Two Loop Partially Quenched and Finite Volume Chiral Perturbation Theory Results

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Overview

- Chiral Perturbation Theory
- $\langle \overline{q}q \rangle$ at Finite Volume JB, Karim Ghorbani
  - $\langle \overline{q}q \rangle$ at Two Loops
  - A Lüscher formula for $\langle \overline{q}q \rangle$
- Partially Quenched Chiral Perturbation Theory at two Loops JB, Niclas Danielsson, Timo Lähde
  - PQChPT at Two Loops: General
  - Papers
  - Long Expressions
  - Some numerical results

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

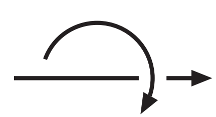
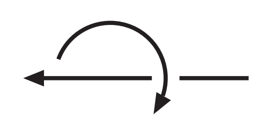
Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Can also see that via  $v < c,\ m_q \neq 0 \Longrightarrow$ / $v = c,\ m_q = 0 \not\Longrightarrow$ 

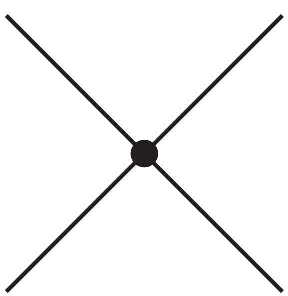
Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$

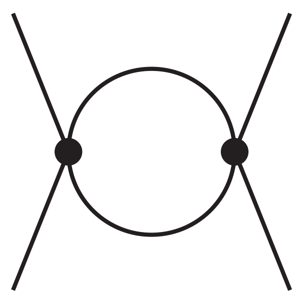
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

Power counting in momenta:



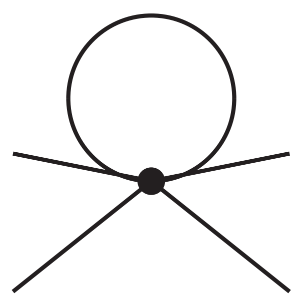
p^2



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

$1/p^2$

$\int d^4p$ $\quad p^4$



$(p^2)\,(1/p^2)\,p^4 = p^4$

Chiral Perturbation Theory

Lagrangian Structure:

	2 flavour		3 flavour		n flavour ChPT	
p^2	F, B	2	F_0, B_0	2	$\hat{F}_0, \hat{B}_0$	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

Note
- All infinities known
- Lagrangians fully known
- Predictions: relates quantities including nonanalytic structure due to Goldstone Boson cuts

$\langle \overline{q}q \rangle$ at Finite Volume

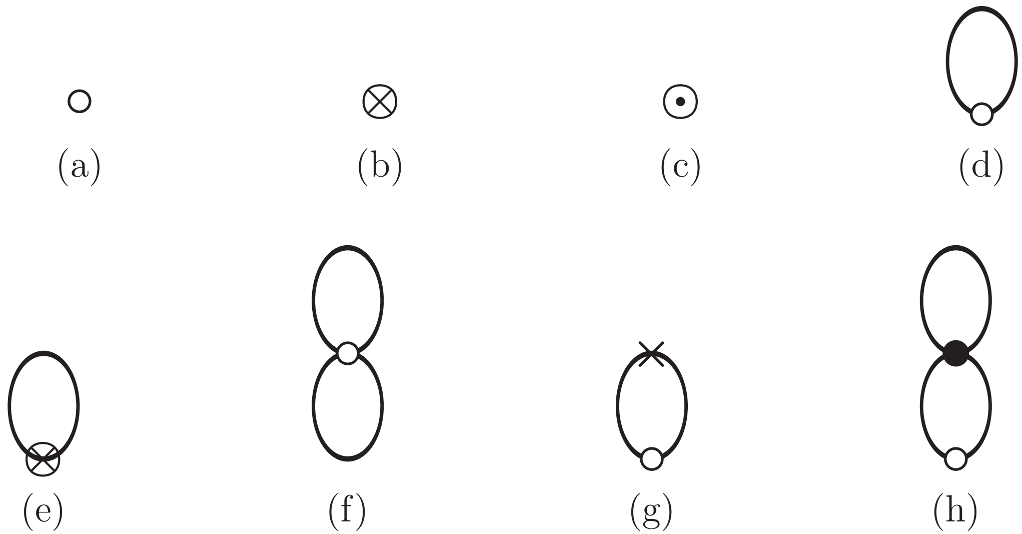
Work done with Karim Ghorbani

- Finite Volume: Particles can travel around the world
- Gasser-Leutwyler: Periodic Boundary conditions: LECs volume independent
- In principle: replace integrals by finite volume integrals
- Problem: full two-loop integrals at finite volume
- Lüscher's method: leading finite volume correction as an integral over another amplitude

$\langle \overline{q}q \rangle$ does not involve irreducible two-loop integrals

Require: $F_\pi L >> 1$ (ChPT) and $m_\pi^2 F_\pi^2 V >> 1$ (p-regime)

$\langle \overline{q}q \rangle$ at Finite Volume



The diagrams up to order p^6 for $\langle \overline{q}q \rangle$

Lines: meson propagators

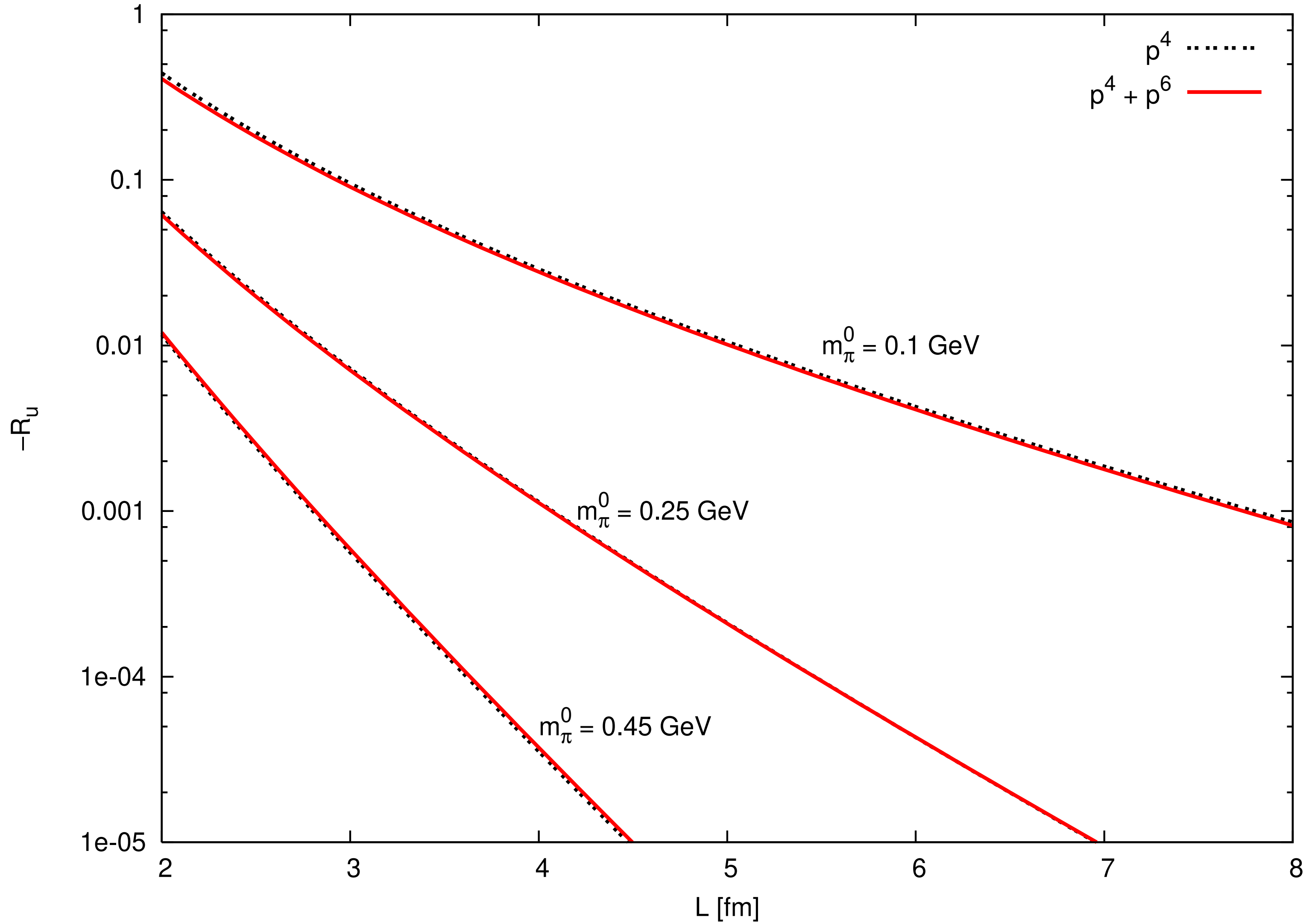
Insertion of $\overline{q}q$: $\circ$ (p^2), $\otimes$ (p^4), $\odot$ (p^6)

