LESSONS FOR HLBL FROM MODEL CALCULATIONS

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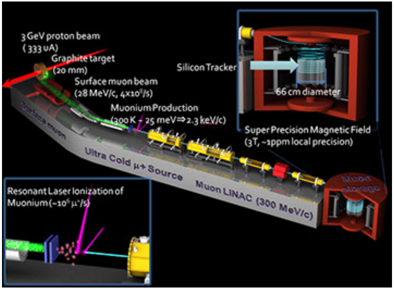
First Workshop of the Muon g-2 Theory Initiative 3-6 June 2017

Why do we do this?

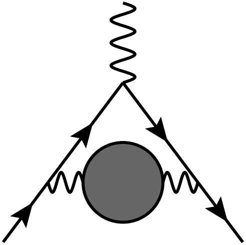
The muon $a_\mu = \dfrac{g-2}{2}$ will be measured more precisely



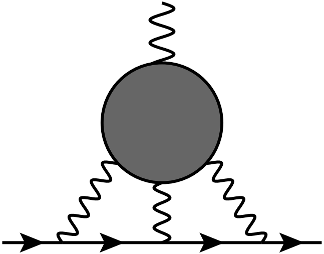
J-PARC

Fermilab

Hadronic contributions



HVP



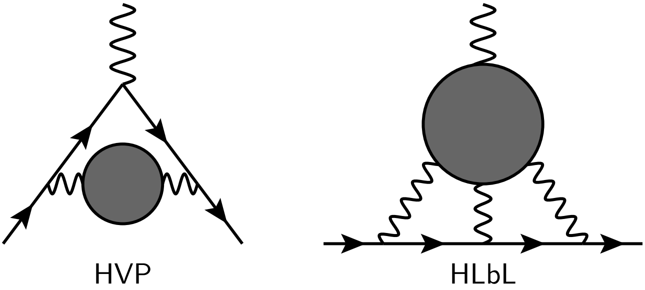
HLbL

- The blobs are hadronic contributions
- There are higher order contributions of both types (with photons outside the blobs)
- Extra photons inside the blobs more tricky (not needed at the moment for HLbL)

To ChPT or not to ChPT

- ChPT = Effective field theory describing the lowest order pseudo-scalar representation
- or the (pseudo) Goldstone bosons from spontaneous breaking of chiral symmetry.
- Describes pions, kaons and etas at low-energies
- It's an effective field theory: new parameters or LECs at each new order
- Recent review of LECs:
  JB, Ecker,Ann.Rev.Nucl.Part.Sci. 64 (2014) 149 [arXiv:1405.6488]
- a_μ is a very low-energy quantity, why not just calculate it in ChPT?

To ChPT or not to ChPT



- Fill the blobs with pions and kaons
- Lowest order for both HVP and HLbL:
  pure pion loop (or scalar QED): well defined answer
- NLO: the blob is nicely finite
  but not after the muon/photon integrations
- Needs a counterterm (NLO LEC) that is the muon $g-2$

To ChPT or not to ChPT

- So need more than ChPT
- Experiment
- Dispersion relations
- lattice QCD
- Models: this talk
- ChPT can be used to put constraints, help understanding results and estimate not evaluated parts,. . .

Why models?

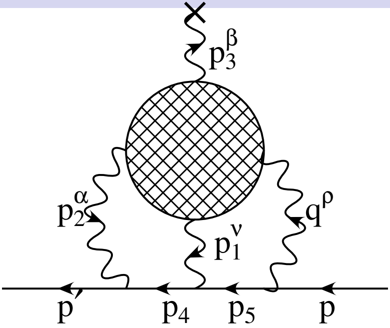
- Pro:
  - Can calculate with them (important in the past)
  - Can use them to understand features of better/more exact calculations
  - Can use them to estimate contributions from regions the other methods do not include
  - Can use them together with better methods to produce better models
- Con:
  - They are not the underlying theory or reality (experiment)
  - hard to estimate errors (guesstimates)
  - Beware: just model quark is different from QCD quark
  - Beware: model pion might not be quite the real pion
- Reminder:
  - HVP: high precision needed
  - HLbL: "just a bit" better than at present,
    but need to make sure the error estimate is not way off

Requirements

Requirements for models: Do as well you can

- Constrain as much as possible from experiment
  - measured states
  - measured form-factors
  - mesaured relevant scattering processes
- Constrain as much as possible from theory
  - include QCD short-distance constraints
  - include long distance constraints from ChPT
- Use common sense
  - Vary model parameters
  - Is your model general enough to describe what you want to describe
  - Different regions treated differently: is there some consistency
- As well as you can should improve with time

