PARTIALLY QUENCHED CHPT AT TWO LOOPS

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Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Overview

- Chiral Perturbation Theory
- Lagrangians
- What is partially quenched?
- PQChPT and some problems
- First results of PQChPT at Two-loops
- What when we cannot simply resum: neutral masses
- Conclusions

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R\right)\right]$$

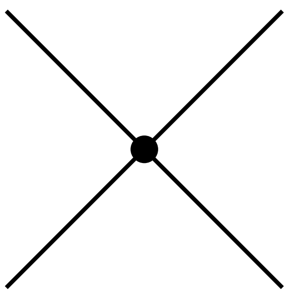
So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

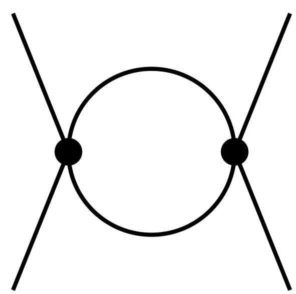
Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

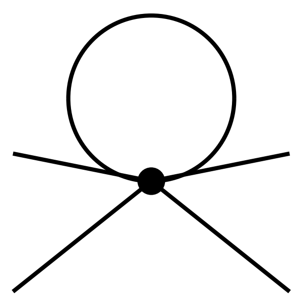
Power counting in momenta:

 p^2

 $(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

 $1/p^2$

$\int d^4p$ p^4

 $(p^2)\,(1/p^2)\,p^4 = p^4$

Two Loop: Lagrangians

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

Note
- replica method $\Longrightarrow$ PQ obtained from N_F flavour
- All infinities known
- 3 flavour is a special case of 3+3 PQ:
  $\hat{L}_i^r, K_i^r \rightarrow L_i^r, C_i^r$

Integrals, Divergences, Subtractions

- Infinities: the general divergence structure can be derived using heat kernel methods and/or background field methods.
  Very useful for checks on calculations
- one-loop: Passarino-Veltman, but need $d-4$ part.
- two-loop: sunset (i.e. two-point) integrals: dispersive method
- two-loop: vertex (i.e. three-point) integrals: Ghinculov-Van der Bij-Yao
- The last need two parameter numerical integrals with possible singularities: (very) slow.
- Subtraction: modified modified minimal subtraction (tadpoles are just a logarithm at NLO)

Usual ChPT two-loop: A list

Review paper on Two-Loops: JB, LU TP 06-16 hep-ph/0604043

Two-Loop Two-Flavour

- Bellucci-Gasser-Sainio: $\gamma\gamma \to \pi^0\pi^0$: 1994
- Bürgi: $\gamma\gamma \to \pi^+\pi^-$, F_π, m_π: 1996
- JB-Colangelo-Ecker-Gasser-Sainio: $\pi\pi$, F_π, m_π: 1996-97
- JB-Colangelo-Talavera: $F_{V\pi}(t)$, $F_{S\pi}$: 1998
- JB-Talavera: $\pi \to \ell\nu\gamma$: 1997
- Gasser-Ivanov-Sainio: $\gamma\gamma \to \pi^0\pi^0$, $\gamma\gamma \to \pi^+\pi^-$: 2005-2006

Two-Loops Three flavours

- $\Pi_{VV\pi}, \Pi_{VV\eta}, \Pi_{VVK}$ — Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera
- $\Pi_{VV\rho\omega}$ — Maltman
- $\Pi_{AA\pi}, \Pi_{AA\eta}, F_\pi, F_\eta, m_\pi, m_\eta$ — Kambor, Golowich; Amorós, JB, Talavera
- Π_{SS} — Moussallam $\boxed{L_4^r, L_6^r}$

Usual ChPT two-loop: A list

- Π_{VVK}, Π_{AAK}, F_K, m_K — Amorós, JB, Talavera
- $K_{\ell 4}$, $\langle \overline{q}q \rangle$ — Amorós, JB, Talavera — L_1^r, L_2^r, L_3^r
- F_M, m_M, $\langle \overline{q}q \rangle$ $(m_u \neq m_d)$ — Amorós, JB, Talavera — $L_{5,7,8}^r, m_u/m_d$
- $F_{V\pi}$, F_{VK+}, F_{VK^0} — Post, Schilcher; JB, Talavera — L_9^r
- $K_{\ell 3}$ — Post, Schilcher; JB, Talavera — V_{us}
- $F_{S\pi}$, F_{SK} (includes σ-terms) — JB, Dhonte — L_4^r, L_6^r
- $K, \pi \to \ell \nu \gamma$ — Geng, Ho, Wu — L_{10}^r
- $\pi\pi$ — JB,Dhonte,Talavera
- πK — JB,Dhonte,Talavera

What is Partially Quenched?

In Lattice gauge theory one calculates

$$\langle 0|(\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)|\rangle$$

$$= \frac{\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}}}{\int [dq][d\overline{q}][dG]e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}}}$$

for Euclidean separations x
Integrals also performed after rotation to Euclidean
(note that I use Minkowski notation throughout)

What is Partially Quenched?

$$\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}(\not{D}^u_G)^{-1}(x,0)(\not{D}^d_G)^{-1}(0,x)\det(\not{D}_G)_{\mathrm{QCD}}$$

What is Partially Quenched?

$$\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0) e^{i\int d^4y \mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG] e^{i\int d^4x (-1/4) G_{\mu\nu}G^{\mu\nu}} (\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x) \det(\not{D}_G)_{\mathrm{QCD}}$$

$\int [dG]$ done via importance sampling

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$$\int [dG] e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}} (\not{D}^u_G)^{-1}(x,0)(\not{D}^d_G)^{-1}(0,x) \,\det\,(\not{D}_G)_{\text{QCD}}$$

$\int [dG]$ done via importance sampling

- Quenched: get distribution from $e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}$ only

What is Partially Quenched?

$$\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\rm QCD}} \propto$$

$$\int [dG]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}(\not{D}_G^u)^{-1}(x,0)(\not{D}_G^d)^{-1}(0,x)\,\det(\not{D}_G)_{\rm QCD}$$

$\int [dG]$ done via importance sampling

- Quenched: get distribution from $e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}$ only
- Unquenched: include $\det(\not{D}_G)_{\rm QCD}$ VERY expensive

What is Partially Quenched?

$$\int [dq][d\overline{q}][dG](\overline{u}\gamma_5 d)(x)(\overline{d}\gamma_5 u)(0)e^{i\int d^4y\mathcal{L}_{\mathrm{QCD}}} \propto$$

$$\int [dG]e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}} (\not{D}^u_G)^{-1}(x,0)(\not{D}^d_G)^{-1}(0,x) \det(\not{D}_G)_{\mathrm{QCD}}$$

$\int [dG]$ done via importance sampling

- Quenched: get distribution from $e^{i\int d^4x(-1/4)G_{\mu\nu}G^{\mu\nu}}$ only
- Unquenched: include $\det(\not{D}_G)_{\mathrm{QCD}}$ VERY expensive
- Partially quenched: $(\not{D}^u_G)^{-1}(x,0)(\not{D}^d_G)^{-1}(0,x)$
  DIFFERENT Quarks then in $\det(\not{D}_G)_{\mathrm{QCD}}$

What is Partially Quenched?

Why do this?

- Is not Quenched: Real QCD is continuous limit from Partially Quenched
- More handles to turn:
  - Allows more systematic studies by varying parameters
  - Sometimes allows to disentangle things from different observables
- $\det\left(\not{D}_G\right)_{\mathrm{QCD}}$: Sea quarks
- $(\not{D}^u_G)^{-1}(x,0)(\not{D}^d_G)^{-1}(0,x)$: Valence Quarks

What is Partially Quenched?

Why not do this?

