Hadronic Decays

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Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Overview

- What do we want to learn about?
- Basic proporties: m_K (and F_K)
- Hadronic decays: $K \to \pi\pi$
  - $\delta_0 - \delta_2$
  - In two-flavour ChPT: hard pion ChPT
- $K \to 3\pi$ in ChPT
- $K \to 2\pi, 3\pi$ isospin breaking in ChPT
- A reminder on the existing analytical work for the $\Delta I = 1/2$ rule

Later talks or not covered

- CP-violation: Prades
- The nonrelativistic calculations of final states including isospin breaking: Colangelo
- the cusp in $K \to \pi^+\pi^0\pi^0$: Colangelo
- Lattice: Sachrajda
- ChPT and $K_{\ell 4}$ at one and two loops: JB, 2000. . .
- Dispersion relations and the various decays: but see Boito,Passemar

What do we want to learn about?

Standard Model Lagrangian has four parts:

$$\underbrace{\mathcal{L}_H(\phi)}_{\text{Higgs}} + \underbrace{\mathcal{L}_G(W,Z,G)}_{\text{Gauge}} \underbrace{\sum_{\psi=\text{fermions}} \bar\psi i \not{D}\, \psi}_{\text{gauge-fermion}} + \underbrace{\sum_{\psi,\psi'=\text{fermions}} g_{\psi\psi'} \bar\psi \phi \psi'}_{\text{Yukawa}}$$

What is tested ?:

gauge-fermion	Very well
Higgs	Real tests coming up
Gauge	Well tested in QCD, partly in electroweak
Yukawa	Rather well tested in B and K (Nobel 2008)

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Discrete symmetries: C, P, T
QCD and QED: C,P,T good; Field theory: CPT good
Weak: breaks P and C, Yukawa breaks CP

What do we want to learn about?

- Yukawa sector
- QCD at low energies
- Possible beyond Standard Model effects

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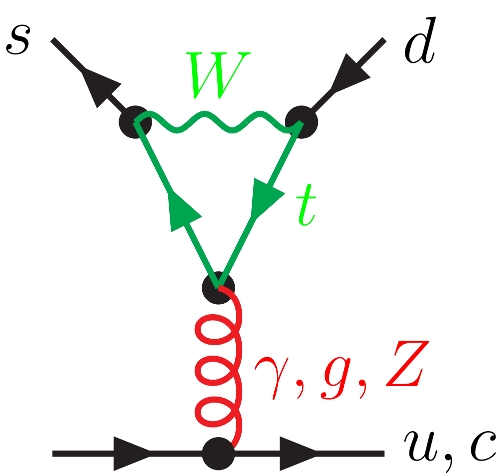
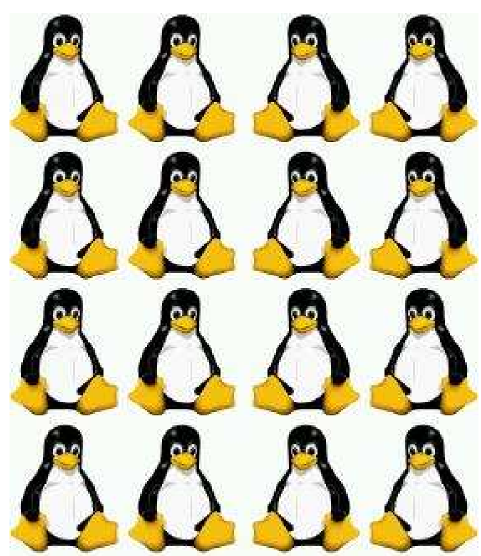
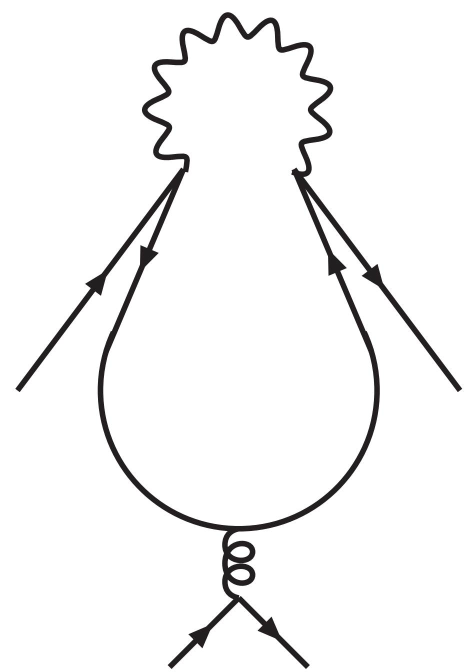
Heavy particles can contribute in loop







The problem

ENERGY SCALE	FIELDS	Effective Theory
M_W	W, Z, γ, g; τ, μ, e, ν_ℓ; t, b, c, s, u, d	Standard Model
	$\Downarrow$ using OPE	
$\lesssim m_c$	γ, g; μ, e, ν_ℓ; s, d, u	QCD,QED,$\mathcal{H}_{\text{eff}}^{\lvert\Delta S\rvert=1,2}$
	$\Downarrow$???	
M_K	γ; μ, e, ν_ℓ; π, K, η	CHPT

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	$\Downarrow$ using OPE	3-loop: Gorbahn et al.
$\lesssim m_c$	γ, g; μ, e, ν_ℓ; s, d, u	QCD,QED,$\mathcal{H}_{\text{eff}}^{\lvert\Delta S\rvert=1,2}$
	$\Downarrow$???	Lattice, Large N_c,...
M_K	γ; μ, e, ν_ℓ; π, K, η	CHPT

Basic properties: Mass

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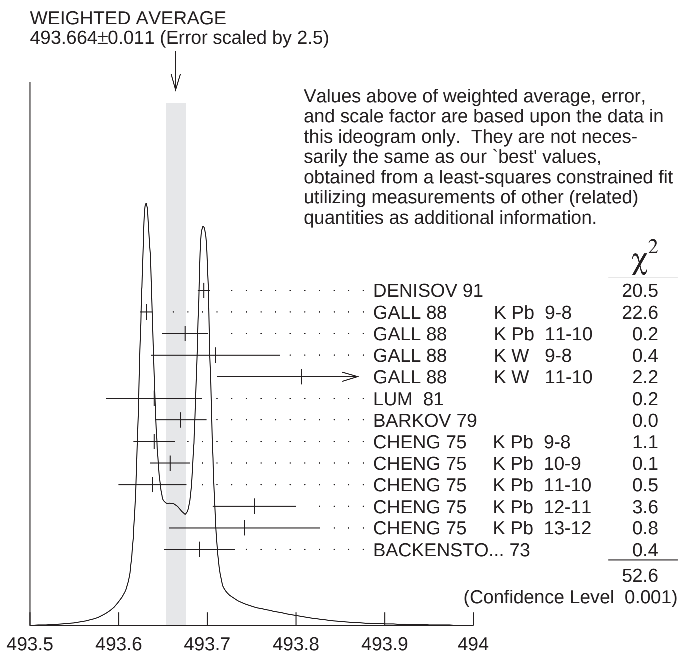

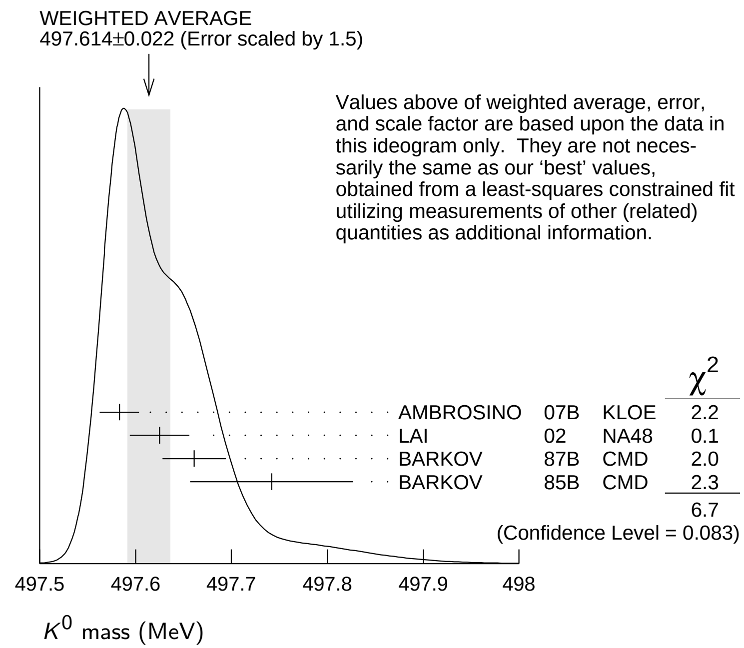

$$m_{K^+} = 493.677 \pm 0.016 \qquad m_{K^0} = 497.614 \pm 0.024$$

PDG2008, our fit

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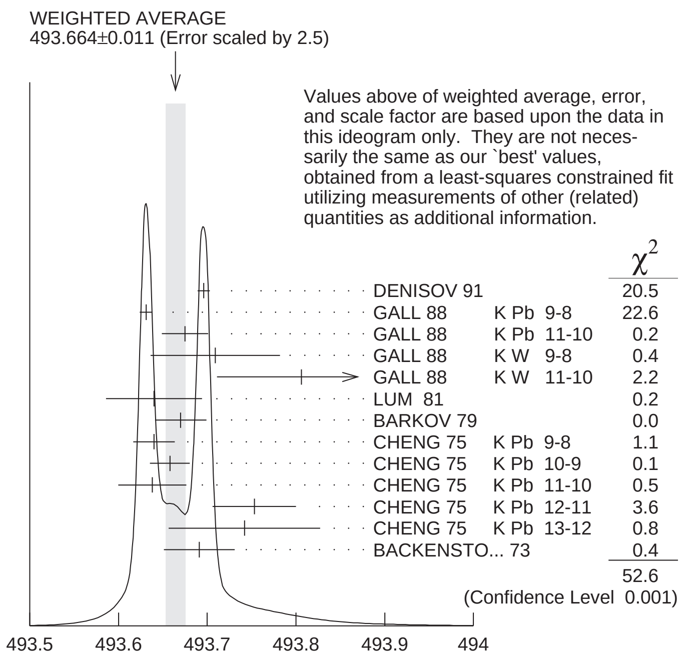

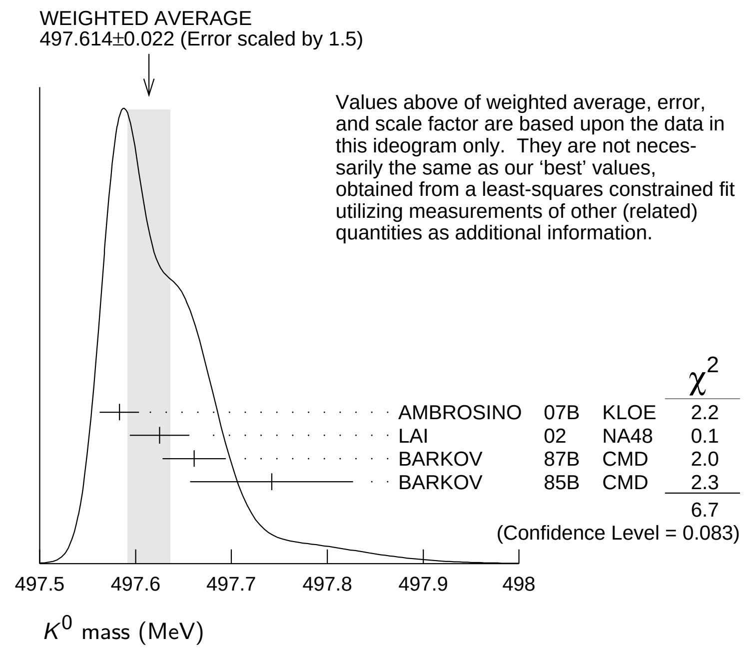

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Question: Can KLOE and NA48/NA62 help with the K^+?

