}(t)$ constant not true at δp^4 (MS: C_i , BG also the rest)

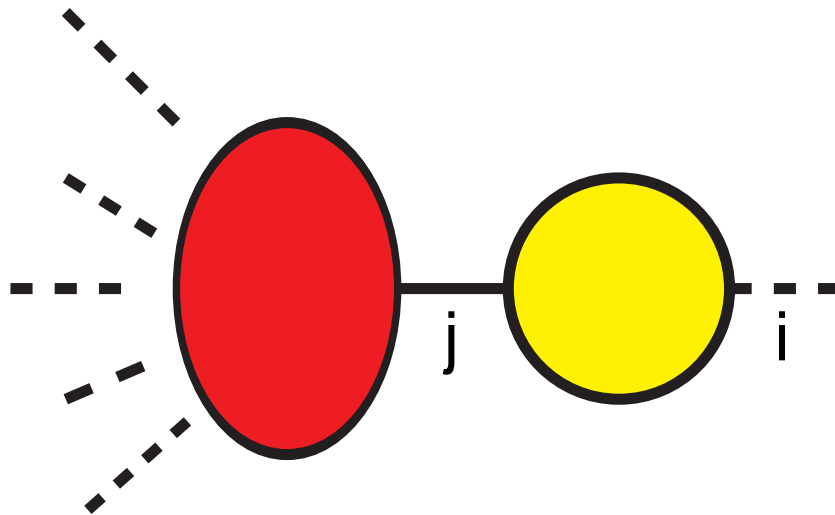
$r_{0-}^0(t), r_{0-}^-(t)$ not true at δp^2

New for isospin breaking

Take LSZ into account properly

Amoros, JB, Talavera, 2001

Matrix element:



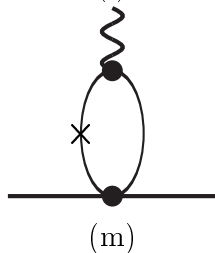
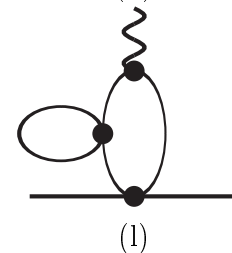
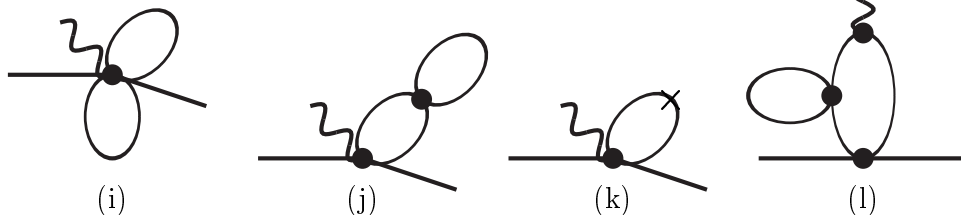
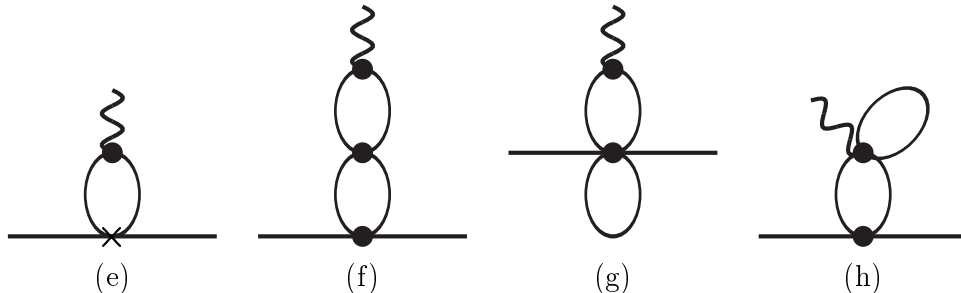
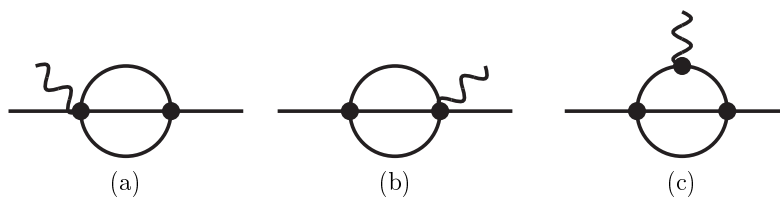
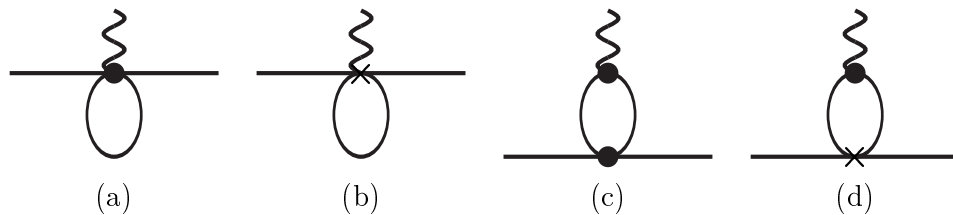
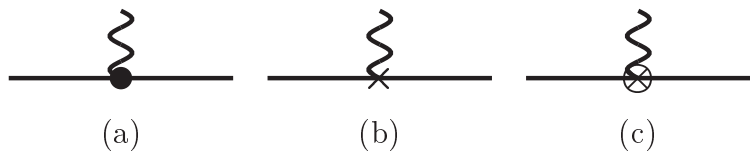
$$\mathcal{A}_{i_1 \dots i_n} = \left(\frac{(-i)^n}{\sqrt{Z_{i_1} \dots Z_{i_n}}} \right) \prod_{i=1}^n \lim_{k_i^2 \rightarrow m_i^2} (k_i^2 - m_i^2) G_{i_1 \dots i_n}(k_1, \dots, k_n)$$

