Chiral Perturbation Theory at Two Loops for Lattice Gauge Theory

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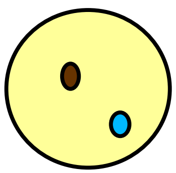
Overview

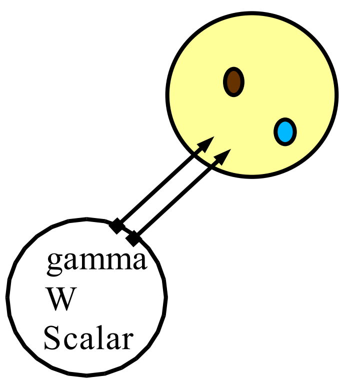
- Introduction
- Why (Effective) Field Theory
- Chiral Perturbation Theory
- ChPT and lattice QCD
- Two Loop: General
- Two Loop: Two Flavours
- Two Loop: PQChPT
- Two Loop: Three Flavours
  - General fitting strategy and some comments
  - $\pi\pi$, πK and scalar form factors
  - $K_{\ell 3}$ and V_{us}
- Conclusions

Introduction

Simplest hadrons : ground state mesons : $\pi^{\pm}$, π^0, K and η.



- Minimum number of constituents
- Lightest state: spatially simplest
- Chiral Symmetry: $m_q \to 0 \Longrightarrow m_M \to 0$
- Masses and Decay Constants Simple



- Form Factors Next Simplest property
- Spatial information via Fourier Transform
- Different probes $\Longrightarrow$ different properties

Why (Effective) Field Theory?

Effective:

- Use right degrees of freedom : essence of (most) physics
- Gap in the spectrum $\Longrightarrow$ separation of scales
- With lower d.o.f.: build most general Lagrangian

➠ $\infty\sharp$ parameters
➠ Where did predictivity go ? $\Big\}\Longrightarrow$ power counting

Why (Effective) Field Theory?

Field Theory

- Only known way to combine QM and special relativity
- Taylor series does not work (convergence radius zero)
- Continuum of excitation states to be taken into account

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- model-independent and systematic: ALL effects at given order included
- Theory $\Longrightarrow$ errors can be estimated

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- Off-shell effects fully under control: these effects are there as new free parameters
- model-independent and systematic: ALL effects at given order included
- Theory $\Longrightarrow$ errors can be estimated
- Many parameters (but possible modelspace is large)
- Expansion might not converge (often still useful for model classification)

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

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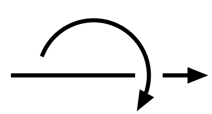
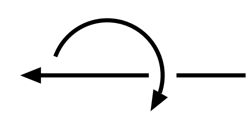
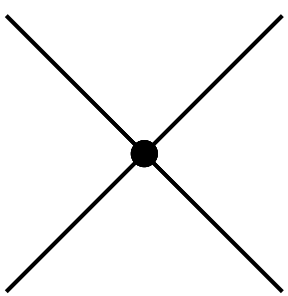
Can also see that via  $v < c,\ m_q \neq 0 \Longrightarrow$ / $v = c,\ m_q = 0 \not\Longrightarrow$ 

Chiral Perturbation Theory

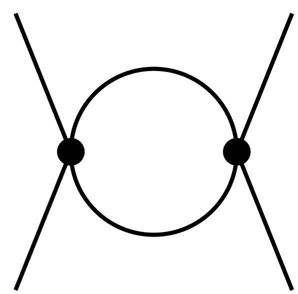
$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

Power counting in momenta:



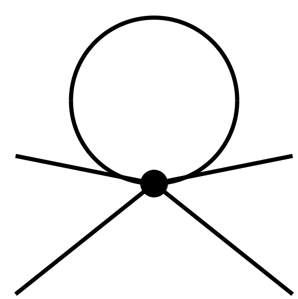
p^2



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

$1/p^2$

$\int d^4p$ $\quad p^4$



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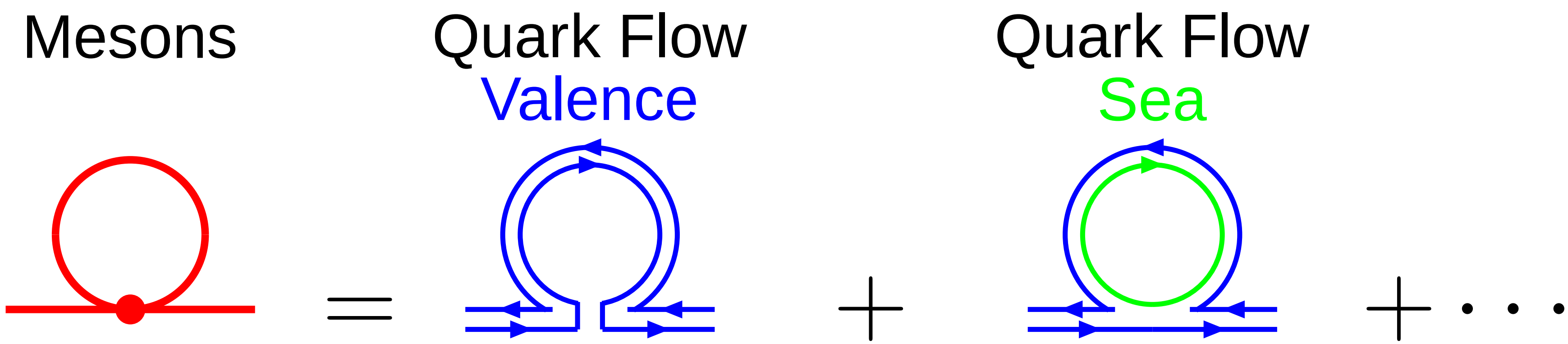
ChPT and Lattice QCD



Valence is easy to deal with in lattice QCD
Sea is very difficult

They can be treated separately: i.e. different quark masses

Partially Quenched ChPT (PQChPT)

Two Loop: General

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

Note
- replica method $\Longrightarrow$ PQ obtained from N_F flavour
- All infinities known
- 3 flavour is a special case of 3+3 PQ:

  $\hat{L}_i^r, K_i^r \to L_i^r, C_i^r$

Two Flavours at Two Loop

Most Calculations Done

$\gamma\gamma \longrightarrow \pi^0\pi^0$ — Bellucci, Gasser, Sainio

$\gamma\gamma \longrightarrow \pi^+\pi^-$, F_π, m_π — Bürgi

$\pi\pi$-scattering, F_π, m_π — JB,Colangelo,Ecker,Gasser,Sainio

$F_{V\pi}(t)$, $F_{S\pi}$ — JB, Colangelo, Talavera

$\pi \longrightarrow \ell\nu\gamma$ — JB, Talavera

- Reasonable convergence
- c_i^r not important for many threshold quantities

$\Longrightarrow$ combined with dispersive methods, Roy equations, etc.

for very precise calculations

PQChPT at Two Loop

Subject just beginning

valence equal mass, 3 sea equal mass:

$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0406017

$F^2_{\pi^+}$: JB,Lähde, soon

Planned: all the other mass combinations

Actual Calculations:

- heavy use of FORM Vermaseren
- use PQ without super Φ_0 in super-symmetric formalism
- Main problem: sheer size of the expressions

PQChPT at Two Loop

Problem: Plotting with many input parameters

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Plot masses as a function of lowest order mass squared

$$\chi_i = 2B_0 m_i = m_M^{2(0)}$$

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Remember: $\chi_i \approx 0.3\ GeV^2 \approx (550\ MeV)^2 \sim$ border ChPT

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1+1 case: Valence: $\chi_1 = \chi_2 = \chi_3$
Sea: $\chi_4 = \chi_5 = \chi_6$