Basic properties: Decay constant

- $K_{\ell 2}$: V_{us} and F_K
- $K_{\ell 3}$: V_{us} and $f_+(0)$.

Two measurements, three quantities: need extra input.
See:

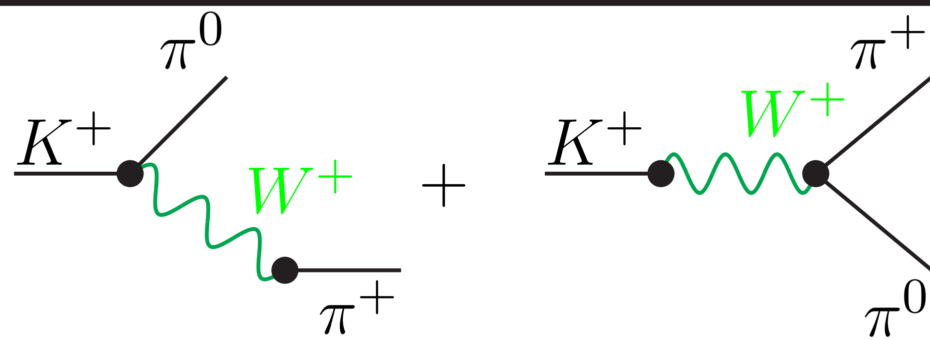
- Flavianet Kaon Working Group
- Flavianet Lattice Averaging Group
- (Semi)leptonic decay group

$F_K/F_\pi = 0.193 \pm 0.002 \pm 0.006 \pm 0.001$ PDG2008
$F_K = 109.96 \pm 0.014 \pm 0.58 \pm 0.14$ MeV

Uncertainty: decay rate, V_{us}, radiative corrections

$K \to \pi\pi$ and the $\Delta I = 1/2$ Rule

$K^+ \longrightarrow \pi^+\pi^0$:



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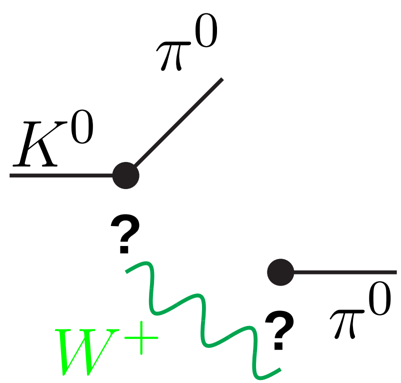

: IMPOSSIBLE

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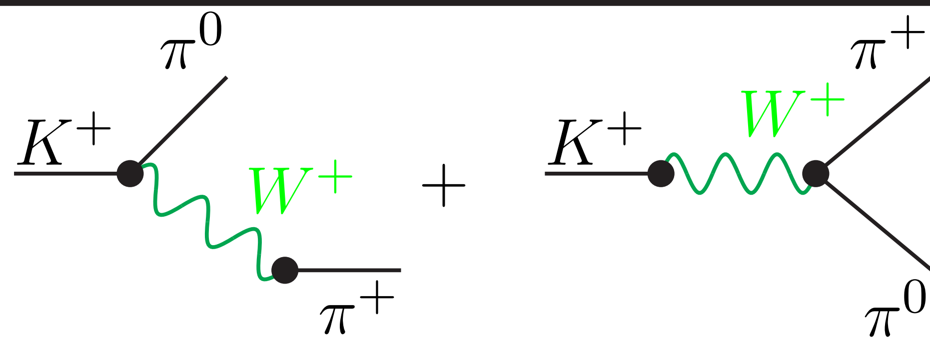

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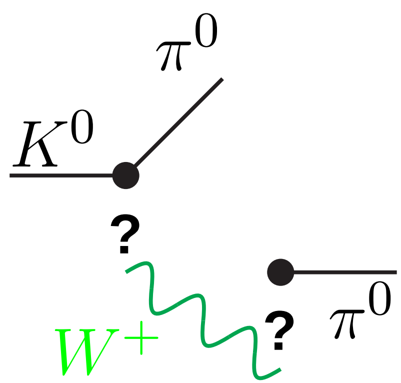

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Experimentally:

$$\Gamma(K^0 \longrightarrow \pi^0\pi^0) = \quad \frac{1}{2}\Gamma(K_S \longrightarrow \pi^0\pi^0) \quad = 2.3\ 10^{-12}\ \text{MeV}$$

$$\Gamma(K^+ \longrightarrow \pi^+\pi^0) = \quad = 1.1\ 10^{-14}\ \text{MeV}$$

So the zero one is the largest !!!

Isospin amplitudes

$$A[K^0 \to \pi^0\pi^0] \equiv \sqrt{\frac{1}{3}}A_0 - \sqrt{\frac{2}{3}}\,A_2$$

$$A[K^0 \to \pi^+\pi^-] \equiv \sqrt{\frac{1}{3}}A_0 + \frac{1}{\sqrt{6}}\,A_2$$

$$A[K^+ \to \pi^+\pi^0] \equiv \frac{\sqrt{3}}{2}A_2$$

$\left|\frac{A_0}{A_2}\right| = 22.4$ and the naive gives $\sqrt{2}$

$\left|\frac{A_0}{A_2}\right| = 18$ (p^2 part) fit to $K \to 2\pi, 3\pi$ up to order p^4 JB, Borg 2005

For later use: $A_I = -i a_I e^{i\delta_I}$

$\delta_0 - \delta_2$

New analysis a_0, a_2 and $\delta_0 - \delta_2$ Cirigilano, Ecker, Pich, 0907.1451

Older:

Cirigliano, Ecker, Neufeld, Pich, hep-ph/0310351

JB, Borg, hep-ph/0405025, hep-ph/0410333, hep-ph/0501163

$\delta_0 - \delta_2 = (60.8 \pm 2.2 \pm 3.1)^o = (57.9 \pm 1.5 \pm ?)^o$ (NLO fit for all)

Now fit to data but only use ChPT at order p^4 for the isospin breaking parts

$\delta_0 - \delta_2 = (52.84 \pm 0.83)^o$

Change 3^o from experiment and 5^o from theory.

$\delta_0 - \delta_2$

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$\delta_0 - \delta_2 = (52.84 \pm 0.83)^o$

$\delta_0 - \delta_2 = (47.7 \pm 1.5)^o$ $\pi\pi$ Colangelo, Gasser, Leutwyler

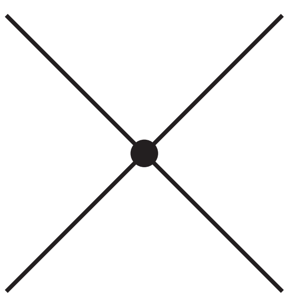
$$\left|\frac{A_0}{A_2}\right| = 21.63 \pm 0.04$$

Chiral Perturbation Theory

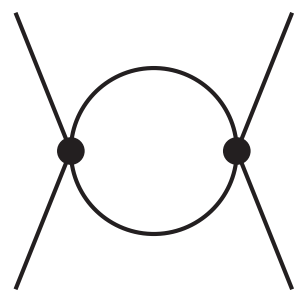
Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting in momenta/masses

Power counting in momenta: Meson loops



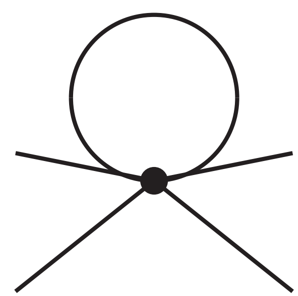
p^2



$$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$$

$1/p^2$

$\int d^4p$ $\quad p^4$



$$(p^2)\,(1/p^2)\,p^4 = p^4$$

Hard pion ChPT?

- Baryon and Heavy Meson ChPT: $p, n, \ldots\ B, B^*$ or D, D^*
  - $p = M_B v + k$
  - Everything else soft
  - Works because baryon or b or c number conserved.
  - Decay constant works: takes away all heavy momentum
  - General idea: M_B dependence can always be reabsorbed in LECs, is analytic in the other parts k.

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- (Heavy) (Vector or other) Meson ChPT:
  - (Vector) Meson: $p = M_V v + k$
  - Everyone else soft or $p = M_V + k$
  - General idea: M_V dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- (Heavy) (Vector) Meson ChPT:
  - $p = M_V v + k$
  - First: only keep diagrams where vectors always present
  - Applied to masses and decay constants
  - Decay constant works: takes away all heavy momentum
  - *It was argued that this could be done, the nonanalytic parts of diagrams with pions at large momenta are reproduced correctly*
  - Done both in relativistic and heavy meson type of formalism

Hard pion ChPT?

- Heavy Kaon ChPT:
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon always goes through
  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,...
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
  - Works like all the previous *heavy* ChPT

Hard pion ChPT?

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  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,...
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
- Flynn-Sachrajda also argued that $K_{\ell 3}$ could be done for q^2 away from q^2_{max}.
- JB-Celis Argument generalizes to other processes with hard/fast pions and applied to $K \to \pi\pi$
- General idea: heavy/fast dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q\to 0}\langle \pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

Hard pion ChPT?

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- soft pion couplings are constrained by current algebra

$$\lim_{q\to 0}\langle\pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

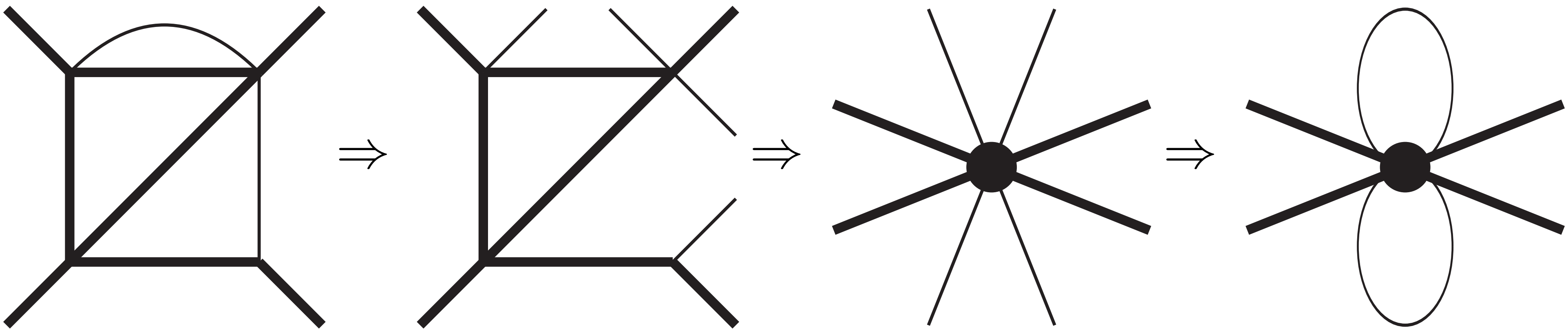
- Nothing prevents hard pions to be in the states α or β
- So by heavily using current algebra I should be able to get the light quark mass nonanalytic dependence

Hard pion ChPT?

Field Theory: a process at given external momenta

- Take a diagram with a particular internal momentum configuration
- Identify the soft lines and cut them
- The result part is analytic in the soft stuff
- So should be describably by an effective Lagrangian with coupling constants dependent on the external given momenta
- If symmetries present, Lagrangian should respect them
- Lagrangian should be complete in *neighbourhood*
- Loop diagrams with this effective Lagrangian *should* reproduce the nonanalyticities in the light masses
  Crucial part of the argument

Hard pion ChPT?



