η and η' decays and what can we learn from them ?

Johan Bijnens

Lund University

bijnens@thep.lu.se

http://www.thep.lu.se/$\sim$bijnens/chpt.html

Overview

- No Production
- No weak decays: Typically: η : BR $\lesssim 10^{-11}$
  η' : BR $\lesssim 10^{-12}$
- Both η and η' decays are suppressed
  $\Longrightarrow$ good laboratories to study nondominant strong interaction effects
- Related theory talks: S. Bass, B. Borasoy, E. Oset, B. Martemyanov
- Related experimental talks: most of the rest of today
- Give an idea of why we look at η and η'
- Overview of the known theory and some puzzles etc

Remember: Eta Physics Handbook

Physica Scripta, Vol. T99, 2002

Contents

Pseudoscalars are special

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $U(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $U(3)_L \times U(3)_R$.

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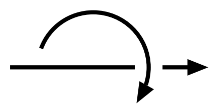
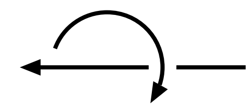
Can also see that via  $v < c$, $m_q \neq 0 \Longrightarrow$ / $v = c$, $m_q = 0 \not\Longrightarrow$ 

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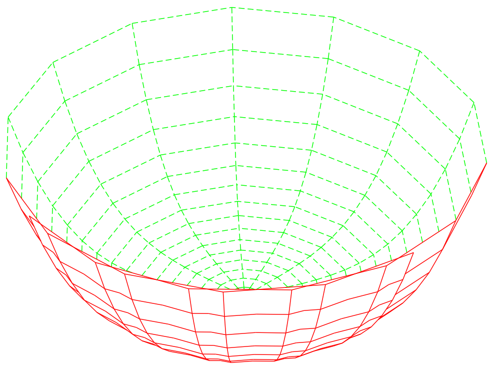
- Hadrons do not come in parity doublets: symmetry must be broken
- A few very light hadrons: $\pi^0 \pi^+ \pi^-$ and also K, η
- Both can be understood from spontaneous Chiral Symmetry Breaking

Goldstone Modes

UNBROKEN: $V(\phi)$



Only massive modes around lowest energy state (=vacuum)

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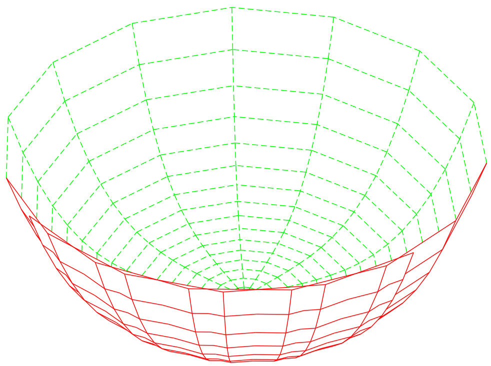

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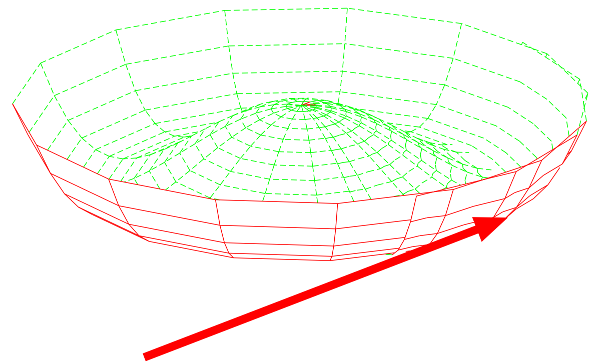

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$\langle\phi\rangle \neq 0$: Breaks symmetry
No parity doublets
Massless mode along ridge

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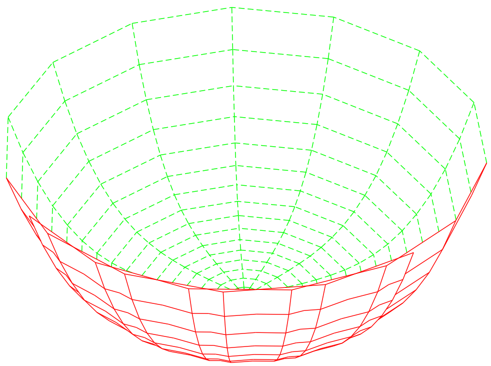

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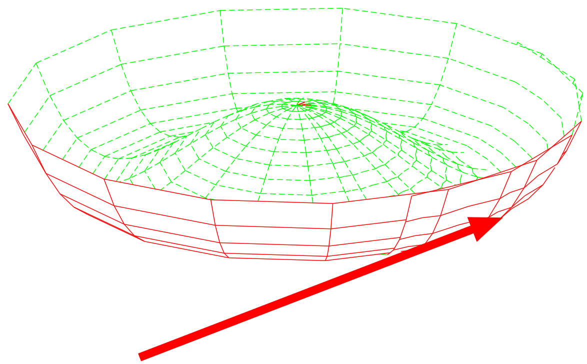

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For QCD: $\langle\phi\rangle \neq 0 \longrightarrow \langle\overline{q}q\rangle \neq 0$ $\quad U(3)_L \times U(3)_R \to U(3)_V$

