Hadronic Contributions to the Muon Anomalous Magnetic Moment

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Overview

- Anomalous Magnetic Moment
- Experiment
- Non hadronic contributions: QED and Weak
- Hadronic Contributions
  - Hadronic Vacuum Polarization
  - Light-by-Light
- Conclusions

Recent Papers: A. Höcker, hep-ph/0410081 (ICHEP Beijing); B. Roberts, hep-ex/0501012, M. Knecht, hep-ph/0307239, M. Passera, hep-ph/0411168

Anomalous magnetic Moment

$\boxed{\text{Classically}}$ $$\mu = \frac{q}{2mc}\vec{L}$$

$\boxed{\text{Quantum Mechanics}}$ $\vec{\mu} = g\frac{q\hbar}{2m_c}\vec{S}$

g: gyromagnetic ratio: intrinsic to particle

$\boxed{\text{Relativistic Quantum Mechanics}}$ Dirac Equation

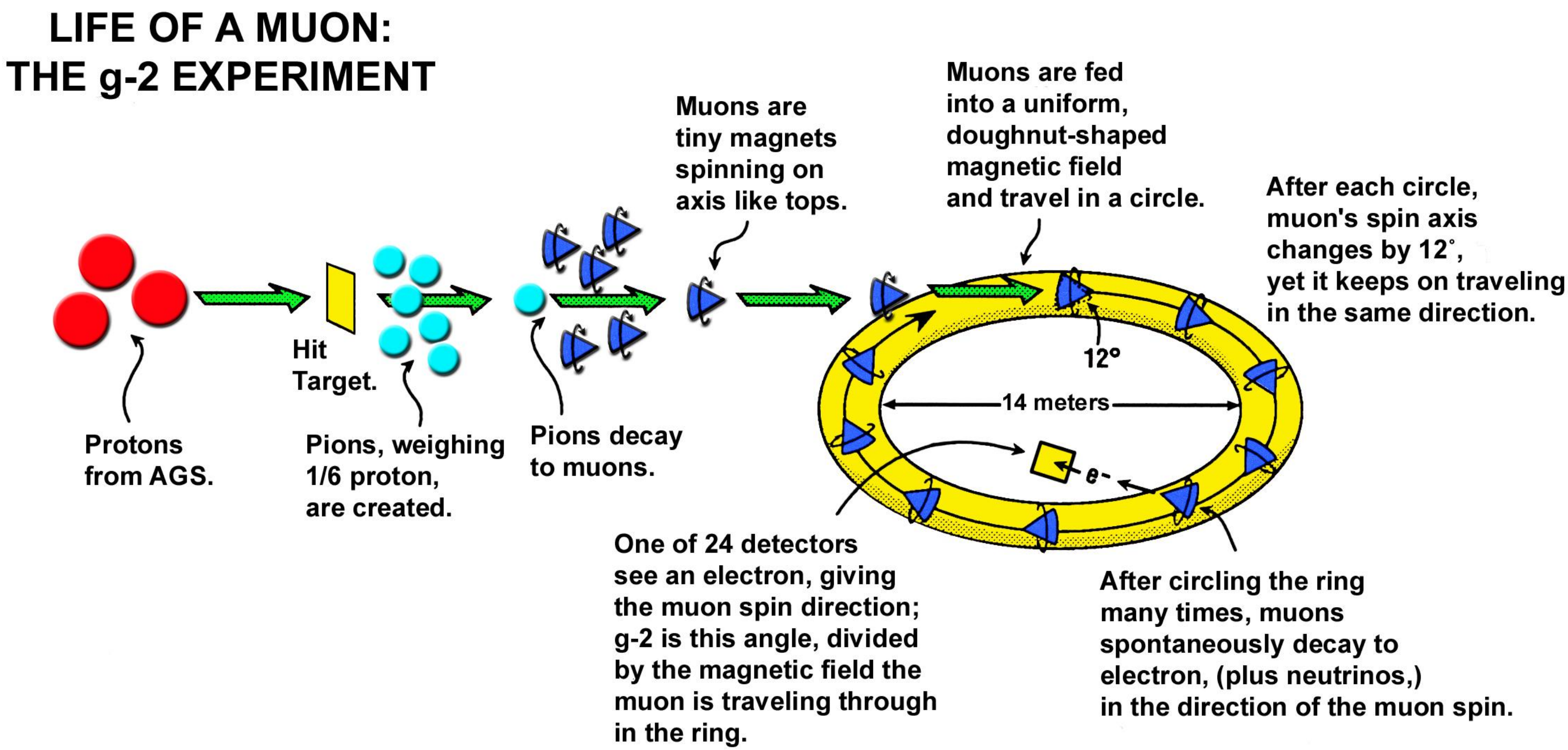
$$g = 2$$

True for e^-, not for $p, n \Longrightarrow$ composite particles

$\boxed{\text{Quantum Field Theory}}$ $g = 2\,(1 + a)$

$a_e = \frac{\alpha}{2\pi}$ Great success for QED (Schwinger)

Experiment



(http://www.g-2.bnl.gov)

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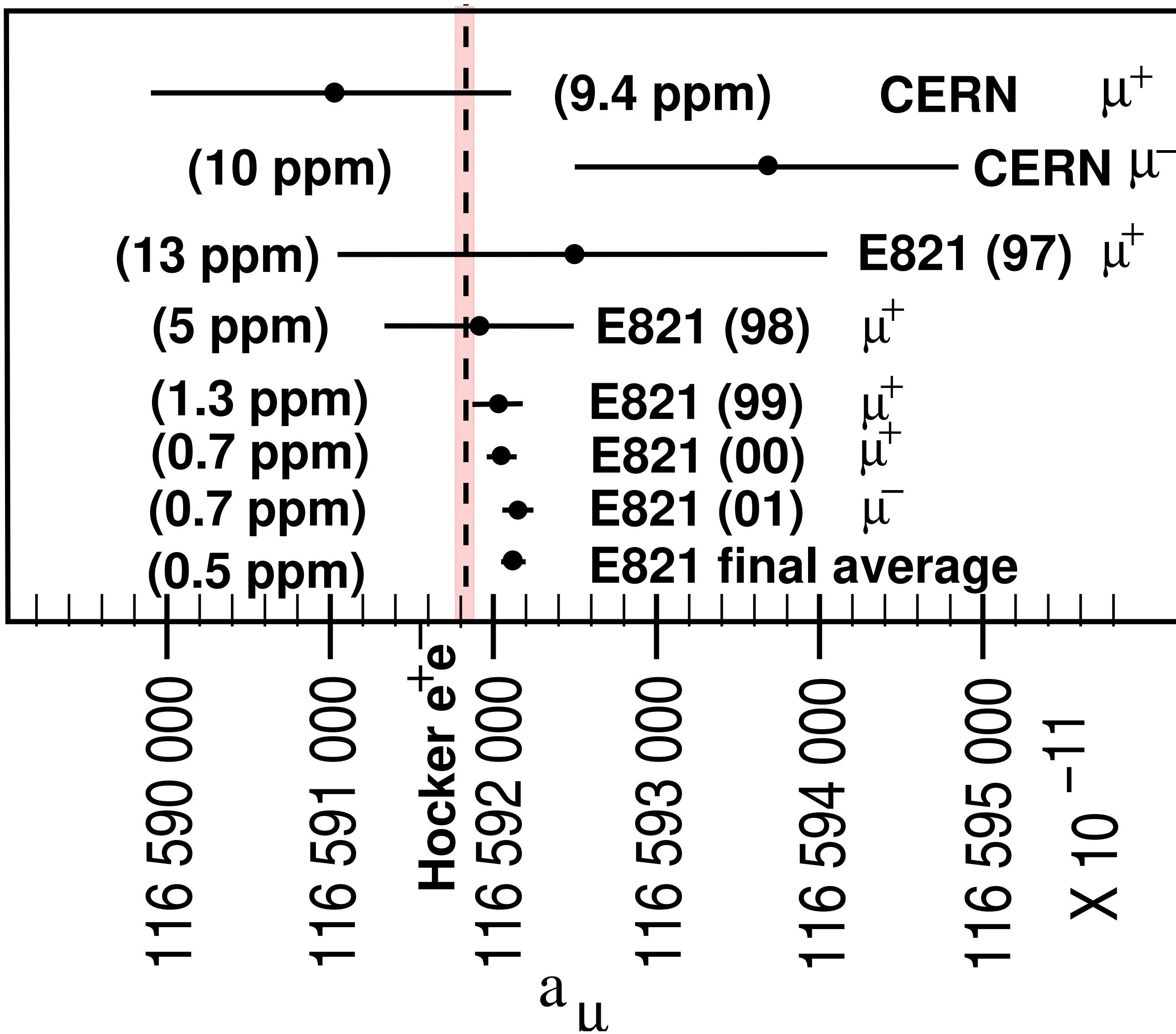

