Chiral Perturbation Theory at Two Loops for Lattice Gauge Theory

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Overview

- Introduction
- Why (Effective) Field Theory
- Chiral Perturbation Theory
- ChPT and lattice QCD
- Two Loop: General
- Two Loop: Two Flavours
- Two Loop: PQChPT
- Two Loop: Three Flavours
  - General fitting strategy and some comments
  - $\pi\pi$, $\pi K$ and scalar form factors
  - $K_{\ell 3}$ and $V_{us}$
- Conclusions
Introduction

Simplest hadrons: ground state mesons: $\pi^\pm$, $\pi^0$, $K$ and $\eta$.

- Minimum number of constituents
- Lightest state: spatially simplest
- Chiral Symmetry: $m_q \to 0 \implies m_M \to 0$
- Masses and Decay Constants Simple

- Form Factors Next Simplest property
- Spatial information via Fourier Transform
- Different probes $\implies$ different properties
Why (Effective) Field Theory?

Effective:  
- Use right degrees of freedom: essence of (most) physics
- Gap in the spectrum $\implies$ separation of scales
- With lower d.o.f.: build most general Lagrangian

$\Rightarrow \infty \#$ parameters
$\Rightarrow$ Where did predictivity go? $\implies$ power counting
Why (Effective) Field Theory?

Field Theory

- Only known way to combine QM and special relativity
- Taylor series does not work (convergence radius zero)
- Continuum of excitation states to be taken into account
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- Continuum of excitation states to be taken into account
- Off-shell effects fully under control: these effects are there as new free parameters
- Model-independent and systematic: ALL effects at given order included
- **Theory** \(\Rightarrow\) errors can be estimated
Why (Effective) Field Theory?

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- Taylor series does not work (convergence radius zero)
- Continuum of excitation states to be taken into account
- Off-shell effects fully under control: these effects are there as new free parameters
- Model-independent and systematic: ALL effects at given order included
- Theory $\implies$ errors can be estimated
- Many parameters (but possible modelspace is large)
- Expansion might not converge (often still useful for model classification)
Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so $M_\rho$ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L D q_L + i\bar{q}_R D q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$. 
Chiral Perturbation Theory

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$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[ i\bar{q}_L \slashed{D} q_L + i\bar{q}_R \slashed{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R) \right]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Can also see that via $v < c$, $m_q \neq 0 \implies m_q = 0 \iff v = c$. 
\[ \langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0 \]

\[ SU(3)_L \times SU(3)_R \] broken spontaneously to \( SU(3)_V \)

8 generators broken \( \implies \) 8 massless degrees of freedom and interaction vanishes at zero momentum

**Power counting in momenta:**

\[ \int d^4p \]

\[ \frac{1}{p^2} \]

\[ p^2 \]

\[ \frac{(p^2)^2}{(1/p^2)^2} p^4 = p^4 \]

\[ p^4 \]

\[ \frac{(p^2)(1/p^2)}{p^4} = p^4 \]
ChPT and Lattice QCD

Mesons \quad = \quad \text{Quark Flow} \quad + \quad \text{Quark Flow}

Valence \quad \text{Valence} \quad + \quad \text{Sea} \quad + \quad \cdots

Valence is easy to deal with in lattice QCD
Sea is very difficult

They can be treated separately: i.e. different quark masses
Partially Quenched ChPT (PQChPT)
Two Loop: General

Lagrangian Structure:

<table>
<thead>
<tr>
<th></th>
<th>2 flavour</th>
<th>3 flavour</th>
<th>3+3 PQChPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^2$</td>
<td>$F, B$</td>
<td>2</td>
<td>$F_0, B_0$</td>
</tr>
<tr>
<td>$p^4$</td>
<td>$l_i^r, h_i^r$</td>
<td>7+3</td>
<td>$L_i^r, H_i^r$</td>
</tr>
<tr>
<td>$p^6$</td>
<td>$c_i^r$</td>
<td>53+4</td>
<td>$C_i^r$</td>
</tr>
</tbody>
</table>