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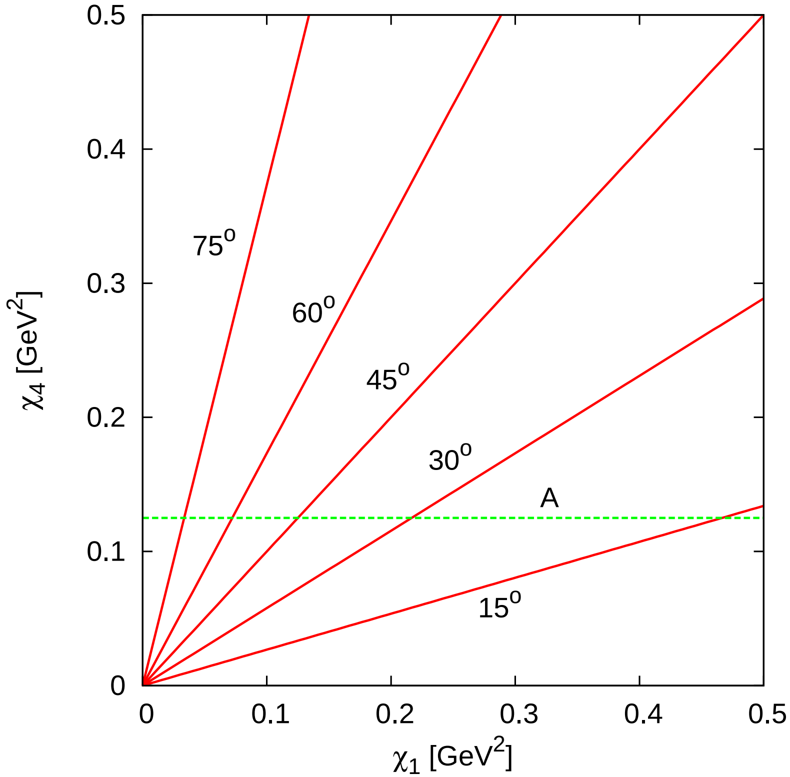
1+1 case: Valence: $\chi_1 = \chi_2 = \chi_3$
Sea: $\chi_4 = \chi_5 = \chi_6$

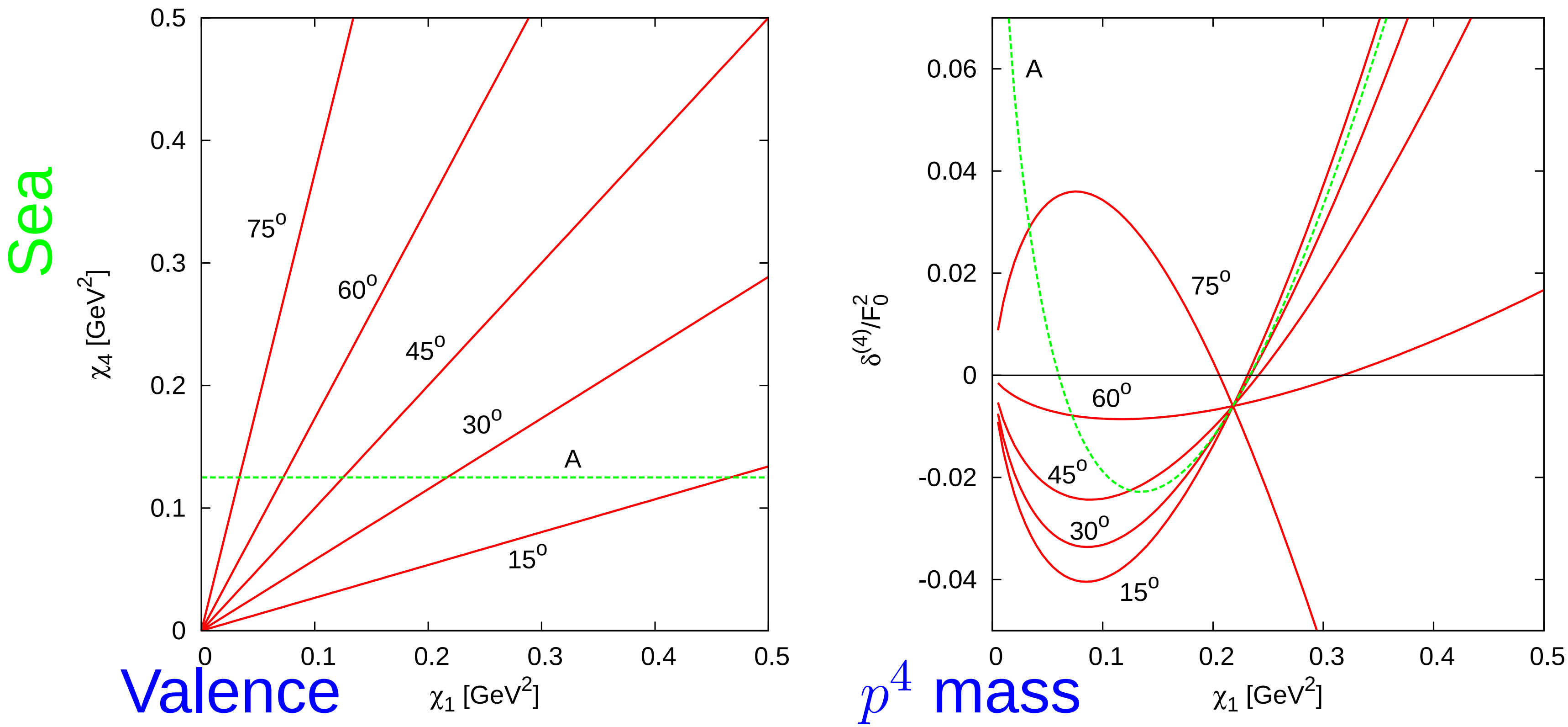
Plot along curves: $\chi_4 = \tan\theta\ \chi_1$ or χ_4 constant