hep-ex/0501012

$a_\mu(E821) = 11659208.0(5.8) \times 10^{-10}$

0.5 ppm

Planned:
E969
0.2 ppm

Later
J-PARC

QED and Weak

- $a_\mu = a_\mu(QED) + a_\mu(hadronic) + a_\mu(weak)$
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- $a_\mu(weak) = 15.4(3) \times 10^{-10}$

  Czarnecki, Marciano, Vaihnstein, Knecht, de Rafael, Peris,. . .

  Notice: $a_\mu(weak\text{ one-loop}) = 19.48 \times 10^{-10}$

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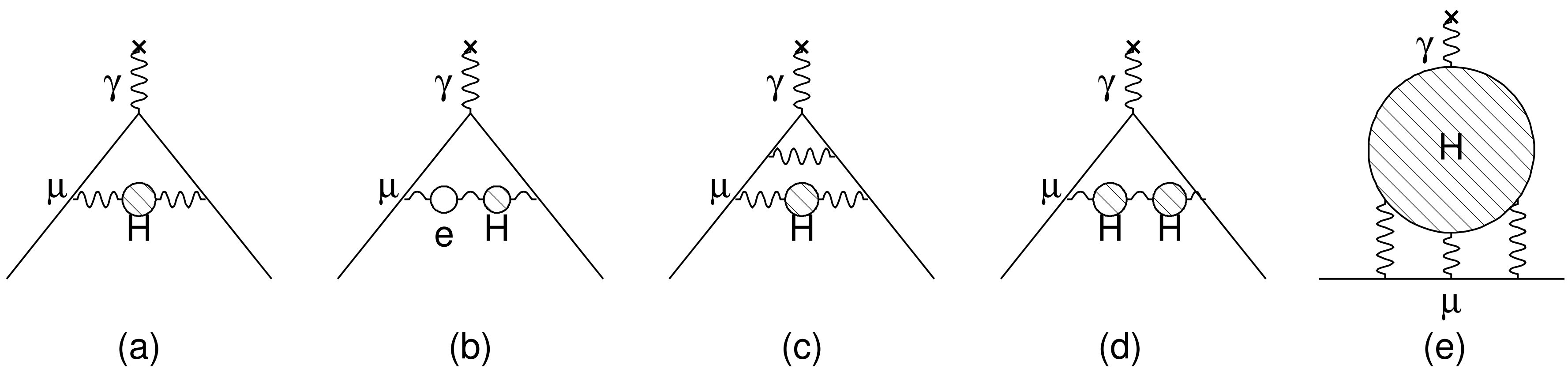
- $$\boxed{\begin{aligned} a_\mu(hadronic) \equiv\ & a_\mu - a_\mu(QED) - a_\mu(weak) \\ =\ & 720.6(5.8) \times 10^{-10} \end{aligned}}$$

Hadronic Contributions

$$\boxed{a_\mu(hadronic\text{-exp}) = \mathbf{720.6(5.8)} \times \mathbf{10^{-10}}}$$



(hep-ex/0501012)

- (a) Hadronic vacuum polarization: HVP
- (b),(c),(d) higher order HVP: HOV
- (e) Light-by-Light scattering: LBL

Hadronic Contributions

- HVP is the largest contribution
- HOVP is about -10×10^{-10}
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THIS WILL BE DIFFICULT

In principle lattice QCD can calulate this

But: difficult observables, large energy ranges, chiral cancellations,...

Hadronic Vacuum Polarization

$$a_\mu(HVP) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) R(s)$$

$$R(s) \equiv \frac{\sigma\left(e^+e^- \rightarrow \text{hadrons}\right)}{\sigma\left(e^+e^- \rightarrow \mu^+\mu^-\right)}$$

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$a_\mu(HOVP) = -10.0(6) \times 10^{-10}$ Krause 1997

$a_\mu(HOVP) = -9.8(1) \times 10^{-10}$ Hagiwara et al., 2004

Hadronic Vacuum Polarization

- $e^+e^- \rightarrow$hadrons (the old fashioned way)
- $\tau \rightarrow$hadrons Davier et al
- Radiative return Kühn et al

Davier, Höcker, Alemany, Eidelmann, Jegerlehner, Yndurain, de Troconiz, Hagiwara, Martin, Nomura, Teubner

$e^+e^- \rightarrow$ hadrons

- Need luminosity normalization at every point
- Correction to the bare process
- exclusive versus inclusive measurements
- final states including extra photons

Main contribution dominated by CMD-2 data
higher energies: combine data from many experiments