$p^2$: Weinberg 1966
$p^4$: Gasser, Leutwyler 84,85
$p^6$: JB, Colangelo, Ecker 99,00

Note

\[ \begin{align*}
\text{replica method} & \implies \text{PQ obtained from } N_F \text{ flavour} \\
\text{All infinities known} \\
\text{3 flavour is a special case of 3+3 PQ:} \\
\hat{L}_i^r, K_i^r & \rightarrow L_i^r, C_i^r
\end{align*} \]
Most Calculations Done

\[ \gamma \gamma \rightarrow \pi^0 \pi^0 \]
\[ \gamma \gamma \rightarrow \pi^+ \pi^-, F_\pi, m_\pi \]
\[ \pi \pi\text{-scattering}, F_\pi, m_\pi \]
\[ F_{V\pi}(t), F_{S\pi} \]
\[ \pi \rightarrow \ell \nu \gamma \]

- Reasonable convergence
- \( c_i^r \) not important for many threshold quantities

\[ \rightarrow \] combined with dispersive methods, Roy equations, etc.
for very precise calculations
Subject just beginning
valence equal mass, 3 sea equal mass:
\[ m^2_{\pi^+} : \text{JB, Danielsson, Lähde, hep-lat/0406017} \]
\[ F^2_{\pi^+} : \text{JB, Lähde, soon} \]

Planned: all the other mass combinations

Actual Calculations:
\[ \text{heavy use of FORM Vermaseren} \]
\[ \text{use PQ without super } \Phi_0 \text{ in supersymmetric formalism} \]
\[ \text{Main problem: sheer size of the expressions} \]
Problem: Plotting with many input parameters

Remember: $\mathcal{M}_i = (\mathcal{M}_0)^2 + \mathcal{O}(m_i^2)$

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Problem: Plotting with many input parameters

Plot masses as a function of lowest order mass squared

\[ \chi_i = 2B_0 m_i = m_{i}^{2(0)} \]
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Remember: \[ \chi_i \approx 0.3 \text{ GeV}^2 \approx (550 \text{ MeV})^2 \sim \text{border ChPT} \]
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1+1 case:

Valence: \( \chi_1 = \chi_2 = \chi_3 \)

Sea: \( \chi_4 = \chi_5 = \chi_6 \)
Problem: Plotting with many input parameters

Plot masses as a function of lowest order mass squared

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1+1 case: Valence: \( \chi_1 = \chi_2 = \chi_3 \)
Sea: \( \chi_4 = \chi_5 = \chi_6 \)

Plot along curves: \( \chi_4 = \tan \theta \chi_1 \) or \( \chi_4 \) constant
Notice the Quenched Chiral Logs:

\[ m^2 \frac{1}{\bar{A}_1} = 1 + a F^2 \bar{A}_4 \log \bar{A}_1 + \ldots \]
PQChPT: 1+1 case

Notice the Quenched Chiral Logs:

\[ m^2 = 1 + a F_0^2 \log \frac{1}{F_0} + \cdots \]
Notice the Quenched Chiral Logs:

\[ \frac{m_\pi^2}{\chi_1} = 1 + \frac{a}{F^2} \chi_4 \log \chi_1 + \cdots \]
$p^4 + p^6$ relative correction mass
PQChPT at Two Loops

\[ p^4 + p^6 \] relative correction mass

decay constant (preliminary)
$\frac{\delta^{(4)}}{F_0^2} + \frac{\delta^{(6)}}{F_0^4}$

$p_4^4$ $p_6^6$

correction
decay constant
fit 10
PQChPT at Two Loops

\[ \delta^{(4)} / F_0^2 + \delta^{(6)} / F_0^4 \]

\[ p^4 + p^6 \]

correction
decay constant

fit 10
Three Flavours at Two Loop

\[ \Pi_{VV\pi}, \Pi_{VV\eta}, \Pi_{VVK} \quad \text{Kambor, Golowich} ; \quad \text{Kambor, Dürr} ; \quad \text{Amorós, JB, Talavera} \]