Isospin Breaking

Deal with the complications of mixing via matrices/determinant

$$\begin{aligned}
 \mathcal{A}_3 = & \mathcal{G}_3^{(2)} + \left\{ \mathcal{G}_3^{(4)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right\} \\
 & + \left[\mathcal{G}_3^{(6)} - \frac{1}{2} Z_{33}^{(6)} \mathcal{G}_3^{(2)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(4)} + \frac{3}{8} \left(Z_{33}^{(4)} \right)^2 \mathcal{G}_3^{(2)} \right. \\
 & + \frac{Z_{38}^{(4)} \Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_3^{(2)} - \frac{1}{2} \left(\frac{\Pi_{38}^{(4)}}{\Delta m^2} \right)^2 \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(4)} \\
 & \left. - \frac{\Pi_{38}^{(6)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{\Pi_{38}^{(4)} \Pi_{88}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{1}{2} Z_{33}^{(4)} \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right]
 \end{aligned}$$

Diagrams



● : p^2 vertex
 × : p^4 vertex
 ⊗ : p^6 vertex

$f_+(0)$: NO isospin violation

	$f_+^{K^+\pi^0}$	$f_+^{K^0\pi^-}$	$f_+^{K^+\pi^+}$	$f_+^{K^0\pi^0}$
order p^2	1.00000	1.00000	1.00000	1.00000
order p^4	-0.02276	-0.02266	-0.02226	-0.02316
order p^6	0.01423	0.01462	0.01406	0.01480
p^6 2-loop	0.01104	0.01130	0.01090	0.01145
$p^6 L_i^r$	0.00320	0.00332	0.00316	0.00336
sum	0.99156	0.99196	0.99180	0.99164

JB-Talavera, JB-Ghorbani

The C_i^r contribution: nothing new here and **not** included

$$f_+(0): m_u/m_d = 0.585(NLO)$$

m_u/m_d NLO, Dashen's theorem

	$f_+^{K^+\pi^0}$	$f_+^{K^0\pi^-}$	$f_+^{K^+\pi^+}$	$f_+^{K^0\pi^0}$
order p^2	1.01702	1.00000	1.00000	0.98288
order p^4	-0.01931	-0.02282	-0.02202	-0.02675
order p^6	0.00986	0.01467	0.01395	0.01919
p^6 2-loop	0.00435	0.01142	0.01084	0.01815
p^6 L_i^r	0.00551	0.00325	0.00311	0.00104
sum	1.00757	0.99186	0.99193	0.97532

$\sin \epsilon = 0.00986$ JB-Ghorbani

The C_i^r contribution: **not** included

$$f_+(0): m_u/m_d = 0.45(NNLO)$$

NNLO and violations of Dashen's theorem

	$f_+^{K^+\pi^0}$	$f_+^{K^0\pi^-}$	$f_+^{K^+\pi^+}$	$f_+^{K^0\pi^0}$
order p^2	1.02465	1.00000	1.00000	0.97514
order p^4	-0.01775	-0.02292	-0.02197	-0.02838
order p^6	0.00809	0.01470	0.01391	0.02095
p^6 2-loop	0.00159	0.01145	0.01081	0.02092
$p^6 L_i^r$	0.00650	0.00325	0.00309	0.00004
sum	1.01499	0.99177	0.99194	0.96772