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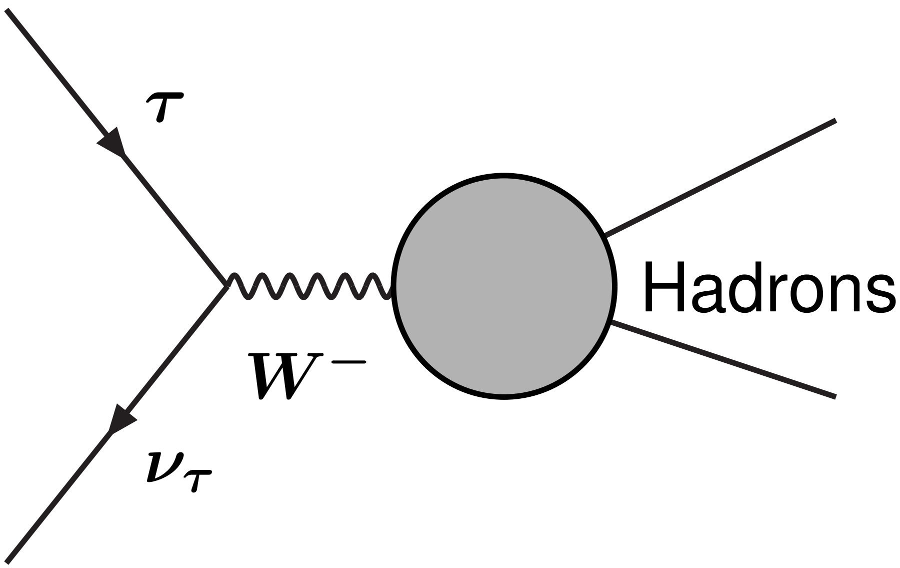
Main contribution dominated by CMD-2 data
higher energies: combine data from many experiments

- Chiral perturbation theory at very low energies
- Dispersion theory in the intermediate regime (under way)
- As much QCD as you can at higher energies

Tau decays

Review: Davier, Höcker, Zhang, hep-ex/0507078 source of many figures



In principle advantage: luminosity is tagging question (and only once), parasitic running

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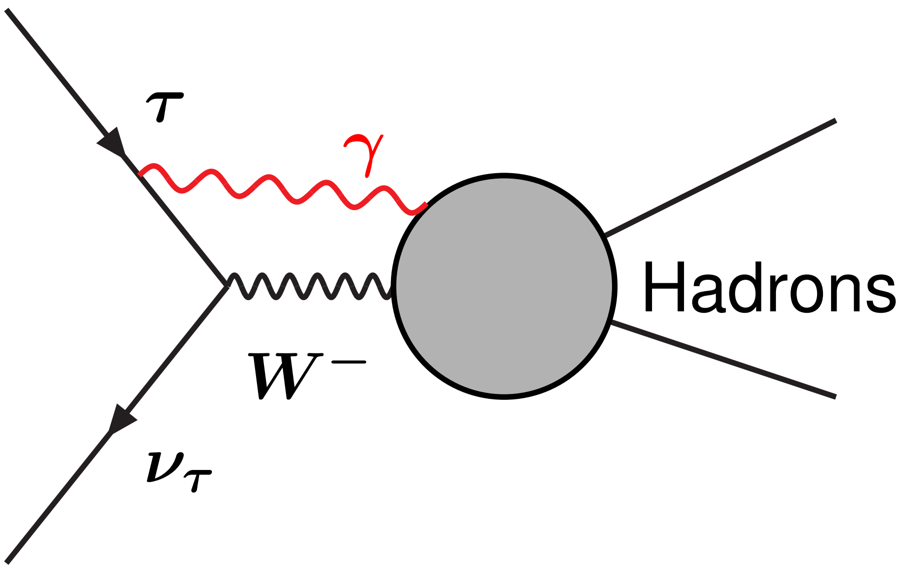

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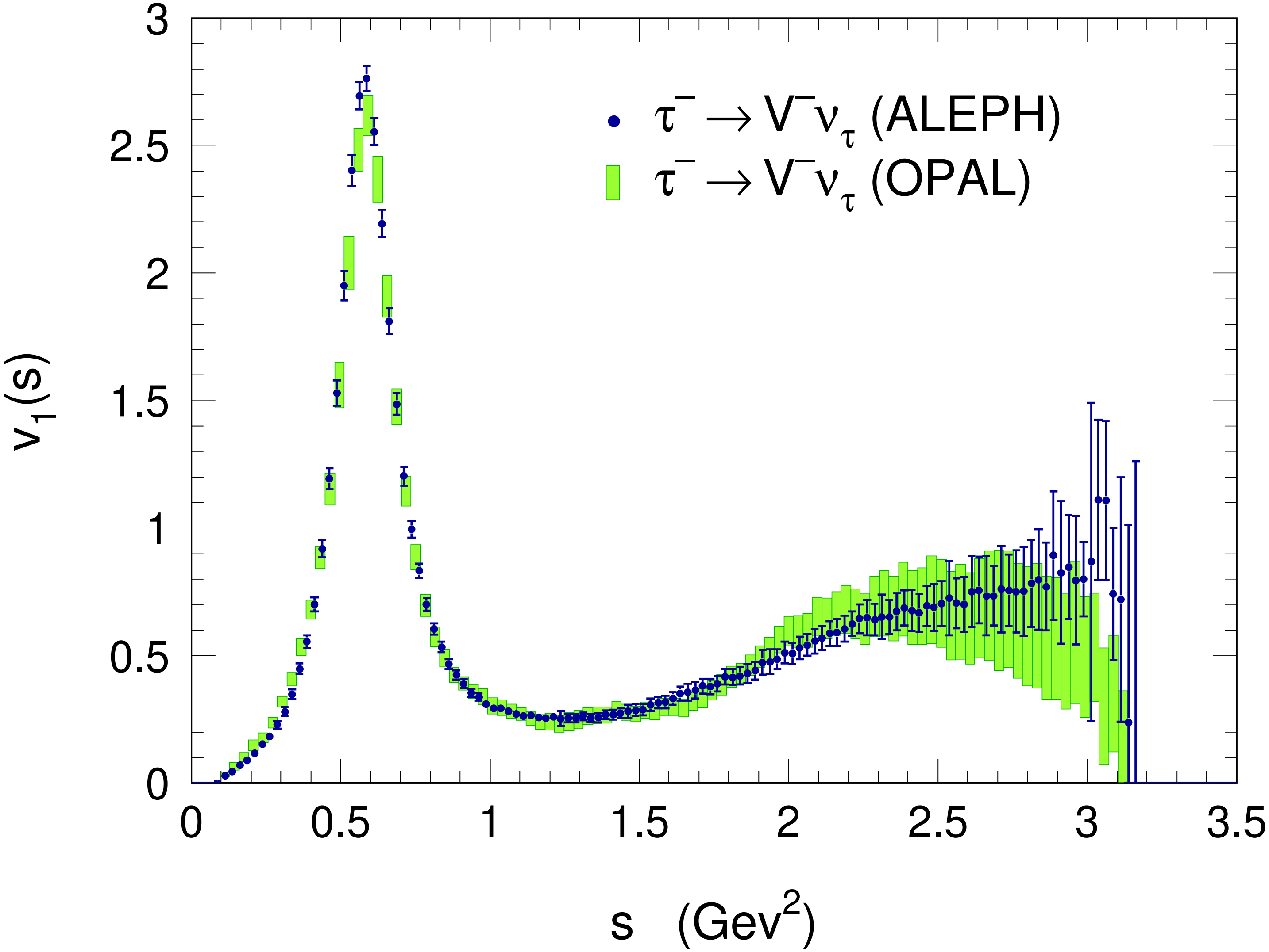
Need to correct for isospin and radiative corrections