\[ \Pi_{AA\pi}, \Pi_{AA\eta}, F_\pi, F_\eta, m_\pi, m_\eta \quad \text{Kambor, Golowich} ; \quad \text{Amorós, JB, Talavera} \]

\[ \Pi_{SS} \quad \text{Moussallam} \]

\[ \Pi_{VVK}, \Pi_{AAK}, F_K, m_K \quad \text{Amorós, JB, Talavera} \]

\[ K_{\ell 4} \quad \text{Amorós, JB, Talavera} \]

\[ F_M, m_M \ (m_u \neq m_d) \quad \text{Amorós, JB, Talavera} \]

\[ F_{V\pi}, F_{VK^+}, F_{VK^0} \quad \text{Post, Schilcher} ; \quad \text{JB, Talavera} \]

\[ K_{\ell 3} \quad \text{Post, Schilcher} ; \quad \text{JB, Talavera} \]

\[ F_{S\pi}, F_{SK} \quad \text{Post, Schilcher} ; \quad \text{JB, Talavera} \]

\[ K, \pi \rightarrow \ell \nu \gamma \quad \text{JB, Dhonte} \]

\[ \pi \pi \quad \text{Geng, Ho, Wu} \]

\[ \pi K \quad \text{JB, Dhonte, Talavera} \]
General Strategy and some comments

- Find enough inputs from experiment
- $C_{i}^{r}$:
  - kinematical dependence: agree well with single resonance saturation
  - quark mass+kinematical: if vector dominated, seems to be OK
  - quark mass+kinematical: if scalar dominated: which scalars? (not $\sigma$)
  - quark masses: which scalars? unrealistically large estimates
- in $p^{6}$ physical or lowest order masses: thresholds in right place requires physical
General Strategy and some comments

Inputs:

\( K_{\ell 4} : F(0), G(0), \lambda \)

\( m_{\pi 0}^2, m_{\eta}^2, m_{K^+}^2, m_{K^0}^2 \)

\( F_{\pi^+} \)

\( F_{K^+} / F_{\pi^+} \)

\( m_s / \hat{m} \)

\( L_4^r, L_6^r \)

\( C_i^r \) from single resonance approximation

\( \hat{m} = (m_u + m_d) / 2 \)

E865 BNL
em with Dashen violation

\[ |q^2| \ll m_{\rho}^2, m_{S}^2 \]

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General Strategy and some comments

<table>
<thead>
<tr>
<th>$10^3 L_1^r$</th>
<th>$10^3 L_2^r$</th>
<th>$10^3 L_3^r$</th>
<th>$10^3 L_4^r$</th>
<th>$10^3 L_5^r$</th>
<th>$10^3 L_6^r$</th>
<th>$10^3 L_7^r$</th>
<th>$10^3 L_8^r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.43 ± 0.12</td>
<td>0.73 ± 0.12</td>
<td>−2.53 ± 0.37</td>
<td>≡ 0</td>
<td>0.97 ± 0.11</td>
<td>≡ 0</td>
<td>−0.31 ± 0.14</td>
<td>0.60 ± 0.18</td>
</tr>
<tr>
<td></td>
<td>0.38</td>
<td>−2.91</td>
<td>≡ 0</td>
<td>1.46</td>
<td>≡ 0</td>
<td>−0.49</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>0.44</td>
<td>−2.31</td>
<td>≡ 0.5</td>
<td>0.82</td>
<td>≡ 0.1</td>
<td>−0.26</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>0.44</td>
<td>−2.33</td>
<td>≡ 0.2</td>
<td>0.88</td>
<td>≡ 0</td>
<td>−0.28</td>
<td>0.54</td>
</tr>
</tbody>
</table>

- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying $C_i^r$ factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6 \times 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and $\pi K$ thresholds
### General Strategy and some comments