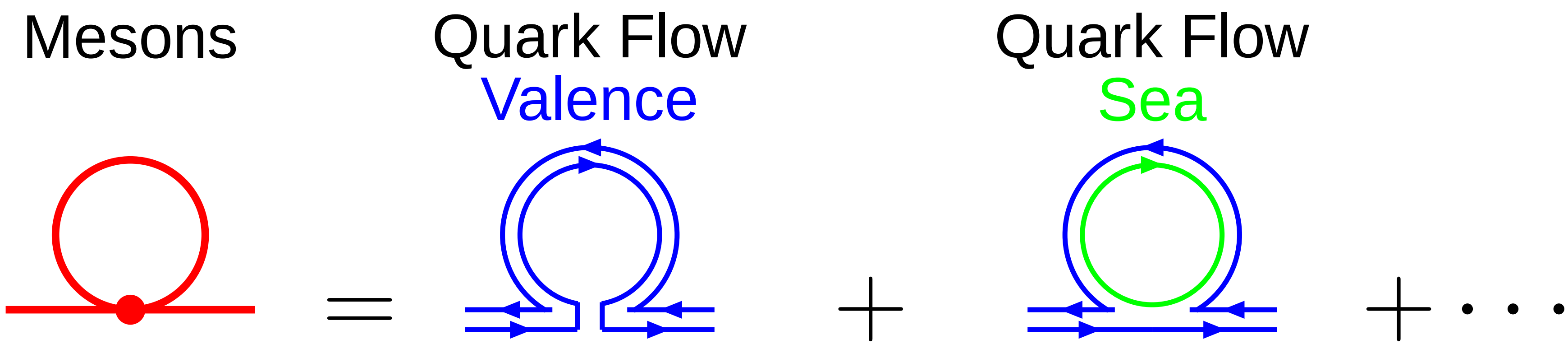
- It is not QCD as soon as Valence$\neq$Sea
- not a Quantum Field Theory
- No unitarity
- No clusterdecomposition
- No CPT theorem
- No spin statistics theorem

What is Partially Quenched?

Do anyway

- Cheap computationally
- Allows extra studies
- Hope it works near real QCD case
- Is a well defined Euclidean statistical model

ChPT and Lattice QCD



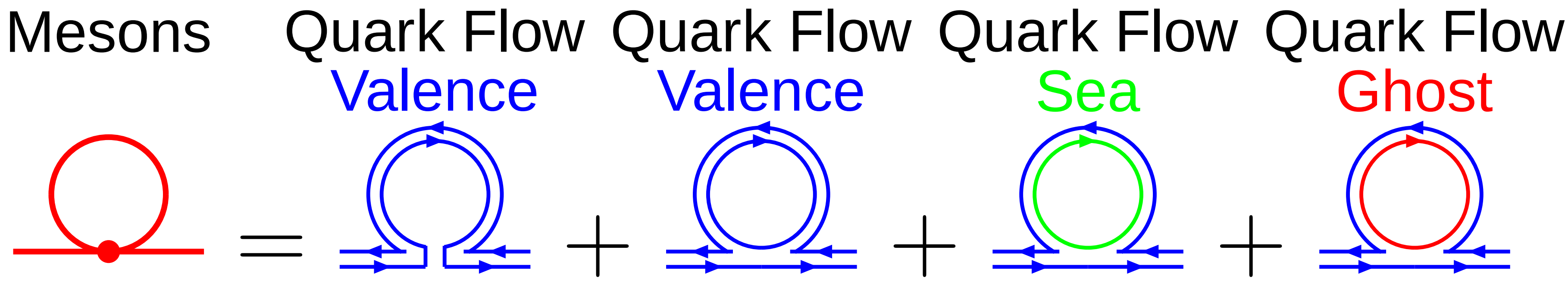
Valence is easy to deal with in lattice QCD
Sea is very difficult

They can be treated separately: i.e. different quark masses

Partially Quenched ChPT (PQChPT)

PQChPT at Two Loops: General

Add ghost quarks: remove the unwanted free valence loops



Possible problem: QCD $\Longrightarrow$ ChPT relies heavily on unitarity

Partially quenched: at least one dynamical sea quark

$\Longrightarrow$ Φ_0 is heavy: remove from PQChPT

Symmetry group becomes $SU(n_v + n_s|n_v) \times SU(n_v + n_s|n_v)$ (approximately)

PQChPT at Two Loops: General

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

Very important note: ChPT is a limit of PQChPT
$\Longrightarrow$ LECs from ChPT are linear combinations of LECs of PQChPT with the same number of sea quarks.

E.g. $L_1^r = L_0^{r(3pq)}/2 + L_1^{r(3pq)}$

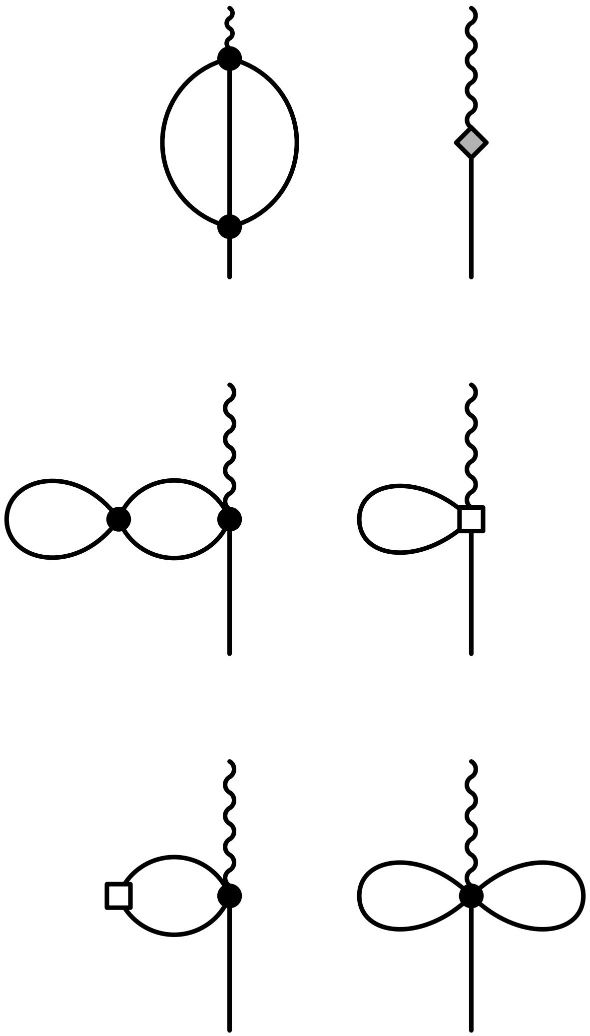
PQChPT at Two Loop

Subject started:
valence equal mass, 3 sea equal mass:
$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0406017
Other mass combinations:
F_{π^+}: JB,Lähde, hep-lat/0501014
F_{π^+}, $m^2_{\pi^+}$ two sea quarks: JB,Lähde, hep-lat/0506004
$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0602003
Neutral masses: JB,Danielsson, hep-lat/0606017

Actual Calculations:
- heavy use of FORM Vermaseren
- use PQ without super Φ_0 in supersymmetric formalism
- Main problem: sheer size of the expressions

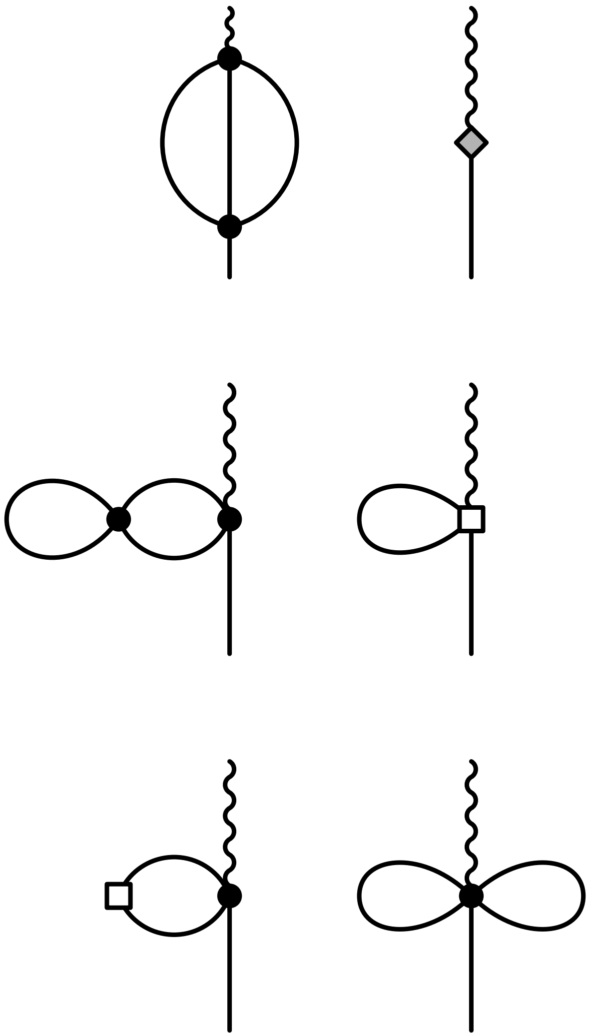
Iso breaking from lattice data: a and L extrapolations needed