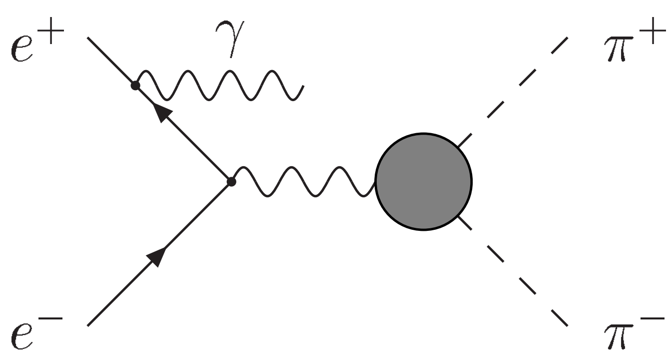
Tau decays

An example of results (ALEPH, OPAL, CLEO in agreement)



Radiative Return

Review: Czyż et al., hep-ph/0312217



So use photon to get to lower E_{cms}

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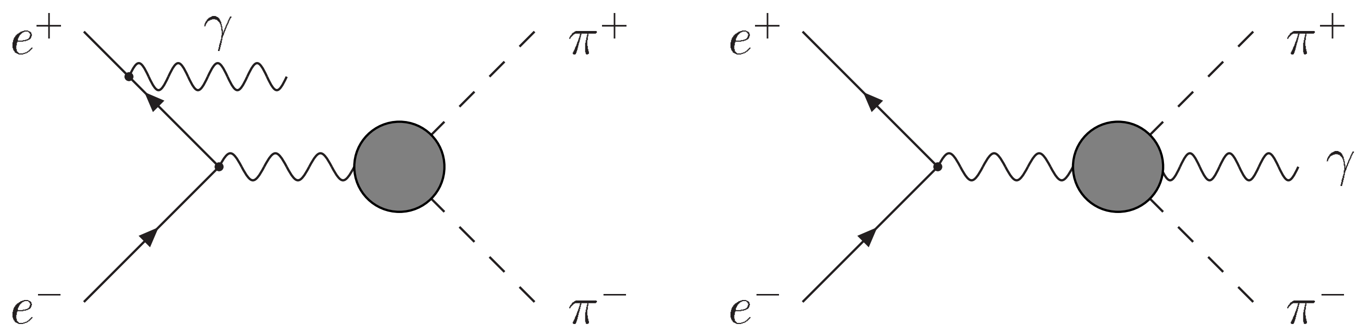

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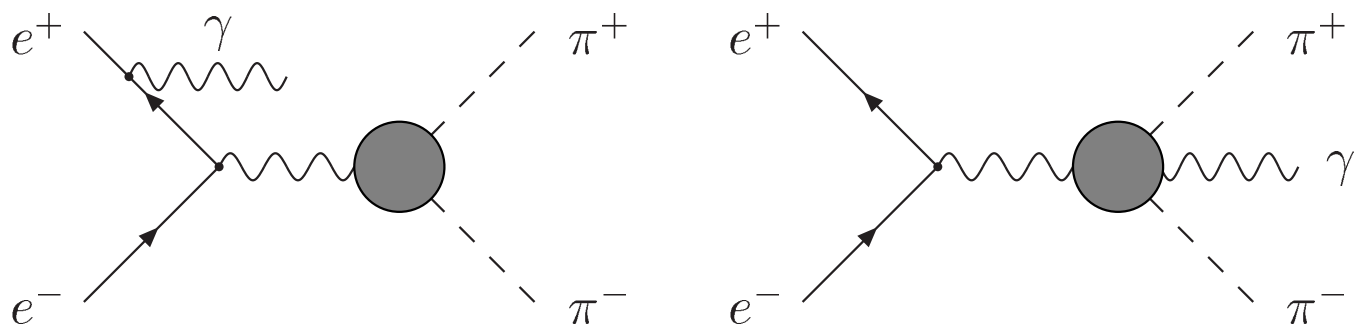

