ISOSPIN BREAKING AT ORDER p^6 in $K_{\ell 3}$ DECAYS

Johan Bijnens

Lund University

bijnens@thep.lu.se
http://www.thep.lu.se/∼bijnens

Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Overview

- Why $K_{\ell 3}$ (and similar things)?
- List of papers or earlier results
- Formfactor relations
- Some aspects of the calculation
- Numerical results
  - $f_+(0)$
  - $f_+(t)$, $r_{0-}(t)$
  - $f_0(t)$, $r^0_{0-}(t)$
  - Δ_{CT}

$K_{\ell 3}$

Why look at $K_{\ell 3}$?

$$\Gamma\left(K^i \to \pi^j \ell^+ \nu_l\right) =$$
$$C_{ij}^2 \frac{G_F^2 S_{EW} m_K^5}{192\pi^3} \left| V_{us} f_+^{K^i\pi^j}(0) \right|^2 \mathcal{I}_\ell^{ij} \left(1 + 2\Delta_{EM}^{ij}\right) .$$

$\mathcal{I}_\ell^{ij}$: depends on measured formfactors
In principle well known

Similar formulas exist for $K \to \pi\nu\bar{\nu}$

$K_{\ell 3}$: Definitions

$$\langle \pi^0(p')|\overline{s}\gamma_\mu u(0)|K^+(p)\rangle = \frac{1}{\sqrt{2}}\left[(p'+p)_\mu f_+^{K^+\pi^0}(t) + (p-p')_\mu f_-^{K^+\pi^0}(t)\right]$$

$$\langle \pi^-(p')|\overline{s}\gamma_\mu u(0)|K^0(p)\rangle = \left[(p'+p)_\mu f_+^{K^0\pi^-}(t) + (p-p')_\mu f_-^{K^0\pi^-}(t)\right]$$

$$\langle \pi^+(p')|\overline{s}\gamma_\mu d(0)|K^+(p)\rangle = \left[(p'+p)_\mu f_+^{K^+\pi^+}(t) + (p-p')_\mu f_-^{K^+\pi^+}(t)\right]$$

$$\langle \pi^0(p')|\overline{s}\gamma_\mu d(0)|K^0(p)\rangle = \frac{-1}{\sqrt{2}}\left[(p'+p)_\mu f_+^{K^0\pi^0}(t) + (p-p')_\mu f_-^{K^0\pi^0}(t)\right]$$

8 form-factors, $t = (p'-p)^2$

$f_+^{K^i\pi^j}(0) = 1$ in the $SU(3)$ limit $m_u = m_d = m_s$

Isospin limit: $f_\pm = f_\pm^{K\pi} = f_\pm^{K^+\pi^0} = f_\pm^{K^0\pi^-} = f_\pm^{K^+\pi^+} = f_\pm^{K^0\pi^0}$

$K_{\ell 3}$ Definitions

Scalar formfactor: $$f_0(t) = f_+(t) + \frac{t}{m^2_{K^i} - m^2_{\pi^j}} f_-(t)$$

$$f_l^{K^i \pi^j} = LO + NLO + NNLO + \cdots$$

$\delta = m_u - m_d$, e^2 and p^2 expansion

$$f_{l=+,0} = 1 + \delta + \delta p^2 + e^2 p^2 + \delta p^4 + \cdots$$

$K_{\ell 3}$ Definitions

Scalar formfactor: $$f_0(t) = f_+(t) + \frac{t}{m^2_{K^i} - m^2_{\pi^j}} f_-(t)$$

$$f_l^{K^i\pi^j} = LO + NLO + NNLO + \cdots$$

$\delta = m_u - m_d$, e^2 and p^2 expansion

$$f_{l=+,0} = 1 + \delta + \delta p^2 + e^2 p^2 + \delta p^4 + \cdots$$

LO is also tree diagrams from p^2-Lagrangian: p^2
NLO is also tree diagrams from p^4-Lagrangian: p^4
NNLO is also tree diagrams from p^6-Lagrangian: p^6

$K_{\ell 3}$: (subset of) papers

- H. Leutwyler and M. Roos, Z.Phys.C25:91,1984. $K_{\ell 3}$ $f_+(0)$ δp^2
- J. Gasser and H. Leutwyler, Nucl.Phys.B250:517-538,1985. $K_{\ell 3}$ $f_+(t)$ δp^2, $f_0(t)$ p^2
- V. Cirigliano et al., hep-ph/0110153, Eur.Phys.J.C23:121-133,2002 $K_{\ell 3}$ $f_+(t), f_0(t)$ δp^2, $e^2 p^2$
- P. Post and K.Schilcher, hep-ph/0112352 Eur.Phys.J.C25,427 2002 $K_{\ell 3}$ $f_+(t), f_0(t)$ p^4 numerics??
- J. Bijnens and P. Talavera, hep-ph/0303103, Nucl. Phys. B669 (2003) 341-362 $K_{\ell 3}$ $f_+(t), f_0(t)$ p^4
- V. Cirigliano et al., hep-ph/0503108, JHEP 0504 (2005) 006 C_i p^4

$K_{\ell 3}$: (subset of) papers

- M. Jamin, J. Oller and A. Pich,
JHEP 0402 (2004) 047 $C_i\ p^4$, dispersive
- F. Mescia and C. Smith, arXiv:0705.2025 $f_+(t)\ \delta p^2$,$e^2 p^2$
- J. Bijnens and K. Ghorbani, arXiv:0711.0148 $f_{+,0}(t)\ \delta p^4$
- V. Bernard, M. Oertel, E.Passemar and J. Stern,
hep-ph/0603202, arXiv:0707.4194[hep-ph]
dispersive, CT
- A. Kastner and H. Neufeld, arXiv:0805.2222[hep-ph]
putting a lot together, see previous talk