Long Expressions



$$\Longrightarrow$$

Long Expressions



$\Longrightarrow$

$$
\begin{aligned}
\delta^{(6)22}_{\rm loops} =\ & \pi_{16}\, L^r_0\, \left[4/9\,\chi_\eta\chi_4 - 1/2\,\chi_1\chi_3 + \chi_{13}^2 - 13/3\,\bar\chi_1\chi_{13} - 35/18\,\bar\chi_2\right] \,-\, 2\,\pi_{16}\, L^r_1\, \chi_{13}^2 \\
& -\, \pi_{16}\, L^r_2\, \left[11/3\,\chi_\eta\chi_4 + \chi_{13}^2 + 13/3\,\bar\chi_2\right] + \pi_{16}\, L^r_3\, \left[4/9\,\chi_\eta\chi_4 - 7/12\,\chi_1\chi_3 + 11/6\,\chi_{13}^2 - 17/6\,\bar\chi_1\chi_{13} - 43/36\,\bar\chi_2\right] \\
& +\, \pi_{16}^2\, \left[-15/64\,\chi_\eta\chi_4 - 59/384\,\chi_1\chi_3 + 65/384\,\chi_{13}^2 - 1/2\,\bar\chi_1\chi_{13} - 43/128\,\bar\chi_2\right] \,-\, 48\, L^r_4 L^r_5\, \bar\chi_1\chi_{13} \,-\, 72\, L^{r2}_4\, \bar\chi_1^2 \\
& -\, 8\, L^{r2}_5\, \chi_{13}^2 \,+\, \bar A(\chi_p)\,\pi_{16}\, \left[-1/24\,\chi_p + 1/48\,\bar\chi_1 - 1/8\,\bar\chi_1\, R^p_{q\eta} + 1/16\,\bar\chi_1\, R^c_p - 1/48\, R^p_{q\eta}\,\chi_p - 1/16\, R^p_{q\eta}\,\chi_q \right.\\
& \left. +\, 1/48\, R^\eta_{pp}\,\chi_\eta + 1/16\, R^c_p\,\chi_{13}\right] \,+\, \bar A(\chi_p)\, L^r_0\, \left[8/3\, R^p_{q\eta}\,\chi_p + 2/3\, R^c_p\,\chi_p + 2/3\, R^d_p\right] \,+\, \bar A(\chi_p)\, L^r_3\, \left[2/3\, R^p_{q\eta}\,\chi_p \right.\\
& \left. +\, 5/3\, R^c_p\,\chi_p + 5/3\, R^d_p\right] \,+\, \bar A(\chi_p)\, L^r_4\, \left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 0} - 2\,\bar\chi_1\, R^p_{q\eta} + 3\,\bar\chi_1\, R^c_p\right] \,+\, \bar A(\chi_p)\, L^r_5\, \left[-2/3\,\bar\chi^{pp}_{\eta\eta 1} - R^p_{q\eta}\,\chi_p \right.\\
& \left. +\, 1/3\, R^p_{q\eta}\,\chi_q + 1/2\, R^c_p\,\chi_p - 1/6\, R^c_p\,\chi_q\right] \,+\, \bar A(\chi_p)^2\, \left[1/16 + 1/72\,(R^p_{q\eta})^2 - 1/72\, R^p_{q\eta}R^c_p + 1/288\,(R^c_p)^2\right] \\
& +\, \bar A(\chi_p)\bar A(\chi_{ps})\, \left[-1/36\, R^p_{q\eta} - 5/72\, R^p_{s\eta} + 7/144\, R^c_p\right] \,-\, \bar A(\chi_p)\bar A(\chi_{qs})\, \left[1/36\, R^p_{q\eta} + 1/24\, R^p_{s\eta} + 1/48\, R^c_p\right] \\
& +\, \bar A(\chi_p)\bar A(\chi_\eta)\, \left[-1/72\, R^p_{q\eta}R^v_{\eta 13} + 1/144\, R^c_p R^v_{\eta 13}\right] \,+\, 1/8\,\bar A(\chi_p)\bar A(\chi_{13}) \,+\, 1/12\,\bar A(\chi_p)\bar A(\chi_{46})\, R^\eta_{pp} \\
& +\, \bar A(\chi_p)\bar B(\chi_p,\chi_p;0)\, \left[1/4\,\chi_p - 1/18\, R^p_{q\eta}R^c_p\,\chi_p - 1/72\, R^p_{q\eta}R^d_p + 1/18\,(R^c_p)^2\,\chi_p + 1/144\, R^c_p R^d_p\right] \\
& +\, \bar A(\chi_p)\bar B(\chi_p,\chi_\eta;0)\, \left[1/18\, R^\eta_{pp}R^c_p\,\chi_p - 1/18\, R^\eta_{13}R^c_p\,\chi_p\right] \,+\, \bar A(\chi_p)\bar B(\chi_q,\chi_q;0)\, \left[-1/72\, R^p_{q\eta}R^d_q + 1/144\, R^c_p R^d_q\right] \\
& -\, 1/12\,\bar A(\chi_p)\bar B(\chi_{ps},\chi_{ps};0)\, R^p_{s\eta}\,\chi_{ps} \,-\, 1/18\,\bar A(\chi_p)\bar B(\chi_1,\chi_3;0)\, R^q_{p\eta}R^c_p\,\chi_p \\
& +\, 1/18\,\bar A(\chi_p)\bar C(\chi_p,\chi_p,\chi_p;0)\, R^c_p R^d_p\,\chi_p \,+\, \bar A(\chi_p;\varepsilon)\,\pi_{16}\, \left[1/8\,\bar\chi_1 R^p_{q\eta} - 1/16\bar\chi_1\, R^c_p - 1/16\, R^c_p\,\chi_p - 1/16\, R^d_p\right] \\
& +\, \bar A(\chi_{ps})\,\pi_{16}\, \left[1/16\,\chi_{ps} - 3/16\,\chi_{qs} - 3/16\,\bar\chi_1\right] \,-\, 2\,\bar A(\chi_{ps})\, L^r_0\,\chi_{ps} \,-\, 5\,\bar A(\chi_{ps})\, L^r_3\,\chi_{ps} \,-\, 3\,\bar A(\chi_{ps})\, L^r_4\,\bar\chi_1 \\
& +\, \bar A(\chi_{ps})\, L^r_5\,\chi_{13} \,+\, \bar A(\chi_{ps})\bar A(\chi_\eta)\, \left[7/144\, R^\eta_{pp} - 5/72\, R^\eta_{ps} - 1/48\, R^\eta_{qq} + 5/72\, R^\eta_{qs} - 1/36\, R^\eta_{13}\right] \\
& +\, \bar A(\chi_{ps})\bar B(\chi_p,\chi_p;0)\, \left[1/24\, R^p_{s\eta}\,\chi_p - 5/24\, R^p_{s\eta}\,\chi_{ps}\right] \,+\, \bar A(\chi_{ps})\bar B(\chi_p,\chi_\eta;0)\, \left[-1/18\, R^\eta_{ps}R^z_{qp\eta}\,\chi_p \right.\\
& \left. -\, 1/9\, R^\eta_{ps}R^z_{qp\eta}\,\chi_{ps}\right] \,-\, 1/48\,\bar A(\chi_{ps})\bar B(\chi_q,\chi_q;0)\, R^d_q \,+\, 1/18\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0)\, R^q_{s\eta}\,\chi_s \\
& +\, 1/9\,\bar A(\chi_{ps})\bar B(\chi_1,\chi_3;0,k)\, R^q_{s\eta} \,+\, 3/16\,\bar A(\chi_{ps};\varepsilon)\,\pi_{16}\, \left[\chi_s + \bar\chi_1\right] \,-\, 1/8\,\bar A(\chi_{p4})^2 \,-\, 1/8\,\bar A(\chi_{p4})\bar A(\chi_{p6}) \\
& +\, 1/8\,\bar A(\chi_{p4})\bar A(\chi_{q6}) \,-\, 1/32\,\bar A(\chi_{p6})^2 \,+\, \bar A(\chi_\eta)\,\pi_{16}\, \left[1/16\,\bar\chi_1\, R^v_{\eta 13} - 1/48\, R^v_{\eta 13}\,\chi_\eta + 1/16\, R^v_{\eta 13}\,\chi_{13}\right] \\
& +\, \bar A(\chi_\eta)\, L^r_0\, \left[4R^\eta_{13}\,\chi_\eta + 2/3\, R^v_{\eta 13}\,\chi_\eta\right] \,-\, 8\,\bar A(\chi_\eta)\, L^r_1\,\chi_\eta \,-\, 2\,\bar A(\chi_\eta)\, L^r_2\,\chi_\eta \,+\, \bar A(\chi_\eta)\, L^r_3\, \left[4R^\eta_{13}\,\chi_\eta + 5/3\, R^v_{\eta 13}\chi_\eta\right] \\