This procedure works at one loop level, matching at tree level, nonanalytic dependence at one loop:

- Toy models and vector meson ChPT JB, Gosdzinsky, Talavera
- Recent work on relativistic meson ChPT Gegelia, Scherer et al.
- Extra terms kept in $K \to 2\pi$: a one-loop check
- Some preliminary two-loop checks

$K \to 2\pi$ in $SU(2)$ ChPT

Add $K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$ Roessl

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{F^2}{4}\left(\langle u_\mu u^\mu\rangle + \langle \chi_+\rangle\right),$$
$$\mathcal{L}^{(1)}_{\pi K} = \nabla_\mu K^\dagger \nabla^\mu K - \overline{M}^2_K K^\dagger K\,,$$
$$\mathcal{L}^{(2)}_{\pi K} = A_1\langle u_\mu u^\mu\rangle K^\dagger K + A_2\langle u^\mu u^\nu\rangle \nabla_\mu K^\dagger \nabla_\nu K + A_3 K^\dagger \chi_+ K + \cdots$$

Add a spurion for the weak interaction $\Delta I = 1/2$, $\Delta I = 3/2$

JB,Celis

$$t^{ij}_k \longrightarrow t^{i'j'}_{k'} = t^{ij}_k (g_L)_{k'}{}^k (g_L^\dagger)_i{}^{i'} (g_L^\dagger)_j{}^{j'}$$
$$t^i_{1/2} \longrightarrow t^{i'}_{1/2} = t^i_{1/2} (g_L^\dagger)_i{}^{i'}.$$

$K \to 2\pi$ **in** $SU(2)$ **ChPT**

The $\Delta I = 1/2$ terms: $\tau_{1/2} = t_{1/2} u^\dagger$

$$\begin{aligned}\mathcal{L}_{1/2} &= iE_1\,\tau_{1/2}K + E_2\,\tau_{1/2}u^\mu\nabla_\mu K + iE_3\langle u_\mu u^\mu\rangle\tau_{1/2}K \\ &\quad + iE_4\tau_{1/2}\chi_+K + iE_5\langle\chi_+\rangle\tau_{1/2}K + E_6\tau_{1/2}\chi_-K \\ &\quad + E_7\langle\chi_-\rangle\tau_{1/2}K + iE_8\langle u_\mu u_\nu\rangle\tau_{1/2}\nabla^\mu\nabla^\nu K + \cdots + h.c.\,.\end{aligned}$$

Note: higher order terms kept in both $\mathcal{L}_{1/2}$ and $\mathcal{L}^{(2)}_{\pi K}$ to check the arguments

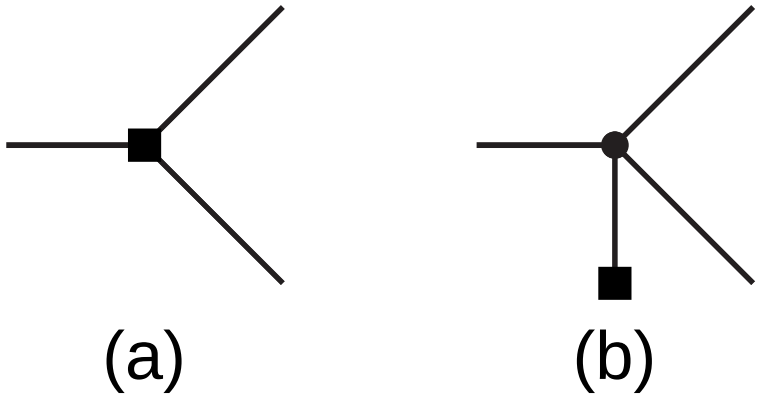
Using partial integration,...:

$\langle\pi(p_1)\pi(p_2)|O|K(p_K)\rangle =$

$f(\overline{M}^2_K)\langle\pi(p_1)\pi(p_2)|\tau_{1/2}K|K(p_K)\rangle + \lambda M^2 + \mathcal{O}(M^4)$

O any operator in $\mathcal{L}_{1/2}$ or with more derivatives. Similar for $\mathcal{L}_{3/2}$

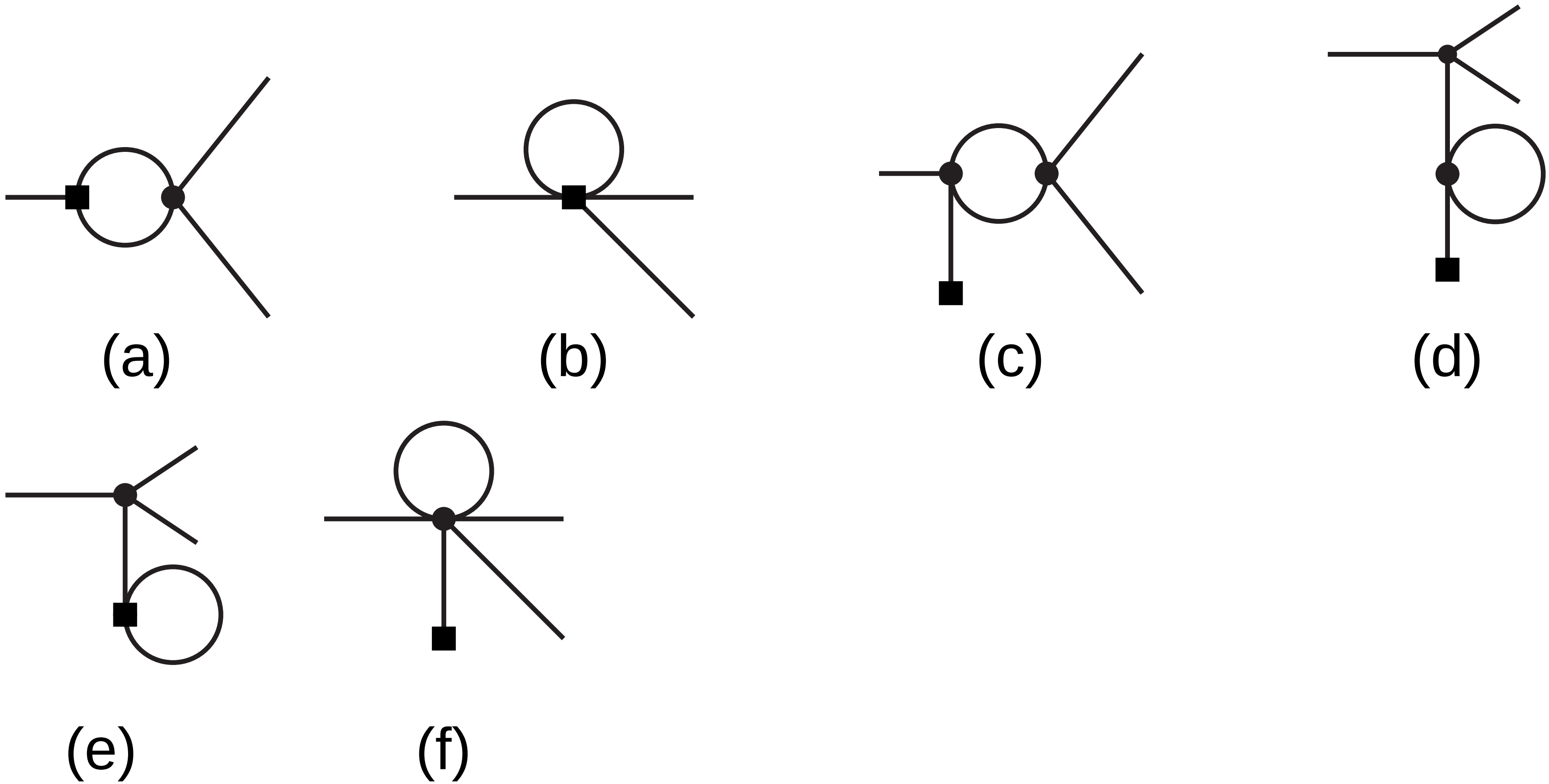
Tree level



$$A_0^{LO} = \frac{\sqrt{3}i}{2F^2}\left[-\frac{1}{2}E_1 + (E_2 - 4E_3)\,\overline{M}_K^2 + 2E_8\overline{M}_K^4 + A_1E_1\right]$$

$$A_2^{LO} = \sqrt{\frac{3}{2}}\,\frac{i}{F^2}\left[(-2D_1 + D_2)\,\overline{M}_K^2\right]$$

One loop



One loop

Diagram	A_0	A_2
Z	$-\frac{2F^2}{3}A_0^{LO}$	$-\frac{2F^2}{3}A_2^{LO}$
(a)	$\sqrt{3}i\left(-\frac{1}{3}E_1+\frac{2}{3}E_2\overline{M}_K^2\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{2}{3}D_2\overline{M}_K^2\right)$
(b)	$\sqrt{3}i\left(-\frac{5}{96}E_1-\left(\frac{7}{48}E_2+\frac{25}{12}E_3\right)\overline{M}_K^2+\frac{25}{24}E_8\overline{M}_K^4\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{61}{12}D_1+\frac{77}{24}D_2\right)\overline{M}_K^2$
(e)	$\sqrt{3}i\frac{3}{16}A_1E_1$	
(f)	$\sqrt{3}i\left(\frac{1}{8}E_1+\frac{1}{3}A_1E_1\right)$	

The coefficients of $\overline{A}(M^2)/F^4$ in the contributions to A_0 and A_2. Z denotes the part from wave-function renormalization.

- $$\overline{A}(M^2) = -\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2}$$
- $K\pi$ intermediate state does not contribute, but did for Flynn-Sachrajda

One-loop

$$
\begin{aligned}
A_0^{NLO} &= A_0^{LO}\left(1+\frac{3}{8F^2}\overline{A}(M^2)\right)+\lambda_0 M^2+\mathcal{O}(M^4)\,,\\
A_2^{NLO} &= A_2^{LO}\left(1+\frac{15}{8F^2}\overline{A}(M^2)\right)+\lambda_2 M^2+\mathcal{O}(M^4)\,.
\end{aligned}
$$

One-loop

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$$A_2^{NLO} = A_2^{LO}\left(1+\frac{15}{8F^2}\overline{A}(M^2)\right)+\lambda_2 M^2+\mathcal{O}(M^4)\,.$$

Match with three flavour $SU(3)$ calculation Kambor, Missimer, Wyler; JB, Pallante, Prades

$$A_0^{(3)LO} = -\frac{i\sqrt{6}CF_0^4}{\overline{F}_K F^2}\left(G_8+\frac{1}{9}G_{27}\right)\overline{M}_K^2\,, \qquad A_2^{(3)LO} = -\frac{i10\sqrt{3}CF_0^4}{9\overline{F}_K F^2}G_{27}\overline{M}_K^2\,,$$

When using $F_\pi = F\left(1+\frac{1}{F^2}\overline{A}(M^2)+\frac{M^2}{F^2}l_4^r\right)\,,\ F_K = \overline{F}_K\left(1+\frac{3}{8F^2}\overline{A}(M^2)+\cdots\right)\,,$

logarithms at one-loop agree with above

A two-loop check

- Similar arguments to JB-Celis, Flynn-Sachrajda work for the pion vector and scalar formfactor
- Therefore at any t the chiral log correction must go like the one-loop calculation.
- But note the one-loop log chiral log is with $t >> m_\pi^2$
- Predicts

$$F_V(t, M^2) = F_V(t, 0)\,\left(1 - \frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

$$F_S(t, M^2) = F_S(t, 0)\,\left(1 - \frac{5}{2}\frac{M^2}{16\pi^2 F^2}\ln\frac{M^2}{\mu^2} + \mathcal{O}(M^2)\right)$$

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- Note that $F_{V,S}(t,0)$ is now a coupling constant and can be complex
- Take the full two-loop ChPT calculation JB,Colangelo,Talavera and expand in $t >> m_\pi^2$.