PQChPT: 1+1 case

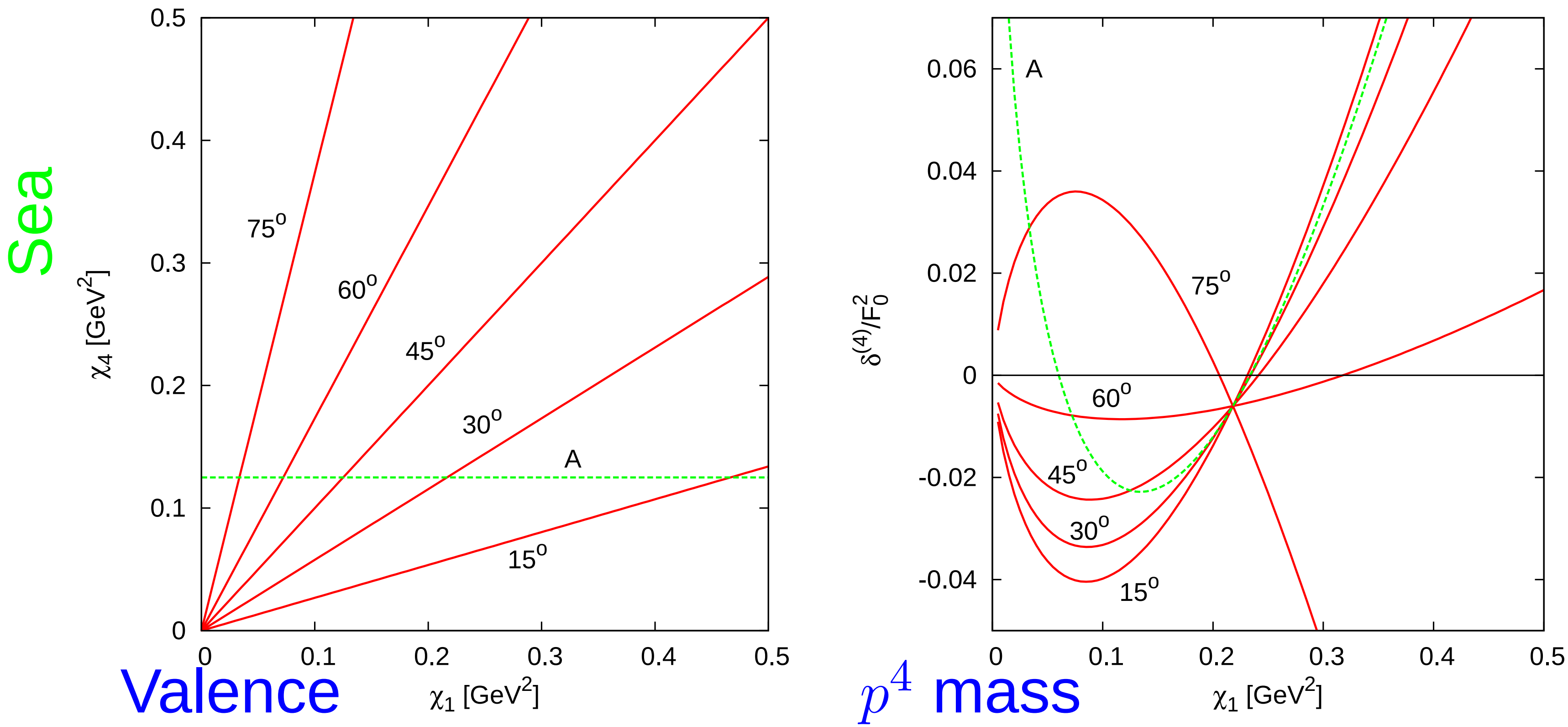
Sea



PQChPT: 1+1 case

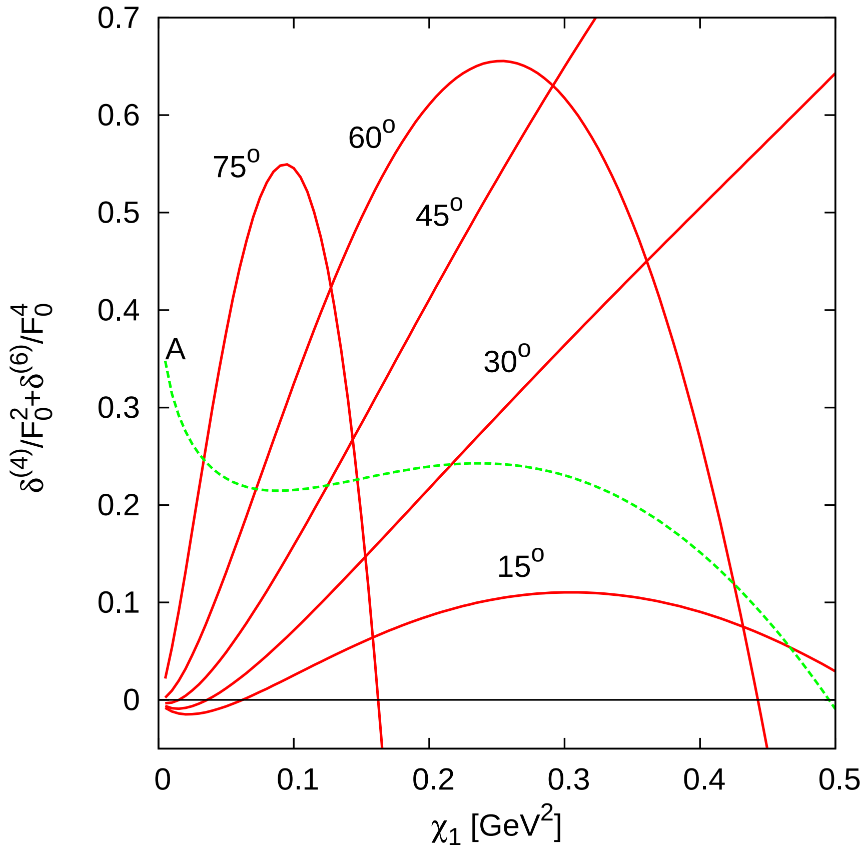


PQChPT: 1+1 case



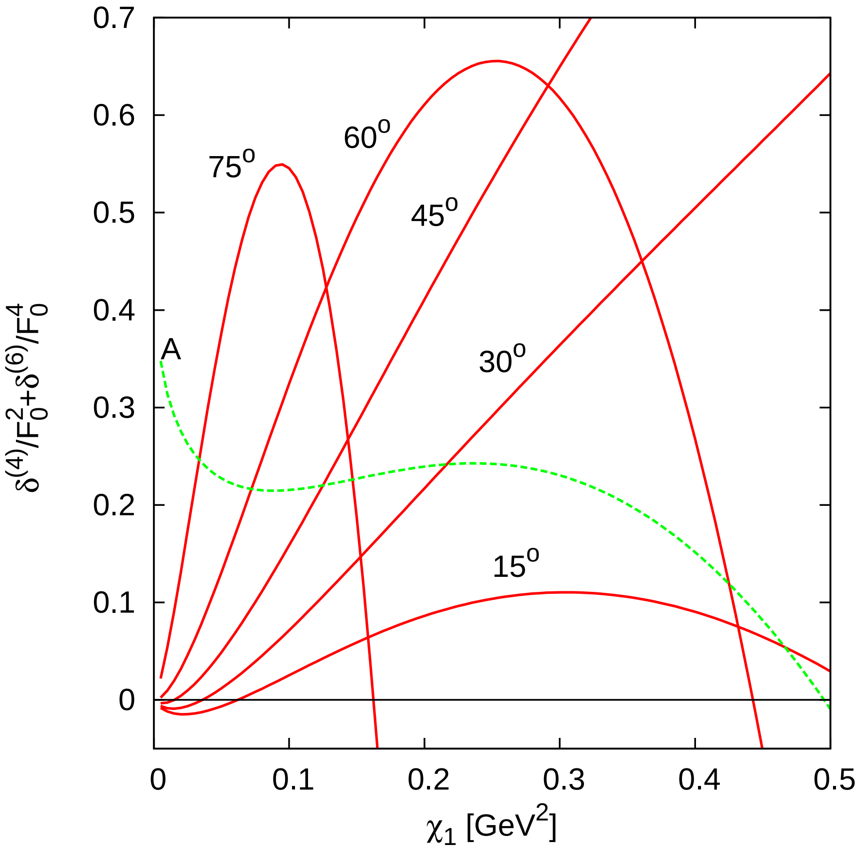
Notice the Quenched Chiral Logs: $\dfrac{m_\pi^2}{\chi_1} = 1 + \dfrac{a}{F^2}\chi_4 \log \chi_1 + \cdots$

