Hadronic Decays

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Various ChPT: http://www.thep.lu.se/~bijnens/chpt.html
Overview

- What do we want to learn about?
- Basic properties: $m_K$ (and $F_K$)
- Hadronic decays: $K \rightarrow \pi\pi$
  - $\delta_0 - \delta_2$
  - In two-flavour ChPT: hard pion ChPT
- $K \rightarrow 3\pi$ in ChPT
- $K \rightarrow 2\pi, 3\pi$ isospin breaking in ChPT
- A reminder on the existing analytical work for the $\Delta I = 1/2$ rule
Later talks or not covered

- CP-violation: Prades
- The nonrelativistic calculations of final states including isospin breaking: Colangelo
- the cusp in $K \rightarrow \pi^+ \pi^0 \pi^0$: Colangelo
- Lattice: Sachrajda
- ChPT and $K_{\ell 4}$ at one and two loops: JB, 2000...
- Dispersion relations and the various decays: but see Boito, Passemar
What do we want to learn about?

Standard Model Lagrangian has four parts:

$$\mathcal{L}_H(\phi) + \mathcal{L}_G(W, Z, G) + \sum_{\psi=\text{fermions}} \bar{\psi} i D \psi + \sum_{\psi, \psi'=\text{fermions}} g_{\psi\psi'} \bar{\psi} \phi \psi'$$

What is tested?:

- gauge-fermion: Very well
- Higgs: Real tests coming up
- Gauge: Well tested in QCD, partly in electroweak
- Yukawa: Rather well tested in $B$ and $K$ (Nobel 2008)
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Discrete symmetries: C, P, T
QCD and QED: C, P, T good; Field theory: CPT good
Weak: breaks P and C, Yukawa breaks CP
What do we want to learn about?

- Yukawa sector
- QCD at low energies
- Possible beyond Standard Model effects
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- Yukawa sector
- QCD at low energies
- Possible beyond Standard Model effects

Heavy particles can contribute in loop
### The problem

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<th>Effective Theory</th>
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Basic properties: Mass

Mass:

\[ m_{K^+} = 493.677 \pm 0.016 \]

PDG2008, our fit

\[ m_{K^0} = 497.614 \pm 0.024 \]
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Mass:

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PDG2008, our fit

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Question: Can KLOE and NA48/NA62 help with the \( K^+ \)?
Basic properties: Decay constant

- $K_{\ell 2}: V_{us}$ and $F_K$
- $K_{\ell 3}: V_{us}$ and $f_+(0)$.

Two measurements, three quantities: need extra input. See:

- Flavianet Kaon Working Group
- Flavianet Lattice Averaging Group
- (Semi)leptonic decay group

$$\frac{F_K}{F_\pi} = 0.193 \pm 0.002 \pm 0.006 \pm 0.001$$
$$F_K = 109.96 \pm 0.014 \pm 0.58 \pm 0.14 \text{ MeV}$$

Uncertainty: decay rate, $V_{us}$, radiative corrections

PDG2008
$K \rightarrow \pi\pi$ and the $\Delta I = 1/2$ Rule

$K^+ \rightarrow \pi^+\pi^0$:

$K^0 \rightarrow \pi^0\pi^0$:

: IMPOSSIBLE
$K \rightarrow \pi\pi$ and the $\Delta I = 1/2$ Rule

$K^+ \rightarrow \pi^+\pi^0$ :

$K^0 \rightarrow \pi^0\pi^0$ :

Experimentally:

\[ \Gamma(K^0 \rightarrow \pi^0\pi^0) = \frac{1}{2} \Gamma(K_S \rightarrow \pi^0\pi^0) = 2.3 \times 10^{-12} \text{ MeV} \]
\[ \Gamma(K^+ \rightarrow \pi^+\pi^0) = 1.1 \times 10^{-14} \text{ MeV} \]

So the zero one is the largest !!!
Isospin amplitudes

\[ A[K^0 \rightarrow \pi^0\pi^0] \equiv \sqrt{\frac{1}{3}} A_0 - \sqrt{\frac{2}{3}} A_2 \]

\[ A[K^0 \rightarrow \pi^+\pi^-] \equiv \sqrt{\frac{1}{3}} A_0 + \frac{1}{\sqrt{6}} A_2 \]

\[ A[K^+ \rightarrow \pi^+\pi^0] \equiv \frac{\sqrt{3}}{2} A_2 \]

\[ \left| \frac{A_0}{A_2} \right| = 22.4 \text{ and the naive gives } \sqrt{2} \]

\[ \left| \frac{A_0}{A_2} \right| = 18 \text{ (} p^2 \text{ part) } \]

For later use: \[ A_I = -i a_I e^{i\delta_I} \]

fit to \( K \rightarrow 2\pi, 3\pi \) up to order \( p^4 \)  
JB, Borg 2005
New analysis $a_0, a_2$ and $\delta_0 - \delta_2$  Cirigilano, Ecker, Pich, 0907.1451

Older:

$\delta_0 - \delta_2 = (60.8 \pm 2.2 \pm 3.1)^o = (57.9 \pm 1.5\pm?)^o$ (NLO fit for all)

Now fit to data but only use ChPT at order $p^4$ for the isospin breaking parts

$\delta_0 - \delta_2 = (52.84 \pm 0.83)^o$

Change $3^o$ from experiment and $5^o$ from theory.
\[ \delta_0 - \delta_2 \]

**New analysis** \( a_0, a_2 \) and \( \delta_0 - \delta_2 \)  Cirigilano, Ecker, Pich, 0907.1451

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\[ \delta_0 - \delta_2 = (52.84 \pm 0.83)^{\circ} \]

\[ \delta_0 - \delta_2 = (47.7 \pm 1.5)^{\circ} \quad \pi \pi \text{ Colangelo, Gasser, Leutwyler} \]

\[ \left| \frac{A_0}{A_2} \right| = 21.63 \pm 0.04 \]
Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting in momenta/masses

Power counting in momenta: Meson loops

\[
\int d^4p \quad p^2 \quad (p^2)^2 \left(\frac{1}{p^2}\right)^2 p^4 = p^4
\]

\[
\frac{1}{p^2} \quad (p^2) \left(\frac{1}{p^2}\right) p^4 = p^4
\]
Baryon and Heavy Meson ChPT: $p, n, \ldots B, B^* \text{ or } D, D^*$

- $p = M_B v + k$
- Everything else soft
- Works because baryon or $b$ or $c$ number conserved.
- Decay constant works: takes away all heavy momentum
- General idea: $M_B$ dependence can always be reabsorbed in LECs, is analytic in the other parts $k$. 
Hard pion ChPT?