Vertex: $\bullet$ (p^2), $\times$ (p^4)

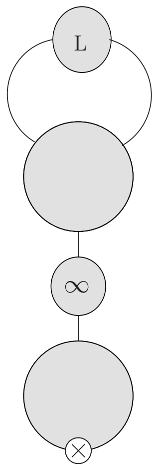
- Is finite
- All finite volume $\mathcal{O}(d-4)$ integrals cancel
- In terms of lowest order masses: p^6 corrections small

$\langle \overline{q}q \rangle$ at Finite Volume

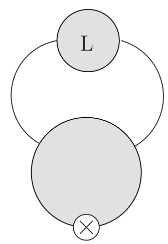
$$R_q = \left(\langle \overline{q}q \rangle_V - \langle \overline{q}q \rangle_\infty \right) / \langle \overline{q}q \rangle_\infty$$



$\langle \overline{q}q \rangle$ at Two Loops at Finite Volume



(a)



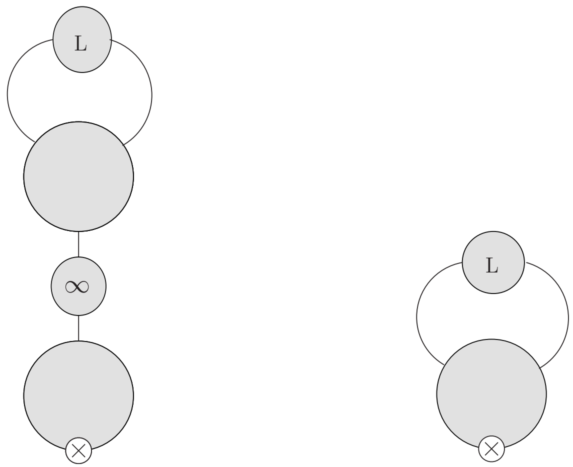
(b)

Lüscher's method

Insertion of $\langle O \rangle$: $\otimes$

(a) not relevant for CHPT and $\langle \overline{q}q \rangle$

$\langle \overline{q}q \rangle$ at Two Loops at Finite Volume



(a) (b)

Lüscher's method

Insertion of $\langle O \rangle$: $\otimes$

(a) not relevant for CHPT and $\langle \overline{q}q \rangle$

$$\zeta(k) = \sqrt{k} m_0 L$$

$$\langle O \rangle_V - \langle O \rangle_\infty = -\sum_{\vec{n}\neq\vec{o}} \frac{1}{16\pi^2} \int_0^\infty \frac{dq^2 q^2}{\sqrt{m_0^2 + q^2}} e^{-\sqrt{\vec{n}^2(m_0^2+q^2)L^2}} \langle \phi | O | \phi \rangle$$

$$= -\langle \phi | O | \phi \rangle \left(\sum_{k=1,\infty} \frac{m(k)}{16\pi^2} \frac{m_0^2}{\sqrt{\zeta(k)}} K_1(\zeta(k)) \right)$$

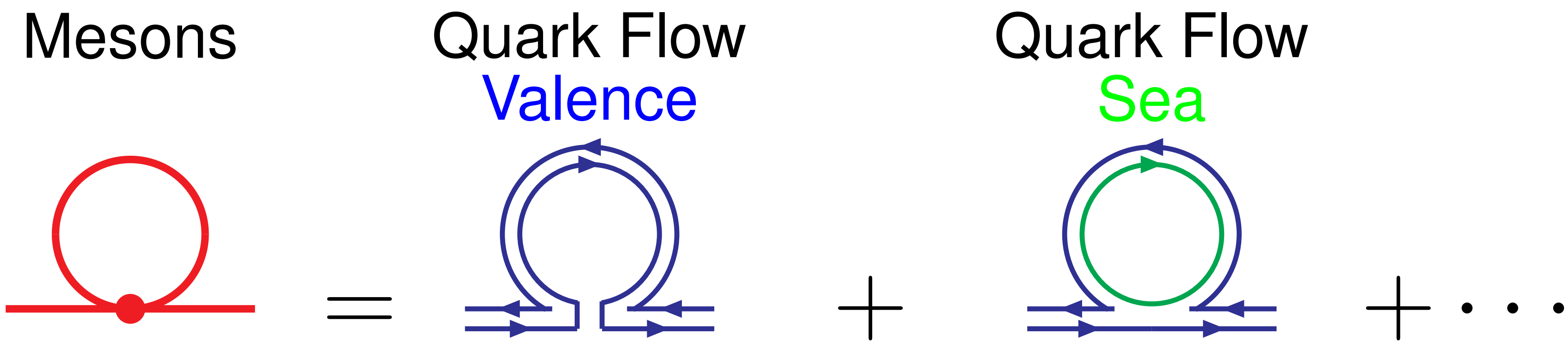
Universal factor per particle species

$\langle \overline{q}q \rangle$ at Finite Volume

Conclusions of this part:

- $\langle \overline{q}q \rangle$ at Two Loops at Finite Volume
- A Lüscher formula for vacuum expectation values
- σ terms obtainable from finite volume vacuum expectation values
- Lüscher formula in terms of overall quantity, not integral
- two-loop test of Lüscher formula not conclusive: too small corrections

Partial Quenching and ChPT



Lattice QCD: Valence is easy to deal with, Sea very difficult

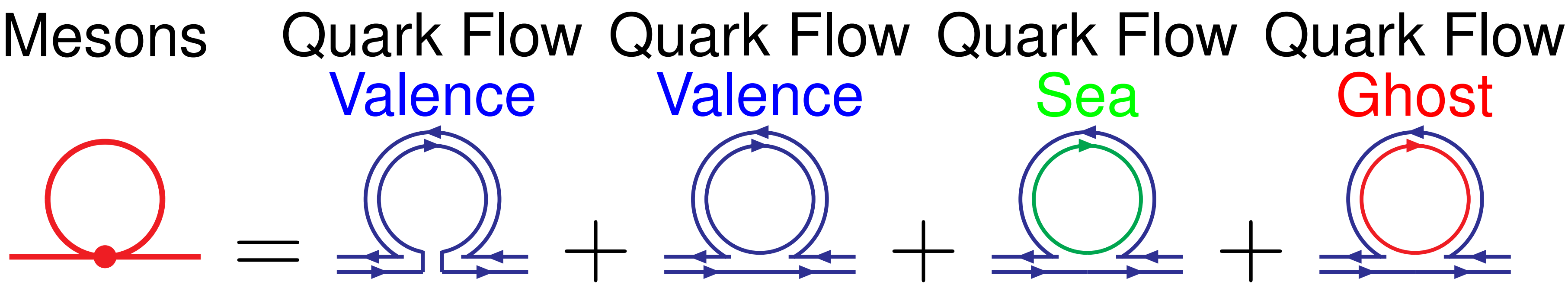
They can be treated separately: i.e. different quark masses
Partially Quenched QCD and ChPT (PQChPT)

One Loop or p^4: Bernard, Golterman, Pallante, Sharpe, Shoresh,. . .

Two Loops or p^6: This talk JB, Niclas Danielsson, Timo Lähde

PQChPT at Two Loops: General

Add ghost quarks: remove the unwanted free valence loops



Possible problem: QCD $\Longrightarrow$ ChPT relies heavily on unitarity

Partially quenched: at least one dynamical sea quark
$\Longrightarrow$ Φ_0 is heavy: remove from PQChPT

Symmetry group becomes $SU(n_v + n_s|n_v) \times SU(n_v + n_s|n_v)$ (approximately)

PQChPT at Two Loops: General

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

Very important note: ChPT is a limit of PQChPT
$\Longrightarrow$ LECs from ChPT are linear combinations of LECs of PQChPT with the same number of sea quarks.