$\sin \epsilon = 0.0143$ JB-Ghorbani

The C_i^r contribution: **not** included

$$C_i^r \text{ from } \eta': f_+^{K^+\pi^0}(0) \Big|_{P_1} = 0.00065 \quad f_+^{K^0\pi^0}(0) \Big|_{P_1} = -0.00065$$

$$f_+(0)$$

$$r_{0-}(0) = 1.02465 + 0.00587 - 0.00711 = 1.02341,$$

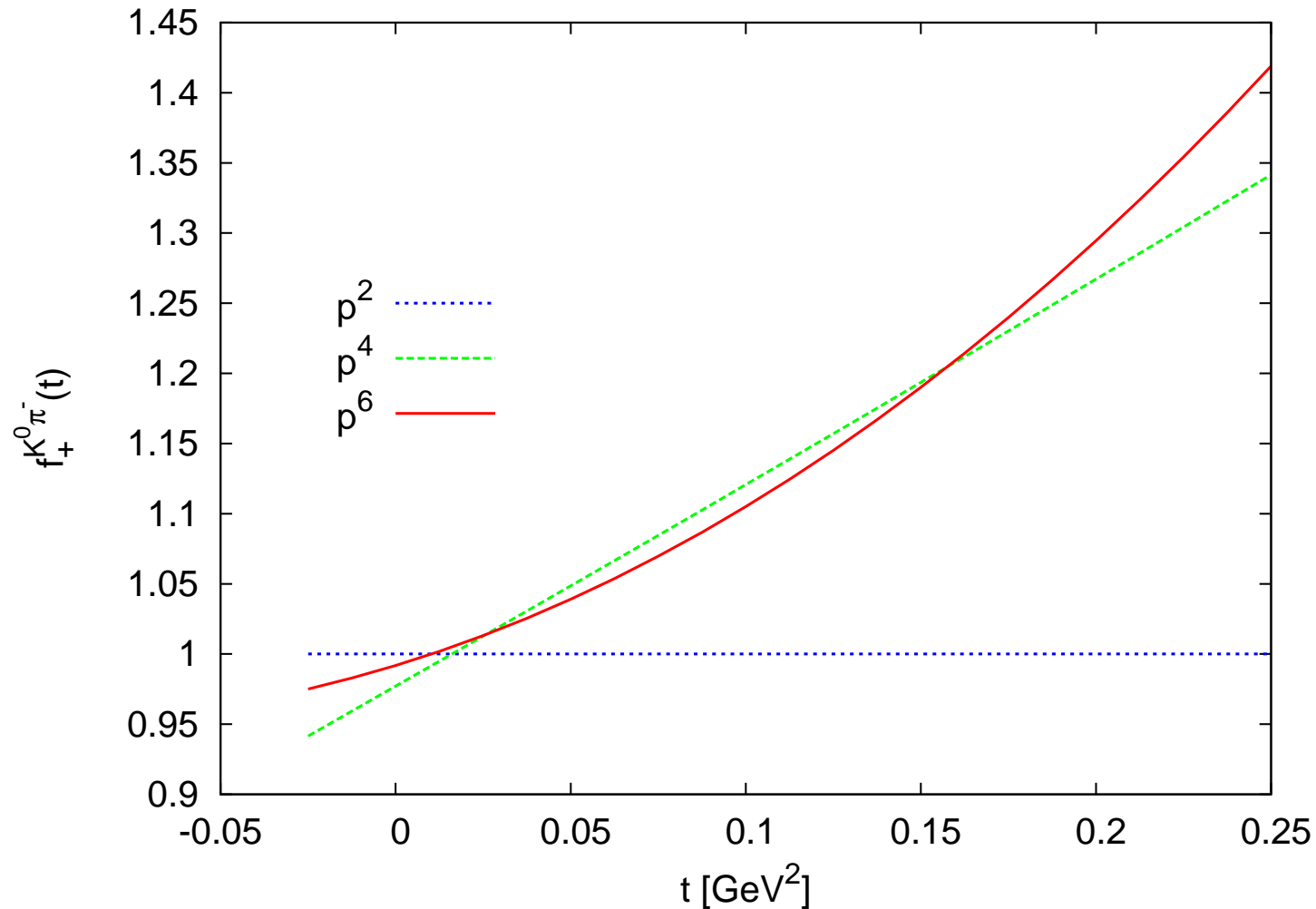
Palutan: KAON07

$$r_{0-exp} = 1 + \Delta_{SU(2)} = 1.0284 \pm 0.0040.$$

As we see, we obtain a reasonable agreement.