Formfactor relation

MS (and us) noticed:

$$f_\ell^{K^+\pi^0}(t) - f_\ell^{K^0\pi^-}(t) - f_\ell^{K^+\pi^+}(t) + f_\ell^{K^0\pi^0}(t) = 0 + \mathcal{O}(\delta^2)\,,$$

$$r(t) \equiv \frac{f_\ell^{K^+\pi^0}(t) f_\ell^{K^0\pi^0}(t)}{f_\ell^{K^0\pi^-}(t) f_\ell^{K^+\pi^+}(t)} = 1 + \mathcal{O}(\delta^2)$$

GL noticed:

$$r_{0-}(t) = \frac{f_+^{K^+\pi^0}(t)}{f_+^{K^0\pi^-}(t)} = \text{constant}$$

Formfactor relations

$r(t) = 1 + \delta^2$ is always true, also for $r^-(t)$ and $r^0(t)$

Vector operator is I=1/2
$(m_u - m_d)(\bar{u}u - \bar{d}d)$ is I =1

$$
\begin{aligned}
f_\ell^{K^+\pi^0}(t) &= f_\ell^A(t) + \delta f_\ell^B(t) + \mathcal{O}(\delta^2)\,, \\
f_\ell^{K^0\pi^-}(t) &= f_\ell^A(t) - \delta f_\ell^D(t) + \mathcal{O}(\delta^2)\,, \\
f_\ell^{K^+\pi^+}(t) &= f_\ell^A(t) + \delta f_\ell^D(t) + \mathcal{O}(\delta^2)\,, \\
f_\ell^{K^0\pi^0}(t) &= f_\ell^A(t) - \delta f_\ell^B(t) + \mathcal{O}(\delta^2)\,,
\end{aligned}
$$

Photon exchange has $I = 0, 1, 2$, $I = 2$ part breaks relation

Relation also true for scalar currents $\bar{s}u$, $\bar{s}d$

Formfactor relations

$r_{0-}(t)$ constant not true at δp^4 (MS: C_i, BG also the rest)

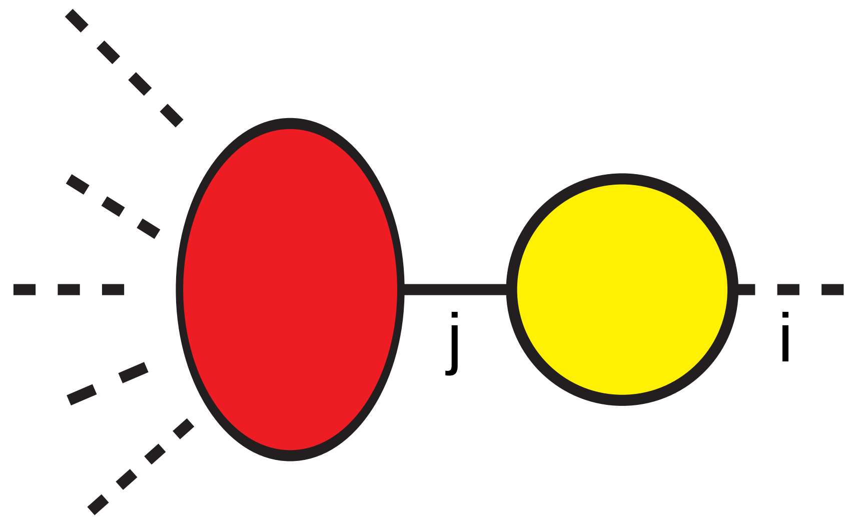
$r^0_{0-}(t)$, $r^-_{0-}(t)$ not true at δp^2

New for isospin breaking

Take LSZ into account properly

Amoros, JB, Talavera, 2001

Matrix element:



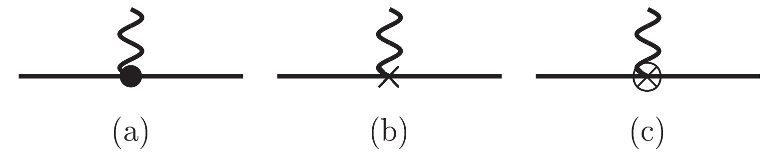
$$\mathcal{A}_{i_1\ldots i_n} = \left(\frac{(-i)^n}{\sqrt{Z_{i_1}\ldots Z_{i_n}}}\right) \prod_{i=1}^n \lim_{k_i^2\to m_i^2}(k_i^2-m_i^2)\, G_{i_1\ldots i_n}(k_1,\ldots,k_n)$$