Explains why pions light

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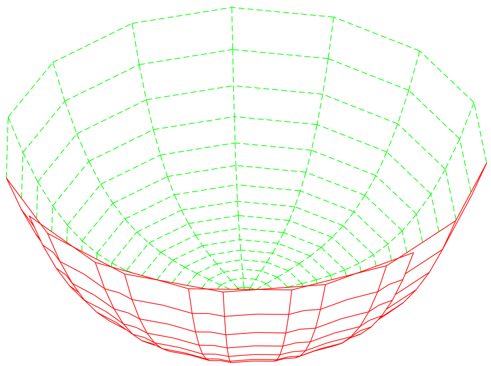

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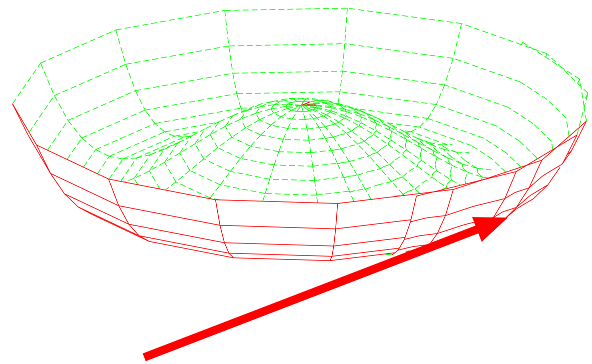

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Explains why pions light but need NINE light particles

So WHY is the η' NOT light?

Anomaly

$$U(3)_L \times U(3)_R = SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$$

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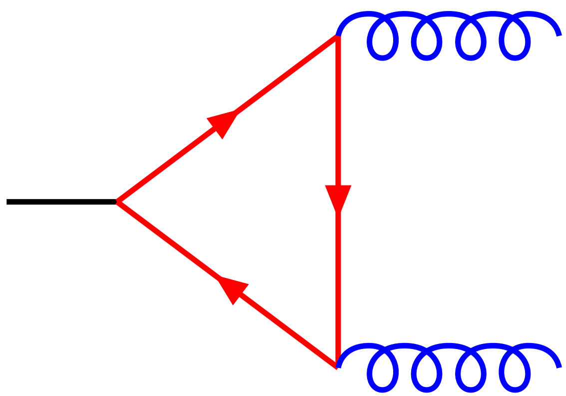

$$\Longrightarrow \quad \partial_\mu A^{0\mu} = 2\sqrt{N_f}\,\omega$$

$$\omega = \frac{1}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\,G_{\mu\nu}G_{\alpha\beta}$$

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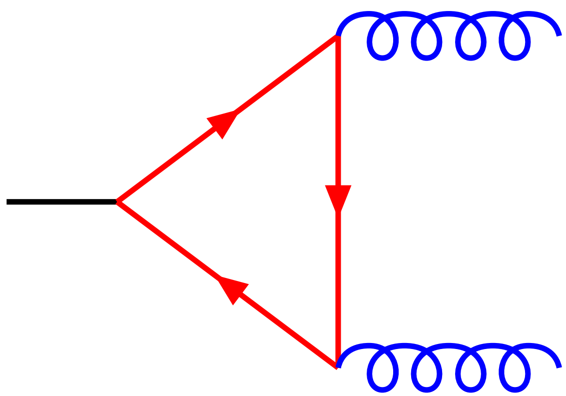

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$$\omega = \frac{1}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\,G_{\mu\nu}G_{\alpha\beta}$$

ω is gluons: strongly interacting: η' heavy

But

So quantum effects break $U(1)_A$

BUT ω is a total derivative $\Longrightarrow$ How does it have an effect?

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η' has possibly large and very interesting nonperturbative effects and interaction with gluons as no other hadron

$m_s \neq \hat{m}$: This also affects η physics

Standard Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

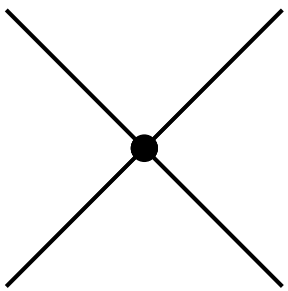
8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

Power counting in momenta possible

Standard Chiral Perturbation Theory

A simple example $\pi\pi \to \pi\pi$

Rules

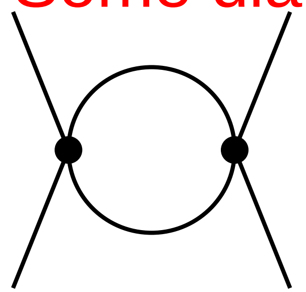


p^2

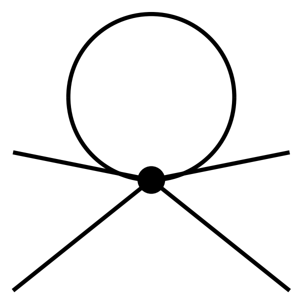
$1/p^2$

$\int d^4p$ $\quad$ p^4

Some diagrams



$$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$$



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Standard ChPT

Lagrangian Structure: (Too?) many parameters

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

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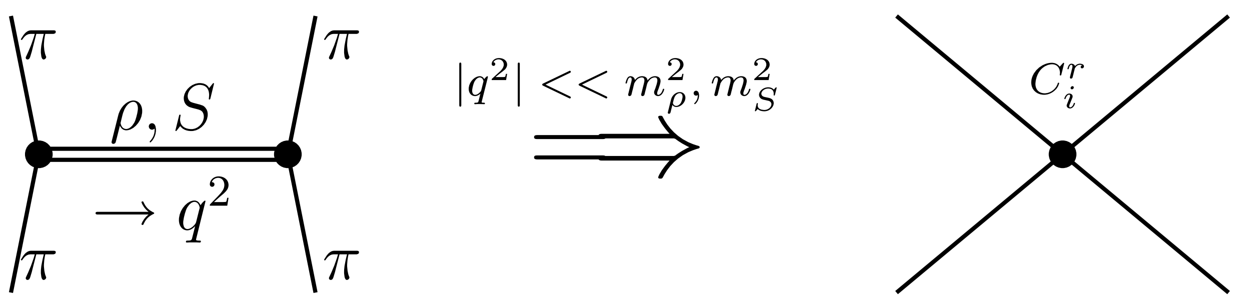