& +\, \bar A(\chi_\eta)\, L^r_4\, \left[4\,\chi_\eta + \bar\chi_1\, R^v_{\eta 13}\right] \,-\, \bar A(\chi_\eta)\, L^r_5\, \left[1/6\, R^\eta_{pp}\,\chi_q + R^\eta_{13}\,\chi_{13} + 1/6\, R^v_{\eta 13}\,\chi_\eta\right] \,+\, 1/288\,\bar A(\chi_\eta)^2\,(R^v_{\eta 13})^2 \\
& +\, 1/12\,\bar A(\chi_\eta)\bar A(\chi_{46})\, R^v_{\eta 13} \,+\, \bar A(\chi_\eta)\bar B(\chi_p,\chi_p;0)\, \left[-1/36\,\bar\chi^{pp\eta}_{\eta\eta 1} - 1/18\, R^p_{q\eta}R^\eta_{pp}\,\chi_p + 1/18\, R^\eta_{pp}R^c_p\,\chi_p \right.\\
& \left. +\, 1/144\, R^d_p R^v_{\eta 13}\right] \,+\, \bar A(\chi_\eta)\bar B(\chi_p,\chi_\eta;0)\, \left[-1/18\,\bar\chi^{\eta p\eta}_{p\eta 1} + 1/18\,\bar\chi^{\eta p\eta}_{q\eta 1} + 1/18\,(R^\eta_{pp})^2 R^z_{qp\eta}\,\chi_p\right] \\
& -\, 1/12\,\bar A(\chi_\eta)\bar B(\chi_{ps},\chi_{ps};0)\, R^\eta_{ps}\,\chi_{ps} \,-\, \bar A(\chi_\eta)\bar B(\chi_\eta,\chi_\eta;0)\, \left[1/216\, R^v_{\eta 13}\,\chi_4 + 1/27\, R^v_{\eta 13}\,\chi_6\right] \\
& -\, 1/18\,\bar A(\chi_\eta)\bar B(\chi_1,\chi_3;0)\, R^1_{\eta\eta}R^3_{\eta\eta}\,\chi_\eta \,+\, 1/18\,\bar A(\chi_\eta)\bar C(\chi_p,\chi_p,\chi_p;0)\, R^\eta_{pp}R^d_p\,\chi_p \,+\, \bar A(\chi_\eta;\varepsilon)\,\pi_{16}\, \left[1/8\,\chi_\eta \right.\\
& \left. -\, 1/16\,\bar\chi_1\, R^v_{\eta 13} - 1/8\, R^\eta_{13}\,\chi_\eta - 1/16\, R^v_{\eta 13}\,\chi_\eta\right] \,+\, \bar A(\chi_1)\bar A(\chi_3)\, \left[-1/72\, R^p_{q\eta}R^c_q + 1/36\, R^1_{3\eta}R^3_{1\eta} + 1/144\, R^c_1 R^c_3\right] \\
& -\, 4\,\bar A(\chi_{13})\, L^r_1\,\chi_{13} \,-\, 10\,\bar A(\chi_{13})\, L^r_2\,\chi_{13} \,+\, 1/8\,\bar A(\chi_{13})^2 \,-\, 1/2\,\bar A(\chi_{13})\bar B(\chi_1,\chi_3;0,k) \\
& +\, 1/4\,\bar A(\chi_{13};\varepsilon)\,\pi_{16}\,\chi_{13} \,+\, 1/4\,\bar A(\chi_{14})\bar A(\chi_{34}) \,+\, 1/16\,\bar A(\chi_{16})\bar A(\chi_{36}) \,-\, 24\,\bar A(\chi_4)\, L^r_1\,\chi_4 \,-\, 6\,\bar A(\chi_4)\, L^r_2\,\chi_4 \\
& +\, 12\,\bar A(\chi_4)\, L^r_4\,\chi_4 \,+\, 1/12\,\bar A(\chi_4)\bar B(\chi_p,\chi_p;0)\,(R^p_{4\eta})^2\,\chi_4 \,+\, 1/6\,\bar A(\chi_4)\bar B(\chi_p,\chi_\eta;0)\, \left[R^p_{4\eta}R^\eta_{p4}\,\chi_4 - R^p_{4\eta}R^\eta_{q4}\,\chi_4\right] \\
& -\, 1/24\,\bar A(\chi_4)\bar B(\chi_\eta,\chi_\eta;0)\, R^v_{\eta 13}\,\chi_4 \,-\, 1/6\,\bar A(\chi_4)\bar B(\chi_1,\chi_3;0)\, R^1_{4\eta}R^3_{4\eta}\,\chi_4 \,+\, 3/8\,\bar A(\chi_4;\varepsilon)\,\pi_{16}\,\chi_4 \\
& -\, 32\,\bar A(\chi_{46})\, L^r_1\,\chi_{46} \,-\, 8\,\bar A(\chi_{46})\, L^r_2\,\chi_{46} \,+\, 16\,\bar A(\chi_{46})\, L^r_4\,\chi_{46} \,+\, \bar A(\chi_{46})\bar B(\chi_p,\chi_p;0)\, \left[1/9\,\chi_{46} + 1/12\, R^\eta_{pp}\,\chi_p \right.\\
& \left. +\, 1/36\, R^\eta_{pp}\,\chi_4 + 1/9\, R^\eta_{p4}\,\chi_6\right] \,+\, \bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0)\, \left[-1/18\, R^\eta_{pp}\,\chi_4 - 1/9\, R^\eta_{p4}\,\chi_6 + 1/9\, R^\eta_{q4}\,\chi_6 + 1/18\, R^\eta_{13}\,\chi_4\right] \\
& -\, 1/6\,\bar A(\chi_{46})\bar B(\chi_p,\chi_\eta;0,k)\, \left[R^\eta_{pp} - R^\eta_{13}\right] \,+\, 1/9\,\bar A(\chi_{46})\bar B(\chi_\eta,\chi_\eta;0)\, R^v_{\eta 13}\,\chi_{46} \,-\, \bar A(\chi_{46})\bar B(\chi_1,\chi_3;0)\, \left[2/9\,\chi_{46} \right.\\
& \left. +\, 1/9\, R^\eta_{p4}\,\chi_6 + 1/18\, R^\eta_{13}\,\chi_4\right] \,-\, 1/6\,\bar A(\chi_{46})\bar B(\chi_1,\chi_3;0,k)\, R^\eta_{13} \,+\, 1/2\,\bar A(\chi_{46};\varepsilon)\,\pi_{16}\,\chi_{46} \\
& +\, \bar B(\chi_p,\chi_p;0)\,\pi_{16}\, \left[1/16\,\bar\chi_1\, R^d_p + 1/96\, R^d_p\,\chi_p + 1/32\, R^d_p\,\chi_q\right] \,+\, 2/3\,\bar B(\chi_p,\chi_p;0)\, L^r_0\, R^d_p\,\chi_p \\
& +\, 5/3\,\bar B(\chi_p,\chi_p;0)\, L^r_3\, R^d_p\,\chi_p \,+\, \bar B(\chi_p,\chi_p;0)\, L^r_4\, \left[-2\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 0}\chi_p - 4\,\bar\chi_1 R^p_{q\eta}\,\chi_p + 4\,\bar\chi_1 R^c_p\,\chi_p + 3\,\bar\chi_1 R^d_p\right] \\
& +\, \bar B(\chi_p,\chi_p;0)\, L^r_5\, \left[-2/3\bar\chi^{pp}_{\eta\eta 1}\chi_p - 4/3\, R^p_{q\eta}\,\chi_p^2 + 4/3\, R^c_p\,\chi_p^2 + 1/2\, R^d_p\,\chi_p - 1/6\, R^d_p\,\chi_q\right] \\
& +\, \bar B(\chi_p,\chi_p;0)\, L^r_6\, \left[4\,\bar\chi_1\bar\chi^{pp}_{\eta\eta 1} + 8\,\bar\chi_1 R^p_{q\eta}\,\chi_p - 8\,\bar\chi_1 R^c_p\,\chi_p\right] \,+\, 4\,\bar B(\chi_p,\chi_p;0)\, L^r_7\,(R^d_p)^2 \\
& +\, \bar B(\chi_p,\chi_p;0)\, L^r_8\, \left[4/3\,\bar\chi^{pp}_{\eta\eta 2} + 8/3\, R^p_{q\eta}\,\chi_p^2 - 8/3\, R^c_p\,\chi_p^2\right] \,+\, \bar B(\chi_p,\chi_p;0)^2\, \left[-1/18\, R^p_{q\eta}R^d_p\,\chi_p + 1/18\, R^c_p R^d_p\,\chi_p \right.\\
& \left. +\, 1/288\,(R^d_p)^2\right] \,+\, 1/18\,\bar B(\chi_p,\chi_p;0)\bar B(\chi_p,\chi_\eta;0)\, \left[R^\eta_{pp}R^d_p\,\chi_p - R^\eta_{13}R^d_p\,\chi_p\right]
\end{aligned}
$$