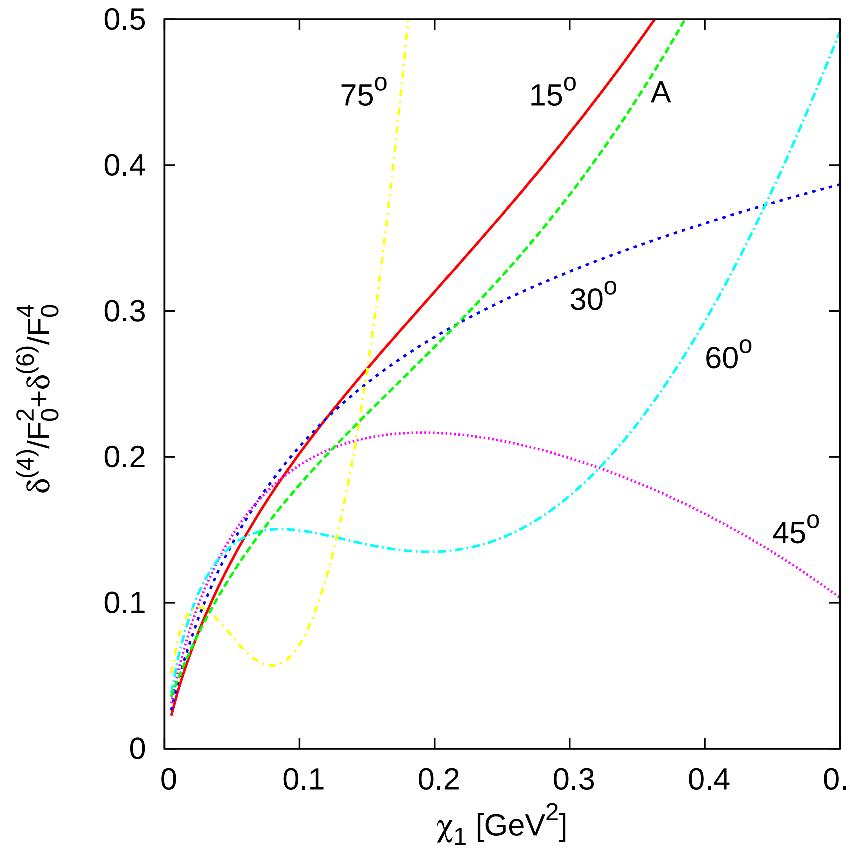
PQChPT at Two Loops



$p^4 + p^6$ relative correction mass

PQChPT at Two Loops



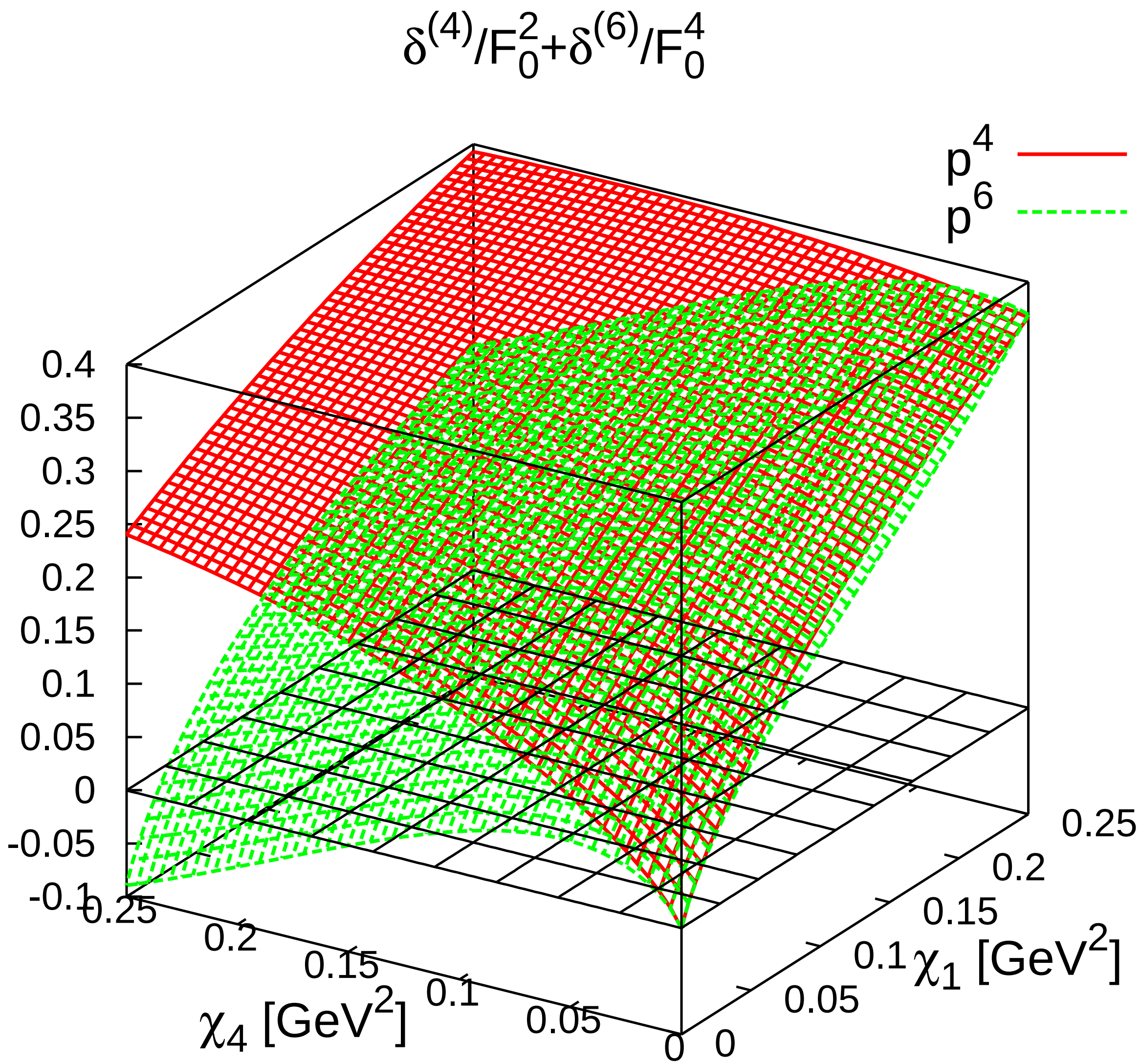


$p^4 + p^6$ relative correction mass

decay constant

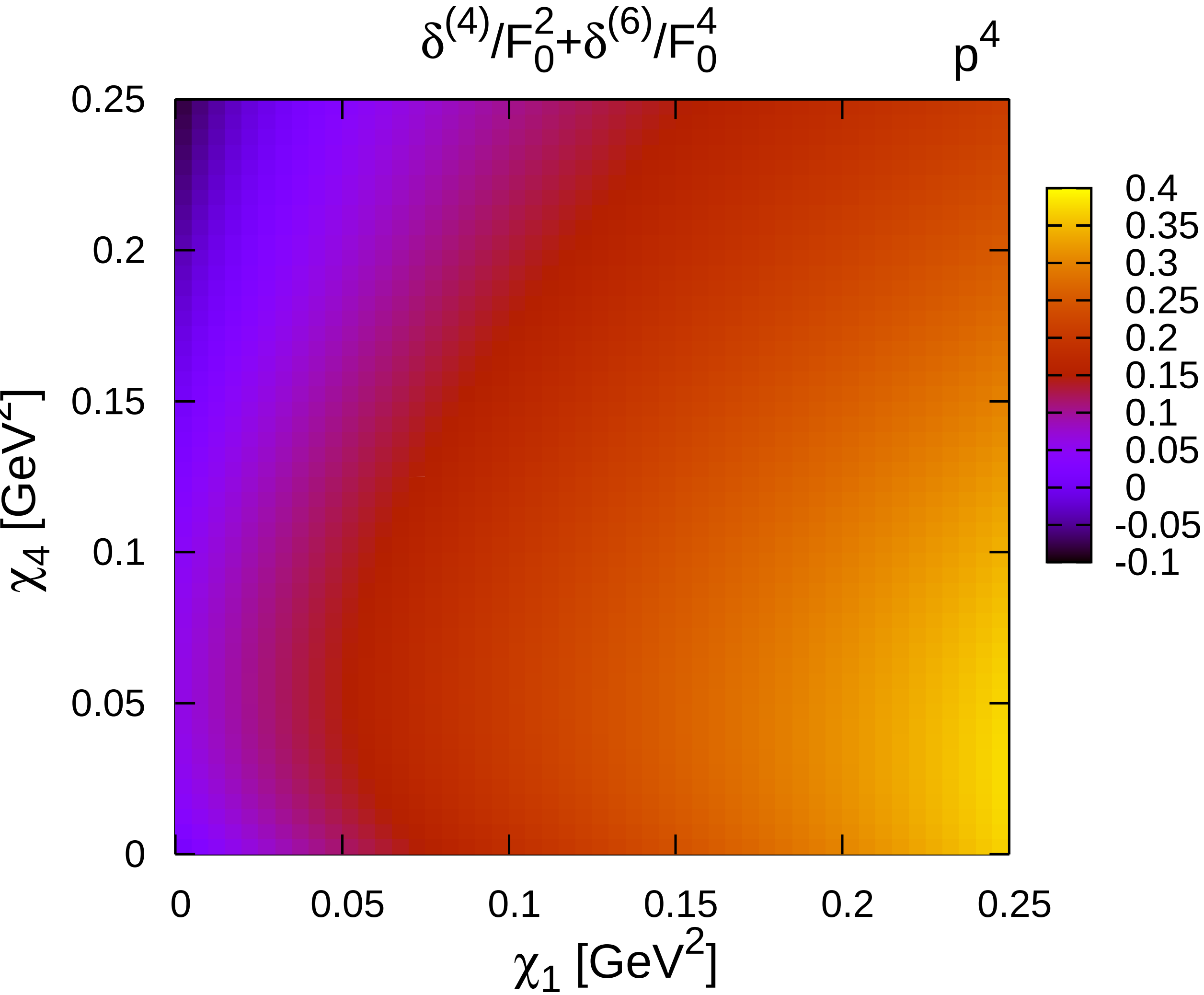
(preliminary)