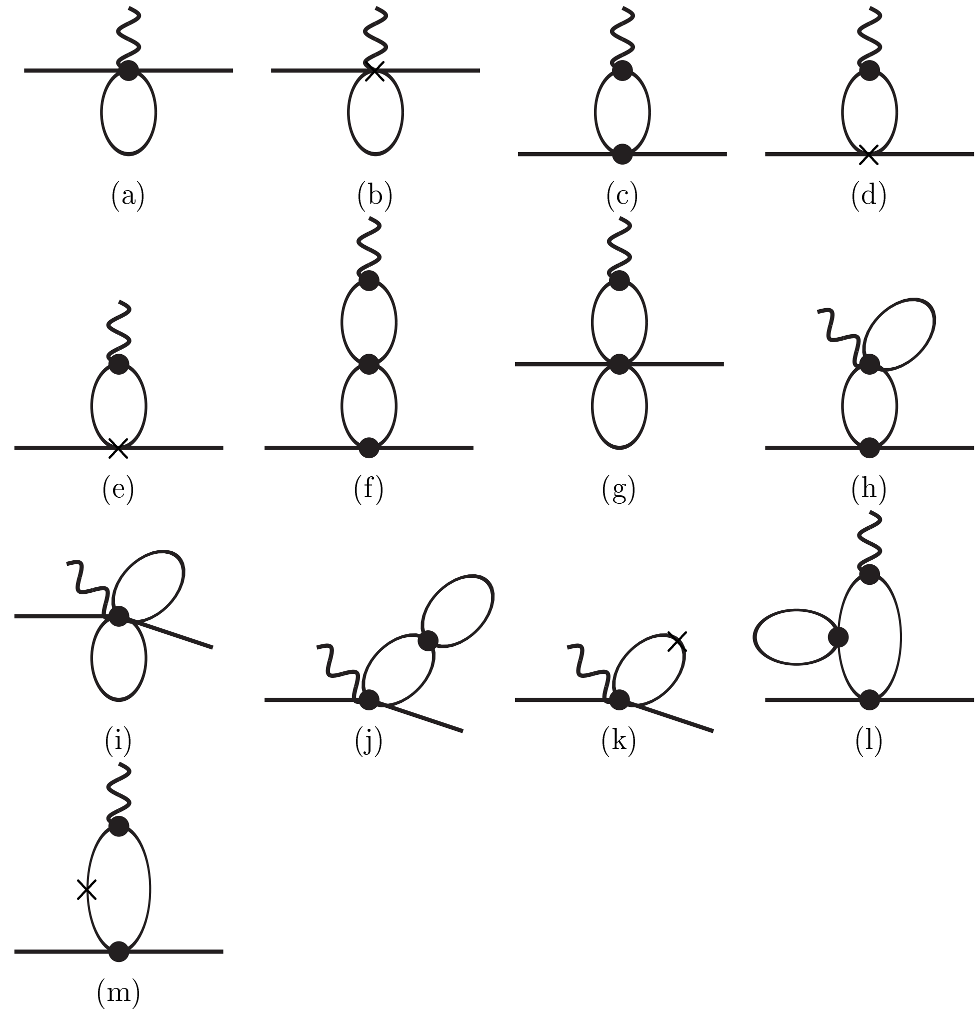
Isospin Breaking

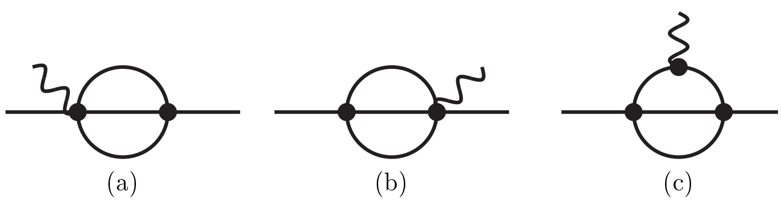
Deal with the complications of mixing via matrices/determinant

$$
\begin{aligned}
\mathcal{A}_3 &= \mathcal{G}_3^{(2)} + \left\{ \mathcal{G}_3^{(4)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right\} \\
&+ \left[\mathcal{G}_3^{(6)} - \frac{1}{2} Z_{33}^{(6)} \mathcal{G}_3^{(2)} - \frac{1}{2} Z_{33}^{(4)} \mathcal{G}_3^{(4)} + \frac{3}{8} \left(Z_{33}^{(4)} \right)^2 \mathcal{G}_3^{(2)} \right. \\
&+ \frac{Z_{38}^{(4)} \Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_3^{(2)} - \frac{1}{2} \left(\frac{\Pi_{38}^{(4)}}{\Delta m^2} \right)^2 \mathcal{G}_3^{(2)} - \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(4)} \\
&\left. - \frac{\Pi_{38}^{(6)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{\Pi_{38}^{(4)} \Pi_{88}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} + \frac{1}{2} Z_{33}^{(4)} \frac{\Pi_{38}^{(4)}}{\Delta m^2} \mathcal{G}_8^{(2)} \right]
\end{aligned}
$$

Diagrams







● : p^2 vertex

× : p^4 vertex

⊗ : p^6 vertex

$f_+(0)$: NO isospin violation

	$f_+^{K^+\pi^0}$	$f_+^{K^0\pi^-}$	$f_+^{K^+\pi^+}$	$f_+^{K^0\pi^0}$
order p^2	1.00000	1.00000	1.00000	1.00000
order p^4	−0.02276	−0.02266	−0.02226	−0.02316
order p^6	0.01423	0.01462	0.01406	0.01480
p^6 2-loop	0.01104	0.01130	0.01090	0.01145
p^6 L_i^r	0.00320	0.00332	0.00316	0.00336
sum	0.99156	0.99196	0.99180	0.99164