Standard ChPT

- Find enough inputs from experiment
- C_i^r:
  - kinematical dependence: agree well with single resonance saturation
  - quark mass+kinematical: if vector dominated, seems to be OK
  - quark mass+kinematical: if scalar dominated: which scalars? (not σ)
  - quark masses: which scalars? unrealistically large estimates
- in p^6 physical or lowest order masses: thresholds in right place requires physical

Standard ChPT

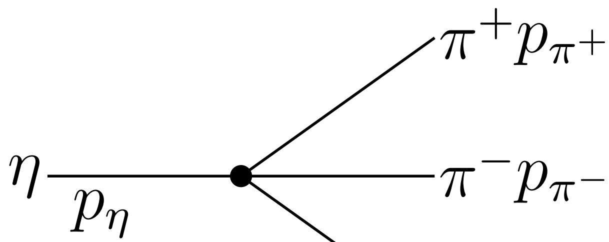
Done to two-loops:

- form-factors
- $\pi\pi$ and πK scattering
- masses, decay constants, several isospin breaking quantities

Combined with dispersion relations for $\pi\pi$ and πK: starting to be precise, $\eta \to 3\pi$ will help.

$\eta \to 3\pi$ beyond p^4: Basic

Review: JB, Gasser, Phys.Scripta T99(2002)34 [hep-ph/0202242]



$$\begin{aligned} s &= (p_{\pi^+}+p_{\pi^-})^2 = (p_\eta - p_{\pi^0})^2 \\ t &= (p_{\pi^-}+p_{\pi^0})^2 = (p_\eta - p_{\pi^+})^2 \\ u &= (p_{\pi^+}+p_{\pi^0})^2 = (p_\eta - p_{\pi^-})^2 \end{aligned}$$

$$s+t+u = m_\eta^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2 \equiv 3s_0\,.$$

$$\langle \pi^0\pi^+\pi^-\text{out}|\eta\rangle = i\,(2\pi)^4\;\delta^4\left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right)\;A(s,t,u)\,.$$

$$\langle \pi^0\pi^0\pi^0\text{out}|\eta\rangle = i\,(2\pi)^4\;\delta^4\left(p_\eta - p_1 - p_2 - p_3\right)\;\overline{A}(s_1,s_2,s_3)$$

$$\overline{A}(s_1,s_2,s_3) = A(s_1,s_2,s_3) + A(s_2,s_3,s_1) + A(s_3,s_1,s_2)\,,$$

$\eta \to 3\pi$ beyond p^4: Lowest order

Pions are in $I = 1$ state $\Longrightarrow A \sim (m_u - m_d)$ or α_{em}

- α_{em} effect is small (but large via $m_{\pi^+} - m_{\pi^0}$)
- $\eta \to \pi^+\pi^-\pi^0\gamma$ needs to be included directly

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ChPT:Cronin 67: $A(s,t,u) = \dfrac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \dfrac{3(s - s_0)}{m_\eta^2 - m_\pi^2}\right\}$

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or with $Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$, $\qquad \hat{m} = \frac{1}{2}(m_u + m_d)$

$$A(s,t,u) = \frac{1}{Q^2}\frac{m_K^2}{m_\pi^2}(m_\pi^2 - m_K^2)\frac{M(s,t,u)}{3\sqrt{3}F_\pi^2},$$

with at lowest order $M(s,t,u) = \frac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$.

$\eta \to 3\pi$ beyond p^4: p^2 and p^4

$\Gamma\left(\eta \to 3\pi\right) \propto |A|^2 \propto Q^{-4}$ allows a PRECISE measurement

$\boxed{Q \approx 24}$ gives lowest order $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 66$ eV .

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Other Source from $m^2_{K^+} - m^2_{K^0} \sim Q^{-2} \Longrightarrow \boxed{Q = 20.0 \pm 1.5}$
Lowest order prediction $\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 140$ eV .

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At order p^4: $$\frac{\int dLIPS|A_2 + A_4|^2}{\int dLIPS|A_2|^2} = 2.4\,,$$

($LIPS$=Lorentz invariant phase-space)

Major source: large S-wave final state rescattering

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Two Loop calculation partially done: stalled since two years

$\eta \to 3\pi$ beyond p^4: Dispersive

Try to resum the S-wave rescattering:

Anisovich-Leutwyler (AL), Kambor,Wiesendanger,Wyler (KWW)

Different method but similar approximations

Here: simplified version of AL

Up to p^8: No absorptive parts from $\ell \geq 2$

$\Longrightarrow M(s,t,u) =$

$$M_0(s) + (s-u)M_1(t) + (s-t)M_1(t) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

M_I: “roughly” contributions with isospin 0,1,2

$\eta \to 3\pi$ beyond p^4: Dispersive

3 body dispersive: difficult: keep only 2 body cuts

start from $\pi\eta \to \pi\pi$ ($m_\eta^2 < 3m_\pi^2$) standard dispersive analysis

analytically continue to physical m_η^2.