HLbL: the main object to calculate



- Muon line and photons: well known
- The blob: fill in with hadrons/QCD
- Trouble: low and high energy very mixed
- Double counting needs to be avoided: hadron exchanges versus quarks

General properties

$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)$:

- In general 138 Lorentz structures (but only 28 contribute to $g-2$)
- Using $q_\rho \Pi^{\rho\nu\alpha\beta} = p_{1\nu}\Pi^{\rho\nu\alpha\beta} = p_{2\alpha}\Pi^{\rho\nu\alpha\beta} = p_{3\beta}\Pi^{\rho\nu\alpha\beta} = 0$ 43 gauge invariant structures
- Bose symmetry relates some of them
- All depend on p_1^2, p_2^2 and q^2, but before derivative and $p_3 \to 0$ also $p_3^2, p_1 \cdot p_2, p_1 \cdot p_3$
- Actually 2 less but singular basis Fischer et al.
- Choice of basis not unique (some more convenient than others, but not always the same)
- Compare HVP: one function, one variable
- Calculation from experiment: difficult: Stoffer
- In four photon measurement: lepton contribution

General properties

$\int \frac{\mathrm{d}^4 p_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 p_2}{(2\pi)^4}$ plus loops inside the hadronic part

- 8 dimensional integral, three trivial,
- 5 remain: $p_1^2, p_2^2, p_1 \cdot p_2, p_1 \cdot p_\mu, p_2 \cdot p_\mu$
- Rotate to Euclidean space:
  - Easier separation of long and short-distance
  - Artefacts (confinement) in models smeared out.
- More recent: can do two more using Gegenbauer techniques Knecht-Nyffeler, Jegerlehner-Nyffeler,JB–Zahiri-Abyaneh–Relefors
- P_1^2, P_2^2 and Q^2 remain
- study $a_\mu^{\mathrm{X}} = \int dl_{P_1} dl_{P_2} a_\mu^{\mathrm{XLL}} = \int dl_{P_1} dl_{P_2} dl_Q a_\mu^{\mathrm{XLLQ}}$ $l_P = \ln\left(P/\mathrm{GeV}\right)$, to see where the contributions are
- Study the dependence on the cut-off for the photons

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A separation proposal: a start

E. de Rafael, "Hadronic contributions to the muon g-2 and low-energy QCD," Phys. Lett. **B322** (1994) 239-246. [hep-ph/9311316].

- Use ChPT p counting and large N_c
- p^4, order 1: pion-loop
- p^8, order N_c: quark-loop and heavier meson exchanges
- p^6, order N_c: pion exchange

Does not fully solve the problem
only short-distance part of quark-loop is really p^8
but it's a start

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Implemented by two groups in the 1990s:

- Hayakawa, Kinoshita, Sanda: meson models, pion loop using hidden local symmetry, quark-loop with VMD, calculation in Minkowski space (HKS)
- JB, Pallante, Prades: Try using as much as possible a consistent model-approach, ENJL, calculation in Euclidean space (BPP)

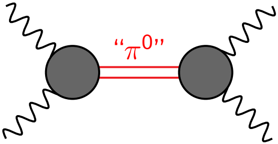
Papers: BPP and HKS

- JB, E. Pallante and J. Prades
  - “Comment on the pion pole part of the light-by-light contribution to the muon g-2,” Nucl. Phys. B **626** (2002) 410 [arXiv:hep-ph/0112255].
  - “Analysis of the Hadronic Light-by-Light Contributions to the Muon $g-2$,” Nucl. Phys. B **474** (1996) 379 [arXiv:hep-ph/9511388].
  - “Hadronic light by light contributions to the muon g-2 in the large N_c limit,” Phys. Rev. Lett. **75** (1995) 1447 [Erratum-ibid. **75** (1995) 3781] [arXiv:hep-ph/9505251].
- Hayakawa, Kinoshita, (Sanda)
  - “Pseudoscalar pole terms in the hadronic light by light scattering contribution to muon g - 2,” Phys. Rev. **D57** (1998) 465-477. [hep-ph/9708227], Erratum-ibid.D66 (2002) 019902[hep-ph/0112102].
  - “Hadronic light by light scattering contribution to muon g-2,” Phys. Rev. **D54** (1996) 3137-3153. [hep-ph/9601310].
  - “Hadronic light by light scattering effect on muon g-2,” Phys. Rev. Lett. **75** (1995) 790-793. [hep-ph/9503463].