- Baryon and Heavy Meson ChPT: \( p, n, \ldots B, B^* \) or \( D, D^* \)
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  - General idea: \( M_B \) dependence can always be reabsorbed in LECs, is analytic in the other parts \( k \).

- (Heavy) (Vector or other) Meson ChPT:
  - (Vector) Meson: \( p = M_V v + k \)
  - Everyone else soft or \( p = M_V + k \)
  - General idea: \( M_V \) dependence can always be reabsorbed in LECs, is analytic in the other parts \( k \).
(Heavy) (Vector) Meson ChPT:

\[ p = M_V v + k \]

First: only keep diagrams where vectors always present

Applied to masses and decay constants

Decay constant works: takes away all heavy momentum

It was argued that this could be done, the nonanalytic parts of diagrams with pions at large momenta are reproduced correctly

Done both in relativistic and heavy meson type of formalism
Hard pion ChPT?

Heavy Kaon ChPT:

- \[ p = M_K v + k \]
- First: only keep diagrams where Kaon always goes through
- Applied to masses and \( \pi K \) scattering and decay constant Roessl, Allton et al.,...
- Applied to \( K_{\ell 3} \) at \( q_{max}^2 \) Flynn-Sachrajda
- Works like all the previous heavy ChPT
Hard pion ChPT?

- **Heavy Kaon ChPT:**
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon always goes through
  - Applied to masses and $\pi K$ scattering and decay constant Roessl, Allton et al., ...
  - Applied to $K\ell_3$ at $q_{max}^2$ Flynn-Sachrajda

- Flynn-Sachrajda also argued that $K\ell_3$ could be done for $q^2$ away from $q_{max}^2$.

- **JB-Celis** Argument generalizes to other processes with hard/fast pions and applied to $K \rightarrow \pi \pi$

- General idea: heavy/fast dependence can always be reabsorbed in LECs, is analytic in the other parts $k$. 
Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q \to 0} \langle \pi^k(q) | O \rangle_{\alpha \beta} = -\frac{i}{F_\pi} \langle \alpha | \left[ Q_5^k, O \right] \beta \rangle,$$
nonanalyticities in the light masses come from soft lines
soft pion couplings are constrained by current algebra
\[
\lim_{q \to 0} \langle \pi^k(q) \alpha | O | \beta \rangle = -\frac{i}{F_\pi} \langle \alpha | [Q_5^k, O] | \beta \rangle,
\]
Nothing prevents hard pions to be in the states \( \alpha \) or \( \beta \)
So by heavily using current algebra I should be able to get the light quark mass nonanalytic dependence
Field Theory: a process at given external momenta

- Take a diagram with a particular internal momentum configuration
- Identify the soft lines and cut them
- The result part is analytic in the soft stuff
- So should be describably by an effective Lagrangian with coupling constants dependent on the external given momenta
- If symmetries present, Lagrangian should respect them
- Lagrangian should be complete in *neighbourhood*
- Loop diagrams with this effective Lagrangian *should* reproduce the nonanalyticities in the light masses

Crucial part of the argument
This procedure works at one loop level, matching at tree level, nonanalytic dependence at one loop:

- Toy models and vector meson ChPT \( \text{JB, Gosdzinsky, Talavera} \)
- Recent work on relativistic meson ChPT \( \text{Gegelia, Scherer et al.} \)
- Extra terms kept in \( K \to 2\pi \): a one-loop check
- Some preliminary two-loop checks
$K \rightarrow 2\pi$ in $SU(2)$ ChPT

Add $K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$, Roessl

\[
\mathcal{L}^{(2)}_{\pi\pi} = \frac{F^2}{4} \left( \langle u_\mu u^\mu \rangle + \langle \chi_+ \rangle \right),
\]

\[
\mathcal{L}^{(1)}_{\pi K} = \nabla_\mu K^\dagger \nabla^\mu K - M_K^2 K^\dagger K,
\]

\[
\mathcal{L}^{(2)}_{\pi K} = A_1 \langle u_\mu u^\mu \rangle K^\dagger K + A_2 \langle u^\mu u^\nu \rangle \nabla_\mu K^\dagger \nabla_\nu K + A_3 K^\dagger \chi_+ K + \cdots
\]

Add a spurion for the weak interaction $\Delta I = 1/2, \Delta I = 3/2$ (JB, Celis)

\[
t^i j_k \rightarrow t^{i'} j^{i'}_k = t^i j_k (g_L)_k^k (g_L)^i_i (g_L)^{i'}_j
\]

\[
t^{i}_{1/2} \rightarrow t^{i'}_{1/2} = t^{i}_{1/2} (g_L)^{i'}_i
\]
The $\Delta I = 1/2$ terms: $\tau_{1/2} = t_{1/2} u^\dagger$

$$\mathcal{L}_{1/2} = i E_1 \tau_{1/2} K + E_2 \tau_{1/2} u^\mu \nabla_\mu K + i E_3 \langle u_\mu u^\mu \rangle \tau_{1/2} K$$
$$+ i E_4 \tau_{1/2} \chi^+ K + i E_5 \langle \chi^+ \rangle \tau_{1/2} K + E_6 \tau_{1/2} \chi^- K$$
$$+ E_7 \langle \chi^- \rangle \tau_{1/2} K + i E_8 \langle u_\mu u_\nu \rangle \tau_{1/2} \nabla^\mu \nabla^\nu K + \cdots + h.c.$$ 