E.g. $L_1^r = L_0^{r(3pq)}/2 + L_1^{r(3pq)}$

PQChPT at Two Loop: Papers

valence equal mass, 3 sea equal mass:

$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0406017

Other mass combinations:

$F^2_{\pi^+}$: JB,Lähde, hep-lat/0501014

$F^2_{\pi^+}$, $m^2_{\pi^+}$ two sea quarks: JB,Lähde, hep-lat/0506004

In progress: the other charged masses

Actual Calculations:
- heavy use of FORM Vermaseren
- Main problem: sheer size of the expressions

No fits to lattice data (yet): a and L extrapolations needed

Why so long expressions

- Many different quark and meson masses (χ_{ij})
- Charged propagators: $-i\,G^c_{ij}(k) = \frac{\epsilon_j}{k^2 - \chi_{ij} + i\varepsilon} \quad (i \neq j)$
- Neutral propagators: $G^n_{ij}(k) = G^c_{ij}(k)\,\delta_{ij} - \frac{1}{n_{\text{sea}}}\,G^q_{ij}(k)$

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$$-i\,G^q_{ii}(k) = \frac{R^d_i}{(k^2-\chi_i+i\varepsilon)^2} + \frac{R^c_i}{k^2-\chi_i+i\varepsilon} + \frac{R^\pi_{\eta ii}}{k^2-\chi_\pi+i\varepsilon} + \frac{R^\eta_{\pi ii}}{k^2-\chi_\eta+i\varepsilon}$$

$$R^i_{jkl} = R^z_{i456jkl}, \qquad R^d_i = R^z_{i456\pi\eta},$$

$$R^c_i = R^i_{4\pi\eta} + R^i_{5\pi\eta} + R^i_{6\pi\eta} - R^i_{\pi\eta\eta} - R^i_{\pi\pi\eta}$$

$$R^z_{ab} = \chi_a - \chi_b, \qquad R^z_{abc} = \frac{\chi_a-\chi_b}{\chi_a-\chi_c}, \qquad R^z_{abcd} = \frac{(\chi_a-\chi_b)(\chi_a-\chi_c)}{\chi_a-\chi_d}$$

$$R^z_{abcdefg} = \frac{(\chi_a-\chi_b)(\chi_a-\chi_c)(\chi_a-\chi_d)}{(\chi_a-\chi_e)(\chi_a-\chi_f)(\chi_a-\chi_g)}$$

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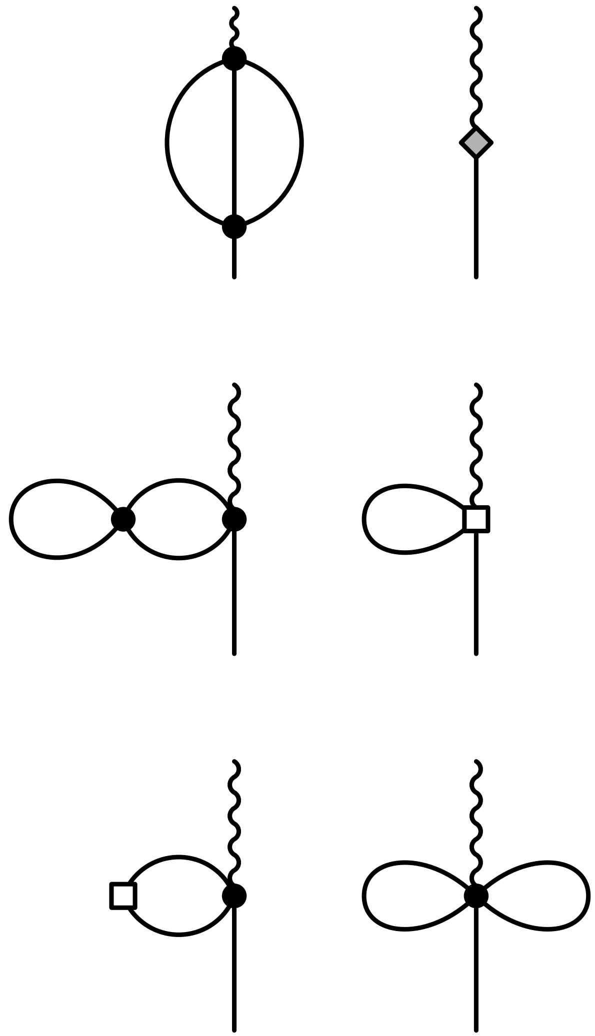
- Relations $\Longrightarrow$ order of magnitude smaller

