$\pi\pi$ and πK at Two Loops

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Overview

- $\pi\pi$ Scattering in Three Flavour ChPT, J. Bijnens, P. Dhonte and P. Talavera, hep-ph/0401039, JHEP 0401(2004)050
- πK Scattering in Three Flavour ChPT, J. Bijnens, P. Dhonte and P. Talavera hep-ph/0404150, JHEP 0405(2004)036

- Introduction
- Why (Effective) Field Theory
- Chiral Perturbation Theory
- Two Loop: General
- Two Loop: Three Flavours
  - General fitting strategy and some comments
  - $\pi\pi$, πK
- Conclusions

Introduction

- $\pi\pi$ and πK scattering are basic strong processes

  Study them as precise as possible
- Earlier: $\pi\pi$ proved that two-flavour case of $\langle \overline{q}q \rangle$
- Three flavour case: works or problems with strange quark loops (in scalar sector)?
- πK excellent place to study this
- Need precise calculations also here

Why (Effective) Field Theory?

Effective:

- Use right degrees of freedom : essence of (most) physics
- Gap in the spectrum $\Longrightarrow$ separation of scales
- With lower d.o.f.: build most general Lagrangian

$$\left.\begin{array}{l}\Rightarrow \infty\sharp \text{ parameters} \\ \Rightarrow \text{Where did predictivity go ?}\end{array}\right\} \Longrightarrow \text{power counting}$$

Why (Effective) Field Theory?

Field Theory

- Only known way to combine QM and special relativity
- Taylor series does not work (convergence radius zero)
- Continuum of excitation states to be taken into account
- Off-shell effects fully under control: these effects are there as new free parameters
- model-independent and systematic: ALL effects at given order included
- Theory $\Longrightarrow$ errors can be estimated
- Many parameters (but possible modelspace is large)
- Expansion might not converge (often still useful for model classification)

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

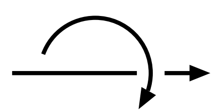
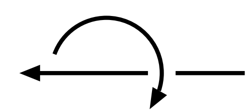
Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Can also see that via  $v < c$, $m_q \neq 0 \Longrightarrow$ / $v = c$, $m_q = 0 \not\Longrightarrow$ 

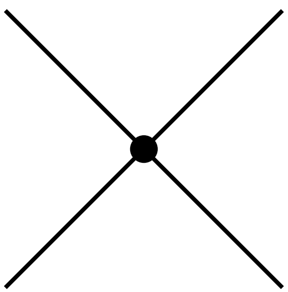
Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$

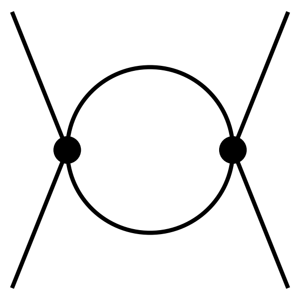
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom and interaction vanishes at zero momentum

Power counting in momenta:



p^2

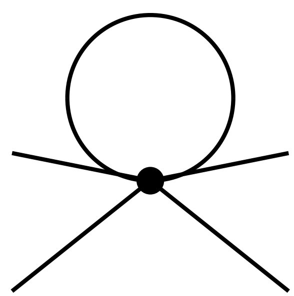


$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

$1/p^2$

$\int d^4p$

p^4



$(p^2)\,(1/p^2)\,p^4 = p^4$

Two Loop: General

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

Note
- PQ $\Longrightarrow$ Talk by Timo Lähde
- All infinities known
- Two Flavour Most Things Done

Three Flavours at Two Loop

$\Pi_{VV\pi}$, $\Pi_{VV\eta}$, Π_{VVK}	Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera	
$\Pi_{VV\rho\omega}$	Maltman	
$\Pi_{AA\pi}$, $\Pi_{AA\eta}$, F_π, F_η, m_π, m_η	Kambor, Golowich; Amorós, JB, Talavera	
Π_{SS}	Moussallam	L_4^r, L_6^r
Π_{VVK}, Π_{AAK}, F_K, m_K	Amorós, JB, Talavera	
$K_{\ell 4}$, $\langle \overline{q}q \rangle$	Amorós, JB, Talavera	L_1^r, L_2^r, L_3^r
F_M, m_M, $\langle \overline{q}q \rangle$ $(m_u \neq m_d)$	Amorós, JB, Talavera	$L_{5,7,8}^r, m_u/m_d$
$F_{V\pi}$, F_{VK^+}, F_{VK^0}	Post, Schilcher; JB, Talavera	L_9^r
$K_{\ell 3}$	Post, Schilcher; JB, Talavera	V_{us}
$F_{S\pi}$, F_{SK} (includes σ-terms)	JB, Dhonte	L_4^r, L_6^r
$K, \pi \to \ell\nu\gamma$	Geng, Ho, Wu	L_{10}^r
$\pi\pi$	JB,Dhonte,Talavera	
πK	JB,Dhonte,Talavera	