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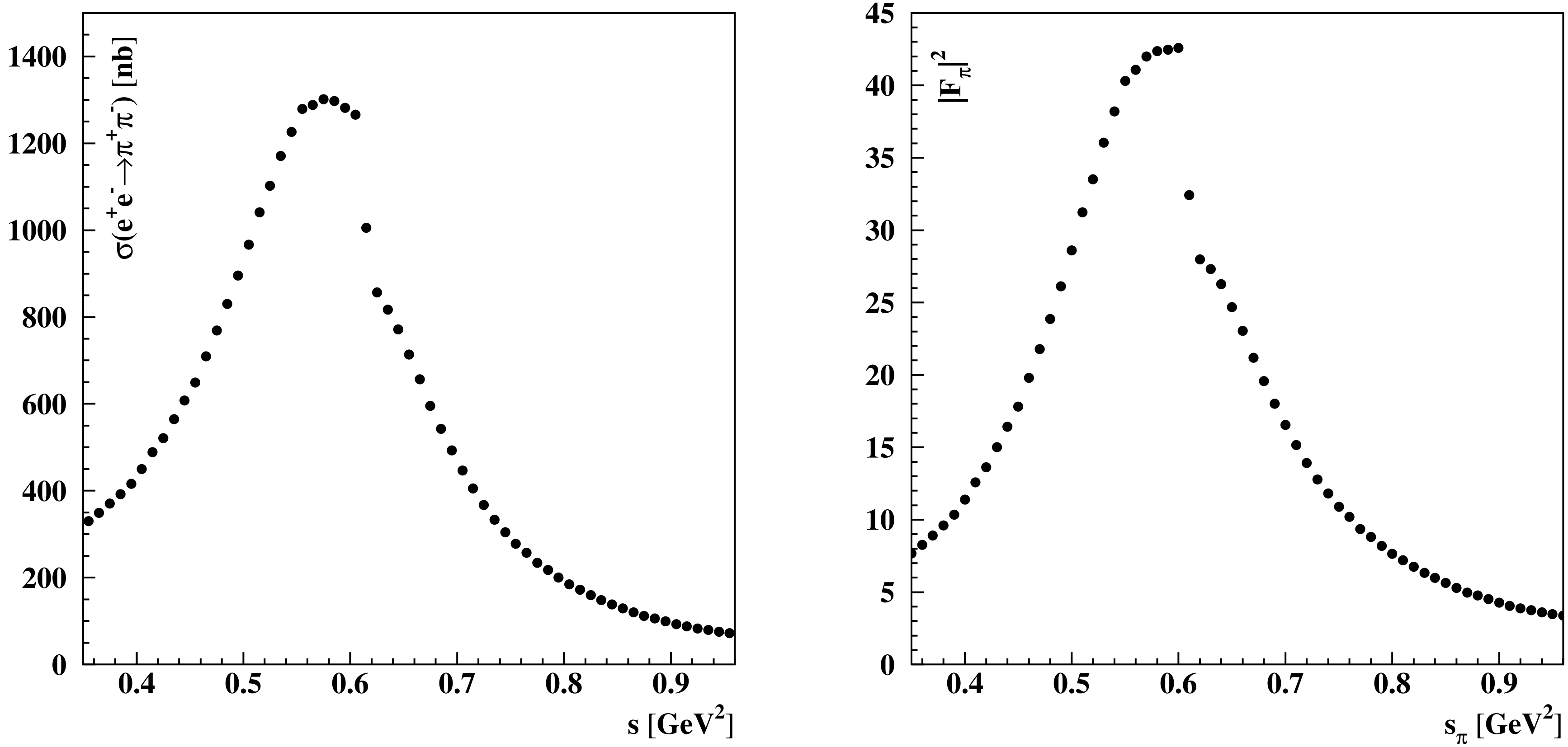
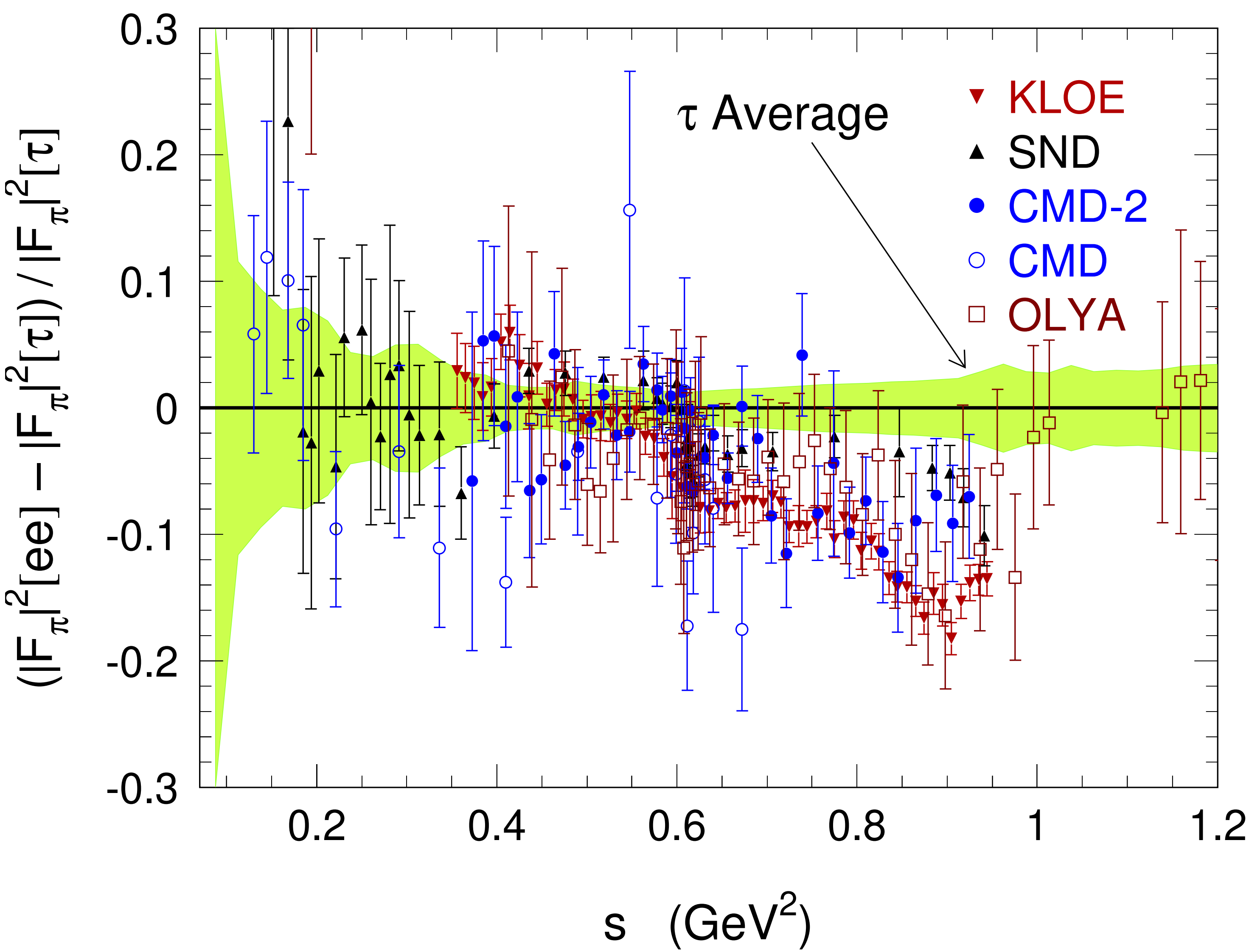
Large experimental advantages: parasitic to factories, luminosity only once

Radiative Return: experiment

Measurements under way at KLOE (hep-ex/0407048), BaBar and Belle(?)



Comparison of the three



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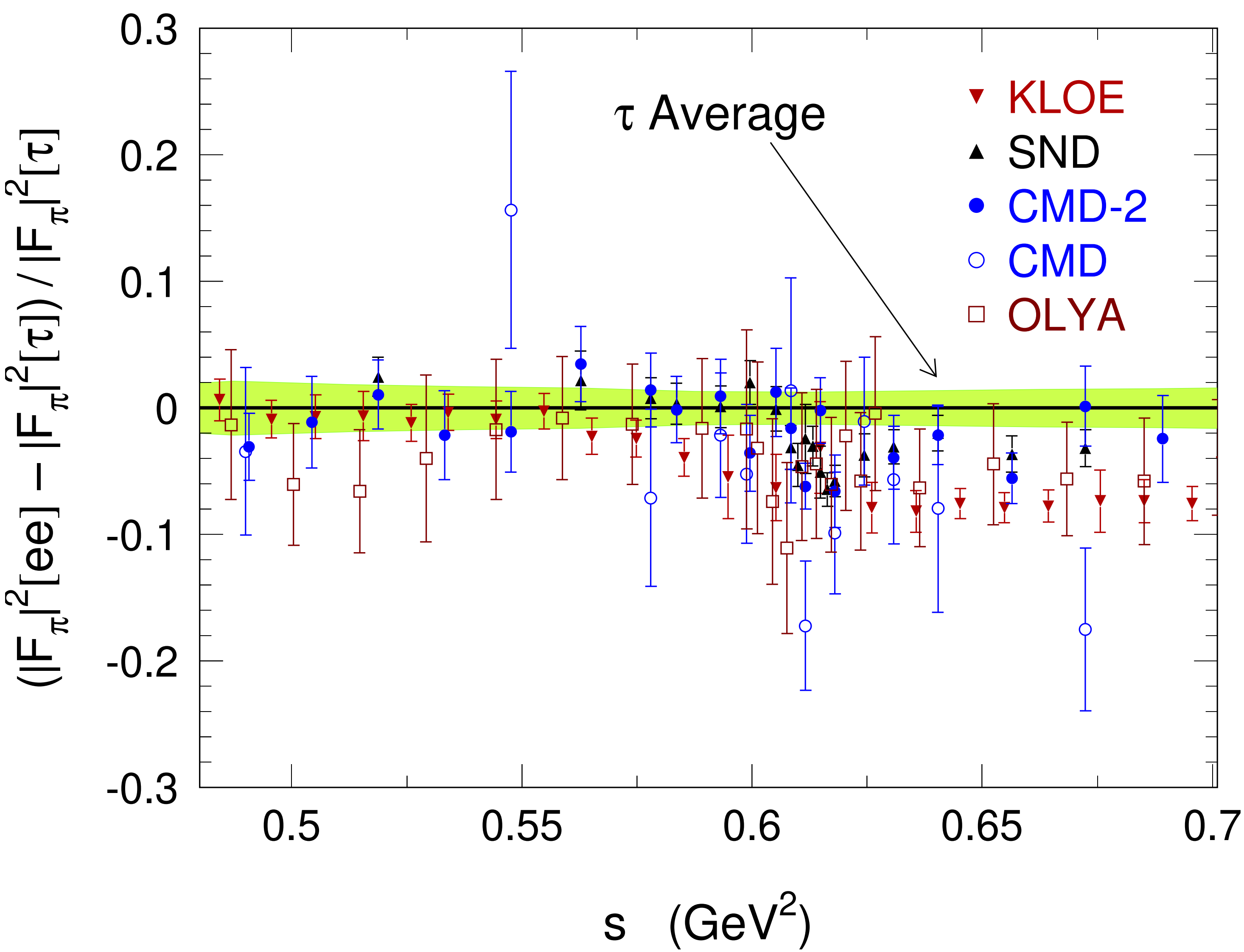