Some main observations

- The largest constribution is π^0 (and η, η') exchange/pole
  - Beware: pole/exchange not quite the same
  - Most evaluations are in reasonable agreement
  - I will use it for an estimate of disconnected/connected on the lattice
  - Took up a large part of yesterday (many speakers)
- The pion loop can be sizable but a large difference between the two evaluations
  - For the pure pion loop part, even larger numbers have been proposed by Engel, Ramsey-Musolf
  - Discussed below
  - Another approach is the dispersive by Colangelo et al. (Stoffer)
- There are other contributions but the sum is smaller than the leading pseudo-scalar exchange
- BPP: $(8.3 \pm 3.2)\ 10^{-10}$ HKS: $(8.96 \pm 1.54)\ 10^{-10}$

π^0 exchange



- "π^0" $= 1/(p^2 - m_\pi^2)$
- The blobs need to be modelled, and in e.g. ENJL contain corrections also to the $1/(p^2 - m_\pi^2)$
- Pointlike has a logarithmic divergence
- Numbers π^0, but also η, η'

π^0 exchange

	$a_\mu \times 10^{10}$				
Cutoff (GeV)	Point-like	ENJL–VMD	Pointlike VMD	Transverse VMD	CELLO-VMD
0.5	4.92(2)	3.29(2)	3.46(2)	3.60(3)	3.53(2)
0.7	7.68(4)	4.24(4)	4.49(3)	4.73(4)	4.57(4)
1.0	11.15(7)	4.90(5)	5.18(3)	5.61(6)	5.29(5)
2.0	21.3(2)	5.63(8)	5.62(5)	6.39(9)	5.89(8)
4.0	32.7(5)	6.22(17)	5.58(5)	6.59(16)	6.02(10)

BPP: All in reasonable agreement $a_\mu^{\pi^0} = 5.9 \times 10^{-10}$

π^0 exchange

- BPP: $a_\mu^{\pi^0} = 5.9(0.9) \times 10^{-10}$
- Nonlocal quark model: $a_\mu^{\pi^0} = 6.27 \times 10^{-10}$

  A. E. Dorokhov, W. Broniowski, Phys.Rev.**D78** (2008)073011. [0805.0760]
- DSE model: $a_\mu^{\pi^0} = 5.75 \times 10^{-10}$

  Goecke, Fischer and Williams, Phys.Rev.D**83**(2011)094006[1012.3886]
- LMD+V: $a_\mu^{\pi^0} = (5.8 - 6.3) \times 10^{-10}$

  M. Knecht, A. Nyffeler, Phys. Rev. **D65**(2002)073034, [hep-ph/0111058]
- Formfactor inspired by AdS/QCD: $a_\mu^{\pi^0} = 6.54 \times 10^{-10}$

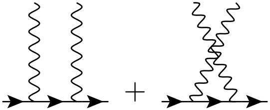
  Cappiello, Cata and D'Ambrosio, Phys.Rev.D**83**(2011)093006 [1009.1161]
- Chiral Quark Model: $a_\mu^{\pi^0} = 6.8 \times 10^{-10}$

  D. Greynat and E. de Rafael, JHEP **1207** (2012) 020 [1204.3029].
- Constraint via magnetic susceptibility: $a_\mu^{\pi^0} = 7.2 \times 10^{-10}$

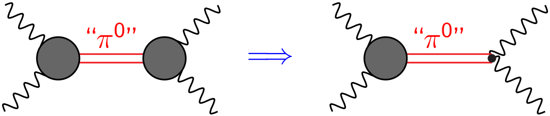
  A. Nyffeler, Phys. Rev. D **79** (2009) 073012 [0901.1172].
- All in reasonable agreement

MV short-distance: π^0 exchange

- K. Melnikov, A. Vainshtein, Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment revisited, Phys. Rev. **D70** (2004) 113006. [hep-ph/0312226]
- take $P_1^2 \approx P_2^2 \gg Q^2$: Leading term in OPE of two vector currents is proportional to axial current
- $\Pi^{\rho\nu\alpha\beta} \propto \frac{P_\rho}{P_1^2} \langle 0 | T\left(J_{A\nu} J_{V\alpha} J_{V\beta}\right) | 0 \rangle$
- J_A comes from

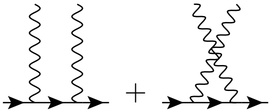


- AVV triangle anomaly: extra info
- Implemented via setting one blob = 1



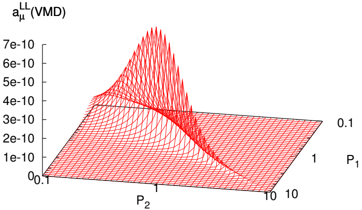
- $a_\mu^{\pi^0} = 7.7 \times 10^{-10}$

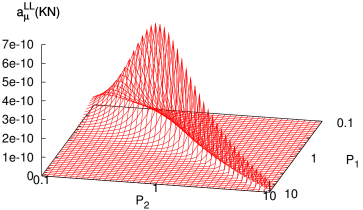
π^0 exchange

- The pointlike vertex implements shortdistance part, not only π^0-exchange
- 
  Are these part of the quark-loop? See also in
  Dorokhov,Broniowski, Phys.Rev. D78(2008)07301
- BPP quarkloop + π^0-exchange $\approx$ MV π^0-exchange