PQChPT at Two Loops



$p^4 + p^6$
correction
decay constant
fit 10

PQChPT at Two Loops



$p^4 + p^6$
correction
decay constant
fit 10

Three Flavours at Two Loop

$\Pi_{VV\pi}$, $\Pi_{VV\eta}$, Π_{VVK}	Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera	
$\Pi_{AA\pi}$, $\Pi_{AA\eta}$, F_π, F_η, m_π, m_η	Kambor, Golowich; Amorós, JB, Talavera	
Π_{SS}	Moussallam	L_4^r, L_6^r
Π_{VVK}, Π_{AAK}, F_K, m_K	Amorós, JB, Talavera	
$K_{\ell 4}$	Amorós, JB, Talavera	L_1^r, L_2^r, L_3^r
F_M, m_M $(m_u \neq m_d)$	Amorós, JB, Talavera	$L_{5,7,8}^r, m_u/m_d$
$F_{V\pi}$, F_{VK^+}, F_{VK^0}	Post, Schilcher; JB, Talavera	L_9^r
$K_{\ell 3}$	Post, Schilcher; JB, Talavera	V_{us}
$F_{S\pi}$, F_{SK}	JB, Dhonte	L_4^r, L_6^r
$K, \pi \to \ell\nu\gamma$	Geng, Ho, Wu	L_{10}^r
$\pi\pi$	JB,Dhonte,Talavera	
πK	JB,Dhonte,Talavera	

General Strategy and some comments

- Find enough inputs from experiment
- C_i^r:
  - kinematical dependence: agree well with single resonance saturation
  - quark mass+kinematical: if vector dominated, seems to be OK
  - quark mass+kinematical: if scalar dominated: which scalars? (not σ)
  - quark masses: which scalars? unrealistically large estimates
- in p^6 physical or lowest order masses: thresholds in right place requires physical

General Strategy and some comments

Inputs:

$K_{\ell 4}$: $F(0)$, $G(0)$, λ E865 BNL

$m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$ em with Dashen violation

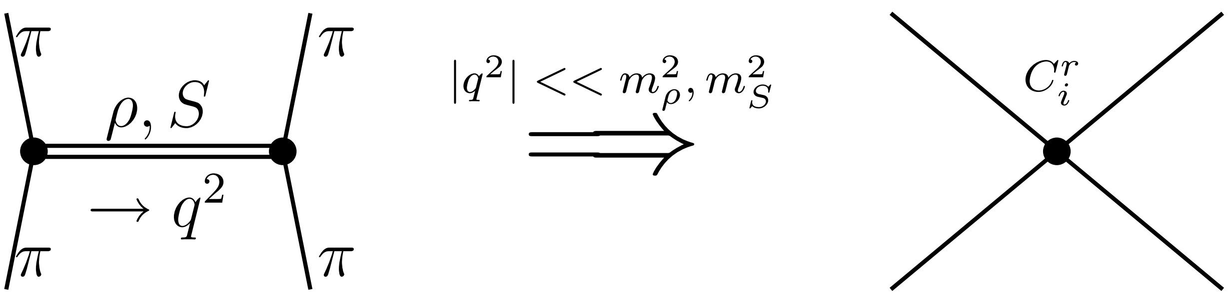
F_{π^+}

F_{K^+}/F_{π^+}

$m_s/\hat{m}$ 24 (26) $\hat{m} = (m_u + m_d)/2$

L^r_4, L^r_6

C^r_i from single resonance approximation



General Strategy and some comments

	fit 10	same p^4	fit B	fit D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.53 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54

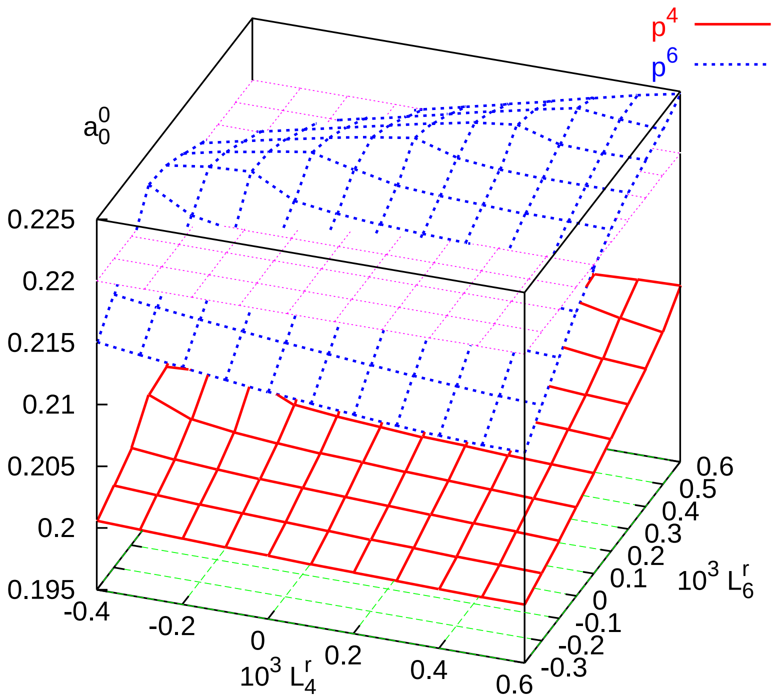
- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying C_i^r factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6\ 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and πK thresholds

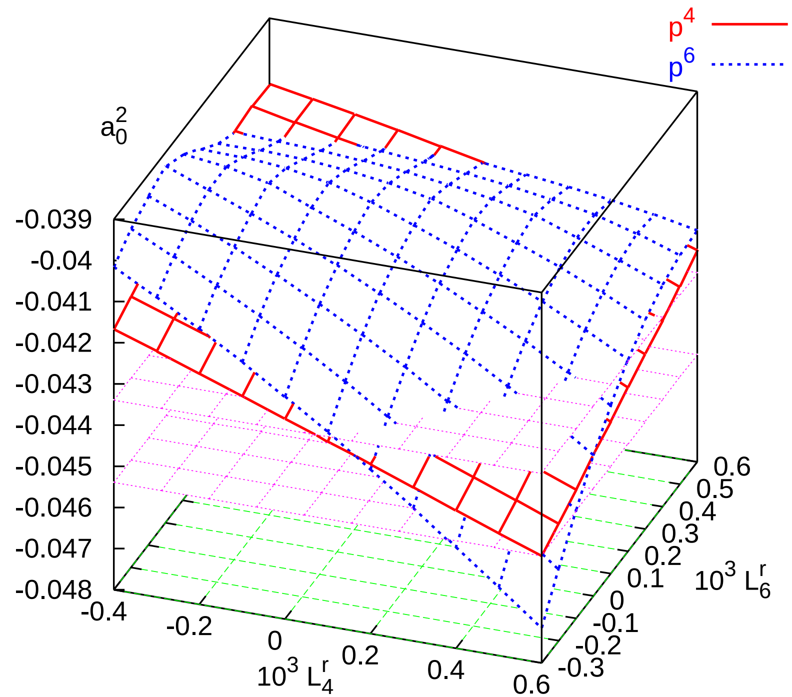
General Strategy and some comments

	fit 10	same p^4	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
m_π^2: p^4, p^6	0.006,0.258	0.009,$\equiv 0$	$-$0.138,0.009	$-$0.091,0.133
m_K^2: p^4, p^6	0.007,0.306	0.075,$\equiv 0$	$-$0.149,0.094	$-$0.096,0.201
m_η^2: p^4, p^6	$-$0.052,0.318	0.013,$\equiv 0$	$-$0.197,0.073	$-$0.151,0.197
m_u/m_d	0.45$\pm$0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}$: p^4, p^6	0.169,0.051	0.22,$\equiv 0$	0.153,0.067	0.159,0.061