Long Expressions



$\Longrightarrow$

$$
\begin{aligned}
\delta^{(6)22}_{\rm loops} =\;& \pi_{16}\, L_0^r\,\left[4/9\,\chi_\eta\chi_4 - 1/2\,\chi_1\chi_3 + \chi_{13}^2 - 13/3\,\bar\chi_1\chi_{13} - 35/18\,\bar\chi_2\right] \;-\; 2\,\pi_{16}\,L_1^r\,\chi_{13}^2 \\
-\;& \pi_{16}\,L_2^r\,\left[11/3\,\chi_\eta\chi_4 + \chi_{13}^2 + 13/3\,\bar\chi_2\right] + \pi_{16}\,L_3^r\,\left[4/9\,\chi_\eta\chi_4 - 7/12\,\chi_1\chi_3 + 11/6\,\chi_{13}^2 - 17/6\,\bar\chi_1\chi_{13} - 43/36\,\bar\chi_2\right] \\
+\;& \pi_{16}^2\,\left[-15/64\,\chi_\eta\chi_4 - 59/384\,\chi_1\chi_3 + 65/384\,\chi_{13}^2 - 1/2\,\bar\chi_1\chi_{13} - 43/128\,\bar\chi_2\right] \;-\; 48\,L_4^rL_5^r\,\bar\chi_1\chi_{13} \;-\; 72\,L_4^{r2}\,\bar\chi_1^2 \\
-\;& 8\,L_5^{r2}\,\chi_{13}^2 \;+\; \bar A(\chi_p)\,\pi_{16}\,\left[-1/24\,\chi_p + 1/48\,\bar\chi_1 - 1/8\,\bar\chi_1\,R^p_{q\eta} + 1/16\,\bar\chi_1\,R^c_p - 1/48\,R^p_{q\eta}\,\chi_p - 1/16\,R^p_{q\eta}\,\chi_q\right. \\
+\;& \left.1/48\,R^\eta_{pp}\,\chi_\eta + 1/16\,R^c_p\,\chi_{13}\right] \;+\; \bar A(\chi_p)\,L_0^r\,\left[8/3\,R^p_{q\eta}\,\chi_p + 2/3\,R^c_p\,\chi_p + 2/3\,R^d_p\right] \;+\; \bar A(\chi_p)\,L_3^r\,\left[2/3\,R^p_{q\eta}\,\chi_p\right. \\
+\;& \left.5/3\,R^c_p\,\chi_p + 5/3\,R^d_p\right] \;+\; \bar A(\chi_p)\,L_4^r\,\left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 0} - 2\,\bar\chi_1\,R^p_{q\eta} + 3\,\bar\chi_1\,R^c_p\right] \;+\; \bar A(\chi_p)\,L_5^r\,\left[-2/3\,\bar\chi^{pp}_{\eta\eta 1} - R^p_{q\eta}\,\chi_p\right. \\
+\;& \left.1/3\,R^p_{q\eta}\,\chi_q + 1/2\,R^c_p\,\chi_p - 1/6\,R^c_p\,\chi_q\right] \;+\; \bar A(\chi_p)^2\,\left[1/16 + 1/72\,(R^p_{q\eta})^2 - 1/72\,R^p_{q\eta}R^c_p + 1/288\,(R^c_p)^2\right] \\
+\;& \bar A(\chi_p)\bar A(\chi_{ps})\,\left[-1/36\,R^p_{q\eta} - 5/72\,R^p_{s\eta} + 7/144\,R^c_p\right] \;-\; \bar A(\chi_p)\bar A(\chi_{qs})\,\left[1/36\,R^p_{q\eta} + 1/24\,R^p_{s\eta} + 1/48\,R^c_p\right] \\
+\;& \bar A(\chi_p)\bar A(\chi_\eta)\,\left[-1/72\,R^p_{q\eta}R^v_{\eta 13} + 1/144\,R^c_pR^v_{\eta 13}\right] \;+\; 1/8\,\bar A(\chi_p)\bar A(\chi_{13}) \;+\; 1/12\,\bar A(\chi_p)\bar A(\chi_{46})\,R^\eta_{pp} \\
+\;& \bar A(\chi_p)\bar B(\chi_p,\chi_p;0)\,\left[1/4\,\chi_p - 1/18\,R^p_{q\eta}R^c_p\,\chi_p - 1/72\,R^p_{q\eta}R^d_p + 1/18\,(R^c_p)^2\,\chi_p + 1/144\,R^c_pR^d_p\right] \\
+\;& \bar A(\chi_p)\bar B(\chi_p,\chi_\eta;0)\,\left[1/18\,R^\eta_{pp}R^c_p\,\chi_p - 1/18\,R^\eta_{13}R^c_p\,\chi_p\right] \;+\; \bar A(\chi_p)\bar B(\chi_q,\chi_q;0)\,\left[-1/72\,R^p_{q\eta}R^d_q + 1/144\,R^c_pR^d_q\right] \\
-\;& 1/12\,\bar A(\chi_p)\bar B(\chi_{ps},\chi_{ps};0)\,R^p_{s\eta}\,\chi_{ps} \;-\; 1/18\,\bar A(\chi_p)\bar B(\chi_1,\chi_3;0)\,R^q_{p\eta}R^c_p\,\chi_p \\
+\;& 1/18\,\bar A(\chi_p)\bar C(\chi_p,\chi_p,\chi_p;0)\,R^c_pR^d_p\,\chi_p \;+\; \bar A(\chi_p;\varepsilon)\,\pi_{16}\,\left[1/8\,\bar\chi_1R^p_{q\eta} - 1/16\bar\chi_1\,R^c_p - 1/16\,R^c_p\,\chi_p - 1/16\,R^d_p\right] \\
+\;& \bar A(\chi_{ps})\,\pi_{16}\,\left[1/16\,\chi_{ps} - 3/16\,\chi_{qs} - 3/16\,\bar\chi_1\right] \;-\; 2\,\bar A(\chi_{ps})\,L_0^r\,\chi_{ps} \;-\; 5\,\bar A(\chi_{ps})\,L_3^r\,\chi_{ps} \;-\; 3\,\bar A(\chi_{ps})\,L_4^r\,\bar\chi_1 \\
+\;& \bar A(\chi_{ps})\,L_5^r\,\chi_{13} \;+\; \bar A(\chi_{ps})\bar A(\chi_\eta)\,\left[7/144\,R^\eta_{pp} - 5/72\,R^\eta_{ps} - 1/48\,R^\eta_{qq} + 5/72\,R^\eta_{qs} - 1/36\,R^\eta_{13}\right] \\
+\;& \bar A(\chi_{ps})\bar B(\chi_p,\chi_p;0)\,\left[1/24\,R^p_{s\eta}\,\chi_p - 5/24\,R^p_{s\eta}\,\chi_{ps}\right] \;+\; \bar A(\chi_{ps})\bar B(\chi_p,\chi_\eta;0)\,\left[-1/18\,R^\eta_{ps}R^z_{qp\eta}\,\chi_p\right. \\
-\;& \left.1/9\,R^\eta_{ps}R^z_{qp\eta}\,\chi_{ps}\right] \;-\; 1/48\,\bar A(\chi_{ps})\bar B(\chi_q,\chi_q;0)\,R^d_q \;+\; 1/18\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0)\,R^q_{s\eta}\,\chi_s \\
+\;& 1/9\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0,k)\,R^q_{s\eta} \;+\; 3/16\,\bar A(\chi_{ps};\varepsilon)\,\pi_{16}\,\left[\chi_s + \bar\chi_1\right] \;-\; 1/8\,\bar A(\chi_{p4})^2 \;-\; 1/8\,\bar A(\chi_{p4})\bar A(\chi_{p6}) \\
+\;& 1/8\,\bar A(\chi_{p4})\bar A(\chi_{q6}) \;-\; 1/32\,\bar A(\chi_{p6})^2 \;+\; \bar A(\chi_\eta)\,\pi_{16}\,\left[1/16\,\bar\chi_1\,R^v_{\eta 13} - 1/48\,R^v_{\eta 13}\,\chi_\eta + 1/16\,R^v_{\eta 13}\,\chi_{13}\right] \\
+\;& \bar A(\chi_\eta)\,L_0^r\,\left[4R^\eta_{13}\,\chi_\eta + 2/3\,R^v_{\eta 13}\,\chi_\eta\right] \;-\; 8\,\bar A(\chi_\eta)\,L_1^r\,\chi_\eta \;-\; 2\,\bar A(\chi_\eta)\,L_2^r\,\chi_\eta \;+\; \bar A(\chi_\eta)\,L_3^r\,\left[4R^\eta_{13}\,\chi_\eta + 5/3\,R^v_{\eta 13}\chi_\eta\right] \\
+\;& \bar A(\chi_\eta)\,L_4^r\,\left[4\,\chi_\eta + \bar\chi_1\,R^v_{\eta 13}\right] \;-\; \bar A(\chi_\eta)\,L_5^r\,\left[1/6\,R^\eta_{pp}\,\chi_q + R^\eta_{13}\,\chi_{13} + 1/6\,R^v_{\eta 13}\,\chi_\eta\right] \;+\; 1/288\,\bar A(\chi_\eta)^2\,(R^v_{\eta 13})^2 \\
+\;& 1/12\,\bar A(\chi_\eta)\bar A(\chi_{46})\,R^v_{\eta 13} \;+\; \bar A(\chi_\eta)\bar B(\chi_p,\chi_p;0)\,\left[-1/36\,\bar\chi^{pp\eta}_{\eta\eta 1} - 1/18\,R^p_{q\eta}R^\eta_{pp}\,\chi_p + 1/18\,R^\eta_{pp}R^c_p\,\chi_p\right. \\
+\;& \left.1/144\,R^d_pR^v_{\eta 13}\right] \;+\; \bar A(\chi_\eta)\bar B(\chi_p,\chi_\eta;0)\,\left[-1/18\,\bar\chi^{\eta p\eta}_{p\eta 1} + 1/18\,\bar\chi^{\eta p\eta}_{q\eta 1} + 1/18\,(R^\eta_{pp})^2R^z_{qp\eta}\,\chi_p\right] \\
-\;& 1/12\,\bar A(\chi_\eta)\bar B(\chi_{ps},\chi_{ps};0)\,R^\eta_{ps}\,\chi_{ps} \;-\; \bar A(\chi_\eta)\bar B(\chi_\eta,\chi_\eta;0)\,\left[1/216\,R^v_{\eta 13}\,\chi_4 + 1/27\,R^v_{\eta 13}\,\chi_6\right] \\
-\;& 1/18\,\bar A(\chi_\eta)\bar B(\chi_1,\chi_3;0)\,R^1_{\eta\eta}R^3_{\eta\eta}\,\chi_\eta \;+\; 1/18\,\bar A(\chi_\eta)\bar C(\chi_p,\chi_p,\chi_p;0)\,R^\eta_{pp}R^d_p\,\chi_p \;+\; \bar A(\chi_\eta;\varepsilon)\,\pi_{16}\,\left[1/8\,\chi_\eta\right. \\
-\;& \left.1/16\,\bar\chi_1\,R^v_{\eta 13} - 1/8\,R^\eta_{13}\,\chi_\eta - 1/16\,R^v_{\eta 13}\,\chi_\eta\right] \;+\; \bar A(\chi_1)\bar A(\chi_3)\,\left[-1/72\,R^p_{q\eta}R^c_q + 1/36\,R^1_{3\eta}R^3_{1\eta} + 1/144\,R^c_1R^c_3\right] \\
-\;& 4\,\bar A(\chi_{13})\,L_1^r\,\chi_{13} \;-\; 10\,\bar A(\chi_{13})\,L_2^r\,\chi_{13} \;+\; 1/8\,\bar A(\chi_{13})^2 \;-\; 1/2\,\bar A(\chi_{13})\bar B(\chi_1,\chi_3;0,k) \\
+\;& 1/4\,\bar A(\chi_{13};\varepsilon)\,\pi_{16}\,\chi_{13} \;+\; 1/4\,\bar A(\chi_{14})\bar A(\chi_{34}) \;+\; 1/16\,\bar A(\chi_{16})\bar A(\chi_{36}) \;-\; 24\,\bar A(\chi_4)\,L_1^r\,\chi_4 \;-\; 6\,\bar A(\chi_4)\,L_2^r\,\chi_4 \\
+\;& 12\,\bar A(\chi_4)\,L_4^r\,\chi_4 \;+\; 1/12\,\bar A(\chi_4)\bar B(\chi_p,\chi_p;0)\,(R^p_{4\eta})^2\,\chi_4 \;+\; 1/6\,\bar A(\chi_4)\bar B(\chi_p,\chi_\eta;0)\,\left[R^p_{4\eta}R^\eta_{p4}\,\chi_4 - R^p_{4\eta}R^\eta_{q4}\,\chi_4\right] \\
-\;& 1/24\,\bar A(\chi_4)\bar B(\chi_\eta,\chi_\eta;0)\,R^v_{\eta 13}\,\chi_4 \;-\; 1/6\,\bar A(\chi_4)\bar B(\chi_1,\chi_3;0)\,R^1_{4\eta}R^3_{4\eta}\,\chi_4 \;+\; 3/8\,\bar A(\chi_4;\varepsilon)\,\pi_{16}\,\chi_4 \\
-\;& 32\,\bar A(\chi_{46})\,L_1^r\,\chi_{46} \;-\; 8\,\bar A(\chi_{46})\,L_2^r\,\chi_{46} \;+\; 16\,\bar A(\chi_{46})\,L_4^r\,\chi_{46} \;+\; \bar A(\chi_{46})\bar B(\chi_p,\chi_p;0)\,\left[1/9\,\chi_{46} + 1/12\,R^\eta_{pp}\,\chi_p\right. \\
+\;& \left.1/36\,R^\eta_{pp}\,\chi_4 + 1/9\,R^\eta_{p4}\,\chi_6\right] \;+\; \bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0)\,\left[-1/18\,R^\eta_{pp}\,\chi_4 - 1/9\,R^\eta_{p4}\,\chi_6 + 1/9\,R^\eta_{q4}\,\chi_6 + 1/18\,R^\eta_{13}\,\chi_4\right] \\
-\;& 1/6\,\bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0,k)\,\left[R^\eta_{pp} - R^\eta_{13}\right] \;+\; 1/9\,\bar A(\chi_{46})\bar B(\chi_\eta,\chi_\eta;0)\,R^v_{\eta 13}\,\chi_{46} \;-\; \bar A(\chi_{46})\bar B(\chi_1,\chi_3;0)\,\left[2/9\,\chi_{46}\right. \\
+\;& \left.1/9\,R^\eta_{p4}\,\chi_6 + 1/18\,R^\eta_{13}\,\chi_4\right] \;-\; 1/6\,\bar A(\chi_{46})\bar B(\chi_1,\chi_3;0,k)\,R^\eta_{13} \;+\; 1/2\,\bar A(\chi_{46};\varepsilon)\,\pi_{16}\,\chi_{46} \\
+\;& \bar B(\chi_p,\chi_p;0)\,\pi_{16}\,\left[1/16\,\bar\chi_1\,R^d_p + 1/96\,R^d_p\,\chi_p + 1/32\,R^d_p\,\chi_q\right] \;+\; 2/3\,\bar B(\chi_p,\chi_p;0)\,L_0^r\,R^d_p\,\chi_p \\
+\;& 5/3\,\bar B(\chi_p,\chi_p;0)\,L_3^r\,R^d_p\,\chi_p \;+\; \bar B(\chi_p,\chi_p;0)\,L_4^r\,\left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 0}\chi_p - 4\,\bar\chi_1R^p_{q\eta}\,\chi_p + 4\,\bar\chi_1R^c_p\,\chi_p + 3\,\bar\chi_1R^d_p\right] \\
+\;& \bar B(\chi_p,\chi_p;0)\,L_5^r\,\left[-2/3\bar\chi^{pp}_{\eta\eta 1}\chi_p - 4/3\,R^p_{q\eta}\,\chi_p^2 + 4/3\,R^c_p\,\chi_p^2 + 1/2\,R^d_p\,\chi_p - 1/6\,R^d_p\,\chi_q\right] \\
+\;& \bar B(\chi_p,\chi_p;0)\,L_6^r\,\left[4\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 1} + 8\,\bar\chi_1R^p_{q\eta}\,\chi_p - 8\,\bar\chi_1R^c_p\,\chi_p\right] \;+\; 4\,\bar B(\chi_p,\chi_p;0)\,L_7^r\,(R^d_p)^2 \\
+\;& \bar B(\chi_p,\chi_p;0)\,L_8^r\,\left[4/3\,\bar\chi^{pp}_{\eta\eta 2} + 8/3\,R^p_{q\eta}\,\chi_p^2 - 8/3\,R^c_p\,\chi_p^2\right] \;+\; \bar B(\chi_p,\chi_p;0)^2\,\left[-1/18\,R^p_{q\eta}R^d_p\,\chi_p + 1/18\,R^c_pR^d_p\,\chi_p\right. \\
+\;& \left.1/288\,(R^d_p)^2\right] \;+\; 1/18\,\bar B(\chi_p,\chi_p;0)\bar B(\chi_p,\chi_\eta;0)\,\left[R^\eta_{pp}R^d_p\,\chi_p - R^\eta_{13}R^d_p\,\chi_p\right]
\end{aligned}
$$