$$M_I(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \,\frac{\mathrm{Im}M_I(s')}{s'-s-i\varepsilon}$$

$$\mathrm{Im}M_I(s') \longrightarrow \mathrm{disc}M_I(s) = \frac{1}{2i}\left(M_I(s+i\varepsilon) - M_I(s-i\varepsilon)\right)$$

$$\begin{aligned}
M_0(s) &= a_0 + b_0 s + c_0 s^2 + \frac{s^3}{\pi}\int \frac{ds'}{s'^3}\,\frac{\mathrm{disc}M_0(s')}{s'-s-i\varepsilon}\,, \\
M_1(s) &= a_1 + b_1 s + \frac{s^2}{\pi}\int \frac{ds'}{s'^2}\,\frac{\mathrm{disc}M_1(s')}{s'-s-i\varepsilon}\,, \\
M_2(s) &= a_2 + b_2 s + c_2 s^2 + \frac{s^3}{\pi}\int \frac{ds'}{s'^3}\,\frac{\mathrm{disc}M_2(s')}{s'-s-i\varepsilon}\,.
\end{aligned}$$

$\eta \to 3\pi$ **beyond** p^4

- Technical complications in solving
- Only 4 relevant constants:

$$M(s,t,u) = a + bs + cs^2 - d(s^2 + tu)$$

$$M_0(s) + \frac{4}{3}M_2(s) \;\; sM_1(s) + M_2(s) + s^2\frac{4L_3 - 1/(64\pi^2)}{F_\pi^2(m_\eta^2 - m_\pi^2)}$$

converge better

$$\begin{aligned} c &= c_0 + \frac{4}{3}c_2 = \frac{1}{\pi}\int \frac{ds'}{s'^3}\left\{\text{disc}M_0(s') + \frac{4}{3}\text{disc}M_2(s')\right\}, \\ d &= -\frac{4L_3 - 1/(64\pi^2)}{F_\pi^2(m_\eta^2 - m_\pi^2)} + \frac{1}{\pi}\int \frac{ds'}{s'^3}\left\{s'\text{disc}M_1(s') + \text{disc}M_2(s')\right\} \end{aligned}$$

Fix a, b by matching to tree level or p^4 amplitude

$\eta \to 3\pi$ beyond p^4: technical trouble

$$M_I(s) = \frac{1}{\pi}\int \frac{ds'}{s'-s-i\varepsilon}\sin\delta_I(s)e^{-i\delta_I(s)}\left\{M_I(s)+\hat{M}_I(s)\right\}$$

$\hat{M}_I(s) = \sum_{n,I'}\int_{-1}^{1} d\cos\theta\ \cos^n\theta c_{nII'}M_{I'}(t)$: t,u channel

need all s in dispersion $\Rightarrow$ t,u outside physical domain, cuts in plane $\Rightarrow$ choose path carefully

$$M_I(s) = \frac{1}{\pi}\int \frac{ds'}{s'-s-i\varepsilon}\sin\delta_I(s)e^{-i\delta_I(s)}\left\{M_I(s)+\hat{M}_I(s)\right\}$$

has nontrivial solutions $\Rightarrow$ need to pick the physically correct one

Disperse in $m_I = M_I/\Omega_I$ $\quad\Omega_I(s) = \exp\left\{\frac{s}{\pi}\int\frac{ds'}{s'}\frac{\delta_I(s')}{s'-s-i\varepsilon}\right\}$

$\eta \to 3\pi$ beyond p^4

$$\frac{M_0(s)}{\Omega_0(s)} = \alpha_0 + \beta_0 s + \gamma_0 s^2 + \frac{s^2}{\pi}\int ds' \frac{\sin\delta_0(s')\,\hat{M}_0(s')}{|\Omega_0(s')|s'^2(s'-s-i\varepsilon)}\,,$$

$$\frac{M_1(s)}{\Omega_1(s)} = \beta_1 s + \frac{s}{\pi}\int ds' \frac{\sin\delta_1(s')\,\hat{M}_1(s')}{|\Omega_1(s')|s'(s'-s-i\varepsilon)}\,,$$

$$\frac{M_2(s)}{\Omega_2(s)} = \frac{s^2}{\pi}\int ds' \frac{\sin\delta_2(s')\,\hat{M}_2(s')}{|\Omega_2(s')|s'^2(s'-s-i\varepsilon)}\,. \qquad (-3)$$

find a $\delta_{0,1,2}(s) \Longrightarrow$ solve for M_1, M_2, M_3.

fix $\alpha_0, \beta_0, \gamma_0, \beta_1$

$$\gamma_0 \approx 0 \qquad \beta_1 \approx -\frac{4L_3 - 1/(64\pi^2)}{F_\pi^2(m_\eta^2 - m_\pi^2)}$$

α_0, β_0 values depend on where in s, t, u plane matching is done

$\eta \to 3\pi$ beyond p^4

AL: Lowest order is $M(s,t,u) = \dfrac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$

zero at $s_4/3\, m_\pi^2$: remains in the neighbourhood:

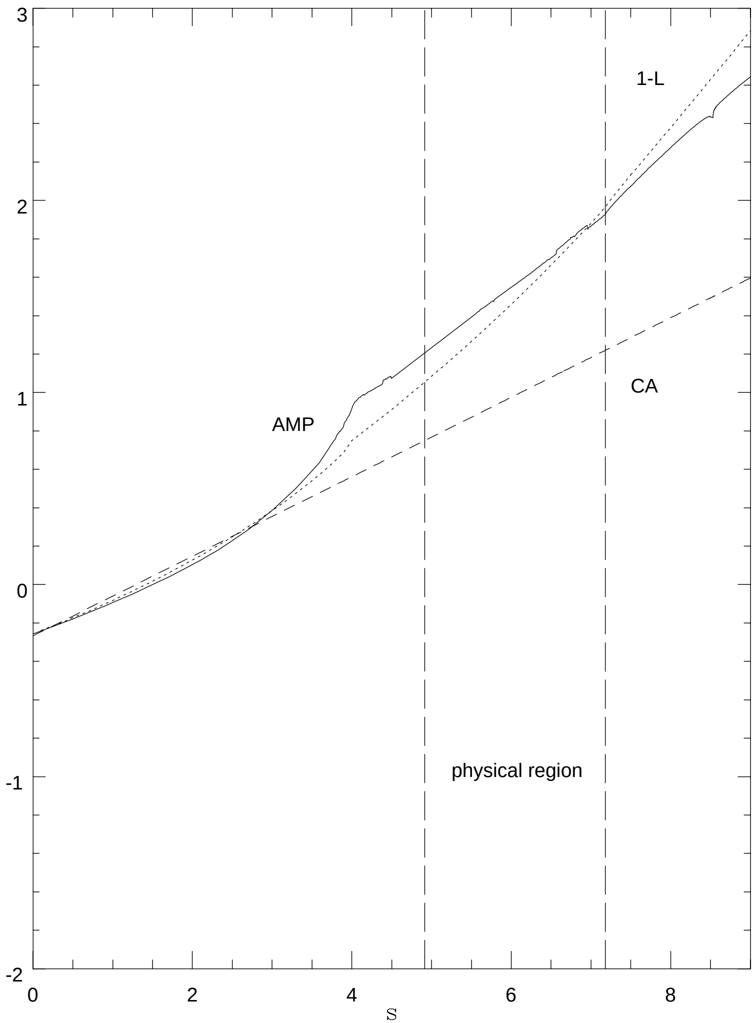
match position of s_A and slope of Adler zero.

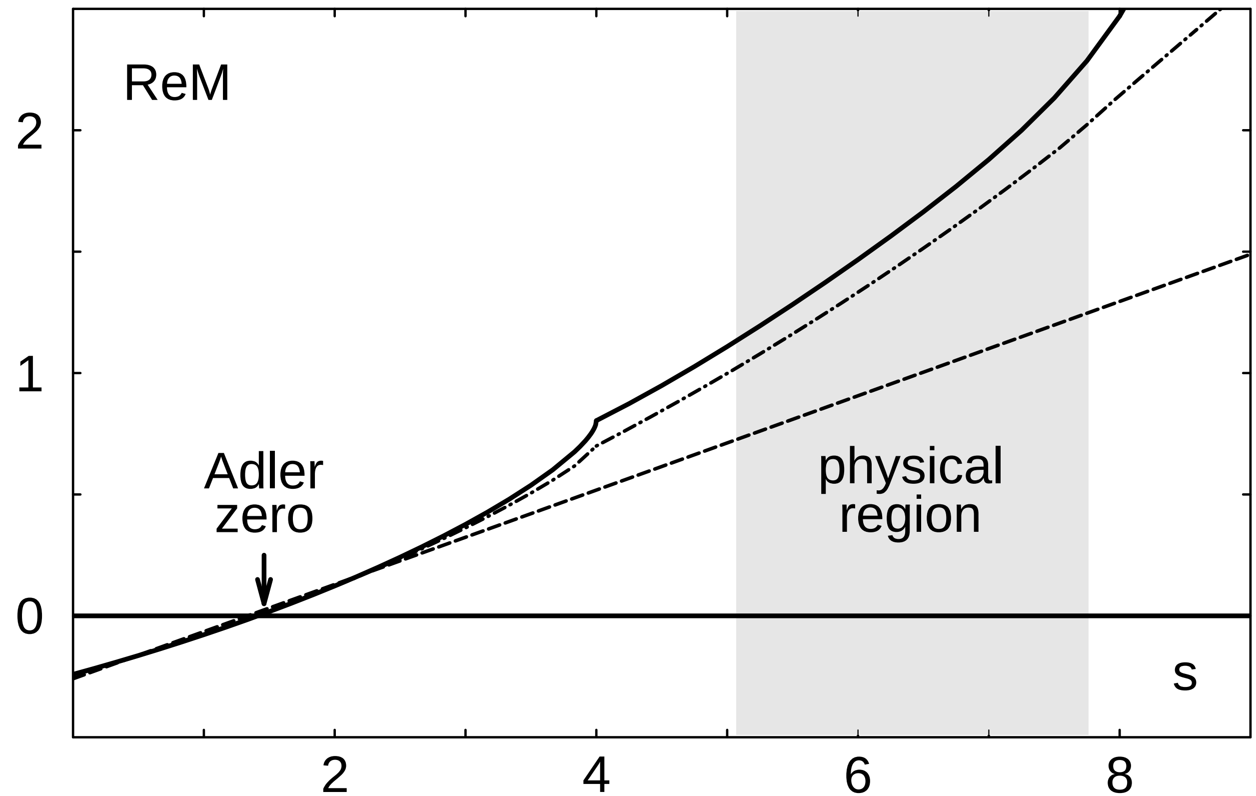
KWW: fix amplitude at some place(s) in s, t, u plane to be equal to p^4