- $m_u = 0$ always very far from the fits
- F_0: pion decay constant in the chiral limit

$\pi\pi$



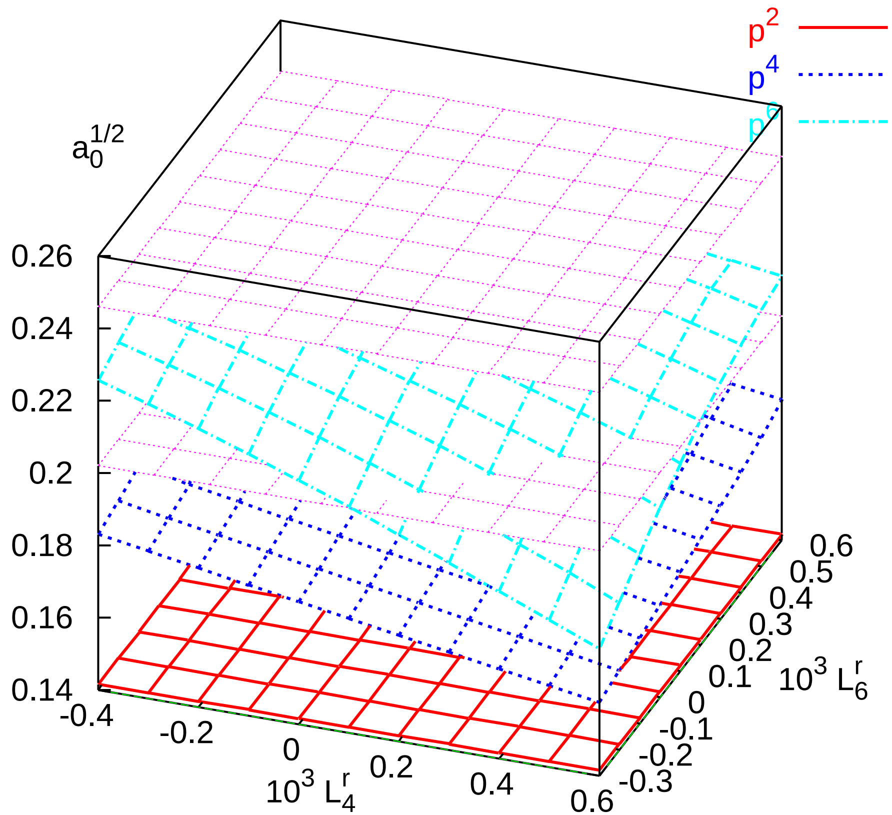


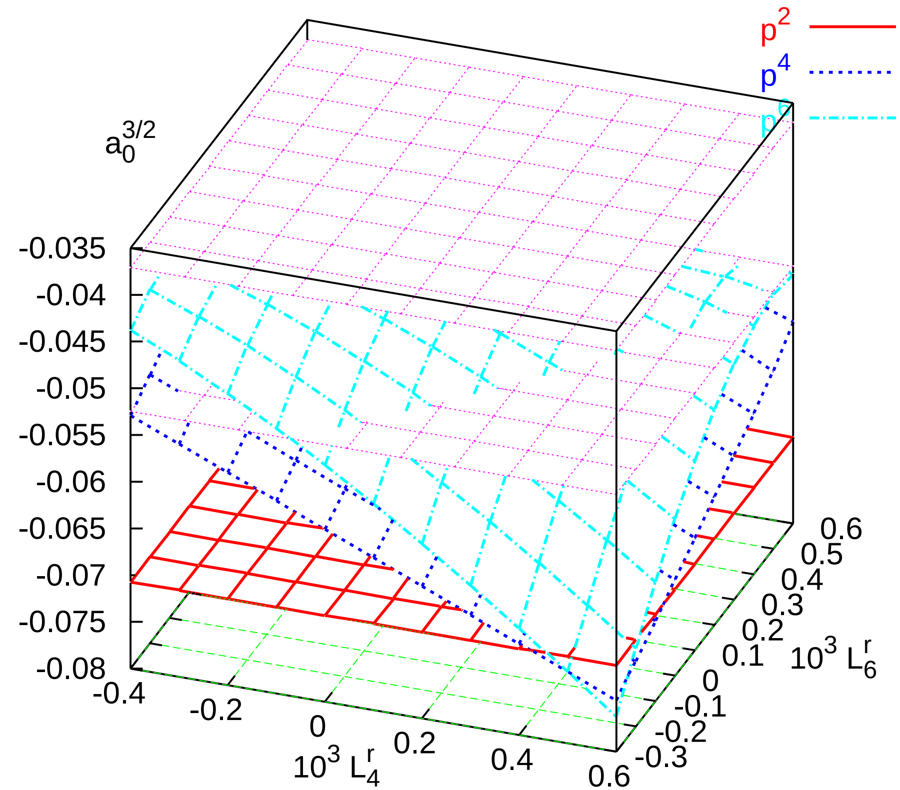
$a_0^0 = 0.220 \pm 0.005$, $a_0^2 = -0.0444 \pm 0.0010$