A two-loop check

Full two-loop ChPT JB,Colangelo,Talavera, expand in $t >> m_\pi^2$:

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with

$$F_V(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(\frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6}\ln\frac{t}{\mu^2}\right)$$
$$F_S(t,0) = 1 + \frac{t}{16\pi^2F^2}\left(1 - 16\pi^2 l_4^r + i\pi - \ln\frac{t}{\mu^2}\right)$$

- The needed coupling constants are complex
- Both calculations have two-loop diagrams with overlapping divergences
- The chiral logs should be valid for any t where a pointlike interaction is a valid approximation

$K \to 3\pi$ in ChPT

- $K \to 3\pi$ Decays in Chiral Perturbation Theory, J. Bijnens, P. Dhonte and F. Persson, hep-ph/0205341, Nucl. Phys. B648 (2003) 317-344.
- Isospin Breaking in $K \to 3\pi$ Decays I: Strong Isospin Breaking, J. Bijnens and F. Borg, hep-ph/0405025, Nuclear Physics B697 (2004) 319-342.
- Isospin Breaking in $K \to 3\pi$ Decays II: Radiative Corrections, J. Bijnens and F. Borg, hep-ph/0410333, accepted in Eur. Phys. J. C.
- Isospin Breaking in $K \to 3\pi$ Decays III: Bremsstrahlung and Fit to Experiment, J. Bijnens and F. Borg, hep-ph/0501163.

Note: Fredrik Borg = Fredrik Persson, obtained PhD 28/1/2005

$K \to 3\pi$: Overview

- ChPT in the nonleptonic mesonic sector
- Lagrangians
- $K \to 3\pi$ kinematics and isospin
- Overview of calculations and results
- Data and Fits

ChPT in the nonleptonic sector

- some earlier work: especially on decays with photons
- Kambor, Missimer, Wyler (KMW) : Constructed $\mathcal{L}$ and ∞ 1990
- G. Esposito-Farese: Checked $\mathcal{L}$ and ∞ 1991
- KMW : Calculated $K \to 2\pi$ and $K \to 3\pi$ 1991
- Donoghue + Holstein + KMW : clarified the relations between observables 1992
- BUT: explicit formulas lost (US Mail)
- Kambor, Ecker, Wyler : simplified octet $\mathcal{L}$ 1993
- $K \to 2\pi$ redone: Bijnens, Pallante, Prades 1998

ChPT in the nonleptonic sector

- First paper: redo $K \to 3\pi$
- Ecker Isidori Muller Neufeld Pich: Electromagnetic octet $\mathcal{L}$ Lagrangian plus ∞ 2000
- applications to $K \to 2\pi$: Several papers
- Isospin breaking in $K \to 3\pi$: remaining papers $\mathcal{O}\left(p^4, p^2(m_u - m_d), e^2p^2\right)$.

Lagrangians: p^2 and e^2

$$\mathcal{L}_2 = \mathcal{L}_{S2} + \mathcal{L}_{W2} + \mathcal{L}_{E2} \qquad \mathcal{L}_{S2} = \frac{F_0^2}{4}\,\langle u_\mu u^\mu + \chi_+ \rangle$$

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$$u_\mu = i u^\dagger D_\mu U\, u^\dagger = u_\mu^\dagger\,, \quad u^2 = U\,, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u\,,$$

U contains the Goldstone boson fields

$$U = \exp\left(\frac{i\sqrt{2}}{F_0} M\right)\,, \qquad M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K^0} & \frac{-2}{\sqrt{6}}\eta_8 \end{pmatrix}.$$

$$\text{Here} \quad \chi = 2B_0 \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} \quad \text{and} \quad D_\mu U = \partial_\mu U - ie\,[Q, U]\,.$$

Lagrangians: p^2 and e^2

$$\mathcal{L}_{W2} = CF_0^4 \left[G_8 \langle \Delta_{32} u_\mu u^\mu \rangle + G_8' \langle \Delta_{32} \chi_+ \rangle + G_{27} t^{ij,kl} \langle \Delta_{ij} u_\mu \rangle \langle \Delta_{kl} u^\mu \rangle \right]$$

$$\Delta_{ij} \equiv u \lambda_{ij} u^\dagger \,, \quad (\lambda_{ij})_{ab} \equiv \delta_{ia}\, \delta_{jb}\,, \quad t^{21,13} = t^{13,21} = 1/3, \ldots$$

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Electromagnetic:

$$\mathcal{L}_{E2} = e^2F_0^4 Z\langle\mathcal{Q}_L\mathcal{Q}_R\rangle + \left(e^2CF_0^6G_E\langle\Delta_{32}\mathcal{Q}_R\rangle\right)$$

$$\mathcal{Q}_L = uQu^\dagger\,, \quad \mathcal{Q}_R = u^\dagger Qu \ \text{ with } \ Q = \mathrm{diag}\,(2/3, -1/3, -1/3)$$

Lagrangians: p^2 and e^2

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Note: F_0 not well known, fit $C F_0^4 G_8$,. . .

use *numerically* $F_0 = F_\pi$ to quote G_8,. . .

Lagrangians: p^4 and e^2p^2

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$\mathcal{L}_{W4}$ thirteen N_i^r (octet) and twelve D_i^r might contribute
(two N_i^r and two D_i^r extra for photon-reducible)

$\mathcal{L}_{S2E2}$ eleven K_i^r can contribute

$\mathcal{L}_{W2E2}$ fourteen Z_i^r can contribute, 27 part not classified

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Done here :

Isospin conserved: 11 combinations of N_i^r, D_i^r relevant: $\tilde{K}_i$

7 coefficients order m_K^4, 4 order $M_K^2 m_\pi^2$

2 (virtually) indistinguishable from G_8 and G_{27}.

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Isospin Broken: 30 combinations of N_i^r, D_i^r, Z_i^r relevant

Results from isospin breaking with $K_i^r = Z_i^r = 0$

$K \to 3\pi$ Kinematics and Isospin

$$\begin{aligned}
K_L(k) &\to \pi^0(p_1)\pi^0(p_2)\pi^0(p_3)\,, \quad [A^L_{000}]\,,\\
K_L(k) &\to \pi^+(p_1)\pi^-(p_2)\pi^0(p_3)\,, \quad [A^L_{+-0}]\,,\\
K_S(k) &\to \pi^+(p_1)\pi^-(p_2)\pi^0(p_3)\,, \quad [A^S_{+-0}]\,,\\
K^+(k) &\to \pi^0(p_1)\pi^0(p_2)\pi^+(p_3)\,, \quad [A_{00+}]\,,\\
K^+(k) &\to \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\,, \quad [A_{++-}]\,,
\end{aligned}$$

plus charge conjugate ones

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\end{array}$$

plus charge conjugate ones
even under $p_1 \leftrightarrow p_2$ except A^S_{+-0} odd (CP and Bose symmetry)

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plus charge conjugate ones

Kinematics:

$$s_1 = (k-p_1)^2\,,\ s_2 = (k-p_2)^2\,,\ s_3 = (k-p_3)^2\,.$$

Dalitz plot variables:

$$y = (s_3 - s_0)/m^2_{\pi^+}\,,\ x = (s_2 - s_1)/m^2_{\pi^+}\,,\ s_0 = (s_1+s_2+s_3)/3\,.$$

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Bremsstrahlung problem: s_i from E_i or $m^2_{\pi\pi}$ different

Kinematics and Isospin

Valid also at p^6

$$
\begin{aligned}
A^L_{000} &= M_0(s_1)+M_0(s_2)+M_0(s_3)\,,\\
A^L_{+-0} &= M_1(s_3)+M_2(s_1)+M_2(s_2)+M_3(s_1)(s_2-s_3)+M_3(s_2)(s_1-s_3)\,,\\
A^S_{+-0} &= M_4(s_1)-M_4(s_2)+M_5(s_1)(s_2-s_3)-M_5(s_2)(s_1-s_3)+M_6(s_3)(s_1-s_2)\,,\\
A_{00+} &= M_7(s_3)+M_8(s_1)+M_8(s_2)+M_9(s_1)(s_2-s_3)+M_9(s_2)(s_1-s_3)\,,\\
A_{++-} &= M_{10}(s_3)+M_{11}(s_1)+M_{11}(s_2)+M_{12}(s_1)(s_2-s_3)+M_{12}(s_2)(s_1-s_3)\,.
\end{aligned}
$$

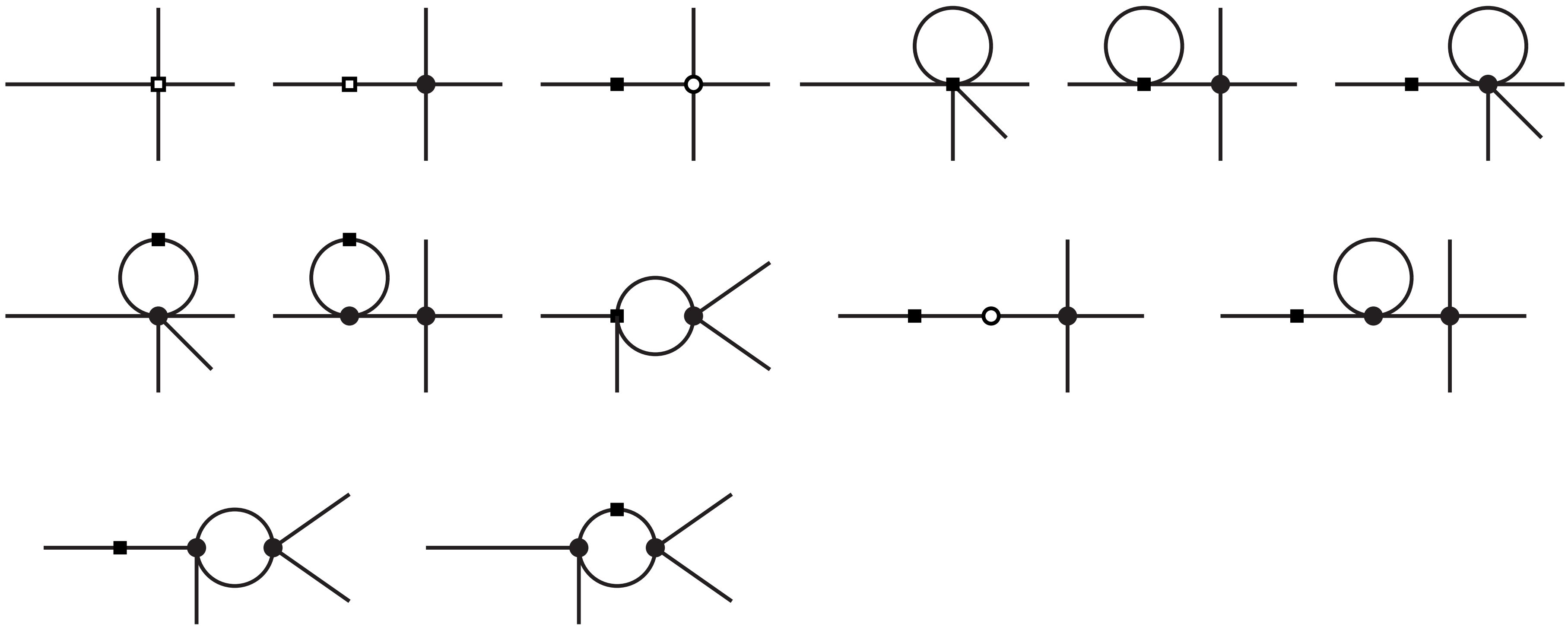
polynomial ambiguity from $s_1 + s_2 + s_3 = \sum_i m_i^2$