JB-Talavera, JB-Ghorbani

The C_i^r contribution: nothing new here and not included

$f_+(0)$: $m_u/m_d = 0.585(NLO)$

m_u/m_d NLO, Dashen's theorem

	$f_+^{K^+\pi^0}$	$f_+^{K^0\pi^-}$	$f_+^{K^+\pi^+}$	$f_+^{K^0\pi^0}$
order p^2	1.01702	1.00000	1.00000	0.98288
order p^4	−0.01931	−0.02282	−0.02202	−0.02675
order p^6	0.00986	0.01467	0.01395	0.01919
p^6 2-loop	0.00435	0.01142	0.01084	0.01815
p^6 L_i^r	0.00551	0.00325	0.00311	0.00104
sum	1.00757	0.99186	0.99193	0.97532

$\sin\epsilon = 0.00986$ JB-Ghorbani

The C_i^r contribution: not included

$f_+(0)$: $m_u/m_d = 0.45(NNLO)$

NNLO and violations of Dashen's theorem

	$f_+^{K^+\pi^0}$	$f_+^{K^0\pi^-}$	$f_+^{K^+\pi^+}$	$f_+^{K^0\pi^0}$
order p^2	1.02465	1.00000	1.00000	0.97514
order p^4	−0.01775	−0.02292	−0.02197	−0.02838
order p^6	0.00809	0.01470	0.01391	0.02095
p^6 2-loop	0.00159	0.01145	0.01081	0.02092
p^6 L_i^r	0.00650	0.00325	0.00309	0.00004
sum	1.01499	0.99177	0.99194	0.96772

$\sin\epsilon = 0.0143$ JB-Ghorbani

The C_i^r contribution: not included

C_i^r from η': $f_+^{K^+\pi^0}(0)\Big|_{P_1} = 0.00065$ $f_+^{K^0\pi^0}(0)\Big|_{P_1} = -0.00065$

$f_+(0)$

$$r_{0-}(0) = 1.02465 + 0.00587 - 0.00711 = 1.02341\,,$$

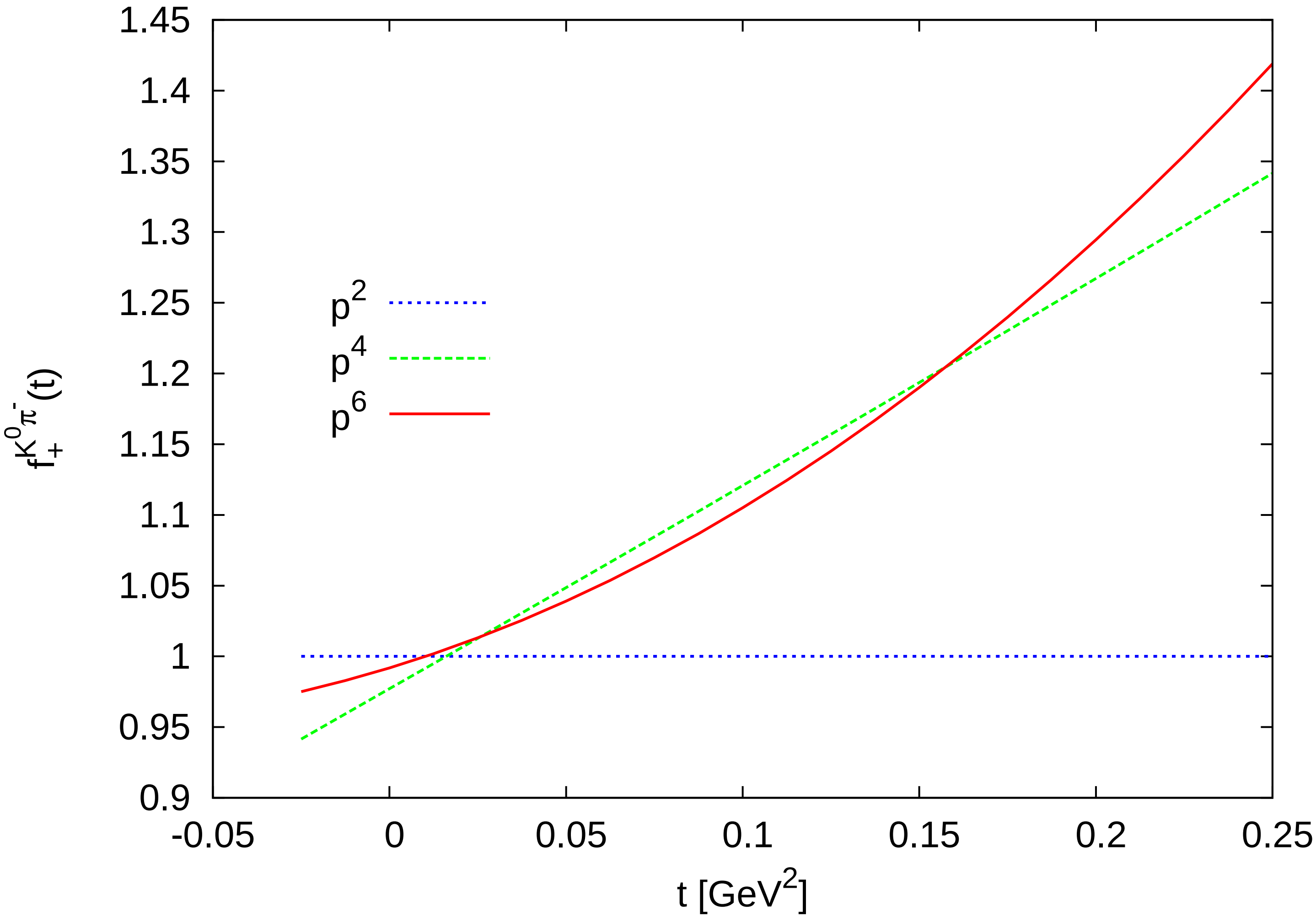
Palutan: KAON07

$$r_{0-exp} = 1 + \Delta_{SU(2)} = 1.0284 \pm 0.0040\,.$$

As we see, we obtain a reasonable agreement.