plus several more pages

Why so long expressions

- Many different quark and meson masses (χ_{ij})
- Charged propagators: $-i\,G^c_{ij}(k) = \frac{\epsilon_j}{k^2-\chi_{ij}+i\varepsilon} \quad (i \neq j)$
- Neutral propagators: $G^n_{ij}(k) = G^c_{ij}(k)\,\delta_{ij} - \frac{1}{n_{\rm sea}}\,G^q_{ij}(k)$

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$$-i\,G^q_{ii}(k) = \frac{R^d_i}{(k^2-\chi_i+i\varepsilon)^2} + \frac{R^c_i}{k^2-\chi_i+i\varepsilon} + \frac{R^\pi_{\eta ii}}{k^2-\chi_\pi+i\varepsilon} + \frac{R^\eta_{\pi ii}}{k^2-\chi_\eta+i\varepsilon}$$

$$R^i_{jkl} = R^z_{i456jkl}, \qquad R^d_i = R^z_{i456\pi\eta},$$

$$R^c_i = R^i_{4\pi\eta} + R^i_{5\pi\eta} + R^i_{6\pi\eta} - R^i_{\pi\eta\eta} - R^i_{\pi\pi\eta}$$

$$R^z_{ab} = \chi_a - \chi_b, \qquad R^z_{abc} = \frac{\chi_a-\chi_b}{\chi_a-\chi_c}, \qquad R^z_{abcd} = \frac{(\chi_a-\chi_b)(\chi_a-\chi_c)}{\chi_a-\chi_d}$$

$$R^z_{abcdefg} = \frac{(\chi_a-\chi_b)(\chi_a-\chi_c)(\chi_a-\chi_d)}{(\chi_a-\chi_e)(\chi_a-\chi_f)(\chi_a-\chi_g)}$$

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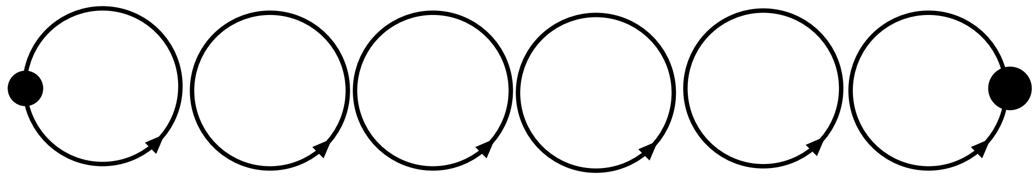
- Relations $\Longrightarrow$ order of magnitude smaller

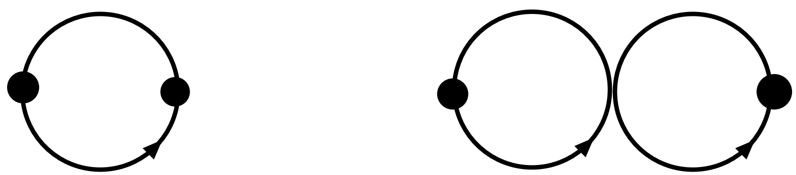
Double poles ?

Think quark lines and add gluons everywhere

Full



Quenched

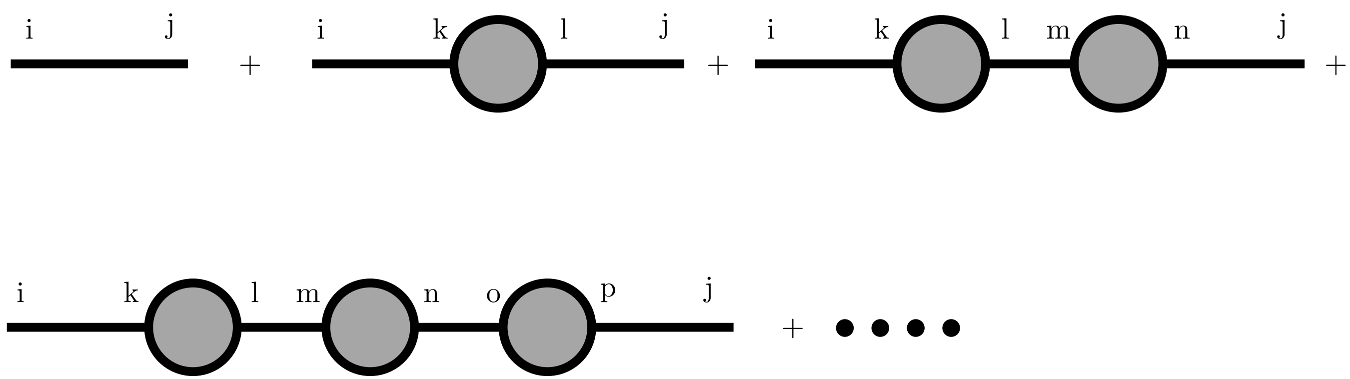


So no resummation at the quark level:

naively a double pole

Same follows from inverting the lowest order kinetic terms

Usual resummation from 1PI



$$G^n_{ij} = G^0_{ij} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lj} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lm}(-i)\Sigma_{mn}G^0_{nj} + \cdots$$
$$= G^0(1 + i\Sigma G^0)^{-1}$$

can be done if Σ and G^0 diagonal: usual resummation

$$\frac{1}{p^2 - m^2} \to \frac{1}{p^2 - m^2 + \Sigma(p^2)}$$

so works in the charged or off-diagonal or $\overline{q}q'$ sector

PQChPT at Two Loop

Masses and decay constants: all possible mass cases worked out for two and three flavours of sea quarks for the charged/off-diagonal case to NNLO.

hep-lat/0406017,hep-lat/0501014,hep-lat/0506004,hep-lat/0602003 all published in Phys. Rev. D

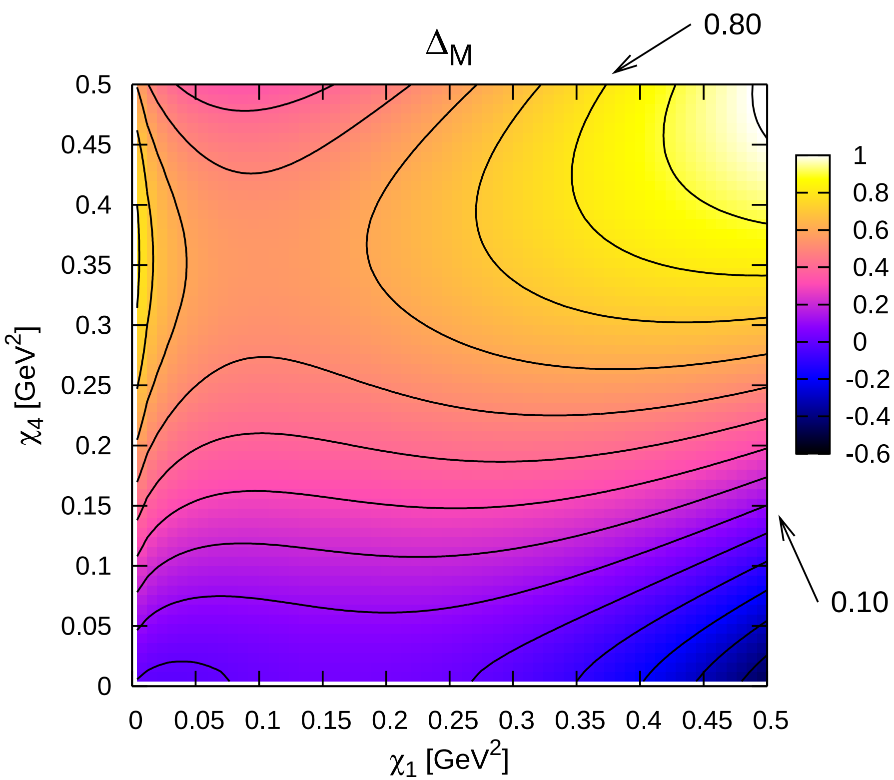
Earlier work at NLO:
Bernard-Golterman-Sharpe-Shoresh-$\cdots$