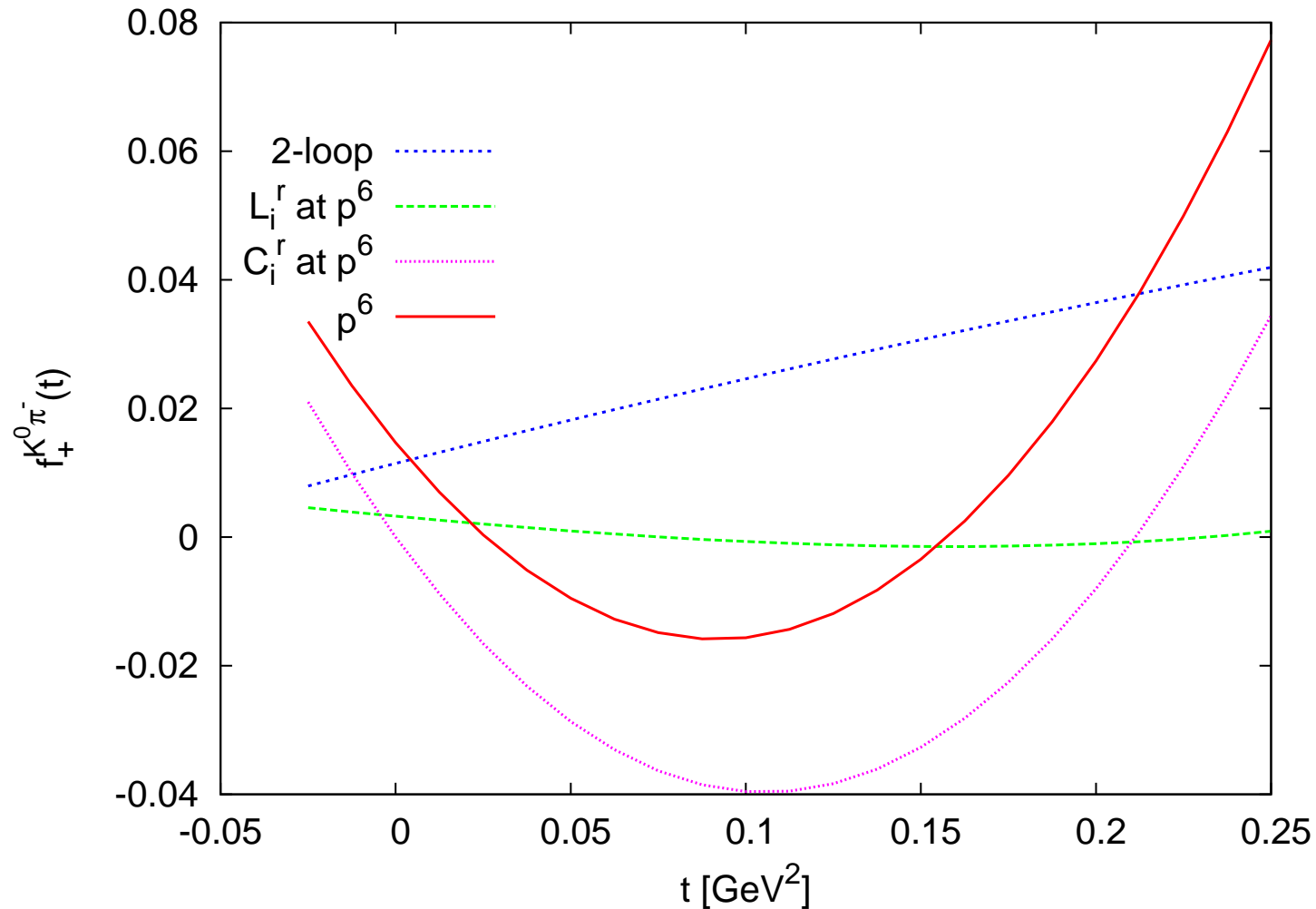
t -dependence

For $K^0 \rightarrow \pi^- \ell^+ \nu$ numerically very little change,

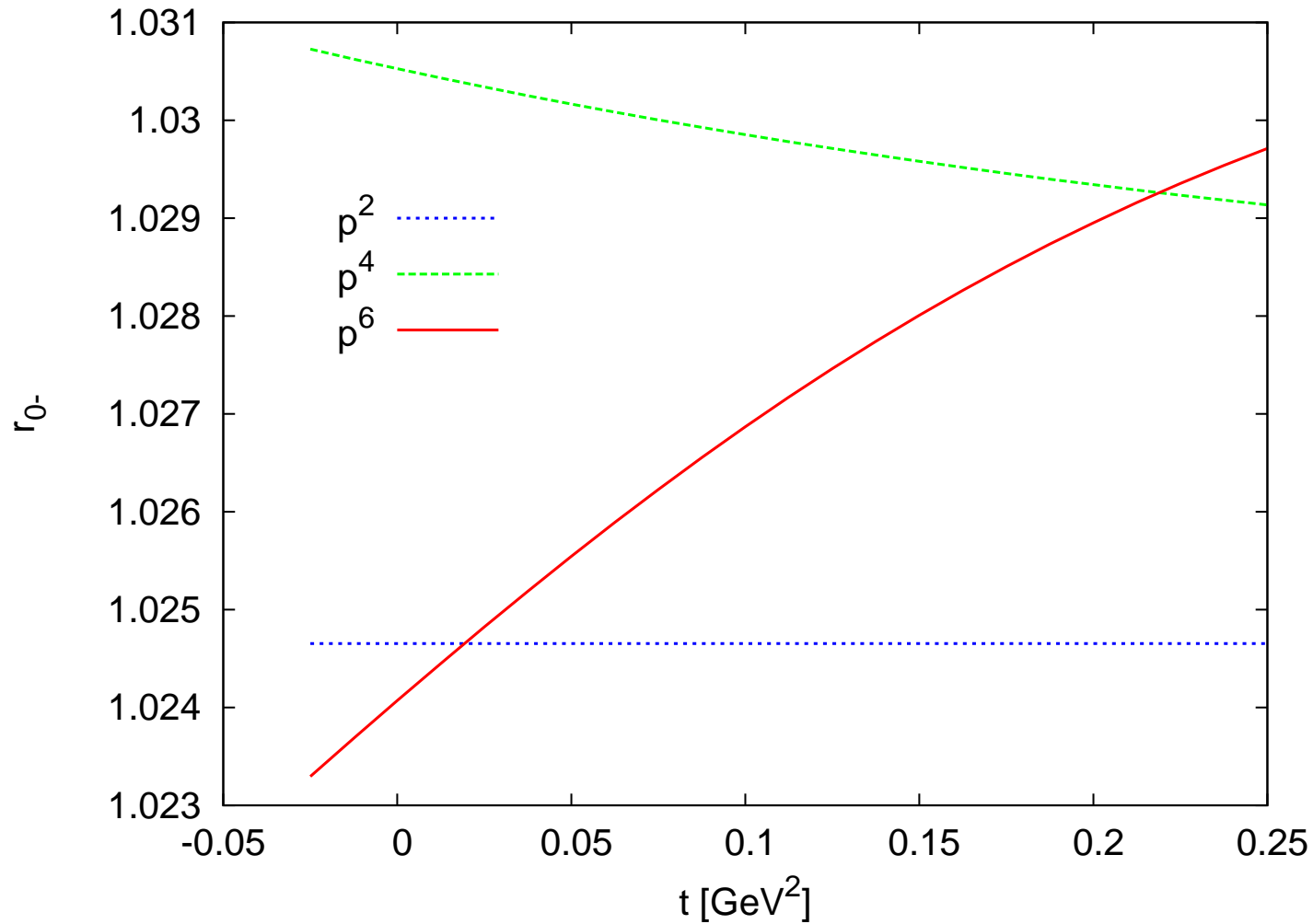


t -dependence: p^6

For $K^0 \rightarrow \pi^- \ell^+ \nu$ numerically very little change,

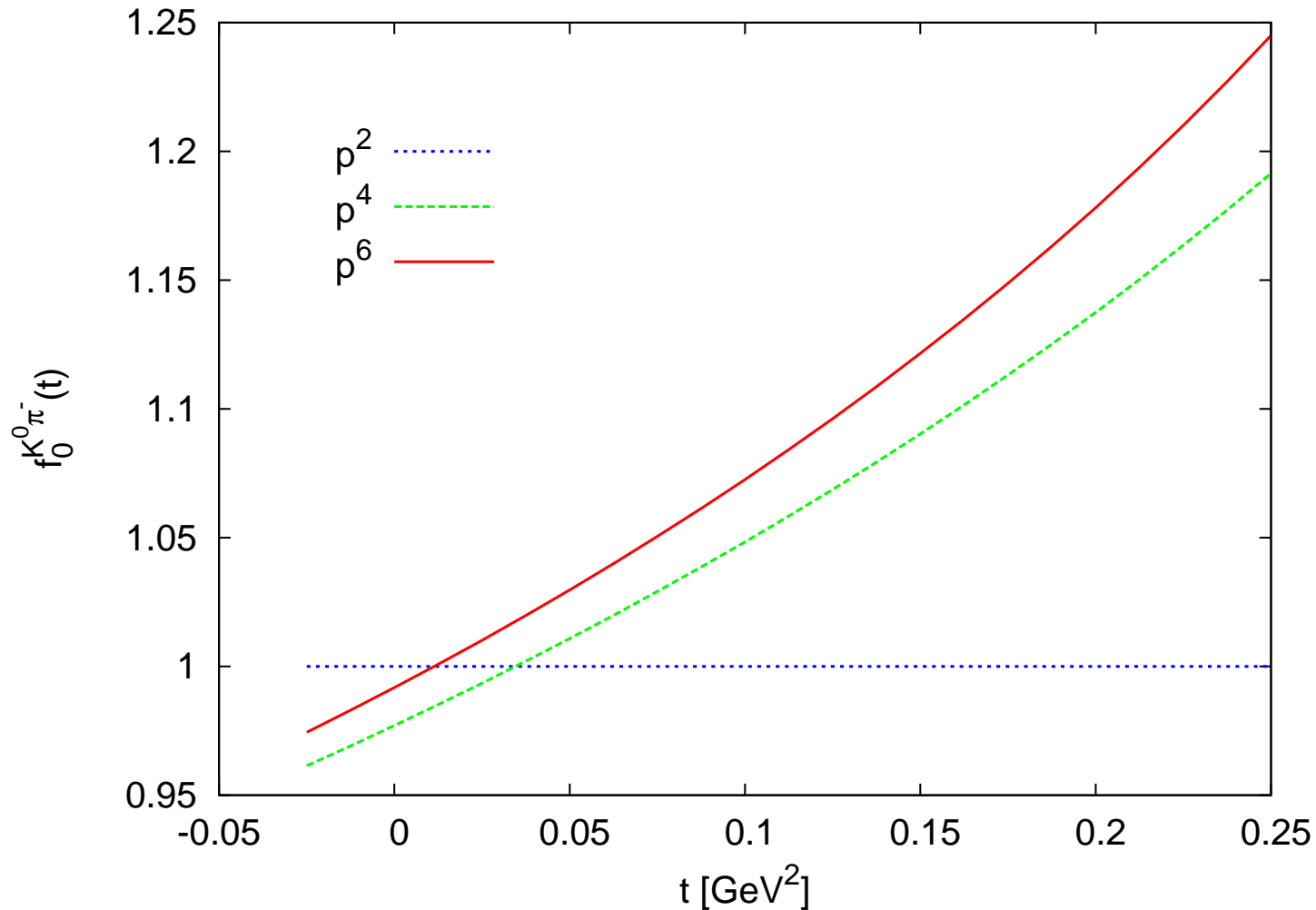