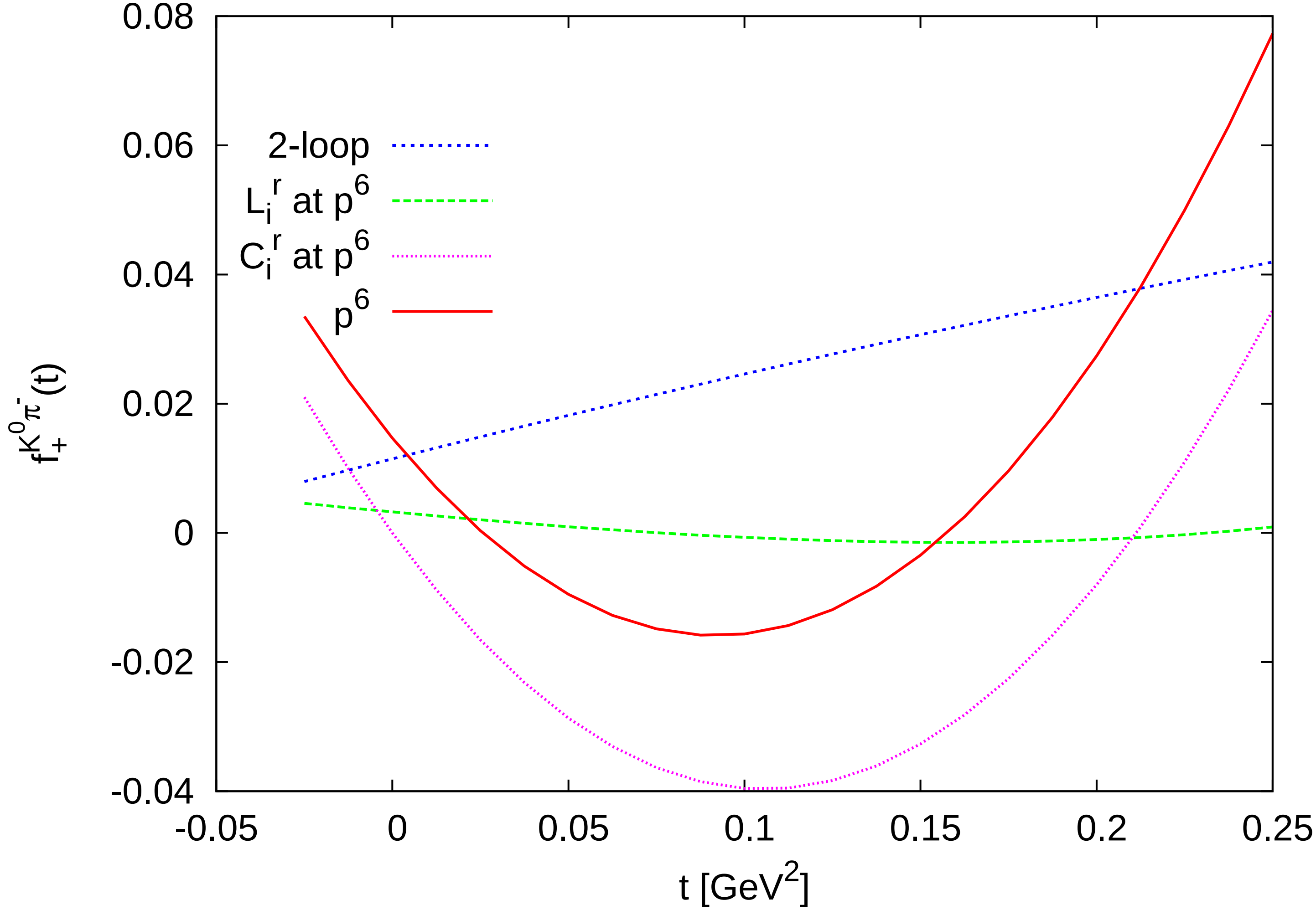
t-dependence

For $K^0 \to \pi^- \ell^+ \nu$ numerically very little change,

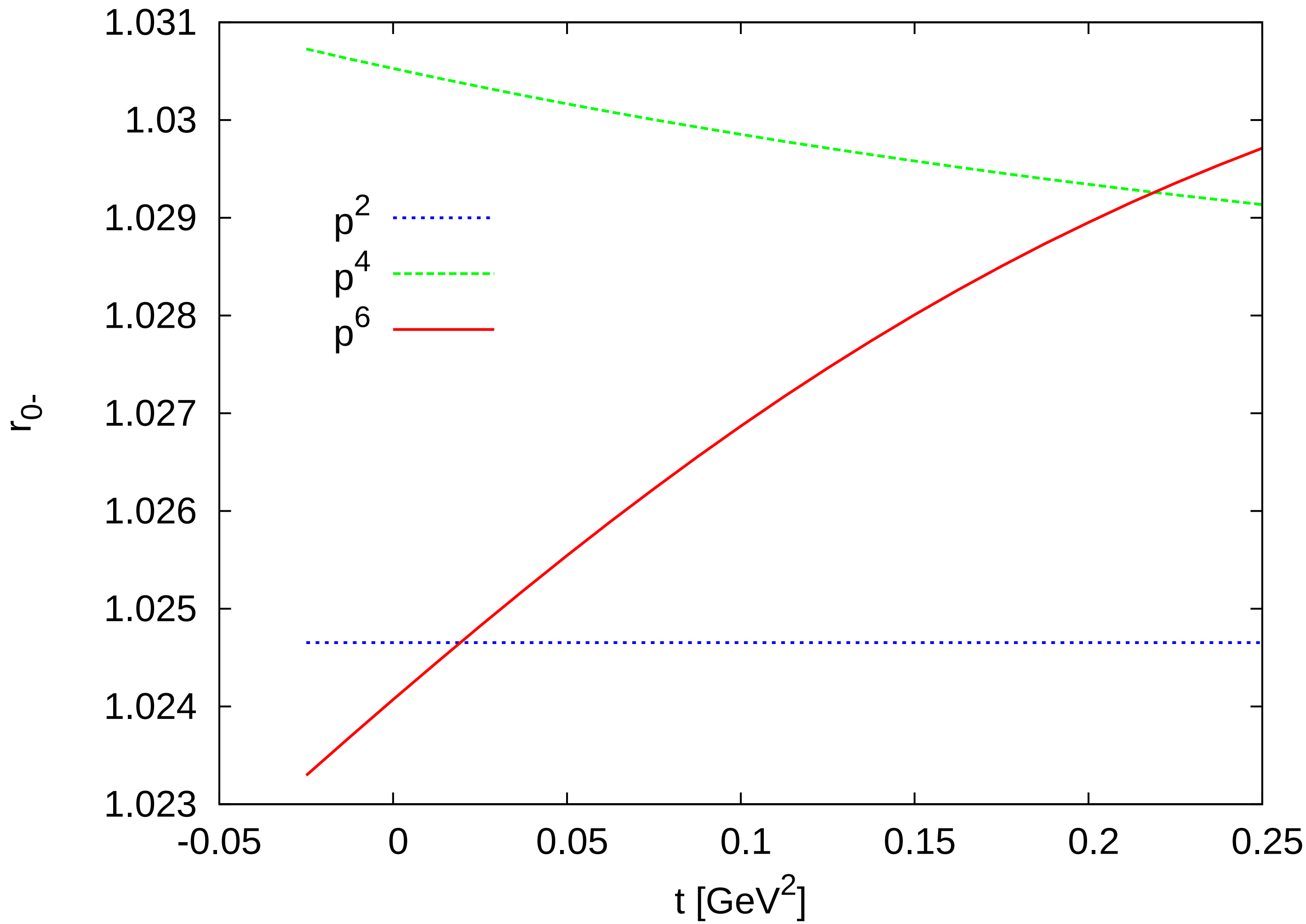


t-dependence: p^6

For $K^0 \to \pi^- \ell^+ \nu$ numerically very little change,

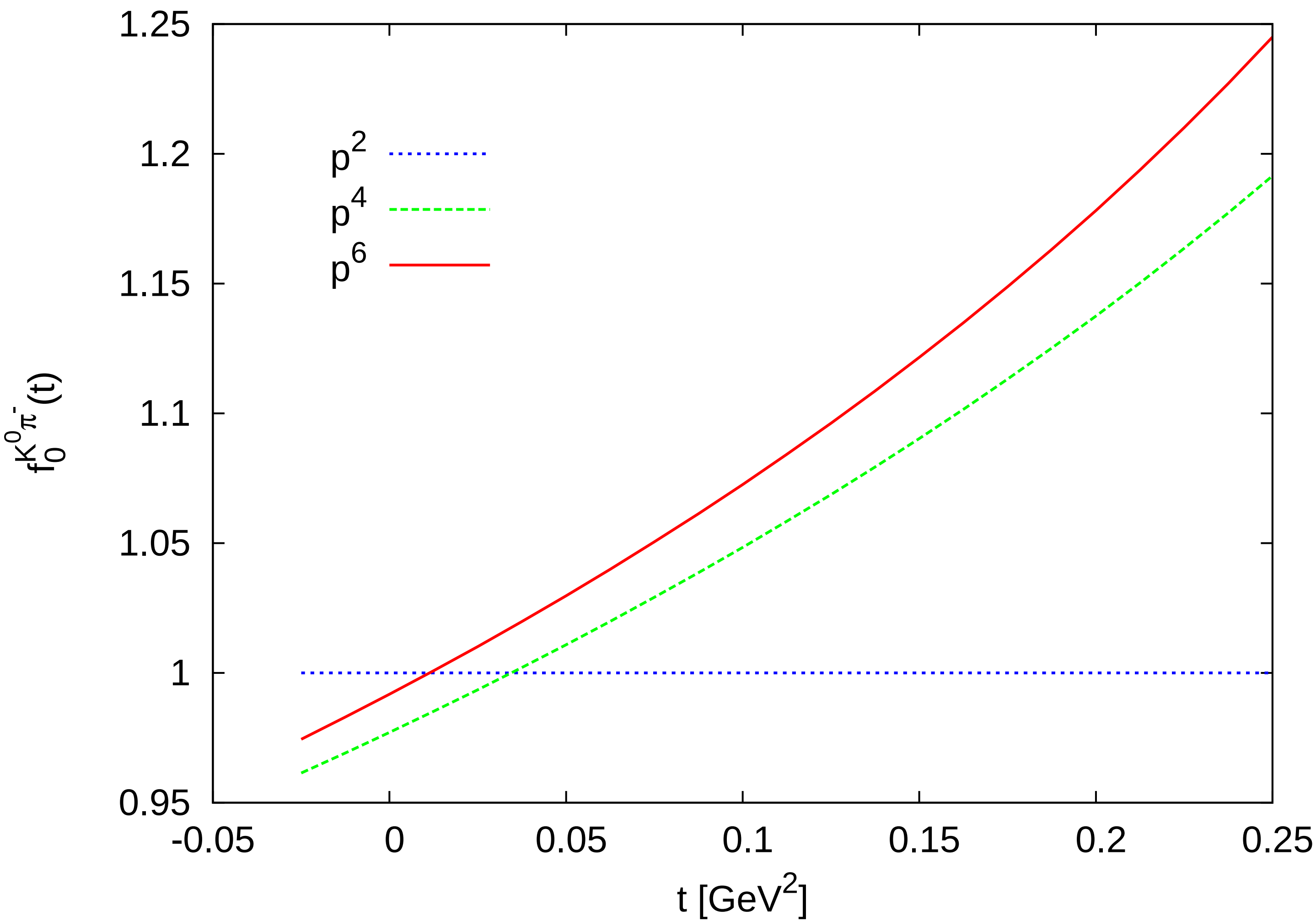


t-dependence: $r_{0-}(t)$