$s = u$ of AL $\Longleftrightarrow$ KWW and simplified with $\hat{M}_I = 0$

Dalitzplot distributions provide a check

$\eta \to 3\pi$ beyond p^4





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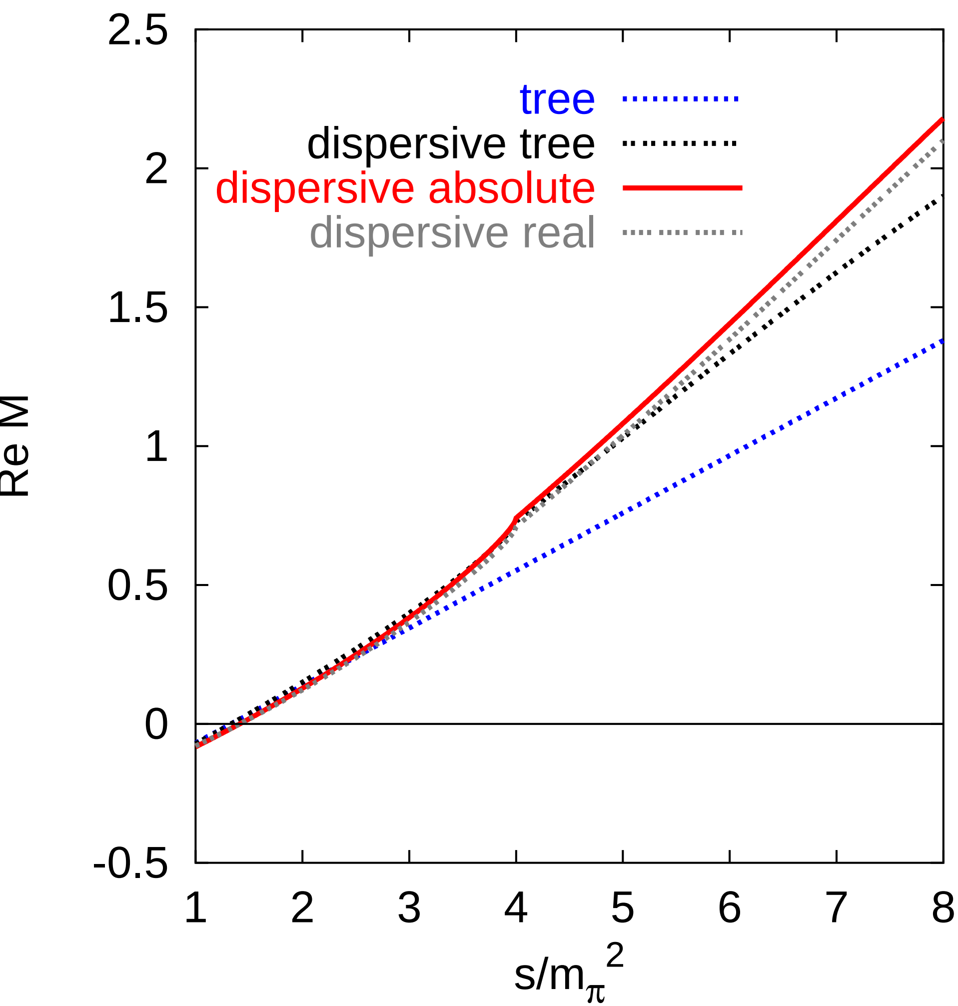

$\eta \to 3\pi$ **beyond** p^4

$$\left\{\begin{array}{lcl} x & = & \sqrt{3}\,\frac{T_+ - T_-}{Q_\eta} = \frac{\sqrt{3}}{2M_\eta Q_\eta}(u-t)\,, \\ y & = & \frac{3T_0}{Q_\eta} - 1 = \frac{3}{2m_\eta Q_\eta}\left\{(m_\eta - m_{\pi^0})^2 - s\right\} - 1\,, \\ Q_\eta & = & m_\eta - 2m_\pi^+ - m_\pi^0 \end{array}\right.$$

$r \equiv \Gamma(\eta \to \pi^0\pi^0\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0) = 1.44 \pm 0.04(PDG2004)\,,$

charged decay: $1 + ay + by^2 + cx^2\,,$ normalized at $x = y = 0$

Neutral decay: $1 + g(x^2 + y^2)$

	a	b	c	g
tree	-1.00	0.25	0.00	0.000
one-loop	-1.33	0.42	0.08	0.03[a]
dispersive (KWW)	-1.16	0.26	0.10	-0.014 — -0.028
tree dispersive	-1.10	0.31	0.001	-0.013
absolute dispersive	-1.21	0.33	0.04	-0.014

$\eta \to 3\pi$ beyond p^4

Experimental results for the charged decay

	a	b	c
Layter	-1.08 ± 0.14	0.034 ± 0.027	0.046 ± 0.031
Gormley	-1.17 ± 0.02	0.21 ± 0.03	0.06 ± 0.04
Crystal Barrel	-0.94 ± 0.15	0.11 ± 0.27	
Crystal Barrel	-1.22 ± 0.07	0.22 ± 0.11	0.06 fixed
KLOE	-1.072 ± 0.009	0.117 ± 0.008	0.047 ± 0.008

neutral decay

	g
Alde	-0.044 ± 0.046
Crystal Barrel	-0.104 ± 0.039
Crystal Ball	-0.062 ± 0.008
SND	-0.020 ± 0.023
KLOE	-0.026 ± 0.014

$\eta' \to \eta\pi\pi$ and $\eta' \to 3\pi$

η' decays: add a ϕ_0 degree of freedom to the usual ChPT

Witten, DiVecchia, Veneziano, Schechter, Rosenzweig,. . .