π^0 exchange

- Which momentum regimes important studied: JB and J. Prades, Mod. Phys. Lett. A **22** (2007) 767 [hep-ph/0702170]
- $a_\mu = \int dl_1 dl_2 a_\mu^{LL}$ with $l_i = \log(P_i/GeV)$



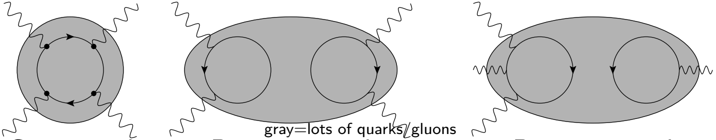


Which momentum regions do what:
volume under the plot $\propto a_\mu$

Pseudoscalar exchange

- Point-like VMD: π^0 η and η' give 5.58, 1.38, 1.04.
- Models that include $U(1)_A$ breaking give similar ratios
- Pure large N_c models use this ratio
- The MV argument should give some enhancement over the full VMD like models
- Total pseudo-scalar exchange is about
  $a_\mu^{PS} = 8 - 10 \times 10^{-10}$
- AdS/QCD estimate (includes excited pseudo-scalars)
  $a_\mu^{PS} = 10.7 \times 10^{-10}$
  D. K. Hong and D. Kim, Phys. Lett. B **680** (2009) 480 [0904.4042]

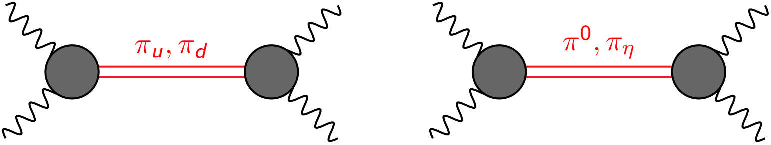
Disconnected/Connected



Connected Disconnected Disconnected

- Estimate the full result with pseudo-scalar exchange
- Connected diagrams only:
  - the gluon exchanges responsible for $U(1)_A$ breaking are not included at all
  - η' becomes light, mainly $(\bar{u}u + \bar{d}d)/\sqrt{2}$ (π_η) and has the same mass as the pion
  - Or the two-light states are π_u $(\bar{u}u)$ and π_d $(\bar{d}d)$
  - η becomes mainly $\bar{s}s$ and much heavier than the pion (and thus small contribution)
- Assume that couplings are not affected (not too bad experimentally)

Disconnected/Connected



- Two flavour case only: up and down quarks (three flavour not more difficult, just more numbers)
- Meson couplings to two-photons is via quark-loop
- Look at charge factors for Connected
  - As "quark-loop": $q_u^4 + q_d^4 = \frac{17}{81}$
  - As π_u, π_d: $q_u^2 q_u^2 + q_d^2 q_d^2 = \frac{17}{81}$
  - As π^0, π_η: $\left(\frac{q_u^2 - q_d^2}{\sqrt{2}}\right)^2 + \left(\frac{q_u^2 + q_d^2}{\sqrt{2}}\right)^2 = \frac{9}{162} + \frac{25}{162} = \frac{17}{81}$
- Include $U(1)_A$ breaking: π_η heavy
  - π^0: $\left(\frac{q_u^2 - q_d^2}{\sqrt{2}}\right)^2 = \frac{9}{162}$

Disconnected/Connected

- So in this limit:
  - Two-flavour case
  - $U(1)_A$ breaking makes π_η infinitely heavy
  - Full result dominated by pseudo-scalar exchange
  - $U(1)_A$ breaking does not affect couplings
  - $$\begin{aligned} \text{Connected:} &\quad \frac{34}{162} \\ \text{Disconnected:} &\quad -\frac{25}{162} \\ \text{Sum:} &\quad \frac{9}{162} \end{aligned}$$
- All assumptions get corrections but final conclusion stays

The disconnected contribution is expected to be large and of opposite sign with significant cancellations

- Argument used to go from large-N_c to π^0, η, η' in JB, Pallante, Prades, Nucl. Phys. B **474** (1996) 379 [arXiv:hep-ph/9511388]
- This form: JB, Relefors, JHEP **1609** (2016) 113 [arXiv:1608.01454]