<table>
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<tr>
<th></th>
<th>fit t 10</th>
<th>same $p^4$</th>
<th>fit t B</th>
<th>fit t D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2B_0 \hat{m}/m_\pi^2$</td>
<td>0.736</td>
<td>0.991</td>
<td>1.129</td>
<td>0.958</td>
</tr>
<tr>
<td>$m_\pi^2: p^4, p^6$</td>
<td>0.006, 0.258</td>
<td>0.009, ≡ 0</td>
<td>-0.138, 0.009</td>
<td>-0.091, 0.133</td>
</tr>
<tr>
<td>$m_K^2: p^4, p^6$</td>
<td>0.007, 0.306</td>
<td>0.075, ≡ 0</td>
<td>-0.149, 0.094</td>
<td>-0.096, 0.201</td>
</tr>
<tr>
<td>$m_\eta^2: p^4, p^6$</td>
<td>-0.052, 0.318</td>
<td>0.013, ≡ 0</td>
<td>-0.197, 0.073</td>
<td>-0.151, 0.197</td>
</tr>
<tr>
<td>$m_u/m_d$</td>
<td>0.45±0.05</td>
<td>0.52</td>
<td>0.52</td>
<td>0.50</td>
</tr>
<tr>
<td>$F_0$ [MeV]</td>
<td>87.7</td>
<td>81.1</td>
<td>70.4</td>
<td>80.4</td>
</tr>
<tr>
<td>$F_K/F_\pi: p^4, p^6$</td>
<td>0.169, 0.051</td>
<td>0.22, ≡ 0</td>
<td>0.153, 0.067</td>
<td>0.159, 0.061</td>
</tr>
</tbody>
</table>

- $m_u = 0$ always very far from the fits
- $F_0$: pion decay constant in the chiral limit
\( a_0^0 = 0.220 \pm 0.005, \quad a_0^2 = -0.0444 \pm 0.0010 \)

Colangelo, Gasser, Leutwyler

\( a_0^0 = 0.159 \quad a_0^2 = -0.0454 \) at order \( p^2 \)
\( a_0^{1/2} = 0.224 \pm 0.022, \ a_0^{2/3} = -0.0448 \pm 0.0077 \)

Büttiker, Descotes-Genon, Moussallam

\( a_0^{1/2} = 0.142 \ a_0^2 = -0.0708 \) at order \( p^2 \)
preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$
Scalar formfactor:

\[ f_0(t) = f_+(t) + \frac{t}{m_K^2 - m^2_\pi} f_-(t) \]

Usual parametrization:

\[ f_{+,0}(t) = f_+(0) \left( 1 + \lambda_{+,0} \frac{t}{m^2_\pi} \right) \]

\(|V_{us}|:\)
**$K_{\ell 3}$ Definitions and $V_{uS}$**

Scalar formfactor:

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$|V_{us}|$:
- Know theoretically $f_+(0) = 1 + \cdots$
- Short distance correction to $G_F$ from $G_{\mu}$ Marciano-Sirlin
- Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$
- Isospin Breaking Leutwyler-Roos $\frac{f_{K^+\pi^0}(0)}{f_{K^0\pi^-}(0)} = 1.022$ In Progress
$K^{\ell 3}$ Definitions and $V_{us}$

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Progress

- Know experimentally $f_+(0)$
- Radiative Corrections: Cirigliano et al., hep-ph/0110153
- Parametrize form-factor: is linear enough for $f_+(t)$?
$K_{\ell 3}$ Definitions and $V_{us}$

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PDG2002:  
|\(V_{ud}\)| = $0.9734 \pm 0.0008$, |\(V_{us}\)| = $0.2196 \pm 0.0026$
|\(V_{ud}\)|$^2$ + |\(V_{us}\)|$^2$ = $(0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) = 0.9957 \pm 0.0019$
\[ f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t) \]

\[ f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops} \]

\[ f_+^{(6)}(t) = -\frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 + \frac{t}{F_\pi^4} R_+^{K\pi} + \frac{t^2}{F_\pi^4} (-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_9^r) \]

Pion electromagnetic Form factor:

JB, Talavera

\[ L_9^r = 0.00593 \pm 0.00043 \]

\[ -4C_{88}^r + 4C_{90}^r = 0.00022 \pm 0.00002 \]