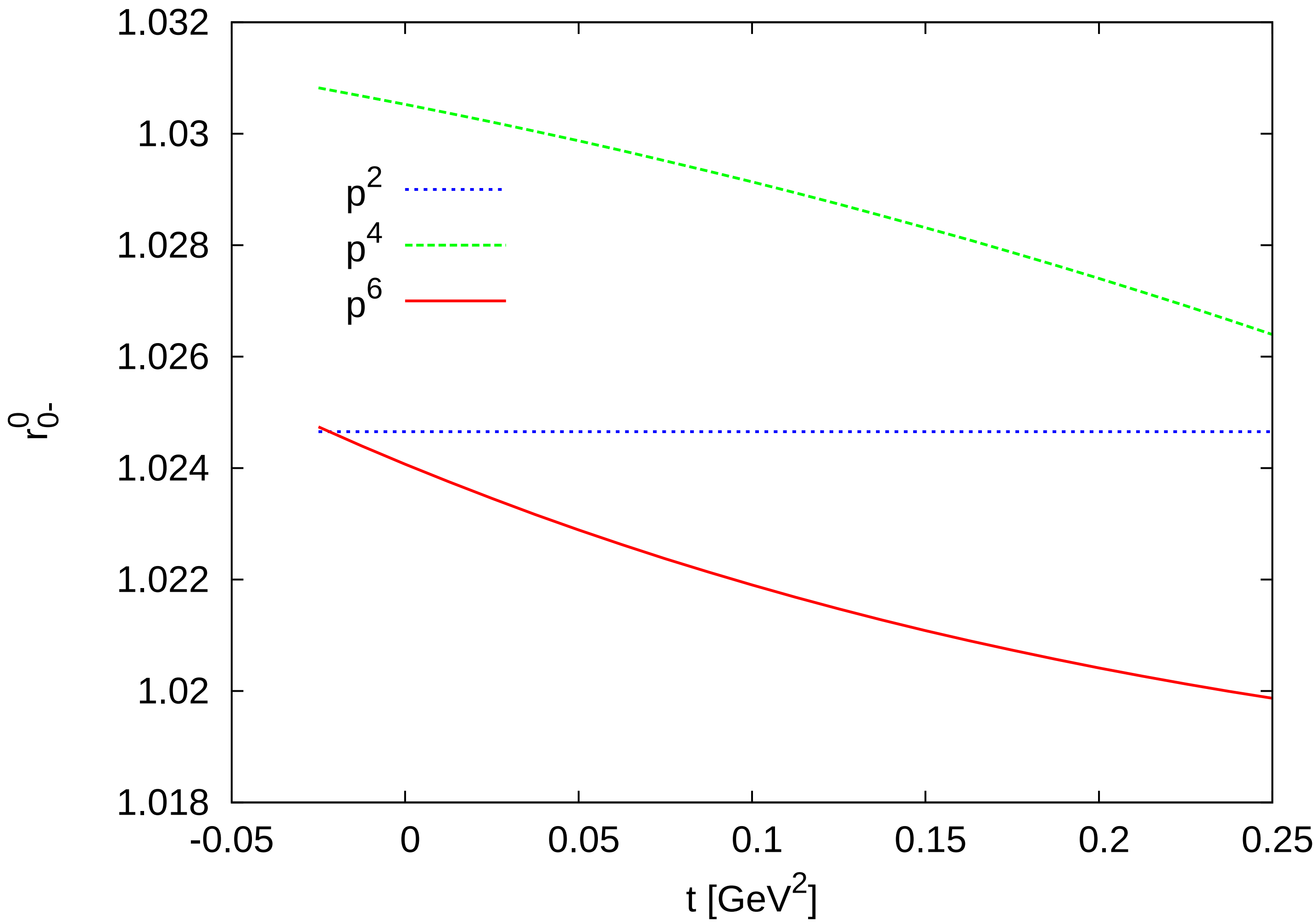


t-dependence: f_0

For $K^0 \to \pi^- \ell^+ \nu$ numerically very little change,



t-dependence: $r_{0-}(t)$



$f_0(t)$ Isospin limit

Old Main Result: JB-Talavera

$$
\begin{aligned}
f_0(t) &= 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 \\
&\quad +8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}\left(F_K/F_\pi - 1\right) \\
&\quad -\frac{8}{F_\pi^4}t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
\end{aligned}
$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO C_i^r and only depend on the L_i^r at order p^6 $\Longrightarrow$
All needed parameters can be determined experimentally