Note: higher order terms kept in both $\mathcal{L}_{1/2}$ and $\mathcal{L}^{(2)}_{\pi K}$ to check the arguments

Using partial integration, . . . :

$$\langle \pi(p_1)\pi(p_2)|O|K(p_K)\rangle =$$
$$f(M_K^2)\langle \pi(p_1)\pi(p_2)|\tau_{1/2} K|K(p_K)\rangle + \lambda M^2 + \mathcal{O}(M^4)$$

$O$ any operator in $\mathcal{L}_{1/2}$ or with more derivatives. Similar for $\mathcal{L}_{3/2}$
Tree level

\[ A_{0}^{LO} = \frac{\sqrt{3}i}{2F^2} \left[ -\frac{1}{2}E_1 + (E_2 - 4E_3) \overline{M}_K^2 + 2E_8 \overline{M}_K^4 + A_1 E_1 \right] \]

\[ A_{2}^{LO} = \sqrt{\frac{3}{2}} \frac{i}{F^2} \left[ (-2D_1 + D_2) \overline{M}_K^2 \right] \]
One loop

(a)  
(b)  
(c)  
(d)  

(e)  
(f)  

### One loop

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<td>$-\frac{2F^2}{3} A_0^{LO}$</td>
<td>$-\frac{2F^2}{3} A_2^{LO}$</td>
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<td>(a)</td>
<td>$\sqrt{3}i \left( -\frac{1}{3} E_1 + \frac{2}{3} E_2 \overline{M}_K^2 \right)$</td>
<td>$\sqrt{3}i \left( -\frac{2}{3} D_2 \overline{M}_K^2 \right)$</td>
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<td>(b)</td>
<td>$\sqrt{3}i \left( -\frac{5}{96} E_1 - \left( \frac{7}{48} E_2 + \frac{25}{12} E_3 \right) \overline{M}_K^2 + \frac{25}{24} E_8 \overline{M}_K^4 \right)$</td>
<td>$\sqrt{3}i \left( -\frac{61}{12} D_1 + \frac{77}{24} D_2 \right) \overline{M}_K^2$</td>
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<tr>
<td>(e)</td>
<td>$\sqrt{3}i \frac{3}{16} A_1 E_1$</td>
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<tr>
<td>(f)</td>
<td>$\sqrt{3}i \left( \frac{1}{8} E_1 + \frac{1}{3} A_1 E_1 \right)$</td>
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The coefficients of $\overline{A}(M^2)/F^4$ in the contributions to $A_0$ and $A_2$. $Z$ denotes the part from wave-function renormalization.

- $\overline{A}(M^2) = -\frac{M^2}{16\pi^2} \log \frac{M^2}{\mu^2}$
- $K\pi$ intermediate state does not contribute, but did for Flynn-Sachrajda
One-loop

\[ A_{0}^{NLO} = A_{0}^{LO} \left(1 + \frac{3}{8F^2} \bar{A}(M^2)\right) + \lambda_0 M^2 + \mathcal{O}(M^4), \]

\[ A_{2}^{NLO} = A_{2}^{LO} \left(1 + \frac{15}{8F^2} \bar{A}(M^2)\right) + \lambda_2 M^2 + \mathcal{O}(M^4). \]
One-loop

\[ A_{0}^{NLO} = A_{0}^{LO} \left(1 + \frac{3}{8F^{2}} \overline{A}(M^{2})\right) + \lambda_{0}M^{2} + \mathcal{O}(M^{4}), \]

\[ A_{2}^{NLO} = A_{2}^{LO} \left(1 + \frac{15}{8F^{2}} \overline{A}(M^{2})\right) + \lambda_{2}M^{2} + \mathcal{O}(M^{4}). \]

Match with three flavour $SU(3)$ calculation Kambor, Missimer, Wyler; JB, Pallante, Prades

\[ A_{0}^{(3)LO} = - \frac{i\sqrt{6}CF_{0}^{4}}{F_{K}F^{2}} \left(G_{8} + \frac{1}{9}G_{27}\right) \overline{M}_{K}^{2}, \quad A_{2}^{(3)LO} = - \frac{i10\sqrt{3}CF_{0}^{4}}{9F_{K}F^{2}}G_{27}\overline{M}_{K}^{2}, \]

When using $F_{\pi} = F \left(1 + \frac{1}{F^{2}} \overline{A}(M^{2}) + \frac{M^{2}}{F^{2}} l_{4}^{\pi}\right)$, $F_{K} = F_{K} \left(1 + \frac{3}{8F^{2}} \overline{A}(M^{2}) + \cdots\right)$, logarithms at one-loop agree with above
A two-loop check

- Similar arguments to JB-Celis, Flynn-Sachrajda work for the pion vector and scalar formfactor
- Therefore at any $t$ the chiral log correction must go like the one-loop calculation.
- But note the one-loop log chiral log is with $t >> m_\pi^2$
- Predicts
  \[
  F_V(t, M^2) = F_V(t, 0) \left( 1 - \frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(M^2) \right)
  \]
  \[
  F_S(t, M^2) = F_S(t, 0) \left( 1 - \frac{5}{2} \frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(M^2) \right)
  \]
- Note that $F_{V,S}(t, 0)$ is now a coupling constant and can be complex
A two-loop check

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- Therefore at any $t$ the chiral log correction must go like the one-loop calculation.
- But note the one-loop log chiral log is with $t >> m^2_{\pi}$.
- Predicts

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\]

- Note that $F_{V,S}(t, 0)$ is now a coupling constant and can be complex.
- Take the full two-loop ChPT calculation JB, Colangelo, Talavera and expand in $t >> m^2_{\pi}$. 