General Strategy and some comments

- Find enough inputs from experiment
- C_i^r:
  - kinematical dependence: agree well with single resonance saturation
  - quark mass+kinematical: if vector dominated, seems to be OK
  - quark mass+kinematical: if scalar dominated: which scalars? (not σ)
  - quark masses: which scalars? unrealistically large estimates
- in p^6 physical or lowest order masses: thresholds in right place requires physical

General Strategy and some comments

Inputs:

$K_{\ell 4}$: $F(0)$, $G(0)$, λ — E865 BNL

$m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$ — em with Dashen violation

F_{π^+}

F_{K^+}/F_{π^+}

General Strategy and some comments

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F_{π^+}

F_{K^+}/F_{π^+}

$m_s/\hat{m}$ 24 (26) $\hat{m} = (m_u + m_d)/2$

L^r_4, L^r_6 Vary $\Rightarrow$ other L^r_i vary correlated

General Strategy and some comments

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$m_s/\hat{m}$ 24 (26) $\hat{m} = (m_u + m_d)/2$

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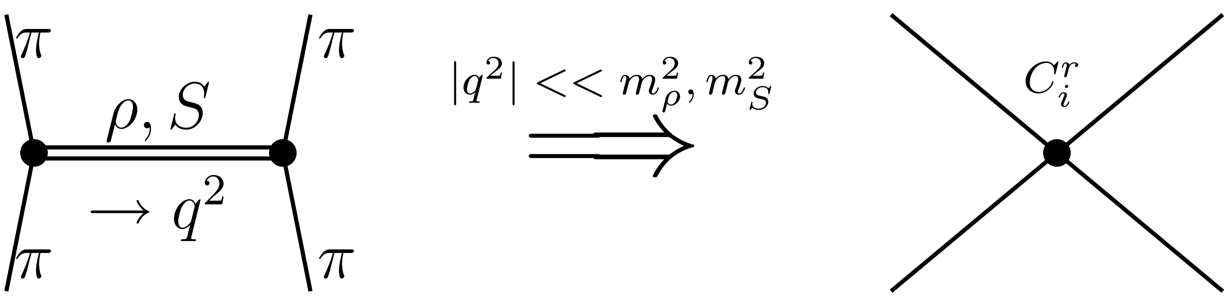
C^r_i from single resonance approximation



General Strategy: fit results

	fit 10	same p^4	fit B	fit D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.53 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54

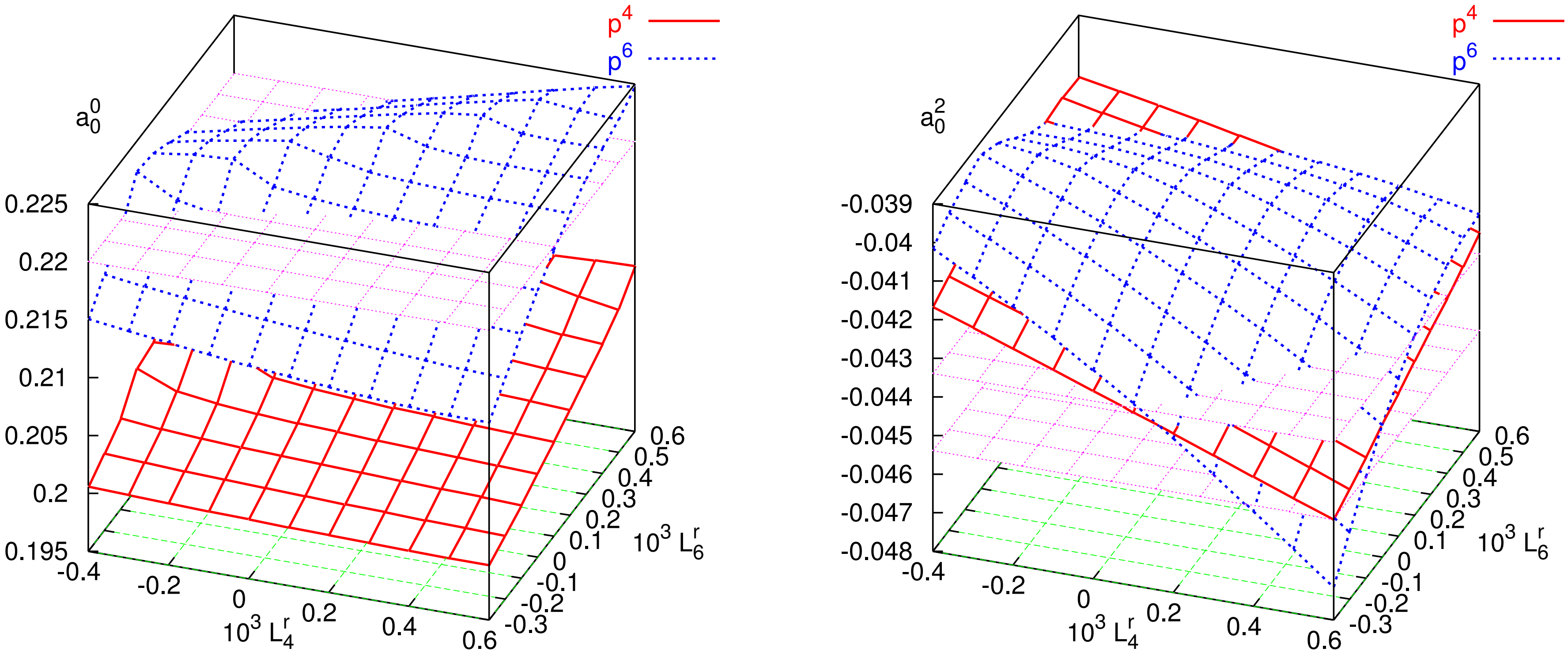
- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying C_i^r factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6\ 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and πK thresholds