Colangelo, Gasser, Leutwyler

$a_0^0 = 0.159$ $a_0^2 = -0.0454$ at order p^2

πK



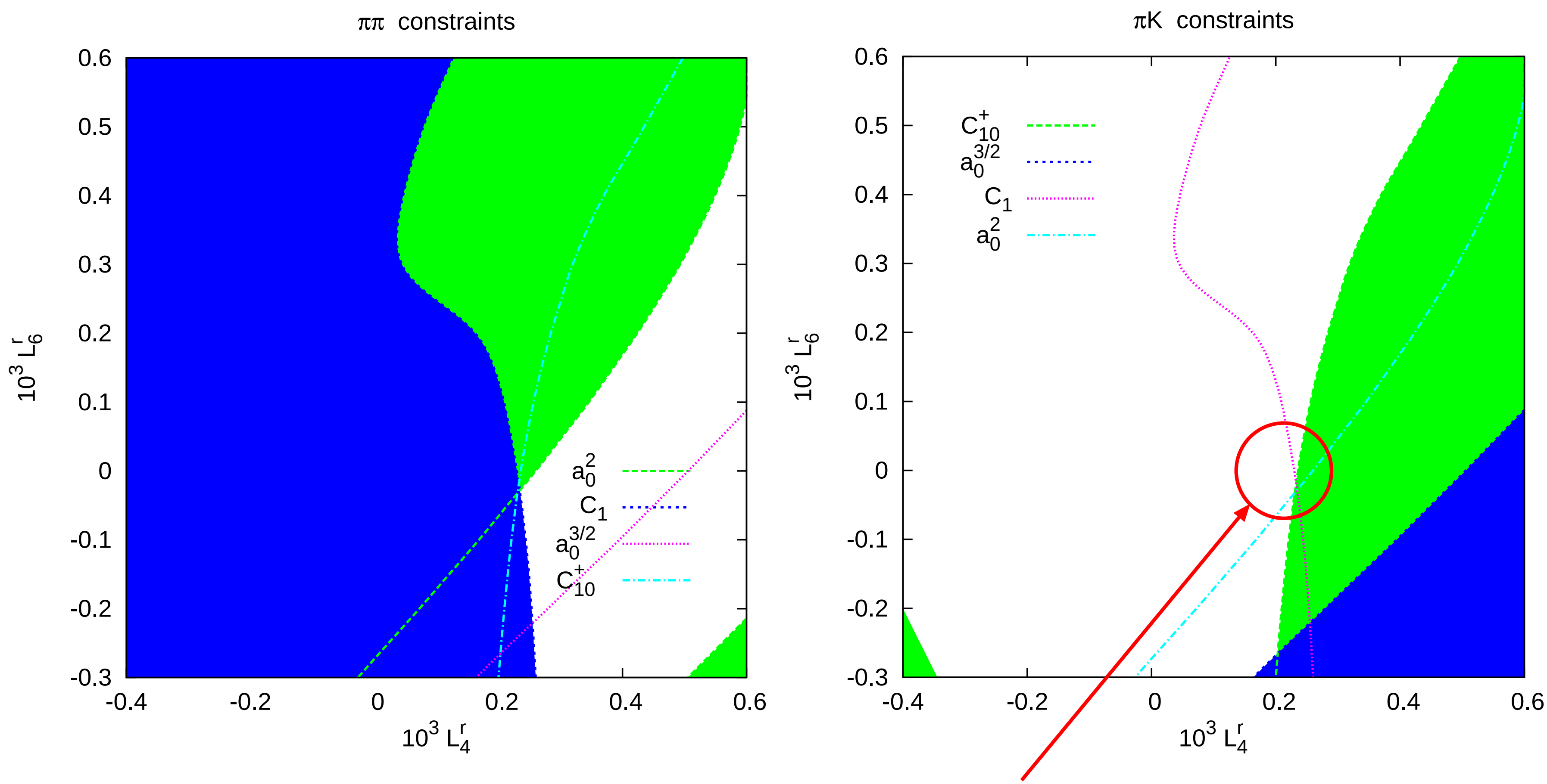


$$a_0^{1/2} = 0.224 \pm 0.022,\ a^2_{3/2} = -0.0448 \pm 0.0077$$

Büttiker, Descotes-Genon, Moussallam

$a_0^{1/2} = 0.142\ a_0^2 = -0.0708$ at order p^2

$\pi\pi$ and πK



preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor: $$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization: $$f_{+,0}(t) = f_+(0)\left(1 + \lambda_{+,0}\frac{t}{m_\pi^2}\right)$$

$|V_{us}|$:

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$|V_{us}|$:

- Know theoretically $f_+(0) = 1 + \cdots$
  - Short distance correction to G_F from G_μ Marciano-Sirlin
  - Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$
  - Isospin Breaking Leutwyler-Roos $\dfrac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.022$ In Progress

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  - Radiative Corrections: Cirigliano et al., hep-ph/0110153
  - Parametrize form-factor: is linear enough for $f_+(t)$?

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PDG2002: $|V_{ud}| = 0.9734 \pm 0.0008$, $|V_{us}| = 0.2196 \pm 0.0026$
$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) = 0.9957 \pm 0.0019$

$f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

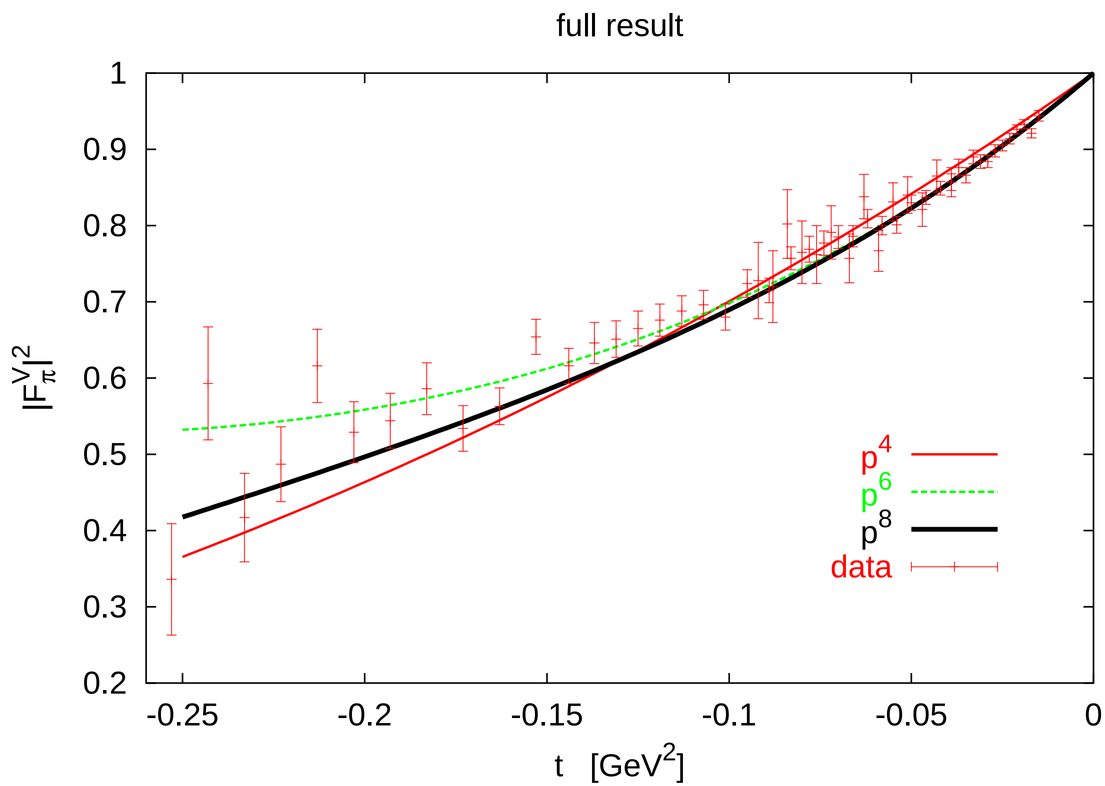
$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4}(-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