Isospin:

$$
\begin{aligned}
M_0(s) &= M_1(s) + 2M_2(s)\,,\\
2M_7(s) + 4M_8(s) &= M_{10}(s) + 2M_{11}(s)\,,\\
M_4(s) &= \frac{1}{3}\left(M_7(s) - M_8(s) + M_{10}(s) - M_{11}(s)\right)\\
M_5(s) - M_6(s) &= M_9(s) + M_{12}(s)\,.
\end{aligned}
$$

Calculations: isospin limit

The diagrams of order p^4



Expressions for the $M_i(s)$: see first paper

Confirmed by Gamiz, Prades, Scimemi, hep-ph/0305164
Lashin, hep-ph/0308200 (how not clear)

I: Strong Isospin Breaking

Includes:

- $m_u - m_d$ and the effects of $\mathcal{L}_{E2}$, $\mathcal{L}_{E2S2}$ and $\mathcal{L}_{W2E2}(G_8)$
- π^0-η mixing (η' via LECs)
- Same diagrams as before
- Formulas immediately a lot longer: see
  `http://www.thep.lu.se/~bijnens/chpt.html`

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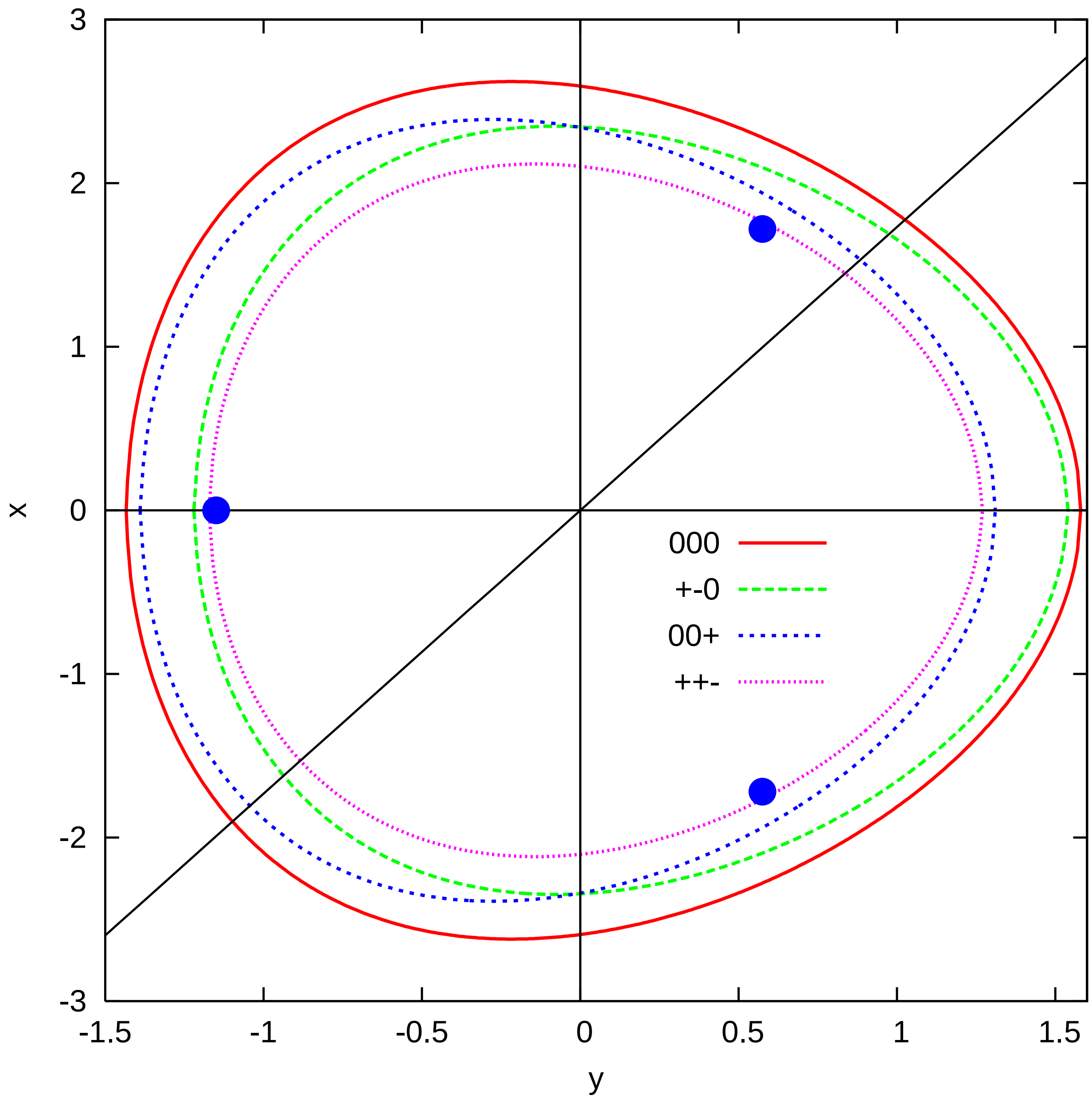
Effects:

- F_{π^+}, F_{K^+} overall factor
- Mass difference π^+-π^0
- Others typically a few %

Phasespace



Curves:

Phasespace boundaries

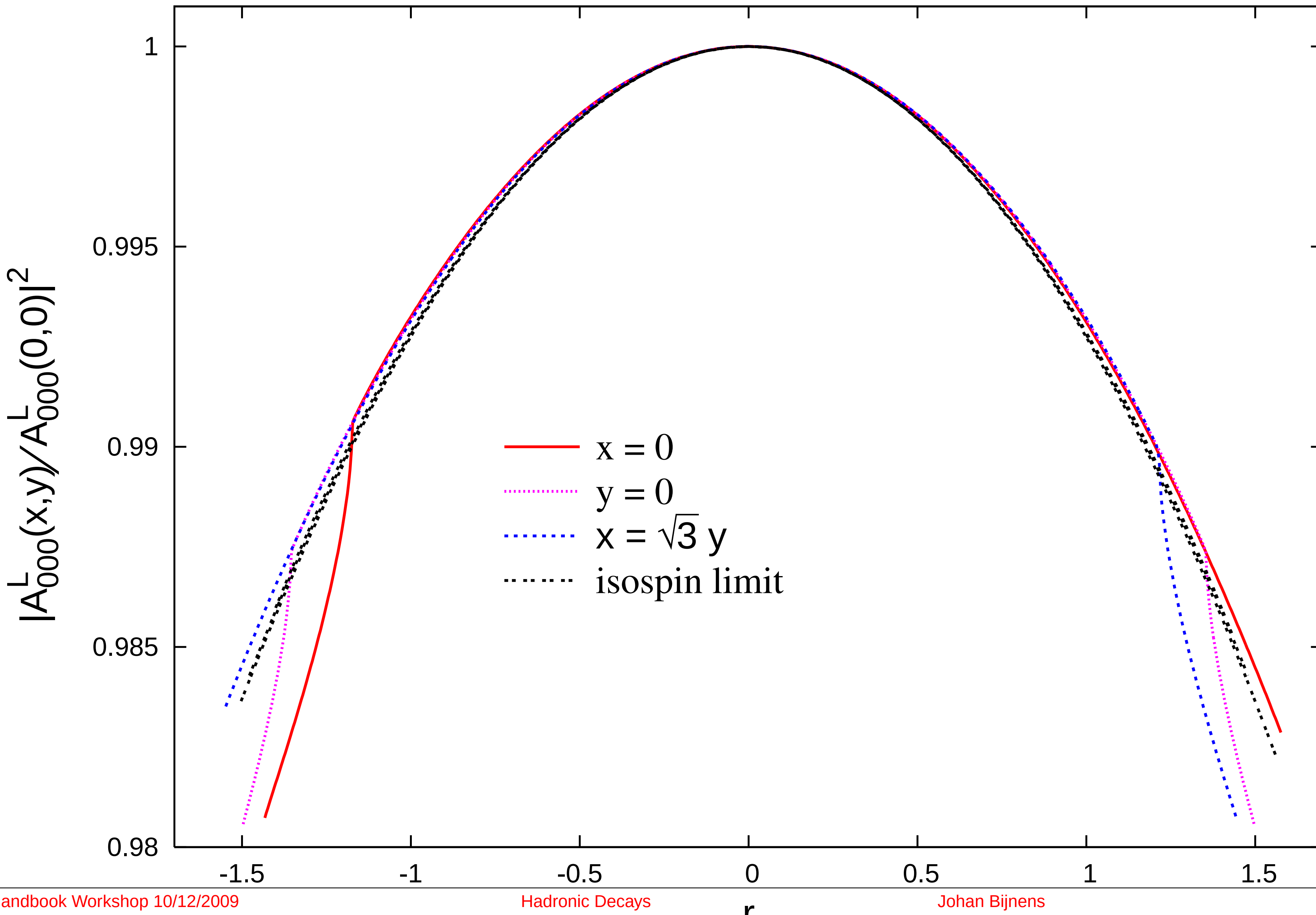
Dots:

$\pi\pi$ pair at rest

Lines:

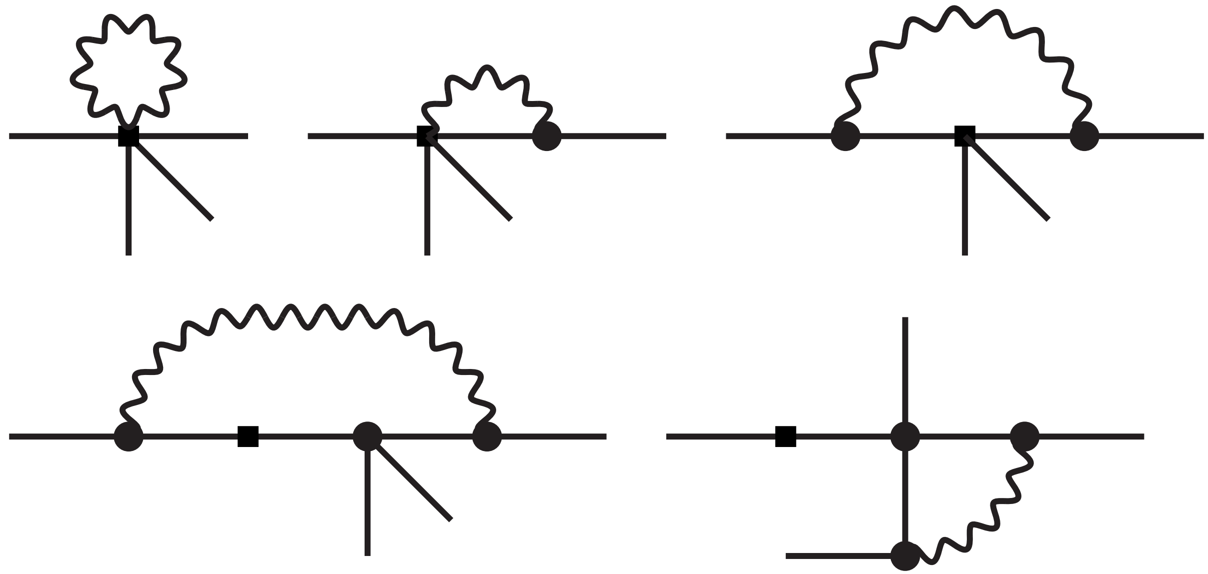
Show $|A|^2$ along these

$K_L \to \pi^0\pi^0\pi^0$



II: Radiative Corrections

- Photon Loops



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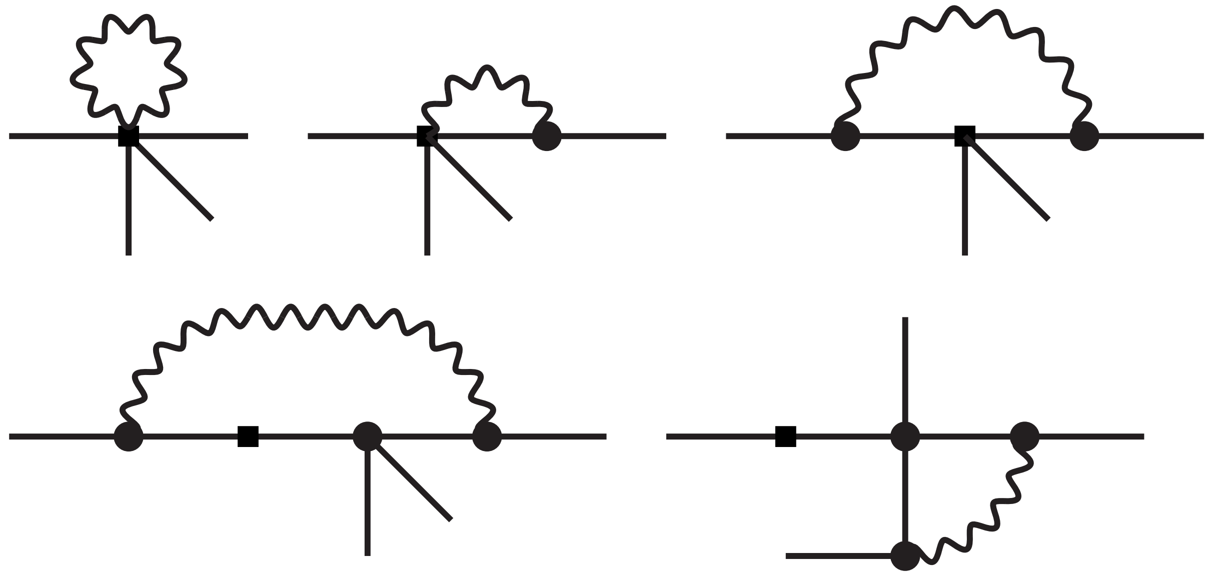
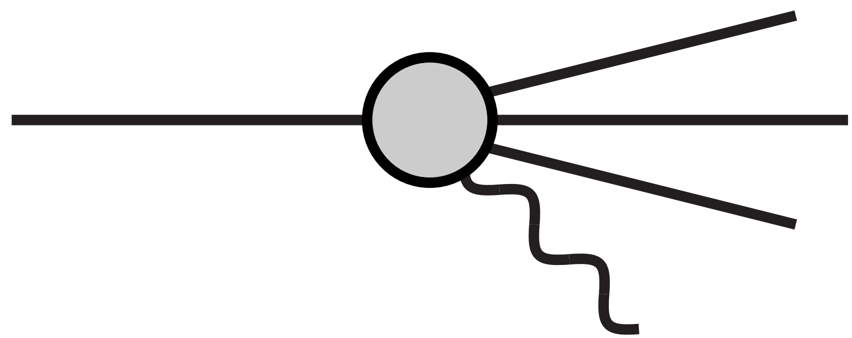
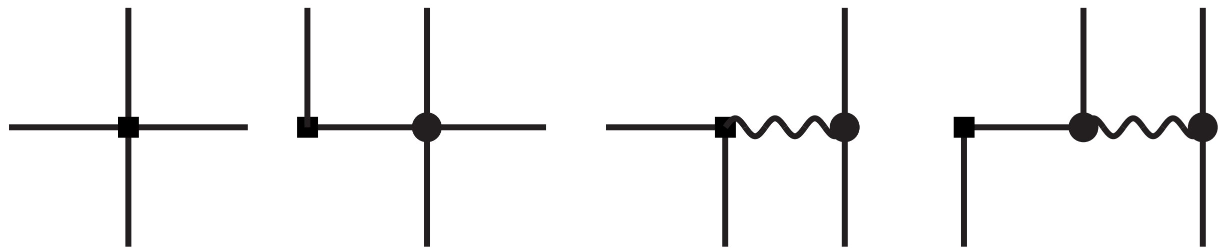


- Soft Bremsstrahlung



II: Radiative Corrections

- Photon reducible diagrams
- Tree Level:



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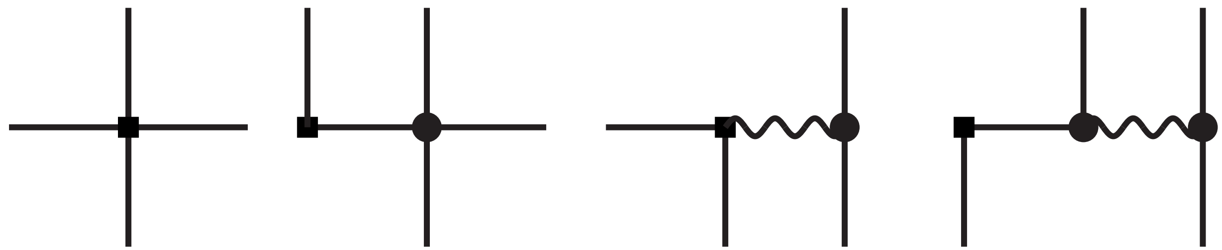

Extra vanish exactly

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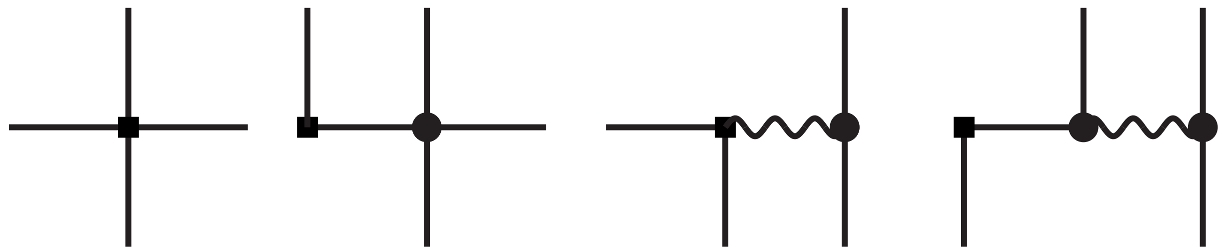

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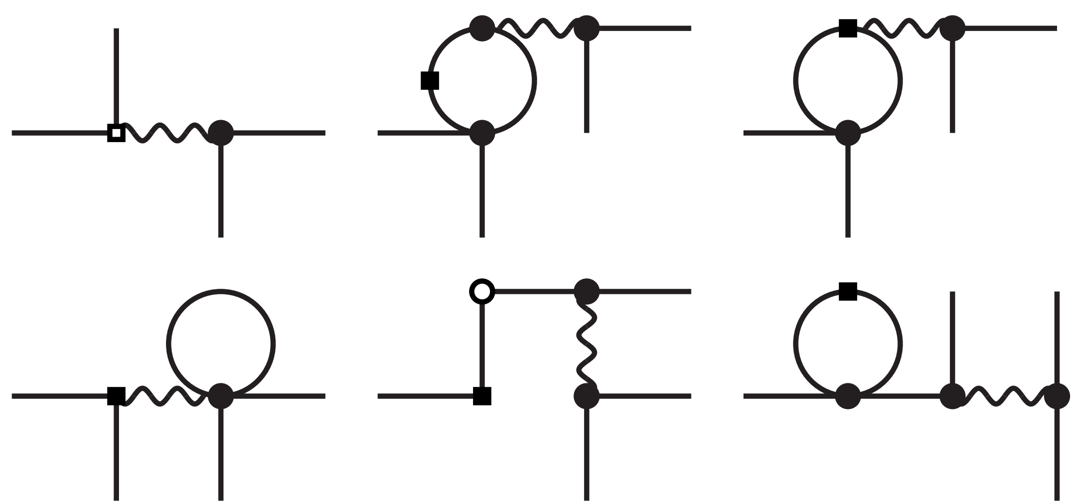
- One-loop Level:



Extra N_i^r, D_i^r from $K \to \pi \ell^+ \ell^-$

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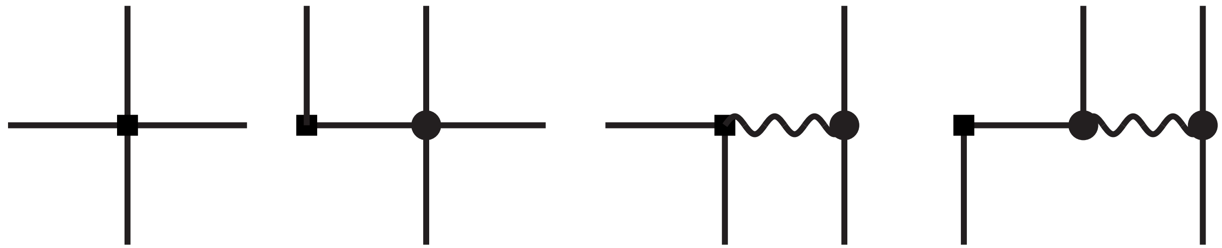