Comparison of the three

$$a_\mu(HVP)(e^+e^-) = (693.4 \pm 5.3 \pm 3.5_{\text{rad}}) \times 10^{-10}$$

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$$a_\mu(HVP)(\tau) = (711.0 \pm 5.0_{\text{exp}} \pm 0.8_{\text{rad}} \pm 2.8_{\text{iso}}) \times 10^{-10}$$

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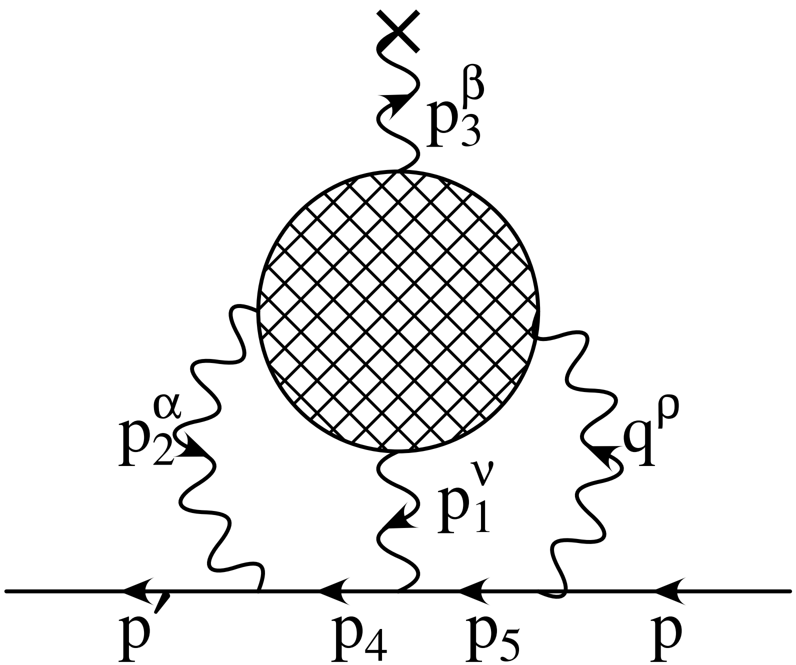
Light-by-Light Contribution



$$\mathcal{M} = |e|^7 A_\beta \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{1}{q^2\, p_1^2\, p_2^2 (p_4^2 - m^2)\, (p_5^2 - m^2)}$$
$$\times \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)\, \bar{u}(p')\gamma_\alpha(\not{p}_4 + m)\gamma_\nu(\not{p}_5 + m)\gamma_\rho u(p)$$

Light-by-Light Contribution

$$\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3) \equiv \sum_{(abcd)} i^3 \int \mathrm{d}^4x\mathrm{d}^4y\mathrm{d}^4z\, e^{i(p_1\cdot x+p_2\cdot y+p_3\cdot z)}$$

$$\times \langle 0|T\left(V_a^\rho(0)V_b^\nu(x)V_c^\alpha(y)V_d^\beta(z)\right)|0\rangle.$$

$V_i^\mu(x) \equiv Q_i\ [\bar{q}_i(x)\gamma^\mu q_i(x)]$

Green function with four electromagnetic currents

138 Lorentz structures $\Longrightarrow$ 32 combinations can contribute

8 dimensional integral: 3 can be done
5 remain: 2 moduli, 3 angles

Many scales problem

Light-by-Light: the scale

Muon Loop: 46.5×10^{-10}

Light-by-Light: the scale

Muon Loop: 46.5×10^{-10}

Λ [GeV]	$10^{10} \times a_\mu$
0.5	24
0.7	26
1.0	38(1)
2.0	45(1)

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Constituent Quark Loop: 24×10^{-10}

But how good is this number?

Light-by-Light Contribution

Hadronic degrees of freedom versus quark picture

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Hadronic degrees of freedom versus quark picture

Separate contributions by chiral powers and large N_c

de Rafael 93

- p^6, N_c : π^0, η, η' exchange
- p^8, N_c : quark loop, “heavy” meson exchanges
- p^4, 1 : meson loops

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04: More short distance put in Melnikov-Vaihnstein (MV)

Light-by-Light: old trick

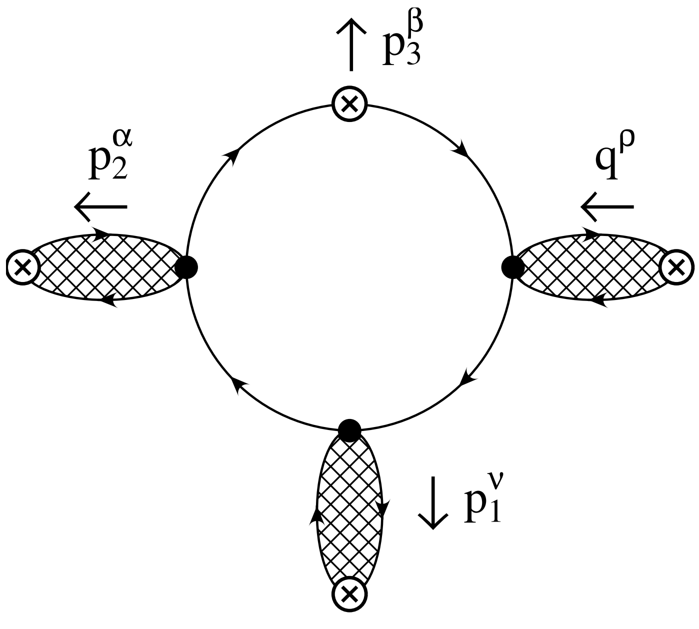
Gauge invariance: $p_{3\beta}\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3) = 0$

$$\Pi^{\rho\nu\alpha\lambda}(p_1, p_2, p_3) = -p_{3\beta}\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}$$

so can calculate at $p_3 = 0$ everywhere

Light-by-Light: Quark Loop

Bijnens, Pallante, Prades



- Low Energy: Extended Nambu-Jona Lasinio model
- High Energy: Bare quark loop with $m_Q = \Lambda$