A two-loop check

Full two-loop ChPT \( \text{JB, Colangelo, Talavera} \), expand in \( t \gg m^2_\pi \):

\[
\begin{align*}
F_V(t, M^2) &= F_V(t, 0) \left( 1 - \frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(M^2) \right) \\
F_S(t, M^2) &= F_S(t, 0) \left( 1 - \frac{5}{2} \frac{M^2}{16\pi^2 F^2} \ln \frac{M^2}{\mu^2} + \mathcal{O}(M^2) \right)
\end{align*}
\]

with

\[
\begin{align*}
F_V(t, 0) &= 1 + \frac{t}{16\pi^2 F^2} \left( \frac{5}{18} - 16\pi^2 l_6^r + \frac{i\pi}{6} - \frac{1}{6} \ln \frac{t}{\mu^2} \right) \\
F_S(t, 0) &= 1 + \frac{t}{16\pi^2 F^2} \left( 1 - 16\pi^2 l_4^r + i\pi - \ln \frac{t}{\mu^2} \right)
\end{align*}
\]

- The needed coupling constants are complex
- Both calculations have two-loop diagrams with overlapping divergences
- The chiral logs should be valid for any \( t \) where a pointlike interaction is a valid approximation
$K \rightarrow 3\pi$ in ChPT


- Isospin Breaking in $K \rightarrow 3\pi$ Decays III: Bremsstrahlung and Fit to Experiment, J. Bijnens and F. Borg, hep-ph/0501163.

Note: Fredrik Borg = Fredrik Persson, obtained PhD 28/1/2005
$K \rightarrow 3\pi$: Overview

- ChPT in the nonleptonic mesonic sector
- Lagrangians
- $K \rightarrow 3\pi$ kinematics and isospin
- Overview of calculations and results
- Data and Fits
ChPT in the nonleptonic sector

- some earlier work: especially on decays with photons
- Kambor, Missimer, Wyler (KMW) : Constructed $\mathcal{L}$ and $\infty$ 1990
- G. Esposito-Farese: Checked $\mathcal{L}$ and $\infty$ 1991
- KMW : Calculated $K \to 2\pi$ and $K \to 3\pi$ 1991
- Donoghue + Holstein + KMW : clarified the relations between observables 1992
- BUT: explicit formulas lost (US Mail)
- Kambor, Ecker, Wyler : simplified octet $\mathcal{L}$ 1993
- $K \to 2\pi$ redone: Bijnens, Pallante, Prades 1998
ChPT in the nonleptonic sector

- First paper: redo $K \rightarrow 3\pi$
- Ecker Isidori Muller Neufeld Pich: Electromagnetic octet $\mathcal{L}$ Lagrangian plus $\propto 2000$
- applications to $K \rightarrow 2\pi$: Several papers
- Isospin breaking in $K \rightarrow 3\pi$: remaining papers $\mathcal{O} \left( p^4, p^2(m_u - m_d), e^2 p^2 \right)$. 
Lagrangians: $p^2$ and $e^2$

\[ \mathcal{L}_2 = \mathcal{L}_{S2} + \mathcal{L}_{W2} + \mathcal{L}_{E2} \]

\[ \mathcal{L}_{S2} = \frac{F_0^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle \]
Lagrangians: $p^2$ and $e^2$

\begin{align*}
\mathcal{L}_2 &= \mathcal{L}_{S2} + \mathcal{L}_{W2} + \mathcal{L}_{E2} \\
\mathcal{L}_{S2} &= \frac{F_0^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle
\end{align*}

\begin{align*}
u_\mu &= i u^\dagger D_\mu U u^\dagger = u_\mu^\dagger, \quad u^2 = U, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u,
\end{align*}

$U$ contains the Goldstone boson fields

\begin{equation}
U = \exp \left( \frac{i \sqrt{2}}{F_0} M \right), \quad M = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi_3 + \frac{1}{\sqrt{6}} \eta_8 \\
\pi^- \\
\frac{-1}{\sqrt{2}} \pi_3 + \frac{1}{\sqrt{6}} \eta_8 \\
K^- \\
K^0 \\
\frac{-2}{\sqrt{6}} \eta_8
\end{pmatrix}.
\end{equation}

Here \( \chi = 2 B_0 \begin{pmatrix} m_u \\ m_d \\ m_s \end{pmatrix} \) and \( D_\mu U = \partial_\mu U - ie [Q, U] \).
Lagrangians: $p^2$ and $e^2$

\[ \mathcal{L}_{W2} = C F_0^4 \left[ G_8 \langle \Delta_{32} u_{\mu} u^{\mu} \rangle + G'_8 \langle \Delta_{32} \chi^+ \rangle + G_{27} t^{ij,kl} \langle \Delta_{ij} u_{\mu} \rangle \langle \Delta_{kl} u^{\mu} \rangle \right] \]

\[ \Delta_{ij} \equiv u \lambda_{ij} u^\dagger, \quad (\lambda_{ij})_{ab} \equiv \delta_{ia} \delta_{jb}, \quad t^{21,13} = t^{13,21} = 1/3, \ldots \]
Lagrangians: $p^2$ and $e^2$

\[
\mathcal{L}_{W^2} = C F_0^4 \left[ G_8 \langle \Delta_{32} u^\mu u^\mu \rangle + G'_8 \langle \Delta_{32} \chi^+ \rangle + G_{27} t^{ij,kl} \langle \Delta_{ij} u^\mu \rangle \langle \Delta_{kl} u^\mu \rangle \right]
\]

\[
\Delta_{ij} \equiv u \lambda_{ij} u^\dagger, \quad (\lambda_{ij})_{ab} \equiv \delta_{ia} \delta_{jb}, \quad t^{21,13} = t^{13,21} = 1/3, \ldots
\]