plus several more pages

PQChPT at Two Loop

Problem: Plotting with many input parameters

Plot masses as a function of lowest order mass squared

I.e. of quark mass: $\boxed{\chi_i = 2B_0 m_i = m_M^{2(0)}}$

Remember: $\chi_i \approx 0.3\ GeV^2 \approx (550\ MeV)^2 \sim$ border ChPT

PQChPT at Two Loop

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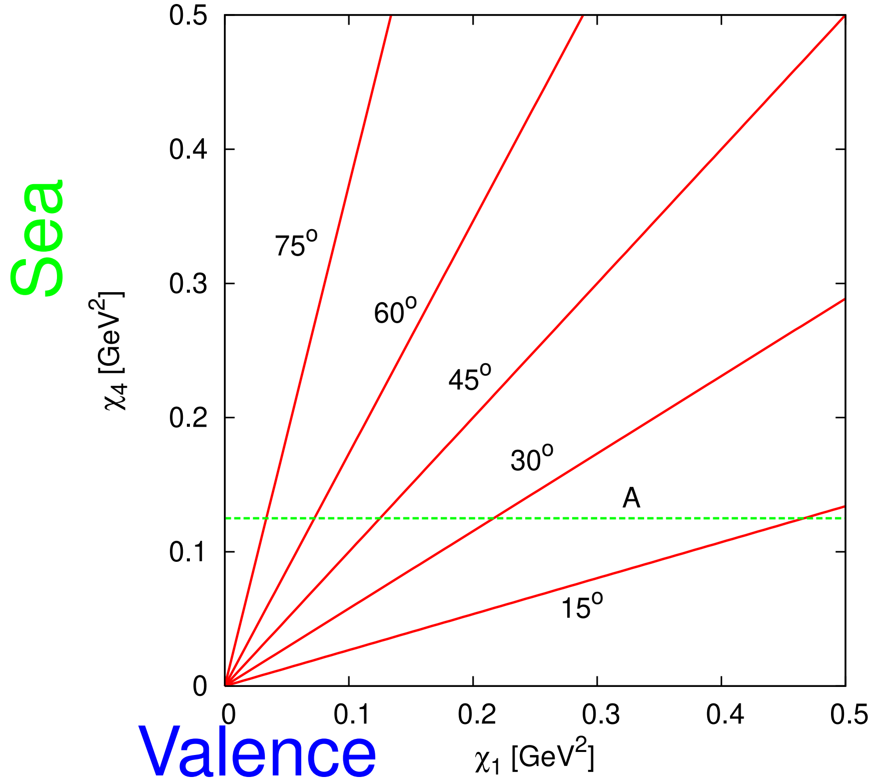
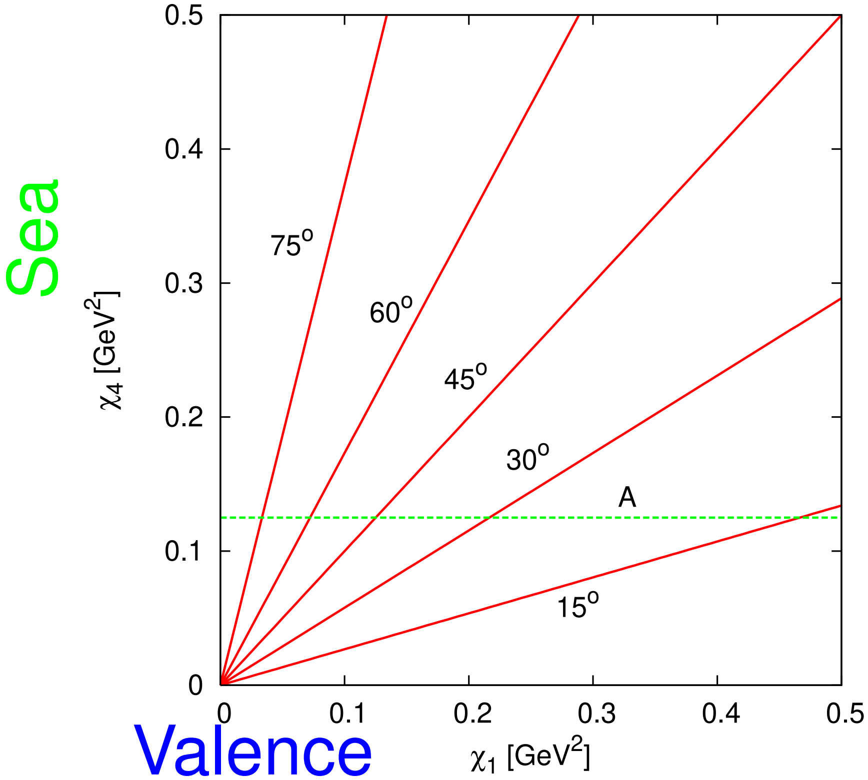
1+1 case: Valence: $\chi_1 = \chi_2 = \chi_3$
Sea: $\chi_4 = \chi_5 = \chi_6$