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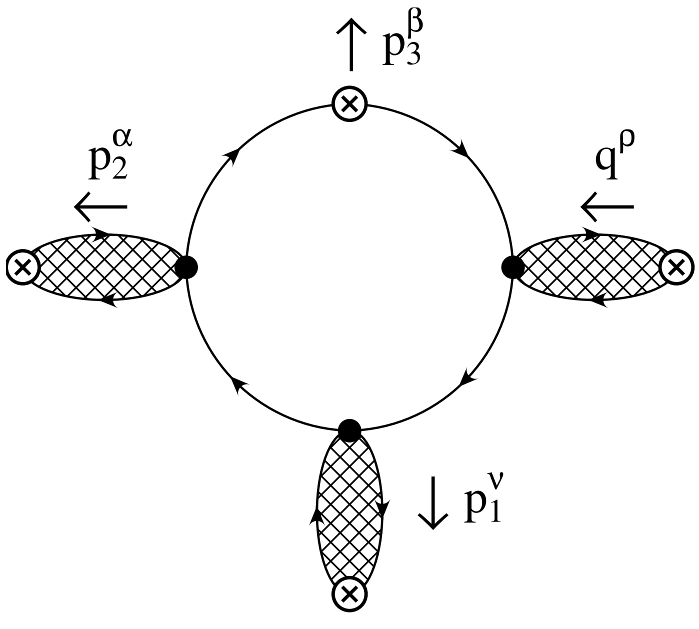
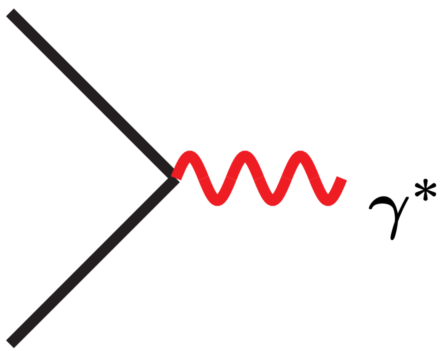


Λ [GeV]	$10^{10} \times a_\mu$
0.7	2.2
1.0	2.0
2.0	1.9
4.0	2.0

- Low Energy: Extended Nambu-Jona Lasinio model
- High Energy: Bare quark loop with $m_Q = \Lambda$
- So OK matching for a first try

Light-by-Light: Pion-Kaon Loop

Leading contribution in chiral counting, suppressed by $1/N_c$

OK: 

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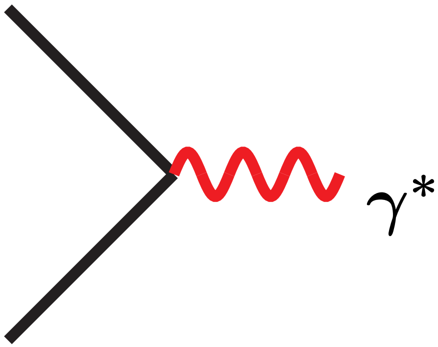
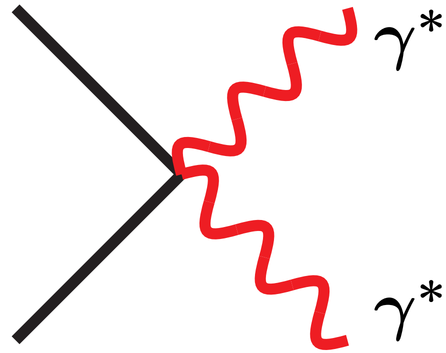


Problem:



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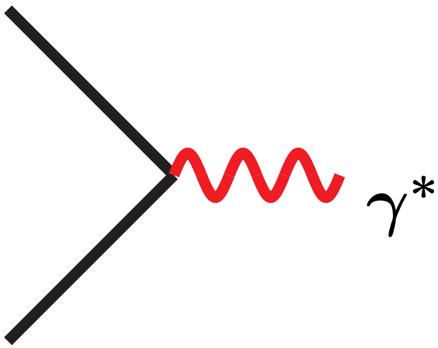
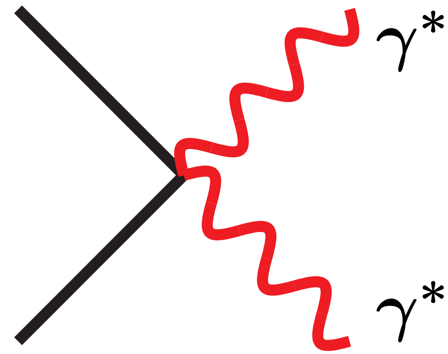
OK: 

Problem: 

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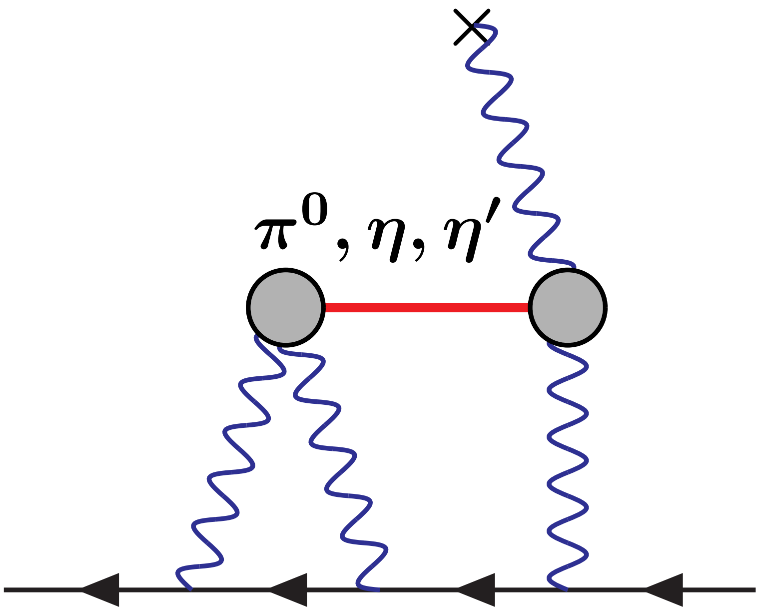
Model for $\pi\pi\gamma(\gamma)$	$10^{10} \times a_\mu$
Point-like	-4.5
VMD BPP	-1.6
ENJL BPP	-1.8
HLS HKS	-0.4

Kaon loop much smaller but order 0.05×10^{-10}

Light-by-Light: π^0, η, η'