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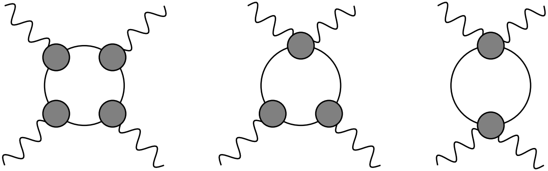
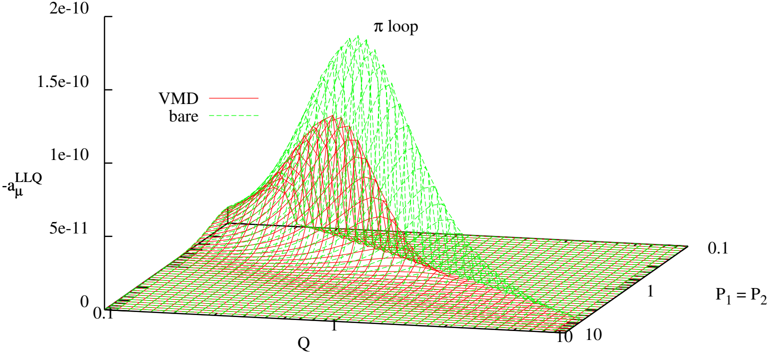
π-loop



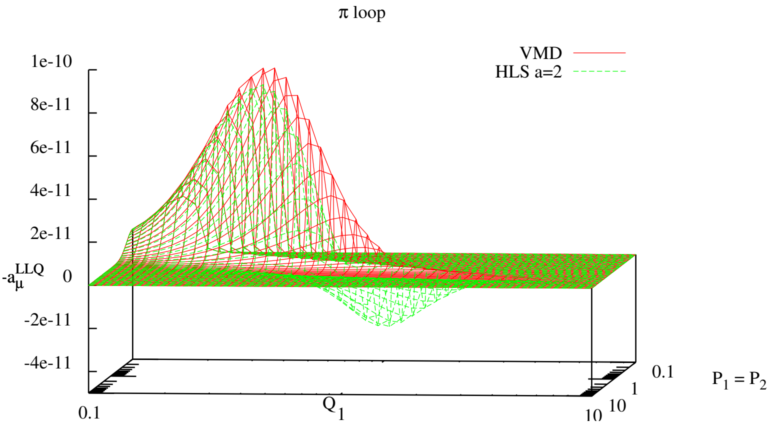
- A bare π-loop (sQED) give about $-4 \cdot 10^{-10}$
- The $\pi\pi\gamma^*$ vertex is always done using VMD
- $\pi\pi\gamma^*\gamma^*$ vertex two choices:
    - Hidden local symmetry model: only one γ has VMD
    - Full VMD
    - Both are chirally symmetric
    - The HLS model used has problems with π^+-π^0 mass difference (due to not having an a_1)
- Final numbers quite different: -0.45 and -1.9 $(\times 10^{-10})$
- For BPP stopped at 1 GeV but within 10% of higher Λ

π loop: Bare vs VMD



- plotted a_μ^{LLQ} for $P_1 = P_2$
- $a_\mu = \int dl_{P_1} dl_{P_2} dl_Q \, a_\mu^{LLQ}$
- $l_Q = \log(Q/1 \text{ GeV})$

π loop: VMD vs HLS



Usual HLS, $a = 2$

π loop

- $\pi\pi\gamma^*\gamma^*$ for $q_1^2 = q_2^2$ has a short-distance constraint from the OPE as well.
- HLS does not satisfy it
- full VMD does: so probably better estimate
- Ramsey-Musolf suggested to do pure ChPT for the π loop
  K. T. Engel and M. J. Ramsey-Musolf, Phys. Lett. B **738** (2014) 123 [arXiv:1309.2225 [hep-ph]].
- Polarizability $(L_9 + L_{10})$ up to 10%, charge radius 30% at low energies, more at higher
- Both HLS and VMD have charge radius effect but not polarizability

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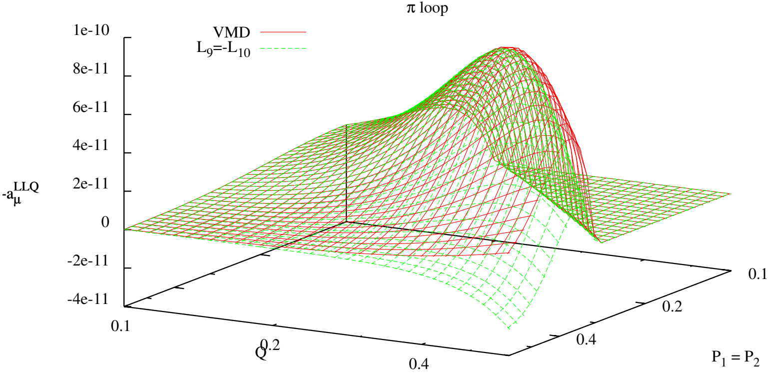
π loop: L_9, L_{10}