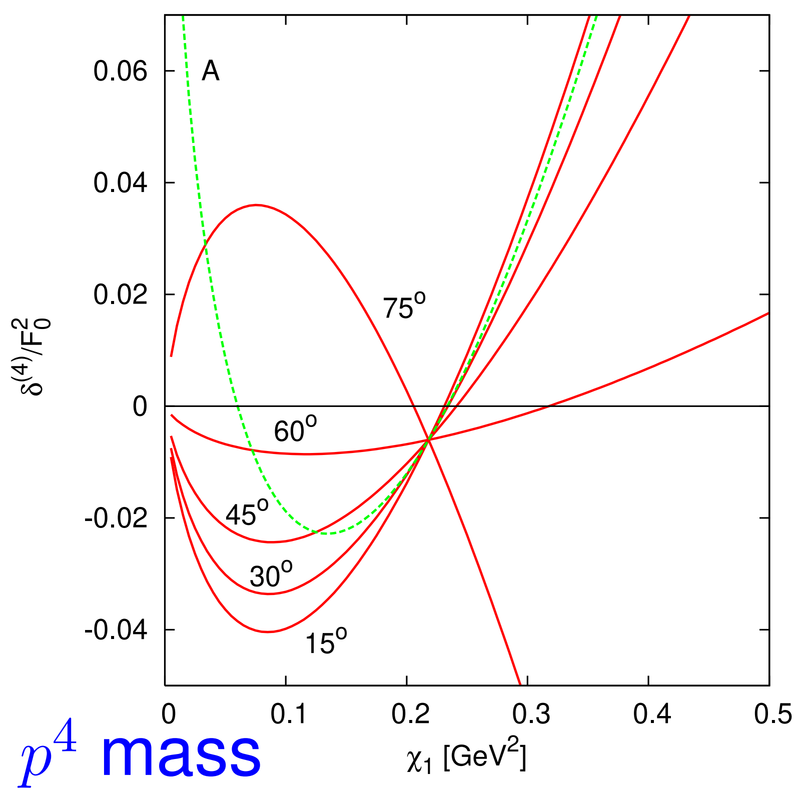
Plot along curves: $\chi_4 = \tan\theta\ \chi_1$ or χ_4 constant