\[
C = -\frac{3}{5} G_F \frac{V_{ud} V_{us}^*}{\sqrt{2}} : \text{chiral and large } N_c \text{ limits } G_8 = G_{27} = 1
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Electromagnetic:

$$\mathcal{L}_{E2} = e^2 F_0^4 Z \langle Q_L Q_R \rangle + \left( e^2 C F_0^6 G_E \langle \Delta_{32} Q_R \rangle \right)$$

$$Q_L = u Q u^\dagger, \quad Q_R = u^\dagger Q u \quad \text{with} \quad Q = \text{diag} \left( 2/3, -1/3, -1/3 \right)$$
Lagrangians: $p^2$ and $e^2$

$$\mathcal{L}_{W_2} = C F_0^4 \left[ G_8 \langle \Delta_{32} u_\mu u^\mu \rangle + G'_8 \langle \Delta_{32} \chi_+ \rangle + G_{27} t^{ij,kl} \langle \Delta_{ij} u_\mu \rangle \langle \Delta_{kl} u^\mu \rangle \right]$$

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Electromagnetic:

$$\mathcal{L}_{E_2} = e^2 F_0^4 Z \langle Q_L Q_R \rangle + (e^2 C F_0^6 G_E \langle \Delta_{32} Q_R \rangle)$$

Note: $F_0$ not well known, fit $C F_0^4 G_8, \ldots$

use numerically $F_0 = F_\pi$ to quote $G_8, \ldots$
Lagrangians: $p^4$ and $e^2 p^2$

\[ \mathcal{L}_4 = \mathcal{L}_{S4} + \mathcal{L}_{W4} + \mathcal{L}_{S2E2} + \mathcal{L}_{W2E2}(G8). \]
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$\mathcal{L}_{S4}$ values of $L^r_i$ use Amoros, Bijnens, Talavera $p^4$ fit

$\mathcal{L}_{W4}$ thirteen $N^r_i$ (octet) and twelve $D^r_i$ might contribute (two $N^r_i$ and two $D^r_i$ extra for photon-reducible)

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Done here:

Isospin conserved: 11 combinations of $N^r_i, D^r_i$ relevant: $\tilde{K}_i$

7 coefficients order $m^4_K$, 4 order $M^2_K m^2_\pi$

2 (virtually) indistinguishable from $G_8$ and $G_{27}$. 

NA62 Handbook Workshop 10/12/2009

Hadronic Decays

Johan Bijnens p.33/55
Lagrangians: \( p^4 \) and \( e^2 p^2 \)

\[ \mathcal{L}_4 = \mathcal{L}_{S4} + \mathcal{L}_{W4} + \mathcal{L}_{S2E2} + \mathcal{L}_{W2E2}(G8). \]

- \( \mathcal{L}_{S4} \) values of \( L_i^r \) use Amoros, Bijnens, Talavera \( p^4 \) fit
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Done here:

**Isospin conserved:** 11 combinations of \( N_i^r, D_i^r \) relevant: \( \tilde{K}_i \)
- 7 coefficients order \( m_K^4, 4 \) order \( M_{K}^2 m_{\pi}^2 \)
- 2 (virtually) indistinguishable from \( G_8 \) and \( G_{27} \).

**Isospin Broken:** 30 combinations of \( N_i^r, D_i^r, Z_i^r \) relevant
- Results from isospin breaking with \( K_i^r = Z_i^r = 0 \)
\[ K \rightarrow 3\pi \] Kinematics and Isospin

\[ K_L(k) \rightarrow \pi^0(p_1)\pi^0(p_2)\pi^0(p_3), \quad [A_{000}^L], \]
\[ K_L(k) \rightarrow \pi^+(p_1)\pi^-(p_2)\pi^0(p_3), \quad [A_{+-0}^L], \]
\[ K_S(k) \rightarrow \pi^+(p_1)\pi^-(p_2)\pi^0(p_3), \quad [A_{+-0}^S], \]
\[ K^+(k) \rightarrow \pi^0(p_1)\pi^0(p_2)\pi^+(p_3), \quad [A_{00+}], \]
\[ K^+(k) \rightarrow \pi^+(p_1)\pi^+(p_2)\pi^-(p_3), \quad [A_{++-}], \]

plus charge conjugate ones
$K \rightarrow 3\pi$ Kinematics and Isospin

\begin{align*}
K_L(k) & \rightarrow \pi^0(p_1)\pi^0(p_2)\pi^0(p_3), \quad [A_{000}^L], \\
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K^+(k) & \rightarrow \pi^+(p_1)\pi^+(p_2)\pi^-(p_3), \quad [A_{+++}],
\end{align*}

plus charge conjugate ones

even under $p_1 \leftrightarrow p_2$ except $A_{+-0}^S$ odd (CP and Bose symmetry)
\( K \to 3\pi \)  

**Kinematics and Isospin**

\[
K_L(k) \to \pi^0(p_1)\pi^0(p_2)\pi^0(p_3), \quad [A^L_{000}],
\]

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Plus charge conjugate ones

**Kinematics:**
\[
s_1 = (k - p_1)^2, \quad s_2 = (k - p_2)^2, \quad s_3 = (k - p_3)^2.
\]

**Dalitz plot variables:**
\[
y = (s_3 - s_0)/m^2_{\pi^+}, \quad x = (s_2 - s_1)/m^2_{\pi^+}, \quad s_0 = (s_1 + s_2 + s_3)/3.
\]
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Bremsstrahlung problem: $s_i$ from $E_i$ or $m_{\pi\pi}^2$ different
Kinematics and Isospin