- ChPT for muon $g-2$ at order p^6 is not powercounting finite so no prediction for a_μ exists.
- But can be used to study the low momentum end of the integral over P_1, P_2, Q
- The four-photon amplitude is finite still at two-loop order (counterterms start at order p^8)
- Add L_9 and L_{10} vertices to the bare pion loop:

  JB, Relefors, Zahiri-Abyaneh, 1208.3548,1208.2554,1308.2575,1510.05796

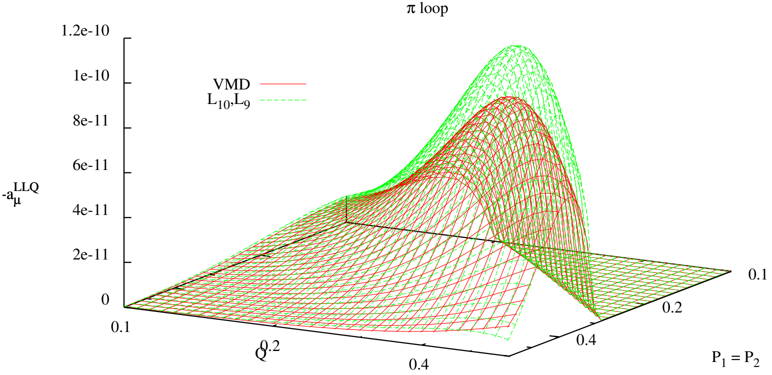
  JB, Relefors, JHEP **1609** (2016) 113 [arXiv:1608.01454 [hep-ph]].

π loop: VMD vs charge radius



low scale, charge radius effect well reproduced

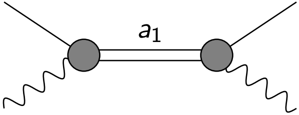
π loop: VMD vs L_9 and L_{10}



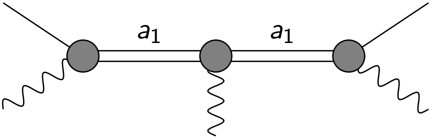
- $L_9 + L_{10} \neq 0$ gives an enhancement of 10-15%
- To do it fully need to get a model: include a_1

Include a_1

- $L_9 + L_{10}$ effect is from



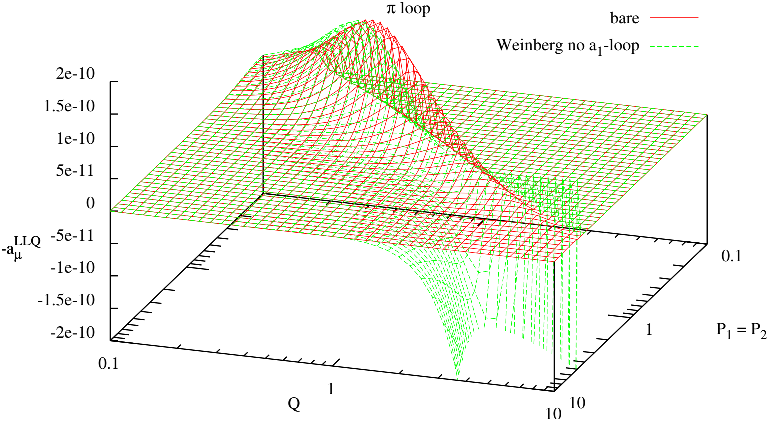
- But to get gauge invariance correctly need



Include a_1

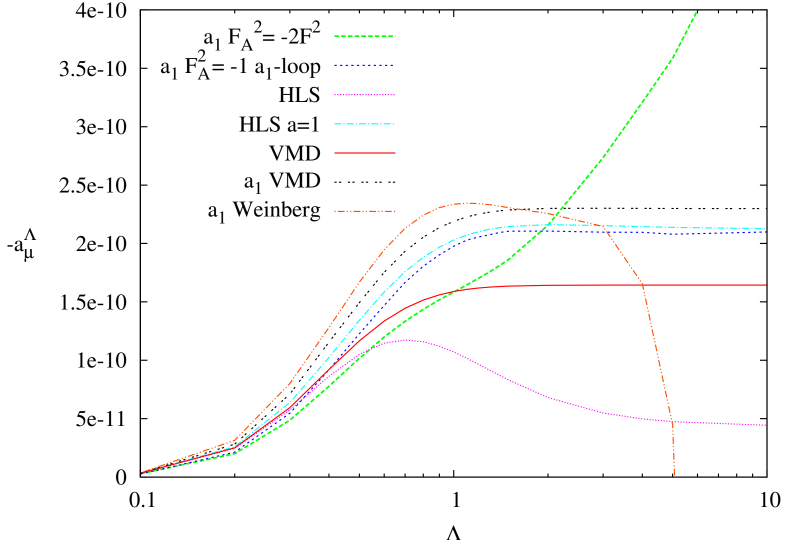
- Consistency problem: full a_1-loop?
- Treat a_1 and ρ classical and π quantum: there must be a π that closes the loop
  Argument: integrate out ρ and a_1 classically, then do pion loops with the resulting Lagrangian
- To avoid problems: representation without a_1-π mixing
- Check for curiosity what happens if we add a_1-loop