VMD: \[ R_+^{K\pi} \approx -4 \times 10^{-5} \text{ GeV}^2 \]
ChPT fit to $f_+(t)$

$$R_{+1}^K = -(4.7 \pm 0.5) \times 10^{-5} \text{ GeV}^2$$

$$c_+ = 3.2 \text{ GeV}^{-4}$$

fixed by ChPT

$$a_+ = 1.009 \pm 0.004$$

$$\lambda_+ = 0.0170 \pm 0.0015$$
\[ R_{+1}^{K\pi} = -2.5 \times 10^{-5}\ \text{GeV}^2 \]

\[
\left( c_+ = 3.2\ \text{GeV}^{-4}\right) \text{ fixed by ChPT}
\]

\[ a_+ = 1.006 \]

\[ \lambda_+ = 0.0214 \pm 0.0018 \]
Main Result:

\[ f_0(t) = 1 - \frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 \]
\[ + 8 \frac{t}{F_\pi^4} (2C_{12}^r + C_{34}^r) (m_K^2 + m_\pi^2) + \frac{t}{m_K^2 - m_\pi^2} (F_K/F_\pi - 1) \]
\[ - \frac{8}{F_\pi^4} t^2 C_{12}^r + \Delta(t) + \Delta(0). \]
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\[ - \frac{8}{F_\pi^4} t^2 C_{12}^r + \Delta(t) + \Delta(0). \]

\( \bar{\Delta}(t) \) and \( \Delta(0) \) contain NO \( C_i^r \) and only depend on the \( L_i^r \) at order \( p^6 \)

\[ \implies \]

All needed parameters can be determined experimentally
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\[ \implies \]

All needed parameters can be determined experimentally

\[ \Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[ L_i^r]. \]

\[ \Delta(0) = -0.0227 (p^4) + 0.0113 (p^6 \text{ pure loop}) + 0.0033 (p^6 L_i^r) \]
More Theory: Dispersion theory relates slopes and curvature in $f_0(t)$ Jamin, Oller, Pich

$$C_{12}^r + C_{34}^r = 3.2 \pm 1.5 \times 10^{-6} \implies f_+(0)p^6 = 0.002 \pm 0.009.$$
**$V_{us}$ present status**

- **More Theory:** Dispersion theory relates slopes and curvature in $f_0(t)$ Jamin, Oller, Pich
  
  $C_{12}^r + C_{34}^r = 3.2 \pm 1.5 \times 10^{-6} \implies f_+(0)p^6 = 0.002 \pm 0.009$.

- **More Experiment:**
  - 2003: E865 $K^+_{\ell 3}$ branching ratio: strong increase
  - 2004: KTeV $K^0_{\ell 3}$ branching ratio: strong increase
  - 2004: formfactor data KTeV ($K^0$) and ISTRA+ ($K^+$):
    - Curvature seen, reasonable agreement with ChPT
    - $\lambda_0$: old discrepancies gone?
$V_{us}$ present status

More Theory: Dispersion theory relates slopes and curvature in $f_0(t)$ Jamin, Oller, Pich

$$C_{12}^r + C_{34}^r = 3.2 \pm 1.5 \times 10^{-6} \implies f_+(0) p^6 = 0.002 \pm 0.009.$$

More Experiment:

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- 2004: KTeV $K_{\ell 3}^0$ branching ratio: strong increase
- 2004: formfactor data KTeV ($K^0$) and ISTRA+ ($K^+$):
  - Curvature seen, reasonable agreement with ChPT
  - $\lambda_0$: old discrepancies gone?
- Expect both more theory and experiment (KLOE,NA48)

the unitarity problem might be on the way out
Conclusions

- 2 flavour ChPT at 2 loops (almost) finished subject
- 3 flavour ChPT at 2 loops
  - many calculations done
  - things seem to work but convergence is fairly slow
  - “kinematical” and “vector” $C_i^r$ seem to be OK
  - $L_4^r, L_6^r$ nonzero but reasonable for large $N_c$
  - $\eta \rightarrow 3\pi$, isobreaking in $K_{\ell 3}$: parts done
- PQChPT at 2 loops: subject just beginning
  - $K_{\ell 3}$ an example of results even with all the $C_i^r$