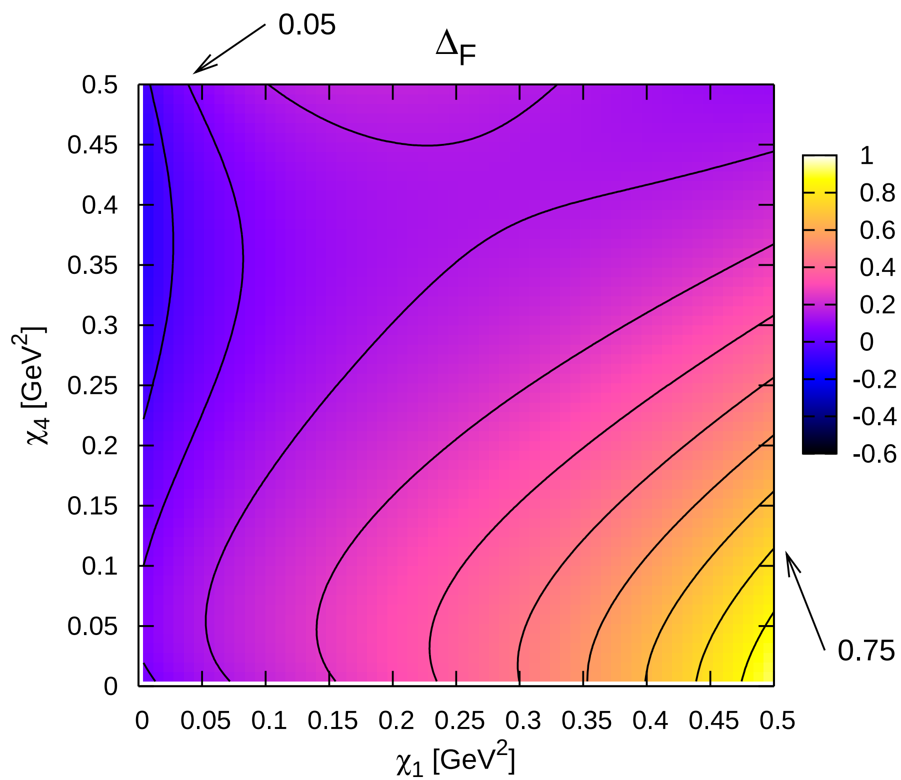
review/lectures: S. Sharpe hep-lat/0607016