Write the most general $U(3)_L \times U(3)_R$ invariant Lagrangian as a function of:

$$\frac{\sqrt{2}\phi_0}{F} + \theta \quad \text{and} \quad U = e^{i\sqrt{2}\phi_0/F} e^{i\sqrt{2}M/F} \quad \text{with} \quad U \to g_R U g_L^\dagger$$

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$\frac{\sqrt{2}\phi_0}{F} + \theta$ and $U = e^{i\sqrt{2}\phi_0/F} e^{i\sqrt{2}M/F}$ with $U \to g_R U g_L^\dagger$

Most general lagrangian: very many terms: $F_i\left(\frac{\sqrt{2}\phi_0}{F} + \theta\right)$

Order p^2, m_q: 5 functions (Gasser-Leutwyler)

Order $p^4, m_q p^2, m_q^2$: 57 functions (Herrera-Siklody et al.)

$\eta' \to \eta\pi\pi$ **and** $\eta' \to 3\pi$

Need something more

$\eta' \to \eta\pi\pi$ and $\eta' \to 3\pi$

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Large number of colours: large N_c

In this limit:

$$\begin{array}{cccc} \partial_\mu A^{0\mu} & = m_q P & + & \omega \\ \mathcal{O}(N_c) & \mathcal{O}(N_c) & & \mathcal{O}(1) \end{array}$$

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So we CAN treat ω as a perturbation

$\Longrightarrow$ Basis of *all* the large N_c Chiral lagrangian predictions for η'

$\eta' \to \eta\pi\pi$ and $\eta' \to 3\pi$

BUT some problems remain:

- There are large $\pi\pi$ rescatterings possible in the S-wave channel ($1/N_c$ suppressed but sizable) $\Longrightarrow$ "σ"
- ρ and ω are present in the final states $\Longrightarrow$ obvious need to go beyond ChPT

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Some attempts at resummation exist: e.g. Borasoy et al.

$\eta' \to \eta\pi\pi$ and $\eta' \to 3\pi$

Both decays have different sources in the Chiral Lagrangian

$$\mathcal{L} = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle + \frac{F^2}{4}\langle \chi U^\dagger + U\chi^\dagger\rangle - \frac{1}{2}m_0^2\phi_0^2$$

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$\eta' \to \eta\pi\pi$: (1) $A = \dfrac{m_\pi^2}{6F}\left(s\sqrt{2}\cos(2\theta) - \sin(2\theta)\right)$

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	$\eta' \to \eta\pi^0\pi^0$	$\eta' \to \eta\pi^+\pi^-$	$\eta' \to \pi^0\pi^0\pi^0$	$\eta' \to \pi^0\pi^+\pi^-$
	1.0 keV	1.9 keV	455 eV	405 eV
Exp	42 ± 6 keV	89 ± 10 keV	311 ± 77 eV	≤ 1005 eV

$\eta' \to \eta\pi\pi$ and $\eta' \to 3\pi$: so different?

$\eta' \to \eta\pi\pi$: no isospin breaking needed, m_π^2 disappears at higher orders $\Rightarrow$ large enhancements possible

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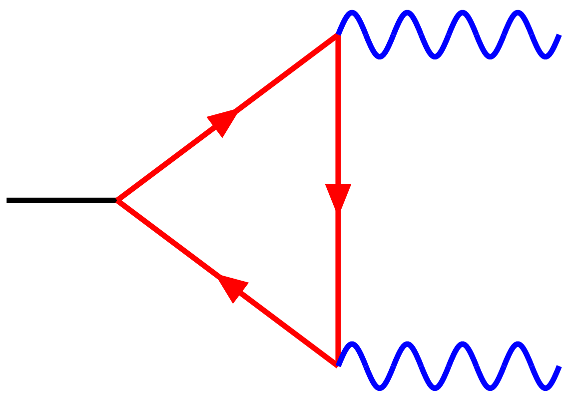
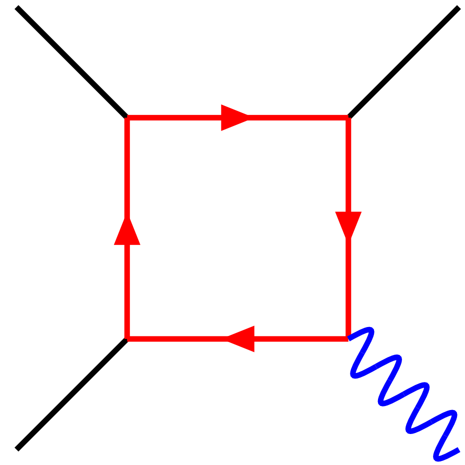
$\boxed{\eta' \to \eta\pi\pi}$ at NLO in $1/N_c$ can add $\eta'\eta\partial_\mu\pi\partial^\mu\pi$: fixes rate but not slope
Study distributions to uncover effects from $a_0, \sigma, f_0, \ldots$ and improve BR precision to check isospin prediction for ratio 2

$\boxed{\eta' \to \pi\pi\pi}$ Detect charged decay mode and study distributions

Anomalies

The famous triangle graph (Now with photons)

 but also  and ...