PQChPT: 1+1 case, 3 sea quarks

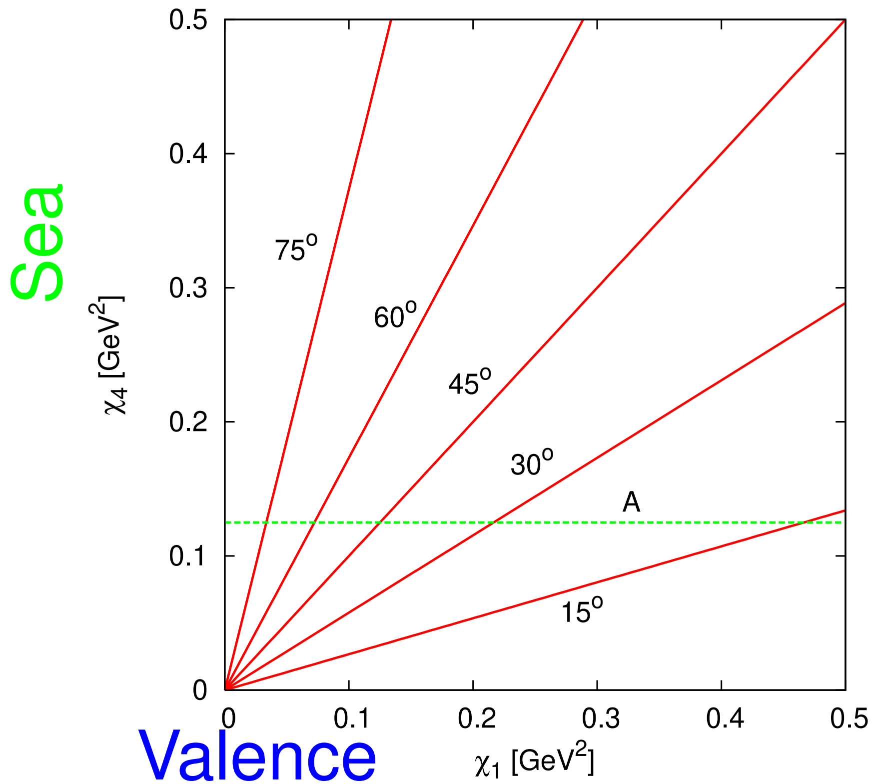


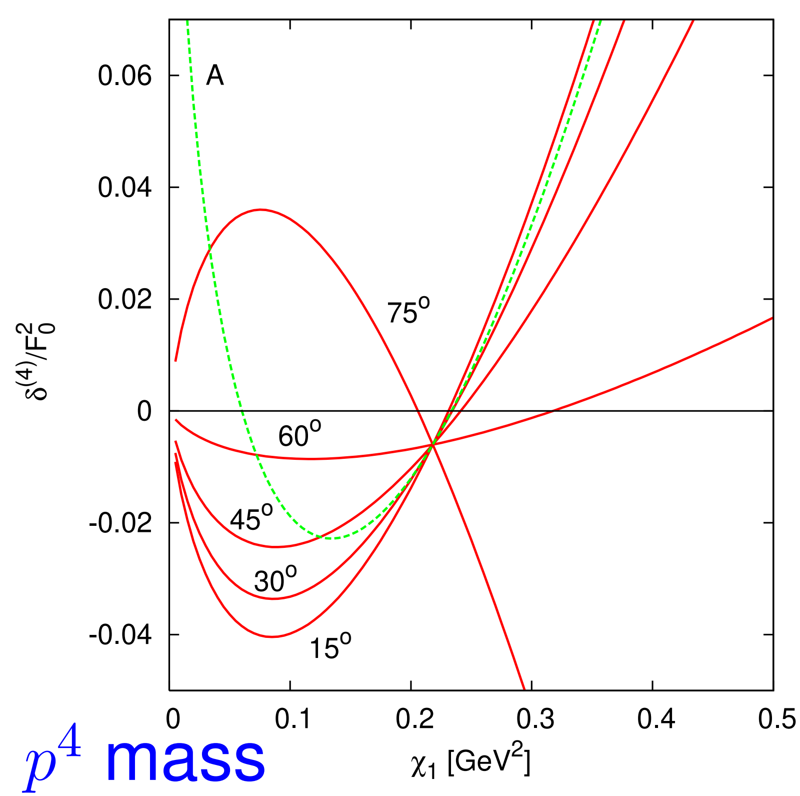
PQChPT: 1+1 case, 3 sea quarks





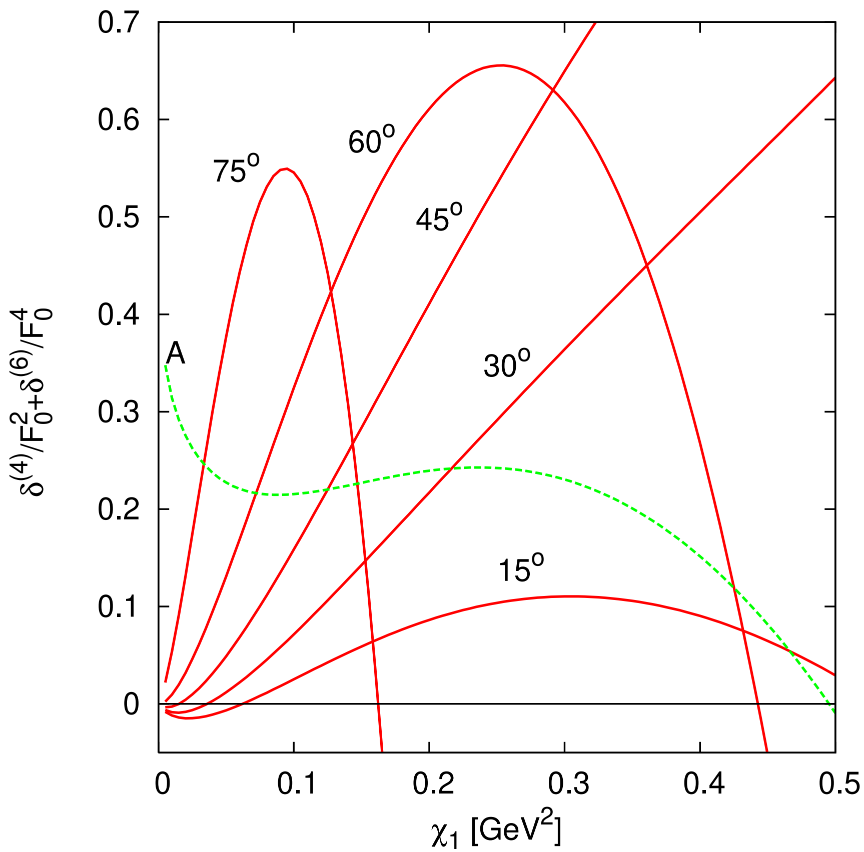
PQChPT: 1+1 case, 3 sea quarks





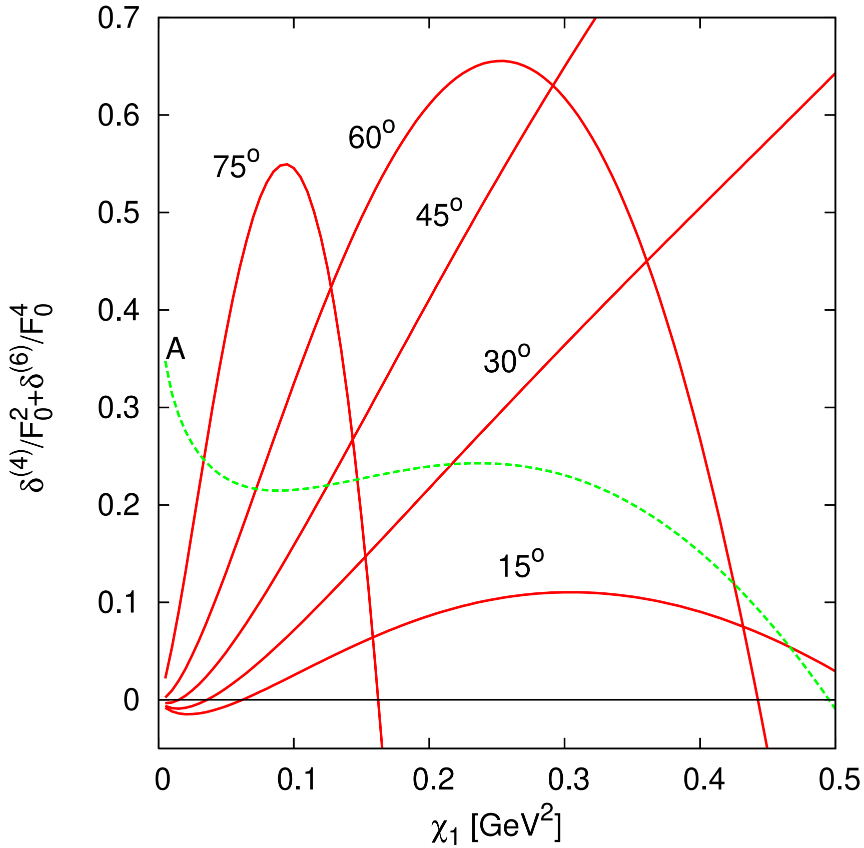
Notice the Quenched Chiral Logs: $\dfrac{m_\pi^2}{\chi_1} = 1 + \dfrac{\alpha}{F^2}\chi_4 \log \chi_1 + \cdots$