Valid also at $p^6$

\[
A^L_{000} = M_0(s_1) + M_0(s_2) + M_0(s_3),
\]
\[
A^L_{++-} = M_1(s_3) + M_2(s_1) + M_2(s_2) + M_3(s_1)(s_2 - s_3) + M_3(s_2)(s_1 - s_3),
\]
\[
A^S_{++-} = M_4(s_1) - M_4(s_2) + M_5(s_1)(s_2 - s_3) - M_5(s_2)(s_1 - s_3) + M_6(s_3)(s_1 - s_2),
\]
\[
A_{00+} = M_7(s_3) + M_8(s_1) + M_8(s_2) + M_9(s_1)(s_2 - s_3) + M_9(s_2)(s_1 - s_3),
\]
\[
A_{++-} = M_{10}(s_3) + M_{11}(s_1) + M_{11}(s_2) + M_{12}(s_1)(s_2 - s_3) + M_{12}(s_2)(s_1 - s_3).
\]

**polynomial ambiguity from** $s_1 + s_2 + s_3 = \sum_i m_i^2$

**Isospin:**

\[
M_0(s) = M_1(s) + 2M_2(s),
\]
\[
2M_7(s) + 4M_8(s) = M_{10}(s) + 2M_{11}(s),
\]
\[
M_4(s) = \frac{1}{3}(M_7(s) - M_8(s) + M_{10}(s) - M_{11}(s))
\]
\[
M_5(s) - M_6(s) = M_9(s) + M_{12}(s).
\]
Calculations: isospin limit

The diagrams of order $p^4$

Expressions for the $M_i(s)$: see first paper

Confirmed by Gamiz, Prades, Scimemi, hep-ph/0305164
Lashin, hep-ph/0308200 (how not clear)
I: Strong Isospin Breaking

Includes:

- $m_u - m_d$ and the effects of $\mathcal{L}_{E2}, \mathcal{L}_{E2S2}$ and $\mathcal{L}_{W2E2}(G8)$
- $\pi^0 - \eta$ mixing ($\eta'$ via LECs)
- Same diagrams as before
- Formulas immediately a lot longer: see
  
  http://www.thep.lu.se/~bijnens/chpt.html
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Effects:

- $F_{\pi^+}$, $F_{K^+}$ overall factor
- Mass difference $\pi^+ - \pi^0$
- Others typically a few %
Curves:
Phasespace boundaries

Dots:
$\pi\pi$ pair at rest

Lines:
Show $|A|^2$ along these
$K_L \rightarrow \pi^0 \pi^0 \pi^0$
II: Radiative Corrections

- Photon Loops
II: Radiative Corrections

- Photon Loops

- Soft Bremsstrahlung
II: Radiative Corrections

- Photon reducible diagrams

- Tree Level:
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  Extra vanish exactly
II: Radiative Corrections

• Photon reducible diagrams

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• One-loop Level:

  \[ \text{Extra } N_i^r, D_i^r \text{ from } K \to \pi \ell^+ \ell^- \]
II: Radiative Corrections

- Photon reducible diagrams

- Tree Level:

- One-loop Level:

Extra $N_i^T, D_i^T$ from $K \rightarrow \pi \ell^+ \ell^-$ Numerically Negligible
\[ K^+ \rightarrow \pi^+ \pi^+ \pi^- \]
$K^+ \rightarrow \pi^0 \pi^0 \pi^+$

Note: Disagree numerically with Nehme, hep-ph/0406209
III: Hard Bremsstrahlung

- Include hard photon Bremsstrahlung to lowest order
  Full amplitude from Low’s theorem
- Do everything also for $K \rightarrow 2\pi$ to have same treatment
  Note: some new pieces, e.g., $e^2 p^2 G_{27}$
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  Note: some new pieces, e.g., $e^2 p^2 G_{27}$

**Checks:**

- Everything done twice indepently
- UV infinities cancel analytically: no $1/(d-4)$
- IR infinities cancel analytically: no $m_\gamma$
- soft-hard Bremsstrahlung match: no $E_\gamma$
  (except for $F_\pi$, $F_K$ parts)
Treatment of Bremsstrahlung

Old experiments: what to do?
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Typically: Decay rate and distribution measured in part of phasespace and then extrapolated to whole decay region
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Typically: Decay rate and distribution measured in part of phasespace and then extrapolated to whole decay region

We adopted:

\[\begin{align*}
\text{• Subtract hard Bremsstrahlung from Decay Rate} \\
\text{• From remainder get } |A(s_0, s_0, s_0)|^2 \\
\text{using experimental } g, h, k \\
\text{• Fit } |A(s_0, s_0, s_0)|^2, g, h, k \text{ with soft Bremsstrahlung and loops, . . .}
\end{align*}\]
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Advantages:

- Mimicks experimental determination (well, sort of)
- Coulomb singularity avoided
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\[
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\end{align*}
\]

We really need to know what Phasespace/Photons covered
$K \to \pi\pi$ only: essentially same as Ecker et al.

Tree level: $G_8 = 10.36 \quad G_{27} = 0.550$

Full: $G_8 = 5.39 \quad G_{27} = 0.359$
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Full fit:

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>0.77 GeV</th>
<th>1.0 GeV</th>
<th>0.6 GeV</th>
<th>0.77 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_8$</td>
<td>5.39(1)</td>
<td>4.60(1)</td>
<td>6.43(1)</td>
<td>5.39(1)</td>
</tr>
<tr>
<td>$G_{27}$</td>
<td>0.359(2)</td>
<td>0.301(1)</td>
<td>0.438(2)</td>
<td>0.359(2)</td>
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<tr>
<td>$\delta_2 - \delta_0$</td>
<td>$-57.9(1.5)^{\circ}$</td>
<td>$-57.3(1.4)^{\circ}$</td>
<td>$-58.9(1.4)^{\circ}$</td>
<td>$-57.9(1.4)^{\circ}$</td>
</tr>
<tr>
<td>$10^3 \tilde{K}_1/G_8$</td>
<td>$\equiv 0$</td>
<td>$\equiv 0$</td>
<td>$\equiv 0$</td>
<td>$\equiv 0$</td>
</tr>
<tr>
<td>$10^3 \tilde{K}_2/G_8$</td>
<td>48.5(2.4)</td>
<td>56.5(2.4)</td>
<td>41.2(1.9)</td>
<td>46.6(1.6)</td>
</tr>
<tr>
<td>$10^3 \tilde{K}_3/G_8$</td>
<td>2.6(1.2)</td>
<td>$-1.7(1.1)$</td>
<td>6.7(1.0)</td>
<td>3.5(0.8)</td>
</tr>
<tr>
<td>$10^3 \tilde{K}<em>4/G</em>{27}$</td>
<td>$\equiv 0$</td>
<td>$\equiv 0$</td>
<td>$\equiv 0$</td>
<td>$\equiv 0$</td>
</tr>
<tr>
<td>$10^3 \tilde{K}<em>5/G</em>{27}$</td>
<td>$-41.2(16.9)$</td>
<td>$-52.0(17.7)$</td>
<td>$-31.1(12.0)$</td>
<td>$-27.0(8.3)$</td>
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<tr>
<td>$10^3 \tilde{K}<em>6/G</em>{27}$</td>
<td>$-102(105)$</td>
<td>$-114(105)$</td>
<td>$-93(76)$</td>
<td>$\equiv 0$</td>
</tr>
<tr>
<td>$10^3 \tilde{K}<em>7/G</em>{27}$</td>
<td>78.6(33)</td>
<td>78.0(33.5)</td>
<td>79.6(22.7)</td>
<td>50.0(13.0)</td>
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<tr>
<td>$\chi^2/DOF$</td>
<td>29.3/10</td>
<td>27.2/10</td>
<td>33.0/10</td>
<td>30.5/11</td>
</tr>
</tbody>
</table>

vary $\tilde{K}_1, \tilde{K}_4$
see first paper
### Octet and $\mu$-variation

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>octet $G_8$</th>
<th>$\mu$ variation $G_{27}$</th>
<th>$\mu$ variation $\delta_2 - \delta_0$</th>
<th>$\tilde{K}_i(G_8)$</th>
<th>$\tilde{K}<em>i(G</em>{27})$</th>
<th>$\tilde{K}_i(0.77 \text{ GeV})$</th>
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<tr>
<td>$G_{27}$</td>
<td>4.84(1)</td>
<td>0.430(1)</td>
<td>$-57.9(0.2)^\circ$</td>
<td>2.0(1)</td>
<td>63.0(1.5)</td>
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<td>$\delta_2 - \delta_0$</td>
<td>0.430(1)</td>
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<td>$10^3 \tilde{K}_1/G_8$</td>
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<tr>
<td>$10^3 \tilde{K}_2/G_8$</td>
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<td>$-6.0(7)$</td>
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<td>$10^3 \tilde{K}_3/G_8$</td>
<td>$-6.0(7)$</td>
<td>$27.0$</td>
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<td>$-25.8$</td>
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<td>$10^3 \tilde{K}<em>5/G</em>{27}$</td>
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<tr>
<td>$10^3 \tilde{K}_8/G_8$</td>
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<tr>
<td>$10^3 \tilde{K}_9/G_8$</td>
<td>20.4(1)</td>
<td>$-0.546$</td>
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<td>$10^3 \tilde{K}_{10}/G_8$</td>
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<td>$-2.92$</td>
<td>2.79</td>
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<td>$10^3 \tilde{K}_{11}/G_8$</td>
<td>0</td>
<td>11.6</td>
<td>$-11.1$</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\chi^2/\text{DOF}$</td>
<td>33.3/10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Octet:** $G_{27} = 0$

keep also $\tilde{K}_i$

with $m_K^2 m_{\pi}^2$ factors

**$\mu$ variation**

$\tilde{K}_i(\mu)$

$\tilde{K}_i(0.77 \text{ GeV})$

Fit about same as before, some experiments fit better, others smaller error
Models

See Ecker, Kambor, Wyler 1993 for explanations

- Vector octet dominance $\tilde{K}_3 = -\frac{1}{2}\tilde{K}_2$: Not at all
See Ecker, Kambor, Wyler 1993 for explanations

- Vector octet dominance \( \Rightarrow \tilde{K}_3 = -\frac{1}{2} \tilde{K}_2: \) Not at all

- Weak Deformation and Factorization

\[
\begin{align*}
N_1^r & = 2k_f (32/3 L_1^r + 4 L_3^r + 2/3 L_9^r), \\
N_2^r & = 2k_f (16/3 L_1^r + 4 L_3^r + 10/3 L_9^r), \\
N_3^r & = 2k_f (8 L_2^r - 2 L_9^r), \\
N_4^r & = 2k_f (-16/3 L_1^r - 8/3 L_3^r - 4/3 L_9^r), \\
N_5^r & = 2k_f (-L_5^r), \\
N_6^r & = 2k_f (2/3 L_5^r), \\
N_7^r & = 2k_f (L_5^r), \\
N_8^r & = 2k_f (4 L_4^r + 2 L_5^r), \\
N_9^r & = N_{10}^r = N_{11}^r = N_{12}^r = N_{13}^r = 0
\end{align*}
\]
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N_9^r &= N_{10}^r = N_{11}^r = N_{12}^r = N_{13}^r = 0
\end{align*}
\]

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>0.77 GeV</th>
<th>0.9 GeV</th>
<th>0.842 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_8$</td>
<td>4.18(1)</td>
<td>4.42</td>
<td>4.22(1)</td>
</tr>
<tr>
<td>$G_{27}$</td>
<td>0.360(2)</td>
<td>0.326(10)</td>
<td>0.339(10)</td>
</tr>
<tr>
<td>$k_F$</td>
<td>2.61(1)</td>
<td>4.94(2)</td>
<td>3.60(5)</td>
</tr>
<tr>
<td>$\chi^2$/DOF</td>
<td>109/14</td>
<td>182/14</td>
<td>60.4/13</td>
</tr>
</tbody>
</table>