Results: $\chi_1 = \chi_2, \chi_4 = \chi_5 = \chi_6$

Use lowest order mass squared: $\chi_i = 2B_0 m_i = m_M^{2(0)}$

Remember: $\chi_i \approx 0.3\ GeV^2 \approx (550\ MeV)^2 \sim$ border ChPT

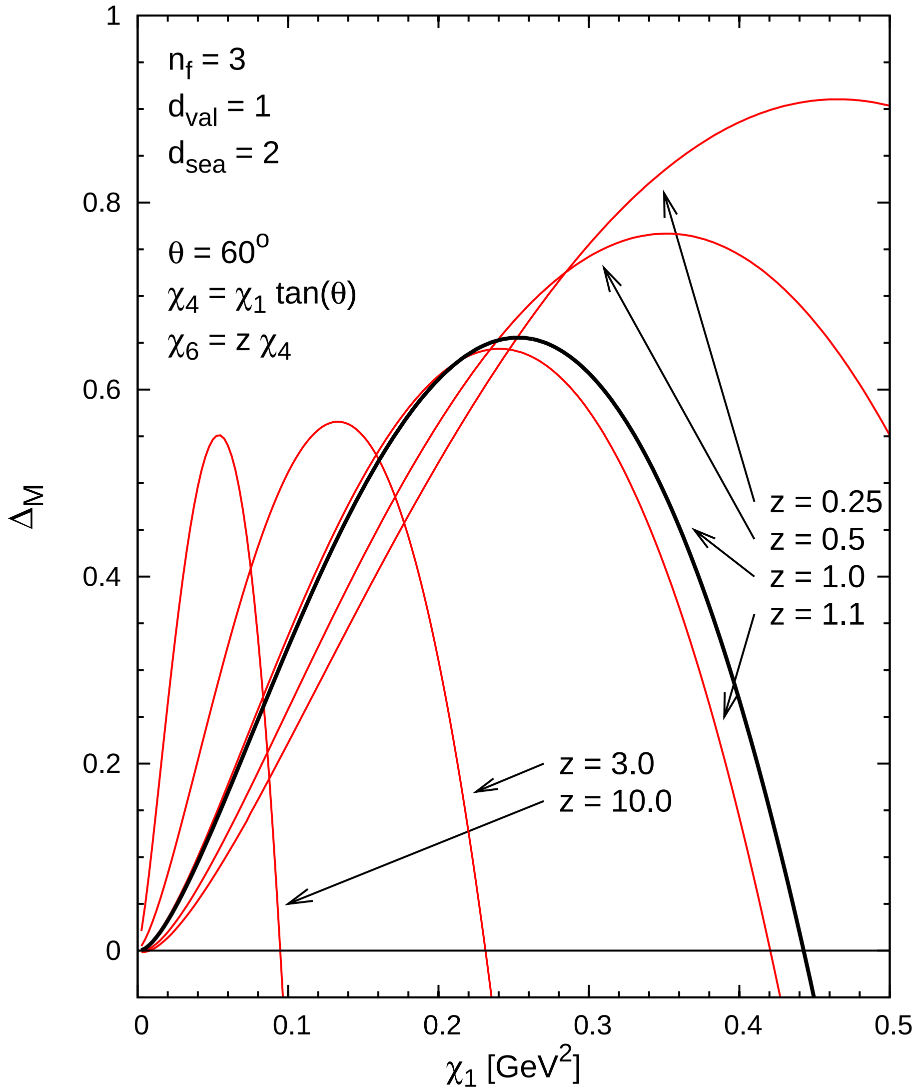


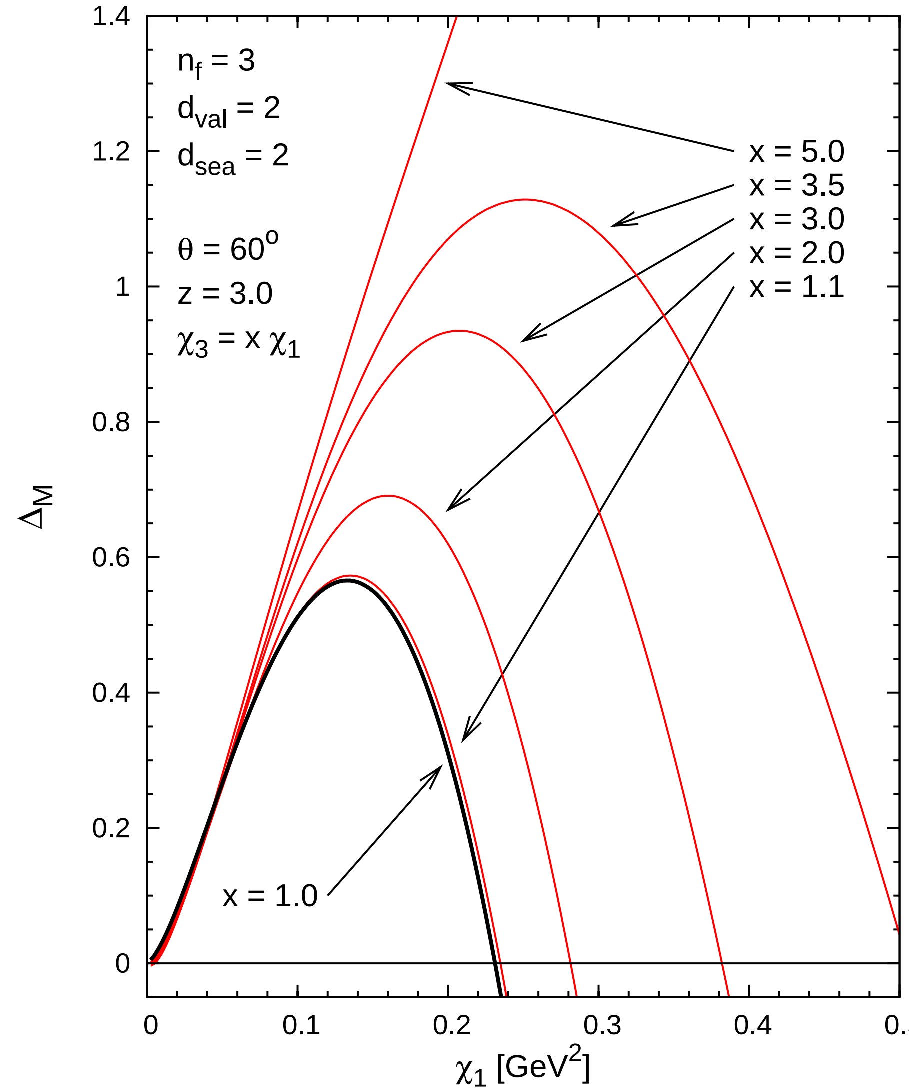


Relative corrections: mass^2 decay constant

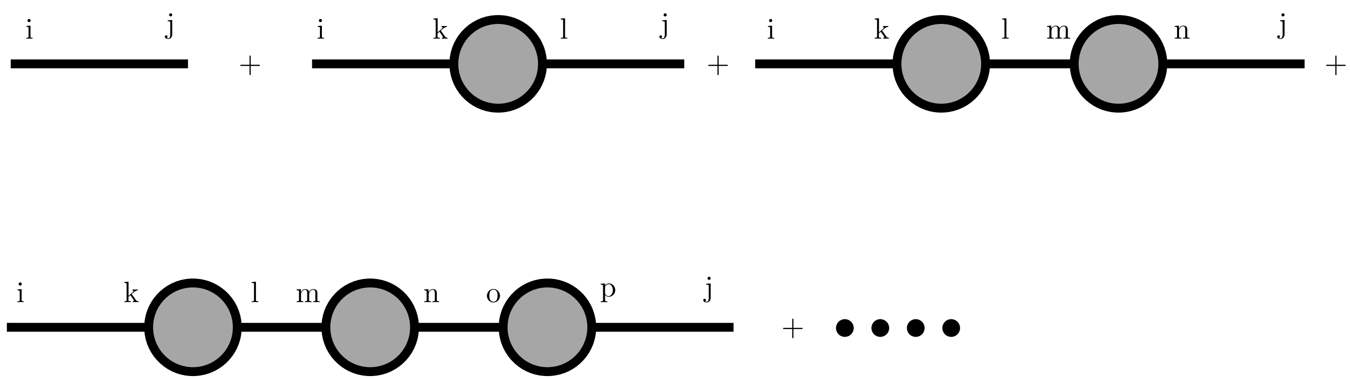
mass²: $\chi_4 = \chi_5 \neq \chi_6$

$\chi_1 = \chi_2$ | $\chi_1 \neq \chi_2$





Neutral masses



$$G^n_{ij} = G^0_{ij} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lj} + G^0_{ik}(-i)\Sigma_{kl}G^0_{lm}(-i)\Sigma_{mn}G^0_{nj} + \cdots$$

ij means: from $\bar{q}_i q_i$ to $\bar{q}_j q_j$ meson

Full resummation done by JB,Danielsson hep-lat/0606017
i.e. full propagator from one-particle-irreducible diagrams

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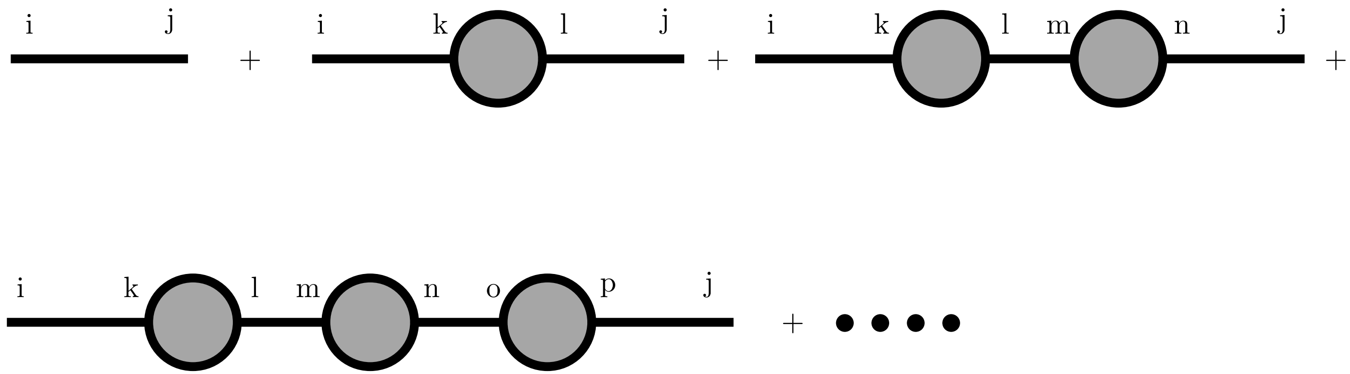

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i.e. full propagator from one-particle-irreducible diagrams

Actually useful: residue of double pole allows to get at all LECs needed for the neutral masses

Neutral masses

$$G^0 = i \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & -\alpha \end{pmatrix} - i\delta \begin{pmatrix} \tilde{\alpha} \\ \tilde{\beta} \\ \tilde{\alpha} \end{pmatrix} \begin{pmatrix} \tilde{\alpha}^T & \tilde{\beta}^T & \tilde{\alpha}^T \end{pmatrix} .$$

$$\begin{aligned}
\alpha &= \mathrm{diag}\left(\alpha_1, \ldots, \alpha_{n_\mathrm{val}}\right) , \beta = \mathrm{diag}\left(\beta_1, \ldots, \beta_{n_\mathrm{sea}}\right) , \\
\tilde{\alpha}^T &= \left(\alpha_1, \ldots, \alpha_{n_\mathrm{val}}\right) , \tilde{\beta}^T = \left(\beta_1, \ldots, \beta_{n_\mathrm{sea}}\right) , \\
\alpha_i &= 1/\left(p^2 - \chi_i\right) , \beta_i = 1/\left(p^2 - \chi_{n_\mathrm{val}+i}\right) , \\
\delta &= \frac{1}{\sum_{j=1,n_\mathrm{sea}} \beta_j} , .
\end{aligned}$$

Neutral masses

$$\Sigma = \begin{pmatrix} R+S & T^T & -R \\ T & W & -T \\ -R & -T^T & R-S \end{pmatrix}$$

R, W Symmetric; S diagonal

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R, W Symmetric; S diagonal

$$G^0(-i)\Sigma = A + B + C + D\,,$$

$$A = \begin{pmatrix} \alpha S & 0 & 0 \\ 0 & \beta W & 0 \\ 0 & 0 & \alpha S \end{pmatrix} \quad B = \begin{pmatrix} \alpha R & 0 & -\alpha R \\ 0 & 0 & 0 \\ \alpha R & 0 & -\alpha R \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & \alpha T^T & 0 \\ \beta T & 0 & -\beta T \\ 0 & \alpha T^T & 0 \end{pmatrix} \quad D = \begin{pmatrix} -\delta\tilde{\alpha} \\ -\delta\tilde{\beta} \\ -\delta\tilde{\alpha} \end{pmatrix} \begin{pmatrix} \tilde{\gamma} & \tilde{\epsilon} & -\tilde{\gamma} \end{pmatrix}.$$

When not diagonal ?