Numerically definitely OK, relation, still checking

Callan-Treiman point

Callan-Treiman: $f_0\left(m_K^2 - m_\pi^2\right) = \frac{F_K}{F_\pi} + \mathcal{O}(m_u, m_d)\,.$

$\Delta_{CT} \equiv f_0\left(m_K^2 - m_\pi^2\right) - \frac{F_K}{F_\pi} = -3.5\,10^{-3}\,.$ (GL, iso)

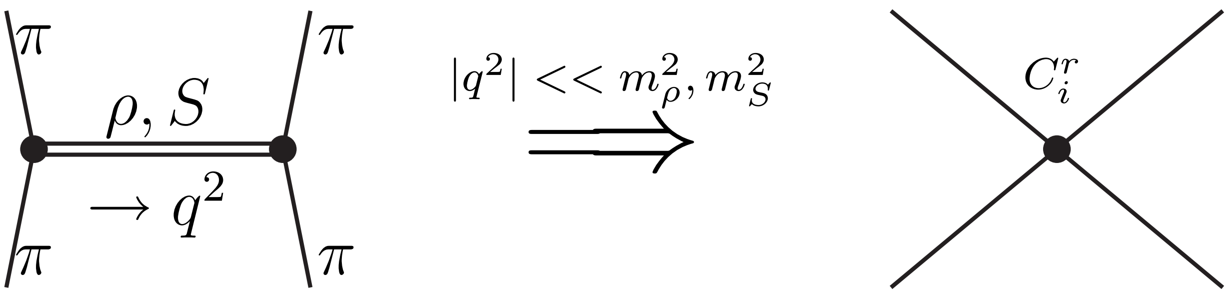
$\Delta_{CT} = -6.2\,10^{-3}\,.$ NNLO using BT formulas

With δp^4 Calculated with $F_K/F_\pi = 1.22$

$$\begin{aligned}
\Delta_{CT}^{K^+\pi^0} &= 15.1\,10^{-3}\,,\\
\Delta_{CT}^{K^0\pi^-} &= -5.6\,10^{-3}\,,\\
\Delta_{CT}^{K^+\pi^+} &= -9.4\,10^{-3}\,,\\
\Delta_{CT}^{K^0\pi^0} &= -26.4\,10^{-3}\,.
\end{aligned}$$

C_i^r

Most analysis use:
C_i^r from (single) resonance approximation



Motivated by large N_c: large effort goes in this

Ananthanarayan, JB, Cirigliano, Donoghue, Ecker, Gamiz, Golterman, Kaiser, Knecht, Peris, Pich, Prades, Portoles, de Rafael,. . .

C_i^r

$$\begin{aligned}
\mathcal{L}_V &= -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \frac{1}{2}m_V^2\langle V_\mu V^\mu\rangle - \frac{f_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle \\
&\quad -\frac{ig_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu,u^\nu]\rangle + f_\chi\langle V_\mu[u^\mu,\chi_-]\rangle \\
\mathcal{L}_A &= -\frac{1}{4}\langle A_{\mu\nu}A^{\mu\nu}\rangle + \frac{1}{2}m_A^2\langle A_\mu A^\mu\rangle - \frac{f_A}{2\sqrt{2}}\langle A_{\mu\nu}f_-^{\mu\nu}\rangle \\
\mathcal{L}_S &= \frac{1}{2}\langle \nabla^\mu S\nabla_\mu S - M_S^2 S^2\rangle + c_d\langle S u^\mu u_\mu\rangle + c_m\langle S\chi_+\rangle \\
\mathcal{L}_{\eta'} &= \frac{1}{2}\partial_\mu P_1\partial^\mu P_1 - \frac{1}{2}M_{\eta'}^2 P_1^2 + i\tilde{d}_m P_1\langle\chi_-\rangle\,.
\end{aligned}$$

$f_V = 0.20$, $f_\chi = -0.025$, $g_V = 0.09$, $c_m = 42$ MeV, $c_d = 32$ MeV, $\tilde{d}_m = 20$ MeV,

$m_V = m_\rho = 0.77$ GeV, $m_A = m_{a_1} = 1.23$ GeV, $m_S = 0.98$ GeV, $m_{P_1} = 0.958$ GeV

f_V, g_V, f_χ, f_A: experiment

c_m and c_d from resonance saturation at $\mathcal{O}(p^4)$

C_i^r

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

C_i^r

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

What we did about it:

- Vary resonance estimate by factor of two
- Vary the scale μ at which it applies: 600-900 MeV
- Check the estimates for the measured ones
- Again: kinematic can be had, quark-mass dependence difficult

Conclusions

- Have calculated all $K \to \pi$ transitions to NNLO, i.e. δp^4
- NNLO contributions lower the isospin breaking compared to NLO
- But m_u/m_d also smaller at NNLO (But see $\eta \to 3\pi$)
- slight increase when all put together but fits experimental ratios well.
- Determination of C_i^r still needed:
- Analytical playing with the amplitude: started
- Future: checking whether we have more nontrivial relations between different observables for the C_i^r contribution.
- New fit of the L_i^r with better C_i^r treatment.