WDM or $k_f = 1/2$ No
Factorization: surprisingly OK
But $k_f$ not naive one
## Data and Fits: NA48/2 missing

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$K^+ \to \pi^+\pi^0$</td>
<td>$(1.1231 \pm 0.0078) \cdot 10^{-17}$</td>
<td>$1.123 \cdot 10^{-17}$</td>
<td>$1.127 \cdot 10^{-17}$</td>
</tr>
<tr>
<td>$K_S\to\pi^0\pi^0$</td>
<td>$(2.2828 \pm 0.0104) \cdot 10^{-15}$</td>
<td>$2.282 \cdot 10^{-15}$</td>
<td>$2.283 \cdot 10^{-15}$</td>
</tr>
<tr>
<td>$K_S\to\pi^+\pi^-$</td>
<td>$(5.0691 \pm 0.0108) \cdot 10^{-15}$</td>
<td>$5.069 \cdot 10^{-15}$</td>
<td>$5.069 \cdot 10^{-15}$</td>
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<tr>
<td>$K_L\to\pi^0\pi^0\pi^0$</td>
<td>$(2.6748 \pm 0.0358) \cdot 10^{-18}$</td>
<td>$2.618 \cdot 10^{-18}$</td>
<td>$2.698 \cdot 10^{-18}$</td>
</tr>
<tr>
<td>$K_L\to\pi^+\pi^-\pi^0$</td>
<td>$(1.5998 \pm 0.0271) \cdot 10^{-18}$</td>
<td>$1.658 \cdot 10^{-18}$</td>
<td>$1.711 \cdot 10^{-18}$</td>
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<tr>
<td>$K^+\to\pi^0\pi^0\pi^+$</td>
<td>$(9.195 \pm 0.0213) \cdot 10^{-19}$</td>
<td>$8.934 \cdot 10^{-19}$</td>
<td>$8.816 \cdot 10^{-19}$</td>
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<tr>
<td>$K^+\to\pi^+\pi^+\pi^-$</td>
<td>$(2.9737 \pm 0.0174) \cdot 10^{-18}$</td>
<td>$2.971 \cdot 10^{-18}$</td>
<td>$2.933 \cdot 10^{-18}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay</th>
<th>Quantity</th>
<th>Experiment</th>
<th>ChPT</th>
<th>Fact.</th>
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</thead>
<tbody>
<tr>
<td>$K_L\to\pi^0\pi^0\pi^0$</td>
<td>$h$</td>
<td>$-0.0050 \pm 0.0014$</td>
<td>$-0.0062$</td>
<td>$-0.0025$</td>
</tr>
<tr>
<td>$K_L\to\pi^+\pi^-\pi^0$</td>
<td>$g$</td>
<td>$0.678 \pm 0.008$</td>
<td>$0.678$</td>
<td>$0.654$</td>
</tr>
<tr>
<td></td>
<td>$h$</td>
<td>$0.076 \pm 0.006$</td>
<td>$0.088$</td>
<td>$0.083$</td>
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<tr>
<td></td>
<td>$k$</td>
<td>$0.0099 \pm 0.0015$</td>
<td>$0.0057$</td>
<td>$0.0068$</td>
</tr>
<tr>
<td>$K_S\to\pi^+\pi^-\pi^0$</td>
<td>$\gamma_S$</td>
<td>$(3.3 \pm 0.5) \cdot 10^{-8}$</td>
<td>$3.0 \cdot 10^{-8}$</td>
<td>$2.9 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>$K^\pm\to\pi^0\pi^0\pi^\pm$</td>
<td>$g$</td>
<td>$0.638 \pm 0.020$</td>
<td>$0.636$</td>
<td>$0.648$</td>
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<tr>
<td></td>
<td>$h$</td>
<td>$0.051 \pm 0.013$</td>
<td>$0.077$</td>
<td>$0.080$</td>
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<tr>
<td></td>
<td>$k$</td>
<td>$0.004 \pm 0.007$</td>
<td>$0.0047$</td>
<td>$0.0069$</td>
</tr>
<tr>
<td>$K^+\to\pi^+\pi^+\pi^-$</td>
<td>$g$</td>
<td>$-0.2154 \pm 0.0035$</td>
<td>$-0.215$</td>
<td>$-0.226$</td>
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<tr>
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<td>$h$</td>
<td>$0.012 \pm 0.008$</td>
<td>$0.012$</td>
<td>$0.019$</td>
</tr>
<tr>
<td></td>
<td>$k$</td>
<td>$-0.0101 \pm 0.0034$</td>
<td>$-0.0034$</td>
<td>$-0.0033$</td>
</tr>
</tbody>
</table>
Estimates of the constants

- I referred already to a factorization model
- Vector meson models: radiative decays
- Real challenge: getting $G_{27}$ and $G_8$
- Gluonic Penguin is the real culprit
Estimates of the constants

- The perturbative part is well known
- Need matching to the hadronic level calculation
- Bardeen, Buras, Gerard 88-01: use large $N_c$
- Almost all analytical work since builds on this
 Estimates of the constants

Estimates of the constants

- Earlier: Bertolini, Fabbrichesi, Eeg Chiral Quark Model
Estimates of the constants

Remember fit: $G_8 = 5.39$ and $G_{27} = 0.36$
Conclusions

- $m_{K^+}$: NA48/62 or KLOE?
- $K \rightarrow \pi\pi$ $SU(2)$ chiral logarithm known
- $K \rightarrow 3\pi$ to first nontrivial order in isospin breaking fully known
- Fit done and tested a few models of the $N_i$
- Fit with/without isospin breaking seems similar
- Need Photon/Phasespace information from experiment: NA48/2 result needs to be included
- CP violation: partly done: Prades
- Fully from first principles: indications are that it will work but no accurate prediction with trustable error bars yet