a_1-loop: cases with good L_9 and L_{10}



- Add F_V, G_V and F_A
- Fix values by Weinberg sum rules and VMD in $\gamma^*\pi\pi$
- no a_1-loop

Integration results



$P_1, P_2, Q \leq \Lambda$

Integration results with a_1

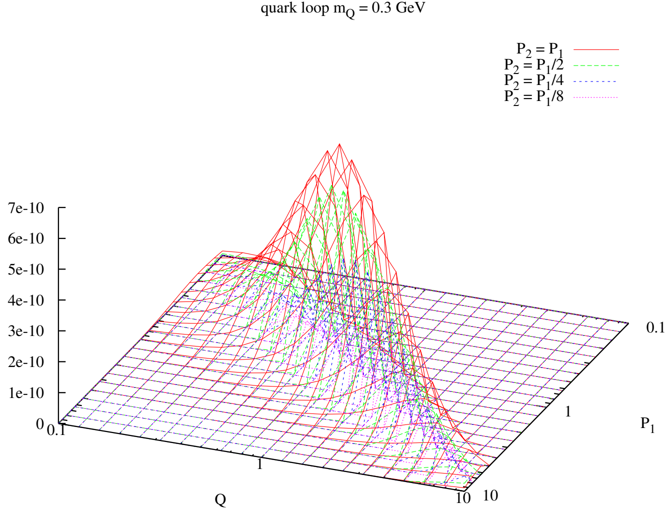
- Problem: get high energy behaviour good enough
- But all models with reasonable L_9 and L_{10} fall way inside the error quoted earlier $(-1.9 \pm 1.3)\ 10^{-10}$
- Conclusion: Use hadrons only below about 1 GeV:
  $a_\mu^{\pi-\mathrm{loop}} = (-2.0 \pm 0.5)\ 10^{-10}$
- Note that Engel and Ramsey-Musolf, arXiv:1309.2225 is a bit more pessimistic quoting numbers from $(-1.1 \text{ to } -7.1)\ 10^{-10}$
- Does not include rescattering

Pure quark loop

Cut-off Λ (GeV)	$a_\mu \times 10^7$ Electron Loop	$a_\mu \times 10^9$ Muon Loop	$a_\mu \times 10^9$ Constituent Quark Loop
0.5	2.41(8)	2.41(3)	0.395(4)
0.7	2.60(10)	3.09(7)	0.705(9)
1.0	2.59(7)	3.76(9)	1.10(2)
2.0	2.60(6)	4.54(9)	1.81(5)
4.0	2.75(9)	4.60(11)	2.27(7)
8.0	2.57(6)	4.84(13)	2.58(7)
Known Results	2.6252(4)	4.65	2.37(16)

- M_Q : 300 MeV
- now known fully analytically
- Us: 5+(3-1) integrals extra are Feynman parameters
- Slow convergence:
  - electron: all at 500 MeV
  - Muon: only half at 500 MeV, at 1 GeV still 20% missing
  - 300 MeV quark: at 2 GeV still 25% missing

Pure quark loop: momentum area



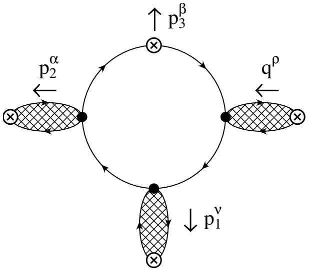
Most from $P_1 \approx P_2 \approx Q$, sizable large momentum part

ENJL quark-loop

Cut-off Λ GeV	$a_\mu \times 10^{10}$ VMD	$a_\mu \times 10^{10}$ ENJL	$a_\mu \times 10^{10}$ masscut	$a_\mu \times 10^{10}$ sum ENJL+masscut
0.5	0.48	0.78	2.46	3.2
0.7	0.72	1.14	1.13	2.3
1.0	0.87	1.44	0.59	2.0
2.0	0.98	1.78	0.13	1.9
4.0	0.98	1.98	0.03	2.0
8.0	0.98	2.00	.005	2.0

- Very stable
- ENJL cuts off slower than pure VMD
- masscut: $M_Q = \Lambda$ to have short-distance and no problem with momentum regions
- Quite stable in region 1-4 GeV