- double VMD: $\frac{m_V^4}{(q_1^2-m_V^2)(q_2^2-m_V^2)}$
- $1/p^2$: $\frac{m_V^4}{(q_1^2+q_2^2-m_V^2)}$
- $\frac{4\pi^2 F_\pi^2}{N_c}\,\frac{q_1^2 q_2^2(q_1^2+q_2^2) - h_2 q_1^2 q_2^2 + h_5(q_1^2+q_2^2) + (N_c M_1^4 M_2^4/4\pi^2 F_\pi^2)}{(q_1^2+M_1^2)(q_1^2+M_2^2)(q_2^2+M_2^2)(q_1^2+M_2^2)}$

  KN,MV, h_2 different

Light-by-Light: π^0, η, η'

BP: Bijnens-Persson, hep-ph/0106130

Model $\pi^0\gamma^*\gamma^*$	$10^{10} a_\mu$ (π^0)	$10^{10} a_\mu$ (π^0, η, η')
Point	33	
ENJL BPP	5.9	8.5(1.3)
HLS HKS	5.7	8.3(6)
double VMD BP	5.6	7.9
$1/p^2$ BP	4.1	7.9
KN	5.8	8.3(1.2)
MV1	6.3	

Light-by-Light: π^0, η, η'

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KN	5.8	8.3(1.2)
MV1	6.3	
MV2	7.7	11.4(1.0)

MV2: a new short-distance contraint on $\Pi^{\mu\nu\alpha\beta}(p_1, p_2)$

Light-by-Light: π^0, η, η'

MV Look at: $\langle 0|T(A_\mu(q_1)A_\nu(q_2)A_\nu(q_3))|\gamma(q_4 \to 0)\rangle$

for $q_3^2 << q_2^2 \sim q_1^2$ thus $\hat{q} = q_1 \approx -q_2$

Light-by-Light: π^0, η, η'

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Use OPE: $T(A_\mu(q_1)A_\nu(q_2)) \sim \frac{1}{\hat{q}^2}\varepsilon_{\mu\nu\alpha\beta}\hat{q}^\alpha j_5^\beta$

and the remainder estimated by π^0 saturation (q_3^2 small)

Light-by-Light: π^0, η, η'

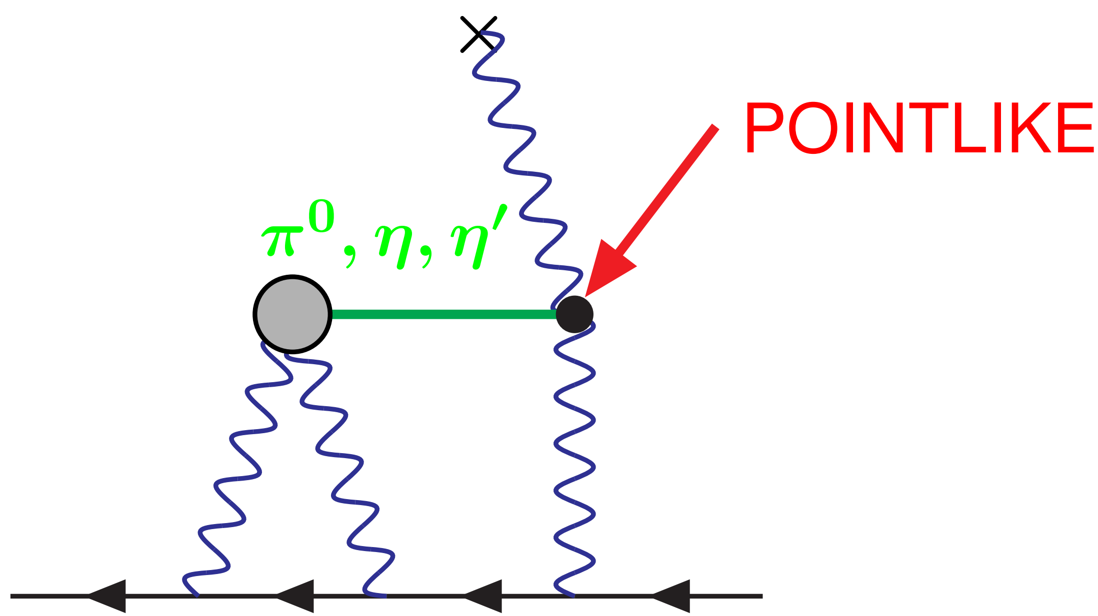
MV Look at: $\langle 0|T(A_\mu(q_1)A_\nu(q_2)A_\nu(q_3))|\gamma(q_4 \to 0)\rangle$

for $q_3^2 << q_2^2 \sim q_1^2$ thus $\hat{q} = q_1 \approx -q_2$

Use OPE: $T(A_\mu(q_1)A_\nu(q_2)) \sim \frac{1}{\hat{q}^2}\varepsilon_{\mu\nu\alpha\beta}\hat{q}^\alpha j_5^\beta$

and the remainder estimated by π^0 saturation (q_3^2 small)

Implemented by:



Light-by-Light: axial vector, scalar

BPP: $\mathbf{0.25 \times 10^{-10}}$

HKS: $\mathbf{0.174 \times 10^{-10}}$

MV: axial vector: different model, get $\mathbf{0.7 \times 10^{-10}}$, pointlike at one end, becomes $\mathbf{2.2 \times 10^{-10}}$

Light-by-Light: Conclusions

$10^{10} a_\mu$	BPP	HKS	MV
quark loop	2.1 ± 0.3	0.97 ± 1.11	-
π, K loop	-1.9 ± 1.3	-0.45 ± 0.81	0 ± 1.0
π^0, η, η'	8.5 ± 1.3	8.27 ± 0.64	11.4 ± 1.0
axial vector	0.25 ± 0.1	0.174	2.2 ± 0.5
scalar	-0.68 ± 0.2	-	-
total			

Note: in BPP scalar and quark loop belong together

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Feeling: pointlike only way to SD?; π, K loop; axial vector

Central value:	9-12
Error:	3-4

Conclusions

$$a_\mu(hadronic) \equiv a_\mu - a_\mu(QED) - a_\mu(weak)$$
$$= 720.6(5.8) \times 10^{-10}$$

	$10^{10} a_\mu$	$10^{10} \delta a_\mu$
HVP (e^+e^-)	693.4±6.4	
HVP (e^+e^- no KLOE)	696.3±7.2	
HVL (τ)	711.0±5.8	
HOVP	−9.8±0.1	
LBL BPP	8.3±3.2	28.7 ± 9.2 (3.1σ)
LBL HKS	9.0±1.5	28.0 ± 8.8 (3.2σ)
LBL MV	13.6±2.5	23.4 ± 9.0 (2.6σ)
LBLfeeling	11±4	26.0 ± 9.5 (2.7σ)

Systematic and statistical errors summed quadratically

Conclusions: some remarks

By adding linearly and taking last number:

$(5.8)_{exp} + (5.3 + 3.5)_{HVP} + (4)_{LBL} + (0.3)_{weak} = 18.9$
$\Longrightarrow (1.4\sigma)$

Note: $\mu^+/\mu^- - 1 = -2.6 \pm 1.6$ for many years after CERN experiment

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$\boxed{\text{2-3 } \sigma \text{ discrepancy still best estimate}}$