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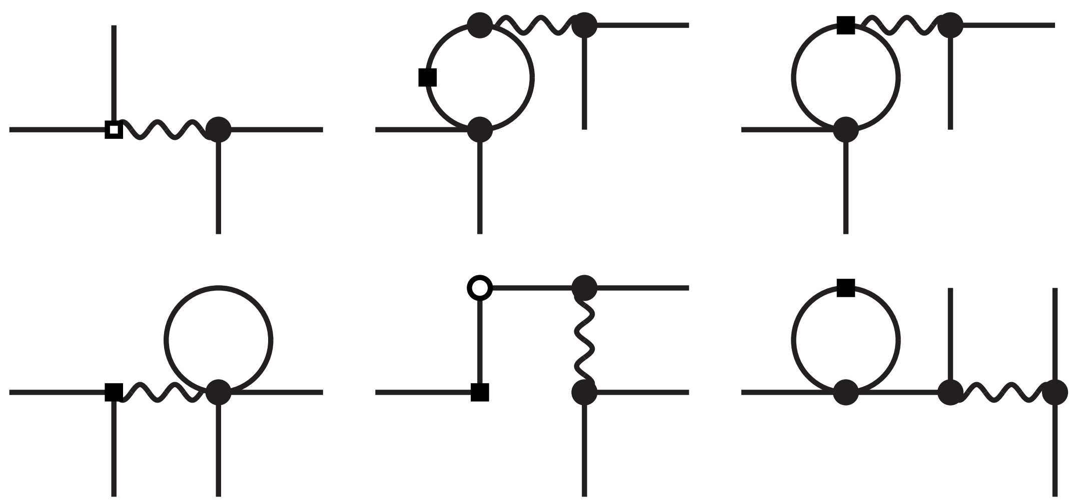
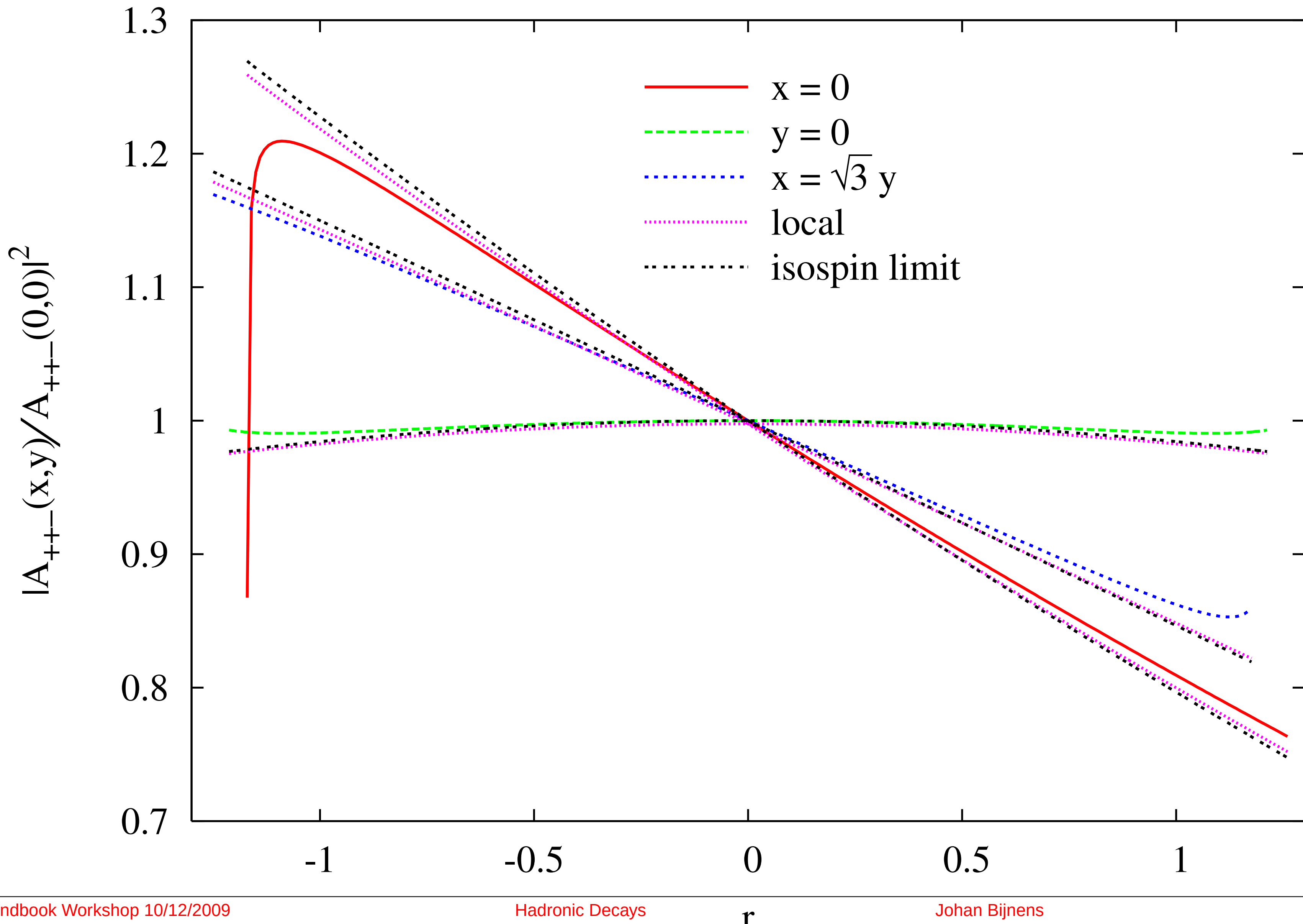


Extra N_i^r, D_i^r from $K \to \pi \ell^+ \ell^-$

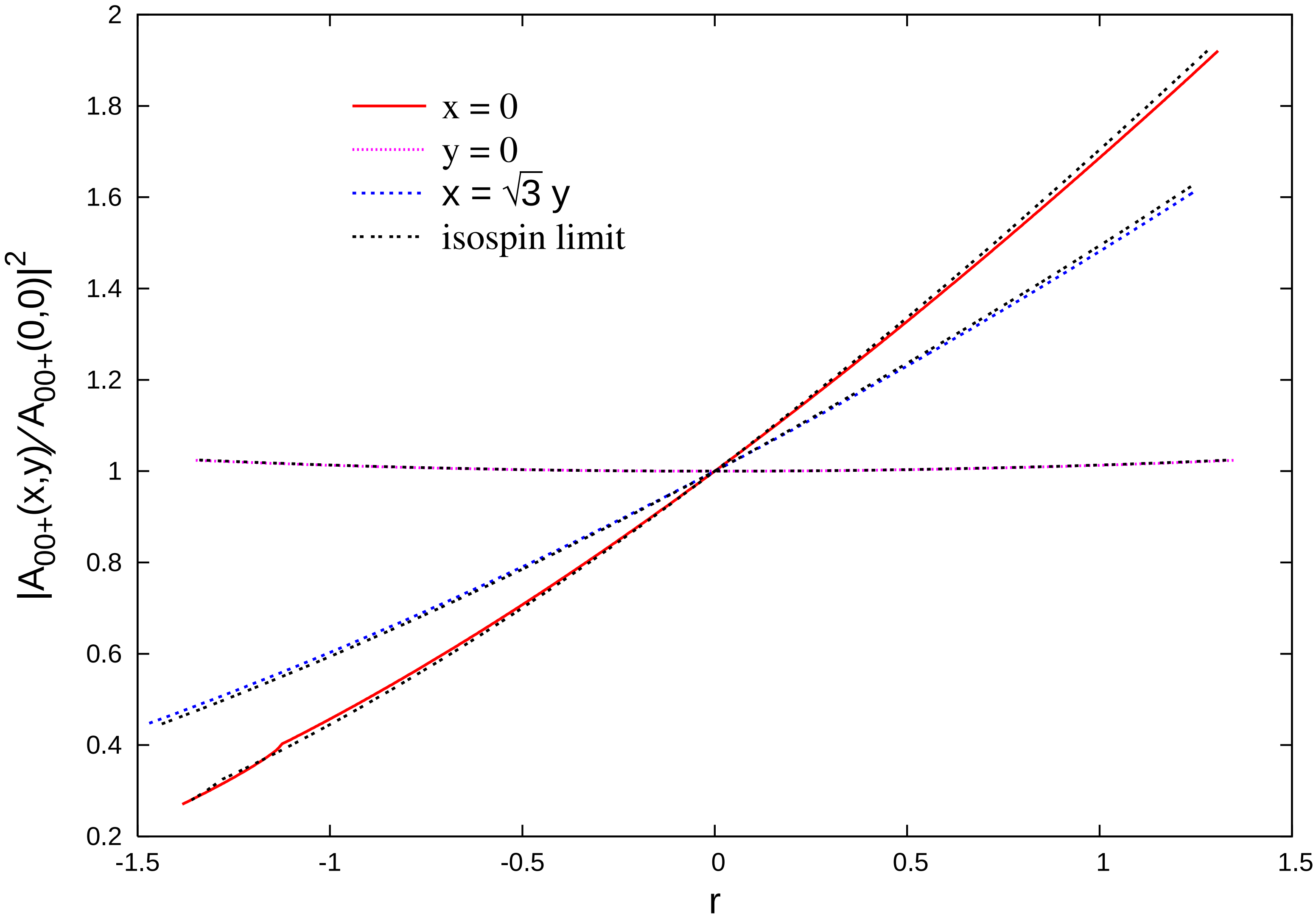
Numerically Negligible

$K^+ \to \pi^+\pi^+\pi^-$



1.3
1.2
1.1
1
0.9
0.8
0.7
-1
-0.5
0
0.5
1
r
$|A_{++-}(x,y)/A_{++-}(0,0)|^2$
x = 0
y = 0
x = √3 y
local
isospin limit

$K^+ \to \pi^0\pi^0\pi^+$



Note: Disagree numerically with Nehme, hep-ph/0406209

III: Hard Bremsstrahlung

- Include hard photon Bremsstrahlung to lowest order
  Full amplitude from Low's theorem
- Do everything also for $K \to 2\pi$ to have same treatment
  Note: some new pieces, e.g., $e^2 p^2 G_{27}$

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Checks:
- ⊛ Everything done twice indepently
- ⊛ UV infinities cancel analytically: no $1/(d-4)$
- ⊛ IR infinities cancel analytically: no m_γ
- ⊛ soft-hard Bremsstrahlung match: no E_γ
  (except for F_π, F_K parts)

Treatment of Bremsstrahlung

Old experiments: what to do?

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We adopted:

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- Fit $|A(s_0, s_0, s_0)|^2$, g, h, k with soft Bremsstrahlung and loops, ...

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We really need to know what Phasespace/Photons covered

Results

$K \to \pi\pi$ only: essentially same as Ecker et al.

Tree level: $G_8 = 10.36 \quad G_{27} = 0.550$

Full: $G_8 = \ 5.39 \quad G_{27} = 0.359$

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Full fit:

μ	0.77 GeV	1.0 GeV	0.6 GeV	0.77 GeV
G_8	5.39(1)	4.60(1)	6.43(1)	5.39(1)
G_{27}	0.359(2)	0.301(1)	0.438(2)	0.359(2)
$\delta_2 - \delta_0$	$-57.9(1.5)^o$	$-57.3(1.4)^o$	$-58.9(1.4)^o$	$-57.9(1.4)^o$
$10^3 \tilde{K}_1/G_8$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$
$10^3 \tilde{K}_2/G_8$	48.5(2.4)	56.5(2.4)	41.2(1.9)	46.6(1.6)
$10^3 \tilde{K}_3/G_8$	2.6(1.2)	$-$1.7(1.1)	6.7(1.0)	3.5(0.8)
$10^3 \tilde{K}_4/G_{27}$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$
$10^3 \tilde{K}_5/G_{27}$	$-$41.2(16.9)	$-$52.0(17.7)	$-$31.1(12.0)	$-$27.0(8.3)
$10^3 \tilde{K}_6/G_{27}$	$-$102(105)	$-$114(105)	$-93(76)$	$\equiv 0$
$10^3 \tilde{K}_7/G_{27}$	78.6(33)	78.0(33.5)	79.6(22.7)	50.0(13.0)
χ^2/DOF	29.3/10	27.2/10	33.0/10	30.5/11

vary $\tilde{K}_1$, $\tilde{K}_4$ see first paper

Octet and μ-variation

μ	octet 0.77 GeV	μ variation 1.0 GeV	μ variation 0.6 GeV
G_8	4.84(1)	–	–
G_{27}	0.430(1)	–	–
$\delta_2 - \delta_0$	$-57.9(0.2)^o$	–	–
$10^3 \tilde{K}_1/G_8$	2.0(1)	−5.88	5.61
$10^3 \tilde{K}_2/G_8$	63.0(1.5)	−2.69	2.57
$10^3 \tilde{K}_3/G_8$	−6.0(7)	0.159	−0.152
$10^3 \tilde{K}_4/G_{27}$	$\equiv 0$	−9.93	9.48
$10^3 \tilde{K}_5/G_{27}$	$\equiv 0$	0	0
$10^3 \tilde{K}_6/G_{27}$	$\equiv 0$	27.0	−25.8
$10^3 \tilde{K}_7/G_{27}$	$\equiv 0$	−21.5	20.5
$10^3 \tilde{K}_8/G_8$	20.4(1)	−0.546	0.521
$10^3 \tilde{K}_9/G_8$	9.1(1)	−2.92	2.79
$10^3 \tilde{K}_{10}/G_8$	$\equiv 0$	11.6	−11.1
$10^3 \tilde{K}_{11}/G_8$	$\equiv 0$	−1.66	1.58
χ^2/DOF	33.3/10	–	–