ENJL: scalar



- $$\Pi^{\rho\nu\alpha\beta} = \overline{\Pi}^{VVS}_{ab}(p_1,r) g_S \left(1+g_S\Pi^S(r)\right)\overline{\Pi}^{SVV}_{cd}(p_2,p_3)\mathcal{V}^{abcd\rho\nu\alpha\beta} + \text{permutations}$$
- $g_S\left(1+g_S\Pi_S\right) = \frac{g_A(r^2)(2M_Q)^2}{2f^2(r^2)}\frac{1}{M_S^2(r^2)-r^2}$
- $\mathcal{V}^{abcd\rho\nu\alpha\beta}$: ENJL VMD legs
- In ENJL only scalar+quark-loop properly chiral invariant

ENJL: scalar/QL

Cut-off Λ GeV	$a_\mu \times 10^{10}$ Quark-loop VMD	$a_\mu \times 10^{10}$ Quark-loop ENJL	$a_\mu \times 10^{10}$ Scalar Exchange
0.5	0.48	0.78	-0.22
0.7	0.72	1.14	-0.46
1.0	0.87	1.44	-0.60
2.0	0.98	1.78	-0.68
4.0	0.98	1.98	-0.68
8.0	0.98	2.00	-0.68

- ENJL only scalar+quark-loop properly chiral invariant
- Note: ENJL+scalar (BPP) $\approx$ Quark-loop VMD (HKS)
- $M_S \approx 620$ MeV certainly an overestimate for real scalars
- If scalar is σ: related to pion loop part?
- quark-loop: $a_\mu^{ql} \approx 1 \times 10^{-10}$

Quark loop DSE/ Nonlocal NJL

- DSE model: $a_\mu^{ql} = 10.7(0.2) \times 10^{-10}$ T. Goecke, C. S. Fischer and R. Williams, arXiv:1210.1759
- Not a full calculation (yet) but includes an estimate of some of the missing parts
- a lot larger than bare quark loop with constituent mass
- DSE model (Maris-Roberts) does reproduces a lot of low-energy phenomenology. My guess was: numbers similar to ENJL.
- Can one find something in between full DSE and ENJL that is easier to handle?
- Nonlocal chiral quark model or nonlocal NJL (but no vector vertex, i.e. no rho) A. E. Dorokhov, A. E. Radzhabov and A. S. Zhevlakov, arXiv:1502.04487 [hep-ph].
  $a_\mu^{ql} = 11.0(0.9) \times 10^{-10}$

Other quark loop

- de Rafael-Greynat 1210.3029 $(7.6 - 8.9)\ 10^{-10}$
- Boughezal-Melnikov 1104.4510 $(11.8 - 14.8)\ 10^{-10}$
- Masjuan-Vanderhaeghen 1212.0357 $(7.6 - 12.5)\ 10^{-10}$
- Various interpretations: the full calculation or not
- All (even DSE) have in common that a low quark mass is used for a large part of the integration range, not shielded by formfactors

Axial-vector exchange

Cut-off Λ (GeV)	$a_\mu \times 10^{10}$ from Axial-Vector Exchange $\mathcal{O}(N_c)$
0.5	0.05(0.01)
0.7	0.07(0.01)
1.0	0.13(0.01)
2.0	0.24(0.02)
4.0	0.59(0.07)

There is some pseudo-scalar exchange piece here as well, off-shell not quite clear what is what.

- $a_\mu^{\rm axial} = 0.6 \times 10^{-10}$
- MV: short distance enhancement + mixing (both enhance about the same)
  $a_\mu^{\rm axial} = 2.2 \times 10^{-10}$
- Jegerlehner (talk Mainz 2014) $(0.76 \pm 0.27)\ 10^{-10}$
- Pauk-Vanderhaeghen $(0.64 \pm 0.20)\ 10^{-10}$

Others

- There are many more estimates around of (heavier) scalars, tensors,. . .
- Typically $\pm 0.3\ 10^{-10}$ or (much) smaller
- But there are many, so need an overall approach

Conclusions

- Present:
  - BPP: $(8.3 \pm 3.2)\ 10^{-10}$
  - HKS: $(8.96 \pm 1.54)\ 10^{-10}$
  - MV: $(13.6 \pm 2.5)\ 10^{-10}$
  - Glasgow: $(10.5 \pm 2.6) \cdot 10^{-10}$
  - JN: $(11.6 \pm 3.9) \cdot 10^{-10}$
- Future:
  - Better approaches are taking over
  - Lattice: we already saw many new numbers
  - Dispersive: one π, η, η' and two pions/kaons will be under control
  - Future for models: are there large other contributions?
- Present for models: so far no sign we were way off