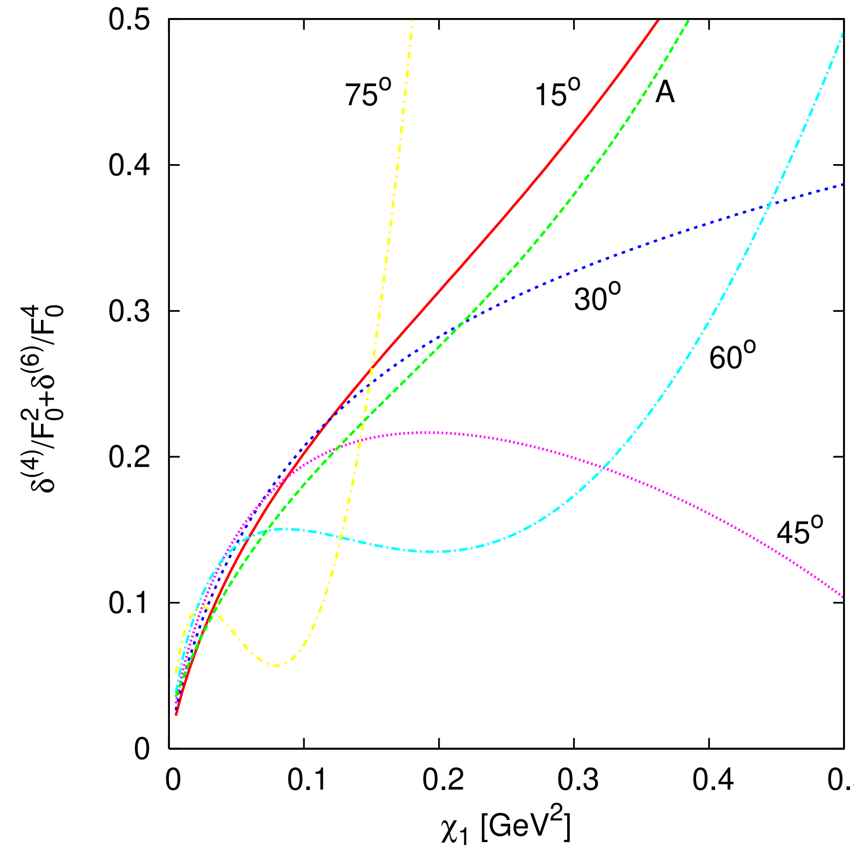
PQChPT: 1+1 case, 3 sea quarks



$p^4 + p^6$ relative correction mass

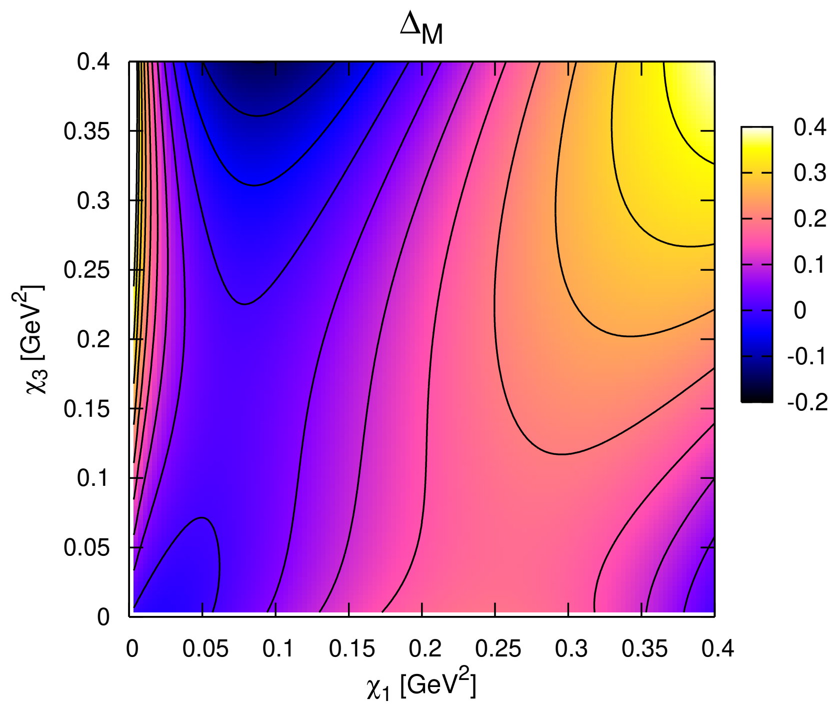
PQChPT: 1+1 case, 3 sea quarks





$p^4 + p^6$ relative correction mass   decay constant

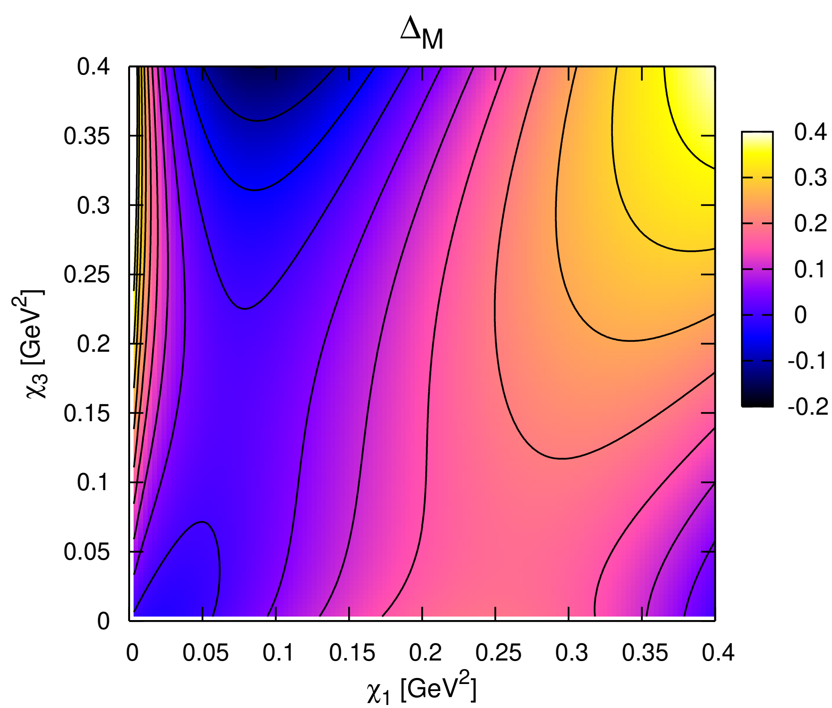
PQChPT: 1+1 case, 2 sea quarks

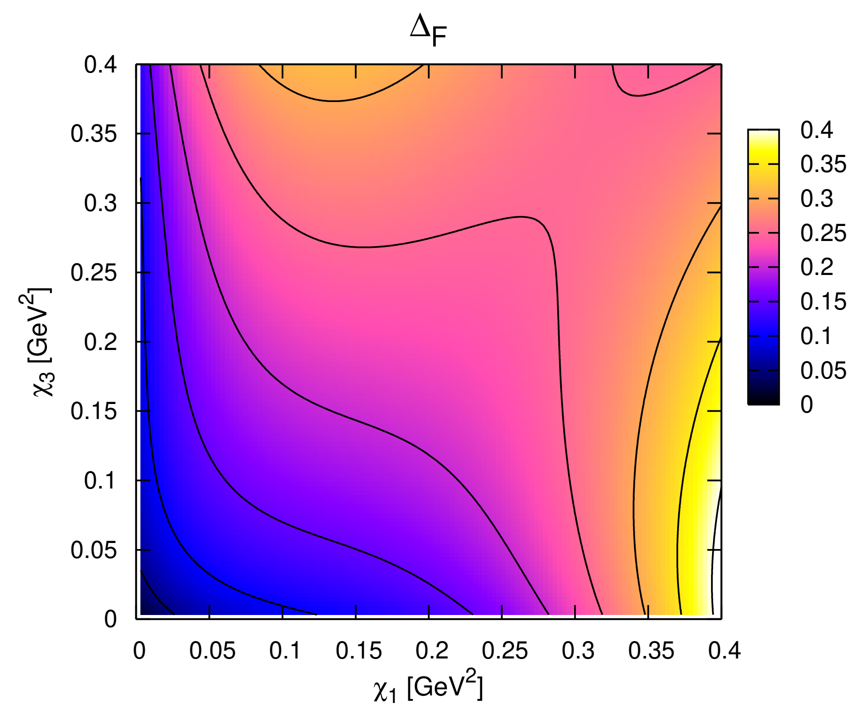


Relative Correction: Mass

χ_1: valence mass, χ_3: sea mass

PQChPT: 1+1 case, 2 sea quarks

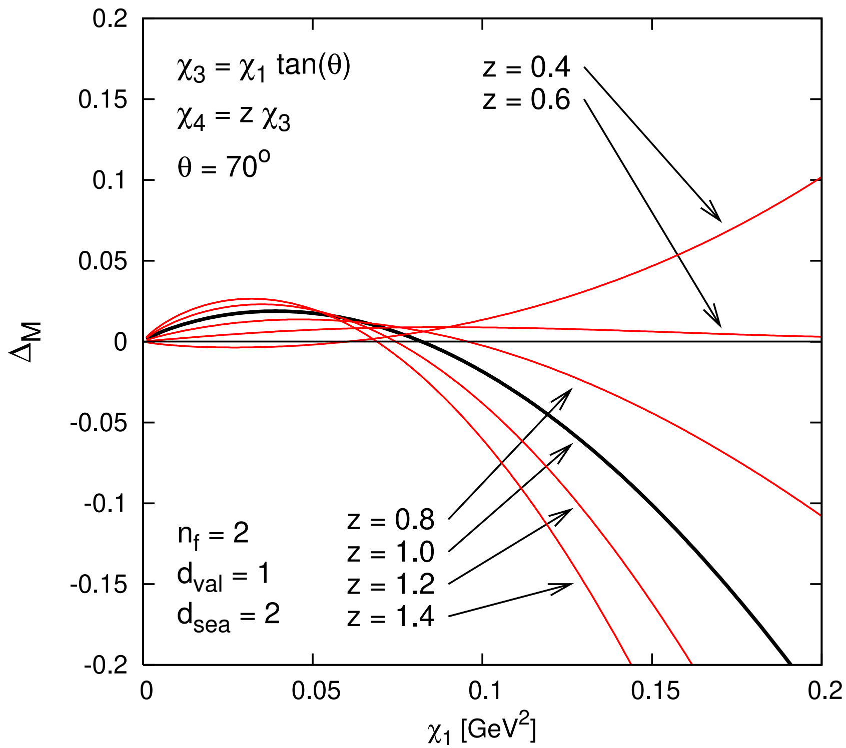




Relative Correction: Mass   Decay Constant

χ_1: valence mass, χ_3: sea mass

PQChPT: 2 sea quarks



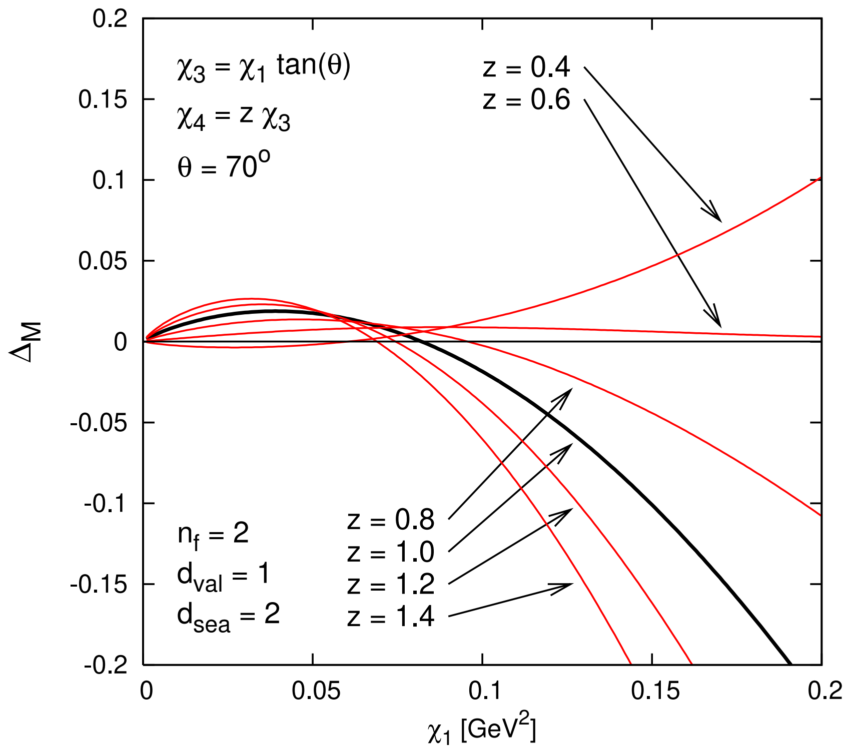
Relative Correction: Mass

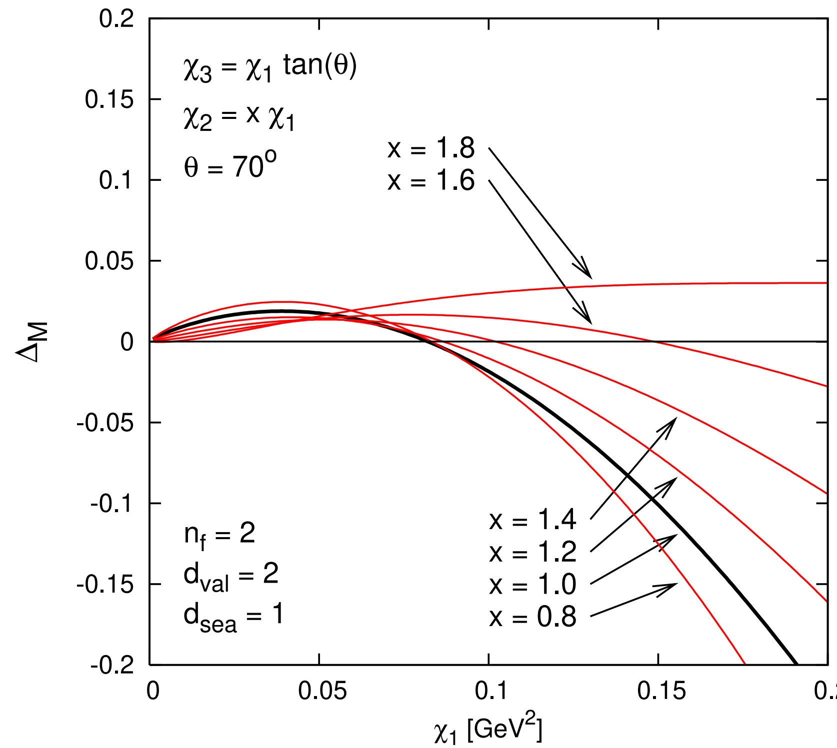
1+2 case

Valence: $\chi_1 \neq \chi_2$

Sea: $\chi_3 = \chi_4$

PQChPT: 2 sea quarks





Relative Correction: Mass

1+2 case

Valence: $\chi_1 \neq \chi_2$

Sea: $\chi_3 = \chi_4$

Mass

2+1 case

Valence: $\chi_1 = \chi_2$

Sea: $\chi_3 \neq \chi_4$

PQChPT: Fitting Strategy

For masses and Decay constants

- At order p^4: $L_4^r, L_5^r, L_6^r, L_8^r$
- At order p^6: all allowed quadratic quark mass combinations show up
- Problem: Need $L_0^r, L_1^r, L_2^r, L_3^r$ from $\pi\pi, \pi K$ scattering but only to order p^4
- Nonanalytic structure at p^6 given (only one included in numerics shown)

Conclusions PQChPT Part

- All relevant mass combinations for masses and decay constants for charged pseudoscalar mesons now known to two loops
- Decay constants and masses for two sea quarks converge nicely
- Masses for three sea quarks convergence slower
- Looking forward to getting lattice data to fit
- Expressions available from `http://www.thep.lu.se/~bijnens/chpt.html`
- For the numerical programs contact the authors