Define $\bar{A} \equiv \sum_{n=0,\infty} A^n$

$$\bar{A} = \begin{pmatrix} \bar{\alpha} & 0 & 0 \\ 0 & \bar{\beta} & 0 \\ 0 & 0 & \bar{\alpha} \end{pmatrix} = \begin{pmatrix} \frac{1}{1-\alpha S} & 0 & 0 \\ 0 & \frac{1}{1-\beta W} & 0 \\ 0 & 0 & \frac{1}{1-\alpha S} \end{pmatrix}$$

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$$\boxed{G = \sum_{n=0,\infty} \left(G^0(-i)\Sigma\right)^n G^0 \,.}$$

$$\sum_{n=0,\infty} \left(-iG^0\Sigma\right)^n = \sum_{n=1,\infty} \left((\bar{A} + \bar{A}C\bar{A})D\right)^n (\bar{A} + \bar{A}C\bar{A})$$
$$+\bar{A} + \bar{A}B\bar{A} + \bar{A}C\bar{A} + \bar{A}C\bar{A}C\bar{A}$$

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$$\left((\bar{A} + \bar{A}C\bar{A})D\right)^2 = \left((\bar{A} + \bar{A}C\bar{A})D\right)\left(-\delta\tilde{\beta}^T W\bar{\beta}\tilde{\beta}\right) \,.$$

Neutral: Final

$$G = i\begin{pmatrix} r+s & t^T & r \\ t & w & t \\ r & t^T & r-s \end{pmatrix} - i\bar{\delta}\begin{pmatrix} \tilde{a} \\ \tilde{b} \\ \tilde{a} \end{pmatrix}\begin{pmatrix} \tilde{a}^T & \tilde{b}^T & \tilde{a}^T \end{pmatrix}.$$

$$s = \bar{\alpha}\alpha\,, \quad w = \bar{\beta}\beta\,, \quad r = \bar{\alpha}\alpha R\bar{\alpha}\alpha + \bar{\alpha}\alpha T^T\bar{\beta}\beta T\bar{\alpha}\alpha\,,$$

$$t = \bar{\beta}\beta T\bar{\alpha}\alpha\,, \quad \tilde{a} = \bar{\alpha}\tilde{\alpha} + \bar{\alpha}T^T\bar{\beta}\tilde{\beta}\,, \quad \tilde{b} = \bar{\beta}\tilde{\beta}\,, \quad \bar{\delta} = \frac{\delta}{1+\delta\tilde{\beta}^T W\bar{\beta}\tilde{\beta}}$$

All poles and double poles get shifted consistently

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All poles and double poles get shifted consistently

BUT valence poles are either charged or sea-quark only neutrals $\Longrightarrow$ neutrals more difficult (L_7^r and η)

The eta mass from PQChPT

Expand around the double pole term

$$G^n_{ij} = \frac{-i\mathcal{Z}\mathcal{D}}{(p^2 - M^2_{ch})^2} + \cdots \, .$$

Measure on the lattice via (Sharpe-Shoresh)

$$R_0(t) \equiv \frac{\langle \pi_{ii}(t, \vec{p} = 0)\pi_{jj}(x = 0)\rangle}{\langle \pi_{ij}(t, \vec{p} = 0)\pi_{ji}(x = 0)\rangle} \quad R_0(t \to \infty) = \frac{\mathcal{D}t}{2M_{ij}}\, ,$$

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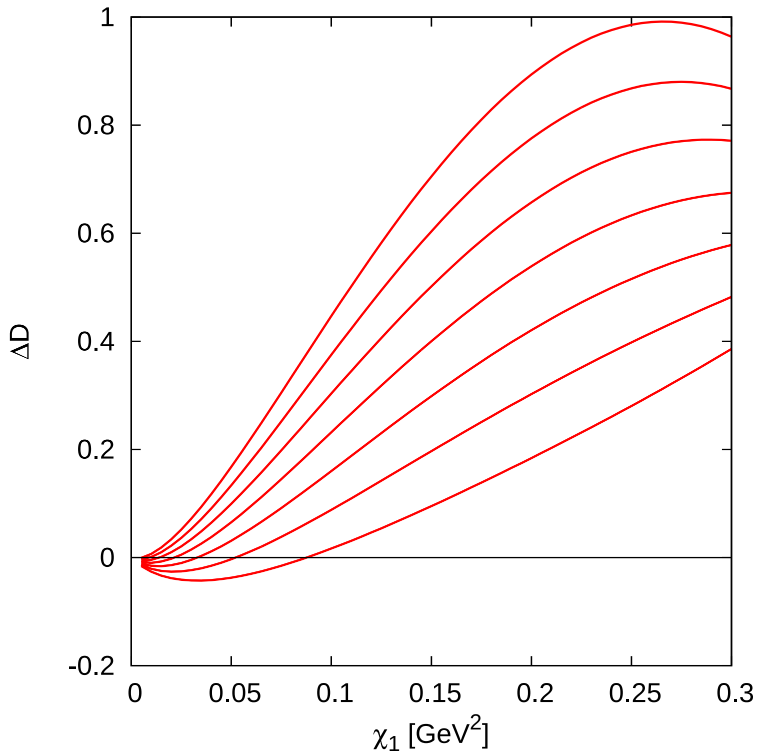
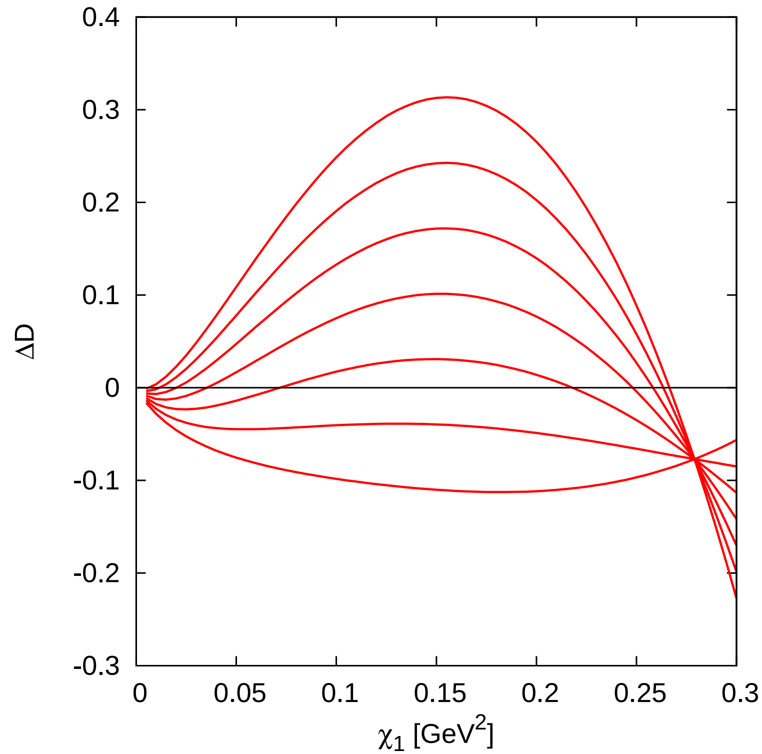
From $\mathcal{D}$ can get L^r_7 (Sharpe-Shoresh) and has been extended JB,Danielsson to NNLO as well that all LECs relevant for m^2_η can be had from $\mathcal{D}$.

Numerics

$\chi_4 = \tan 60^o \chi_1$, $\Delta\mathcal{D} = \mathcal{D}/\mathcal{D}^{(2)} - 1$





vary L_7^r, other LECs=0, left $\mu = 0.77$ GeV, right $\mu = 0.6$ GeV

$\mathcal{D}^{(2)} = (1/3)\left\{\chi_1 - \chi_4\right\}$

$\mathcal{D}$ vanishes for $\chi_1 = \chi_4$ numerically and analytically

Conclusions

- PQChPT at two loops: basic calculations done
- `http://www.thep.lu.se/~bijnens/chpt.html` for formulas
- Numerical programs: ask me (or a collaborator)
- $\mathcal{D}$ is a useful quantity
- Wait for lattice fits
- For formfactors: extend the vertex integrals
- For formfactors: even bigger expressions
- Twisted mass ChPT: no obvious obstacle towards two-loop
- Please use (and cite ⌣̈) our work