t -dependence: $r_{0-}(t)$

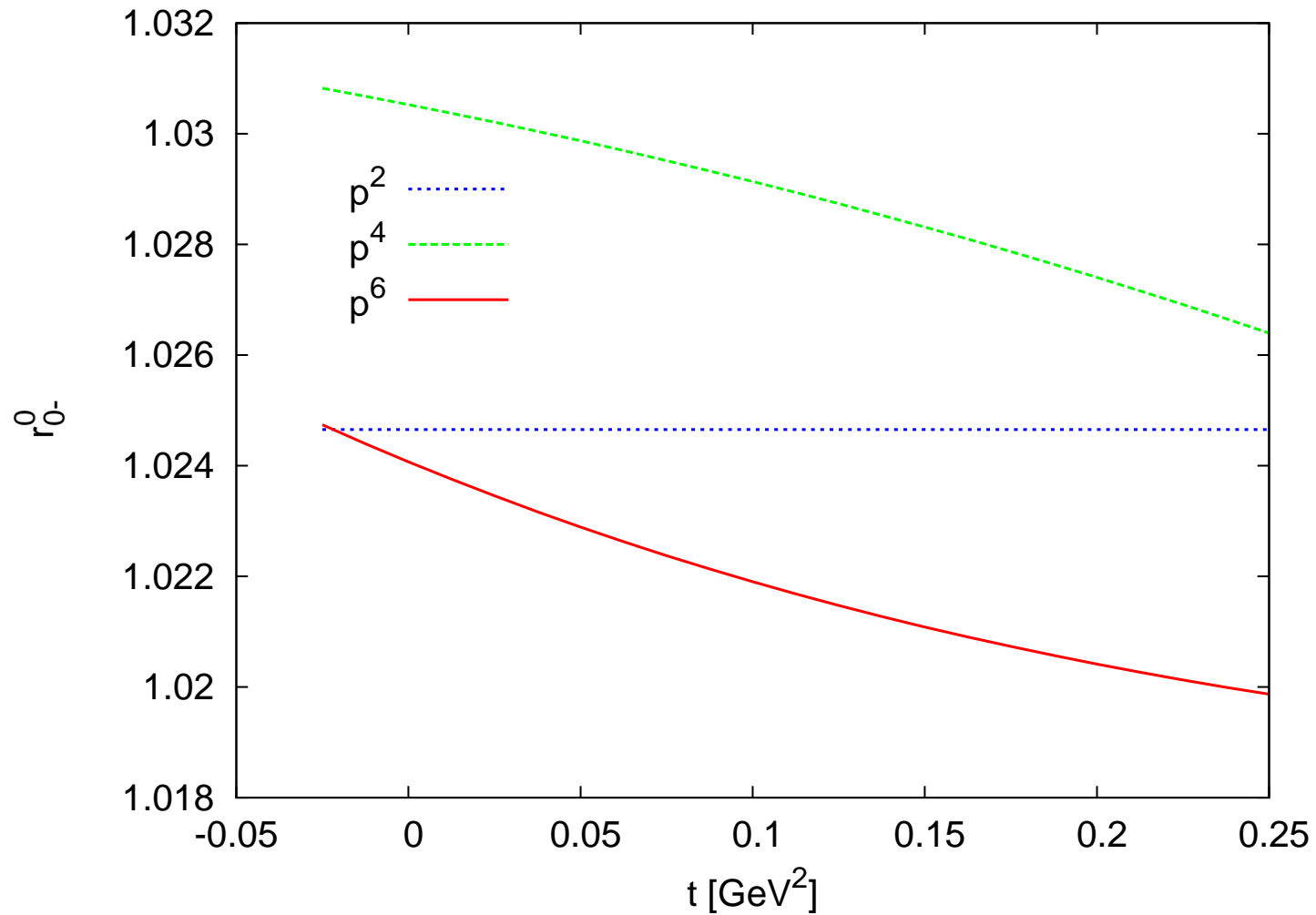


t -dependence: f_0

For $K^0 \rightarrow \pi^- \ell^+ \nu$ numerically very little change,



t -dependence: $r_{0-}(t)$



$f_0(t)$ Isospin limit

Old Main Result: JB-Talavera

$$f_0(t) = 1 - \frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 \\ + 8 \frac{t}{F_\pi^4} (2C_{12}^r + C_{34}^r) (m_K^2 + m_\pi^2) + \frac{t}{m_K^2 - m_\pi^2} (F_K/F_\pi - 1) \\ - \frac{8}{F_\pi^4} t^2 C_{12}^r + \bar{\Delta}(t) + \Delta(0).$$

$\bar{\Delta}(t)$ and $\Delta(0)$ contain **NO** C_i^r and only depend on the L_i^r at order $p^6 \implies$

All needed parameters can be determined experimentally

Numerically definitely OK, relation, still checking

Callan-Treiman point

$$\text{Callan-Treiman: } f_0 (m_K^2 - m_\pi^2) = \frac{F_K}{F_\pi} + \mathcal{O}(m_u, m_d).$$

$$\Delta_{CT} \equiv f_0 (m_K^2 - m_\pi^2) - \frac{F_K}{F_\pi} = -3.5 \cdot 10^{-3}. \text{ (GL, iso)}$$