Wess-Zumino-Witten term, Adler-Bardeen theorem, Anomaly, and much more

$$\partial_\mu A^{0\mu} = 2\sqrt{N_f}\omega + \quad F_{\mu\nu}\tilde{F}^{\mu\nu} + \ldots$$

anomaly with the other interactions

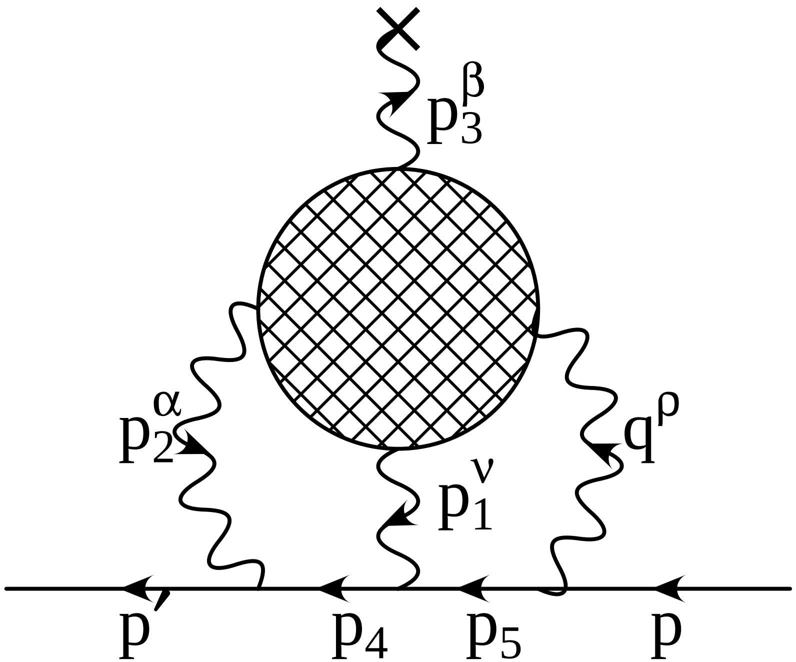
Anomalies: What processes

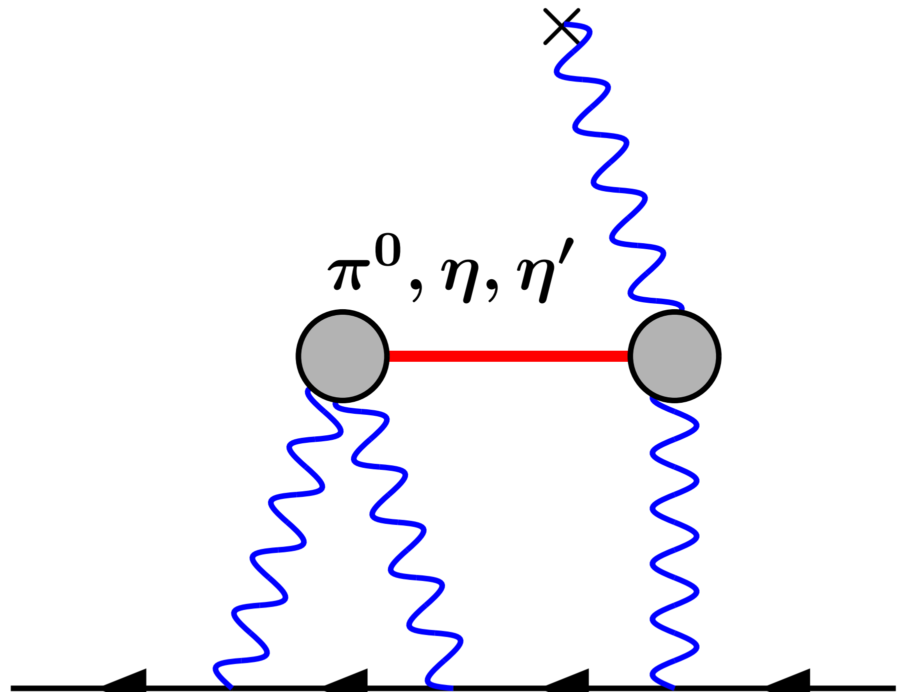
List of experiments *obviously* incomplete

$\pi^0 \to \gamma\gamma$	Primex, e^+e^-	
$\eta \to \gamma\gamma$	Primex, e^+e^-	
$\eta' \to \gamma\gamma$	e^+e^-	
$\gamma\pi^0\pi^+\pi^-$	Primakoff	
$\eta \to \pi^+\pi^-\gamma^{(*)}$	WASA,KLOE	
$\eta' \to \pi^+\pi^-\gamma^{(*)}$	WASA	
$\eta \to \gamma^{(*)}\gamma^{(*)}$	WASA,KLOE, CLEOc	g-2
$\eta' \to \gamma^{(*)}\gamma^{(*)}$	WASA,KLOE, CLEOc	g-2
$\eta' \to \rho^0\gamma$	WASA	

$\gamma^{(*)}$ stands for (if allowed) γ, e^+e^-, $\mu^+\mu^-$ or an off-shell photon in tagged $\gamma\gamma$ collisions

Muon g-2





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- Understanding physics: π^0 versus η versus η'
- In all channels: are the flavour singlet degrees of freedom significantly different from the nonsinglet ones.
- Glue is important for η' in its mass
- Can we detect glue also in other places or is the rest *merely* a problem of final state interactions
- requires getting at the mechanisms behind η, η' decays
- High quality distributions are a must