Octet: $G_{27} = 0$
keep also $\tilde{K}_i$
with $m_K^2 m_\pi^2$ factors

μ variation
$\tilde{K}_i(\mu) - \tilde{K}_i(0.77\ GeV)$

Fit about same as before, some experiments fit better, others smaller error

Models

See Ecker, Kambor, Wyler 1993 for explanations

- Vector octet dominance $\Longrightarrow \tilde{K}_3 = -\frac{1}{2}\tilde{K}_2$: Not at all

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$$
\begin{aligned}
N_1^r &= 2k_f(32/3\,L_1^r + 4\,L_3^r + 2/3\,L_9^r)\,,\\
N_2^r &= 2k_f(16/3L_1^r + 4L_3^r + 10/3\,L_9^r)\,,\\
N_3^r &= 2k_f(8L_2^r - 2L_9^r)\,,\\
N_4^r &= 2k_f(-16/3\,L_1^r - 8/3\,L_3^r - 4/3\,L_9^r)\,,\\
N_5^r &= 2k_f(-L_5^r)\,,\\
N_6^r &= 2k_f(2/3\,L_5^r)\,,\\
N_7^r &= 2k_f(L_5^r)\,,\\
N_8^r &= 2k_f(4\,L_4^r + 2\,L_5^r)\,,\\
N_9^r &= N_{10}^r = N_{11}^r = N_{12}^r = N_{13}^r = 0
\end{aligned}
$$

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N_3^r &= 2k_f\,(8 L_2^r - 2 L_9^r)\,,\\
N_4^r &= 2k_f\,(-16/3\,L_1^r - 8/3\,L_3^r - 4/3\,L_9^r)\,,\\
N_5^r &= 2k_f\,(-L_5^r)\,,\\
N_6^r &= 2k_f\,(2/3\,L_5^r)\,,\\
N_7^r &= 2k_f\,(L_5^r)\,,\\
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\end{aligned}
$$

μ	0.77 GeV	0.9 GeV	0.842 Ge
G_8	4.18(1)	4.42	4.22(1)
G_{27}	0.360(2)	0.326(10)	0.339(10
k_F	2.61(1)	4.94(2)	3.60(5)
χ^2/DOF	109/14	182/14	60.4/13

WDM or $k_f = 1/2$ No

Factorization: surprisingly OK

But k_f not naive one

Data and Fits: NA48/2 missing

Decay	Width [GeV]	ChPT [GeV]	Fact. [GeV]
$K^+ \to \pi^+\pi^0$	$(1.1231 \pm 0.0078) \cdot 10^{-17}$	$1.123 \cdot 10^{-17}$	$1.127 \cdot 10^{-17}$
$K_S \to \pi^0\pi^0$	$(2.2828 \pm 0.0104) \cdot 10^{-15}$	$2.282 \cdot 10^{-15}$	$2.283 \cdot 10^{-15}$
$K_S \to \pi^+\pi^-$	$(5.0691 \pm 0.0108) \cdot 10^{-15}$	$5.069 \cdot 10^{-15}$	$5.069 \cdot 10^{-15}$
$K_L \to \pi^0\pi^0\pi^0$	$(2.6748 \pm 0.0358) \cdot 10^{-18}$	$2.618 \cdot 10^{-18}$	$2.698 \cdot 10^{-18}$
$K_L \to \pi^+\pi^-\pi^0$	$(1.5998 \pm 0.0271) \cdot 10^{-18}$	$1.658 \cdot 10^{-18}$	$1.711 \cdot 10^{-18}$
$K^+ \to \pi^0\pi^0\pi^+$	$(9.195 \pm 0.0213) \cdot 10^{-19}$	$8.934 \cdot 10^{-19}$	$8.816 \cdot 10^{-19}$
$K^+ \to \pi^+\pi^+\pi^-$	$(2.9737 \pm 0.0174) \cdot 10^{-18}$	$2.971 \cdot 10^{-18}$	$2.933 \cdot 10^{-18}$

Decay	Quantity	Experiment	ChPT	Fact.
$K_L \to \pi^0\pi^0\pi^0$	h	-0.0050 ± 0.0014	-0.0062	-0.0025
$K_L \to \pi^+\pi^-\pi^0$	g	0.678 ± 0.008	0.678	0.654
	h	0.076 ± 0.006	0.088	0.083
	k	0.0099 ± 0.0015	0.0057	0.0068
$K_S \to \pi^+\pi^-\pi^0$	γ_S	$(3.3 \pm 0.5) \cdot 10^{-8}$	$3.0 \cdot 10^{-8}$	$2.9 \cdot 10^{-8}$
$K^\pm \to \pi^0\pi^0\pi^\pm$	g	0.638 ± 0.020	0.636	0.648
	h	0.051 ± 0.013	0.077	0.080
	k	0.004 ± 0.007	0.0047	0.0069
$K^+ \to \pi^+\pi^+\pi^-$	g	-0.2154 ± 0.0035	−0.215	−0.226
	h	0.012 ± 0.008	0.012	0.019
	k	-0.0101 ± 0.0034	−0.0034	−0.0033

Estimates of the constants

- I referred already to a factorization model
- Vector meson models: radiative decays
- Real challenge: getting G_{27} and G_8
- Gluonic Penguin is the real culprit

Estimates of the constants

- The perturbative part is well known
- Need matching to the hadronic level calculation
- Bardeen, Buras, Gerard 88-01: use large N_c
- Almost all analytical work since builds on this

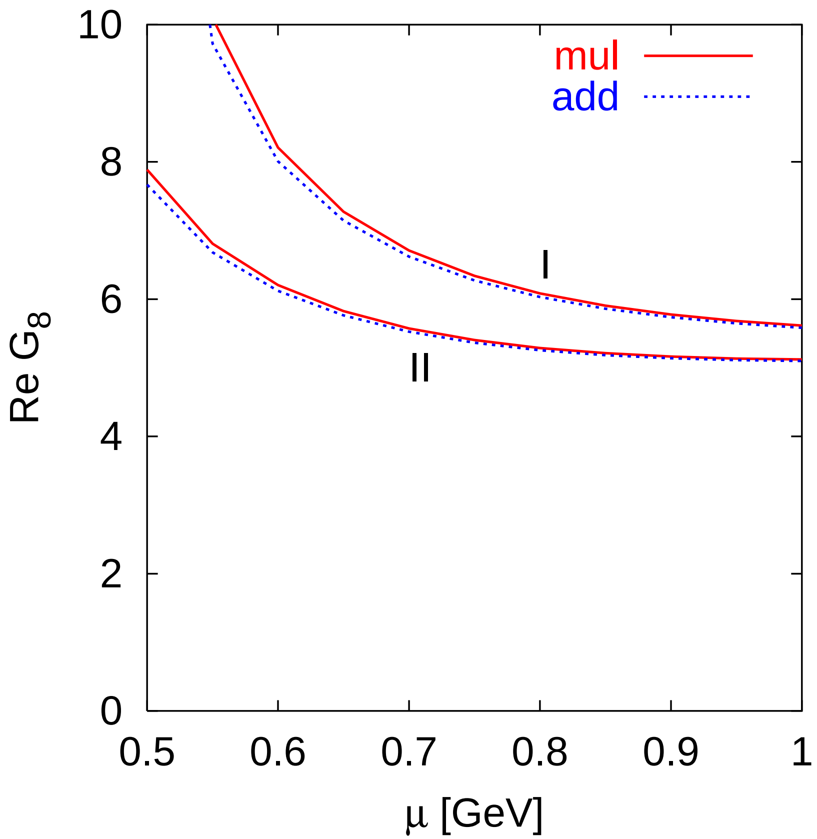
Estimates of the constants

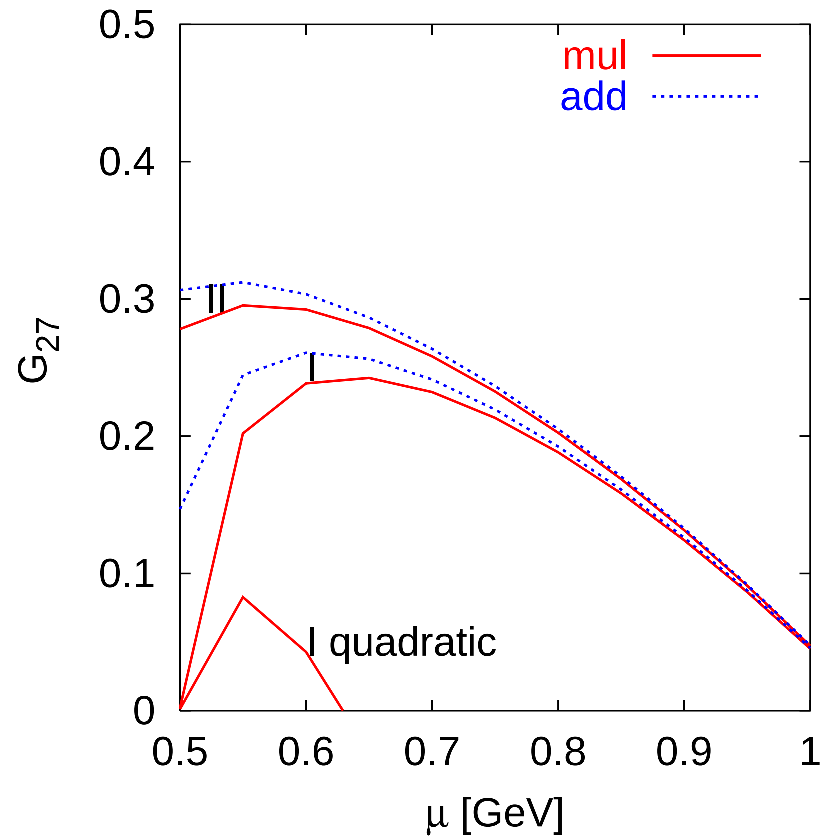
- T. Hambye et al. BBG approach hep-ph/9802300, hep-ph/9906434
- JB,Prades: consistent matching to models via a fictitious X-boson exchange so match currents only, not four-quark operators. Low Q^2: ChPT, Intermediate: ENJL plus for B_K and Q_8 better models and data. Still lacking: beyond chiral limit and improving intermediate Q^2. hep-ph/9811472, hep-ph/9911392, hep-ph/0005189, hep-ph/0108240, hep-ph/0304222, hep-ph/0601197
- Peris, de Rafael Use a MHA approach for each needed Green function, i.e. single resonance saturation per channel hep-ph/9805442, hep-ph/9812471, hep-ph/0006146, hep-ph/0305104
- Hambye et al. AdS/QCD . Essentially MHA but with a tower of (axial-)vector mesons hep-ph/0512089, hep-ph/0612010

Estimates of the constants

- Pallante, Pich Large N_c plus FSI hep-ph/0007208, hep-ph/0105011
- Earlier: Bertolini, Fabbrichesi, Eeg Chiral Quark Model hep-ph/9511255, hep-ph/9705244, hep-ph/0002234

Estimates of the constants





Remember fit: $G_8 = 5.39$ and $G_{27} = 0.36$

Conclusions

- m_{K^+}: NA48/62 or KLOE?
- $K \to \pi\pi$ $SU(2)$ chiral logarithm known
- $K \to 3\pi$ to first nontrivial order in isospin breaking fully known
- Fit done and tested a few models of the N_i
- Fit with/without isospin breaking seems similar
- Need Photon/Phasespace information from experiment: NA48/2 result needs to be included
- CP violation: partly done: Prades
- Fully from first principles: indications are that it will work but no accurate prediction with trustable error bars yet