Pion electromagnetic Form factor:
JB, Talavera

$L_9^r = 0.00593 \pm 0.00043$

$-4C_{88}^r + 4C_{90}^r = 0.00022 \pm 0.00002$

VMD: $R_{+1}^{K\pi} \approx -4\,10^{-5}$ GeV^2



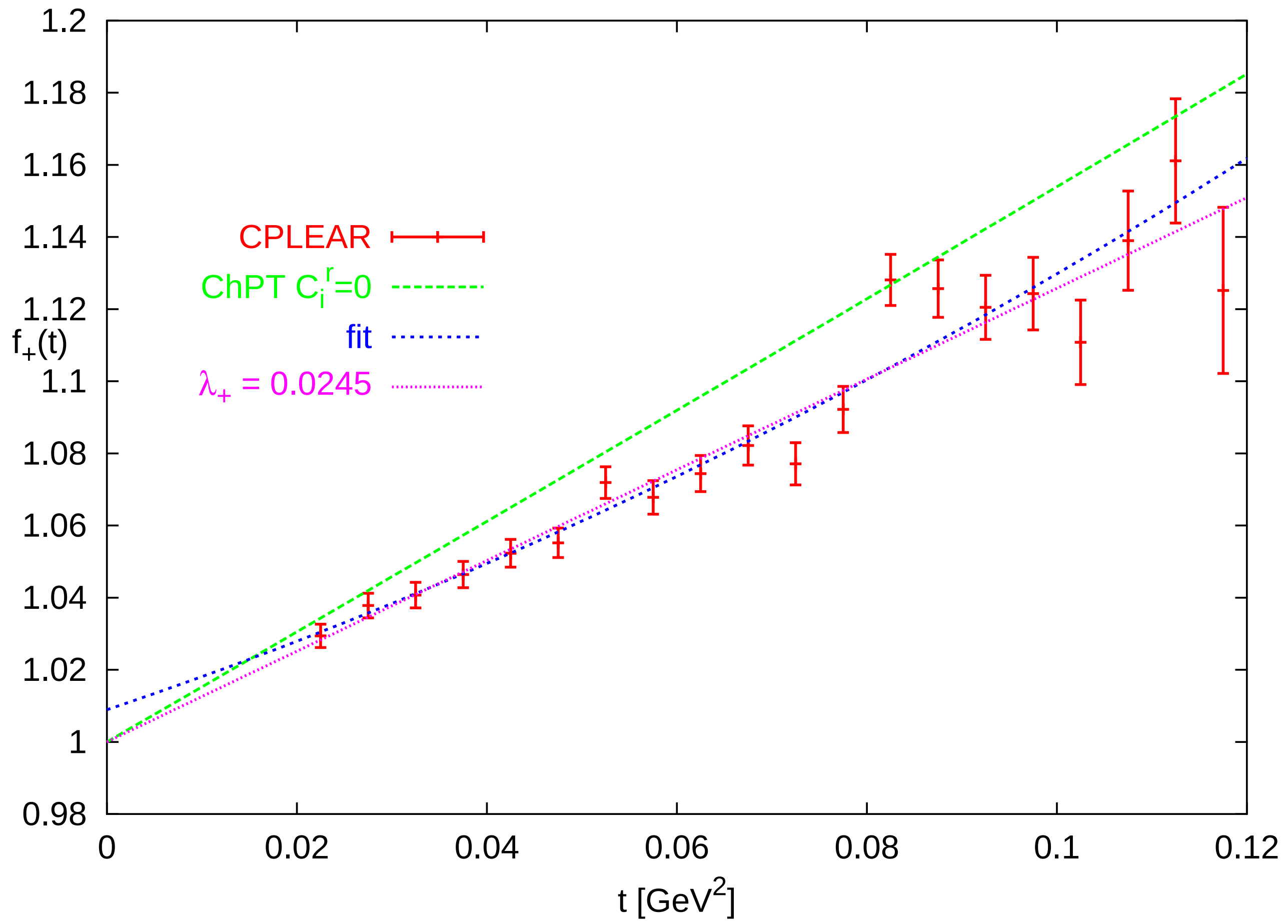
ChPT fit to $f_+(t)$

$\Rightarrow R^{K\pi}_{+1} =$
$-(4.7 \pm 0.5)\,10^{-5}$ GeV^2

$\left(c_+ = 3.2 \text{ GeV}^{-4}\right)$
fi xed by ChPT

$\Rightarrow a_+ = 1.009 \pm 0.004$

$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$



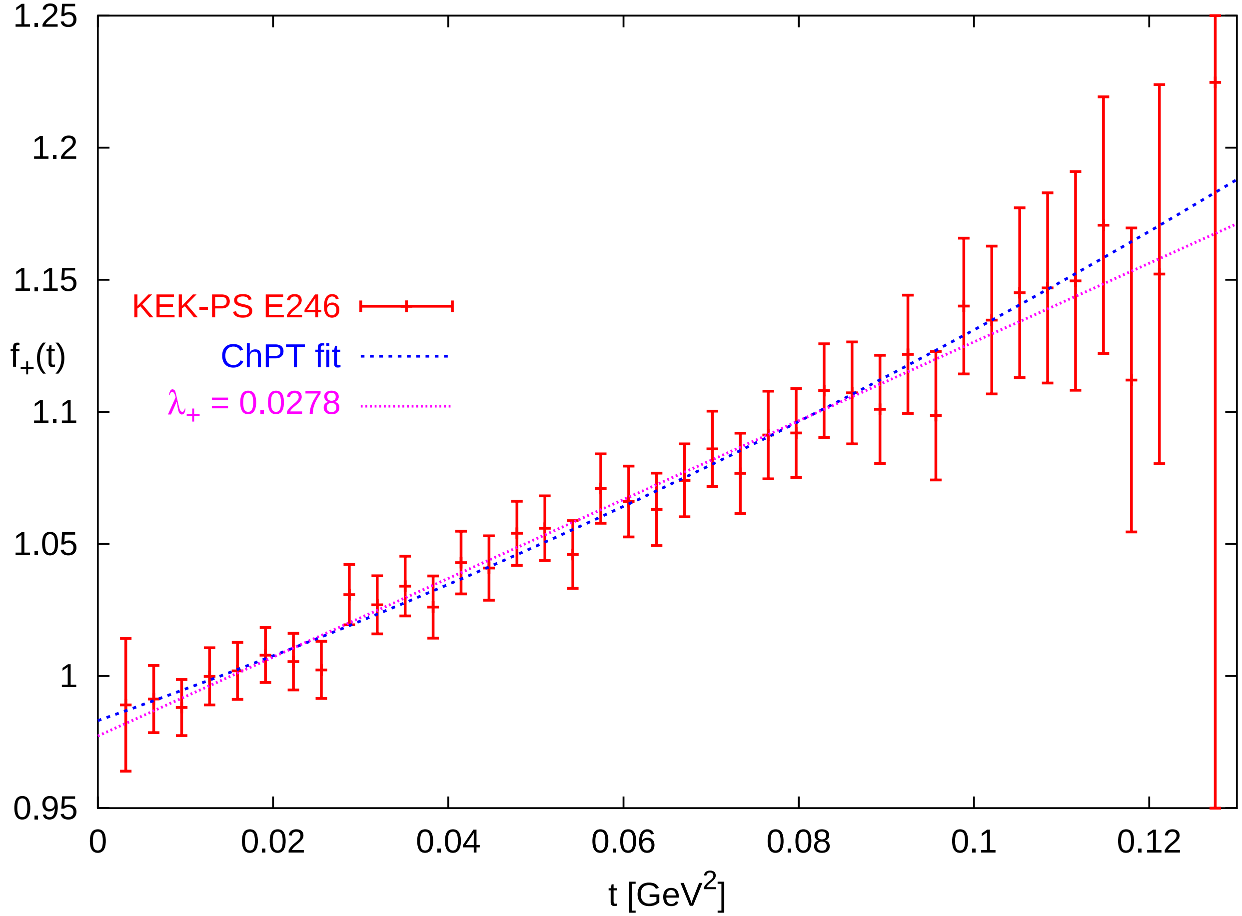
ChPT fit to $f_+(t)$

$\Rightarrow R_{+1}^{K\pi} =$
$-2.5\ 10^{-5}$ GeV^2

$\left(c_+ = 3.2 \text{ GeV}^{-4}\right)$
fixed by ChPT

$\Rightarrow a_+ = 1.006$

$\Rightarrow \lambda_+ = 0.0214 \pm 0.0018$



$f_0(t)$

Main Result:

$$
\begin{aligned}
f_0(t) \quad = \quad & 1 - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\left(m_K^2 - m_\pi^2\right)^2 \\
& + 8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}\left(F_K/F_\pi - 1\right) \\
& - \frac{8}{F_\pi^4} t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
\end{aligned}
$$

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$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO C_i^r and only depend on the L_i^r at order p^6

$\Longrightarrow$

All needed parameters can be determined experimentally

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$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r]\,.$$

$$\Delta(0) = -0.0227\,(p^4) + 0.0113\,(p^6 \text{ pure loop}) + 0.0033\,(p^6 L_i^r)$$

V_{us} present status

- More Theory: Dispersion theory relates slopes and curvature in $f_0(t)$ Jamin, Oller, Pich
  $C^r_{12} + C^r_{34} = 3.2 \pm 1.5\ 10^{-6} \Longrightarrow f_+(0)_{p^6} = 0.002 \pm 0.009$.

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    - Curvature seen, reasonable agreement with ChPT
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- Expect both more theory and experiment (KLOE,NA48)

the unitarity problem might be on the way out

Conclusions

- 2 flavour ChPT at 2 loops (almost) finished subject
- 3 flavour ChPT at 2 loops
  - many calculations done
  - things seem to work but convergence is fairly slow
  - "kinematical" and "vector" C_i^r seem to be OK
  - L_4^r, L_6^r nonzero but reasonable for large N_c
  - $\eta \to 3\pi$, isobreaking in $K_{\ell 3}$: parts done
- PQChPT at 2 loops: subject just beginning
- $K_{\ell 3}$ an example of results even with all the C_i^r