$$\Delta_{CT} = -6.2 \cdot 10^{-3}. \text{ NNLO using BT formulas}$$

With δp^4 Calculated with $F_K/F_\pi = 1.22$

$$\Delta_{CT}^{K^+\pi^0} = 15.1 \cdot 10^{-3},$$

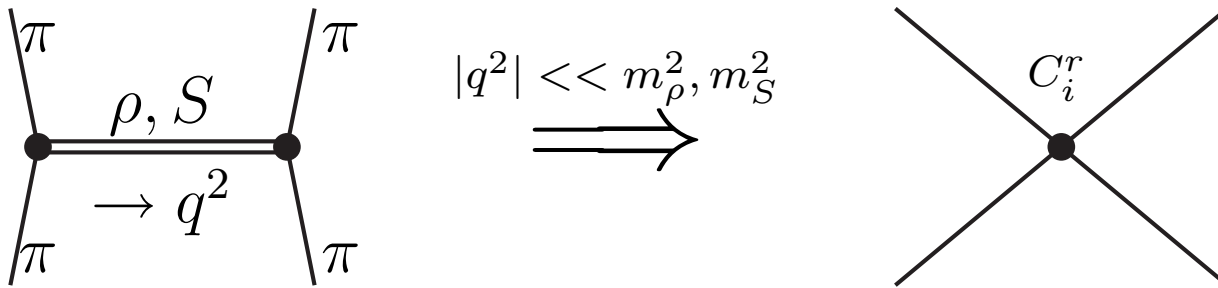
$$\Delta_{CT}^{K^0\pi^-} = -5.6 \cdot 10^{-3},$$

$$\Delta_{CT}^{K^+\pi^+} = -9.4 \cdot 10^{-3},$$

$$\Delta_{CT}^{K^0\pi^0} = -26.4 \cdot 10^{-3}.$$

Most analysis use:

C_i^r from (single) resonance approximation



Motivated by large N_c : large effort goes in this

Ananthanarayan, JB, Cirigliano, Donoghue, Ecker, Gamiz, Golterman, Kaiser, Knecht, Peris, Pich, Prades, Portoles, de Rafael, . . .

$$\begin{aligned}
 \mathcal{L}_V &= -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \frac{1}{2}m_V^2\langle V_\mu V^\mu\rangle - \frac{f_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle \\
 &\quad - \frac{ig_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + f_\chi\langle V_\mu[u^\mu, \chi_-]\rangle \\
 \mathcal{L}_A &= -\frac{1}{4}\langle A_{\mu\nu}A^{\mu\nu}\rangle + \frac{1}{2}m_A^2\langle A_\mu A^\mu\rangle - \frac{f_A}{2\sqrt{2}}\langle A_{\mu\nu}f_-^{\mu\nu}\rangle \\
 \mathcal{L}_S &= \frac{1}{2}\langle \nabla^\mu S \nabla_\mu S - M_S^2 S^2 \rangle + c_d\langle S u^\mu u_\mu \rangle + c_m\langle S \chi_+ \rangle \\
 \mathcal{L}_{\eta'} &= \frac{1}{2}\partial_\mu P_1 \partial^\mu P_1 - \frac{1}{2}M_{\eta'}^2 P_1^2 + i\tilde{d}_m P_1 \langle \chi_- \rangle.
 \end{aligned}$$

$$f_V = 0.20, \quad f_\chi = -0.025, \quad g_V = 0.09, \quad c_m = 42 \text{ MeV}, \quad c_d = 32 \text{ MeV}, \quad \tilde{d}_m = 20 \text{ MeV},$$

$$m_V = m_\rho = 0.77 \text{ GeV}, \quad m_A = m_{a_1} = 1.23 \text{ GeV}, \quad m_S = 0.98 \text{ GeV}, \quad m_{P_1} = 0.958 \text{ GeV}$$

f_V, g_V, f_χ, f_A : experiment

c_m and c_d from resonance saturation at $\mathcal{O}(p^4)$

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

What we did about it:

- Vary resonance estimate by factor of two
- Vary the scale μ at which it applies: 600-900 MeV
- Check the estimates for the measured ones
- Again: kinematic can be had, quark-mass dependence difficult

Conclusions

- Have calculated all $K \rightarrow \pi$ transitions to NNLO, i.e. δp^4
- NNLO contributions lower the isospin breaking compared to NLO
- But m_u/m_d also smaller at NNLO (But see $\eta \rightarrow 3\pi$)
- slight increase when all put together but fits experimental ratios well.
- Determination of C_i^r still needed:
- Analytical playing with the amplitude: started
- Future: checking whether we have more nontrivial relations between different observables for the C_i^r contribution.
- New fit of the L_i^r with better C_i^r treatment.