General Strategy: some outputs

	fit 10	same p^4	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
m_π^2: p^4, p^6	0.006,0.258	0.009,$\equiv 0$	$-$0.138,0.009	$-$0.091,0.133
m_K^2: p^4, p^6	0.007,0.306	0.075,$\equiv 0$	$-$0.149,0.094	$-$0.096,0.201
m_η^2: p^4, p^6	$-$0.052,0.318	0.013,$\equiv 0$	$-$0.197,0.073	$-$0.151,0.197
m_u/m_d	0.45$\pm$0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
F_K/F_π: p^4, p^6	0.169,0.051	0.22,$\equiv 0$	0.153,0.067	0.159,0.061

- Pattern of mass corrections can vary a lot
- F_K/F_π always OK expansion
- $m_u = 0$ always very far from the fits

- F_0: pion decay constant in the chiral limit

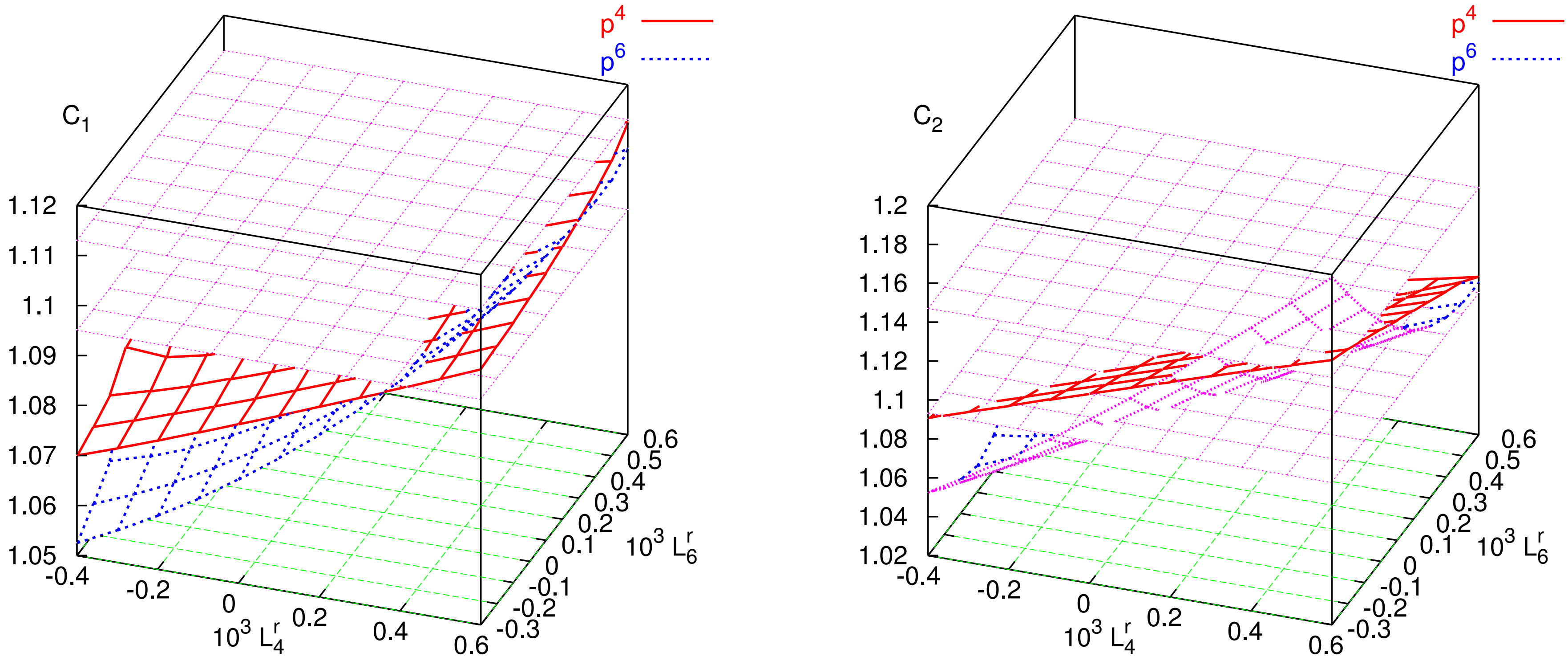


$a_0^0 = 0.220 \pm 0.005$, $a_0^2 = -0.0444 \pm 0.0010$

Colangelo, Gasser, Leutwyler

$a_0^0 = 0.159$ $a_0^2 = -0.0454$ at order p^2

$\pi\pi$ subthreshold parameters

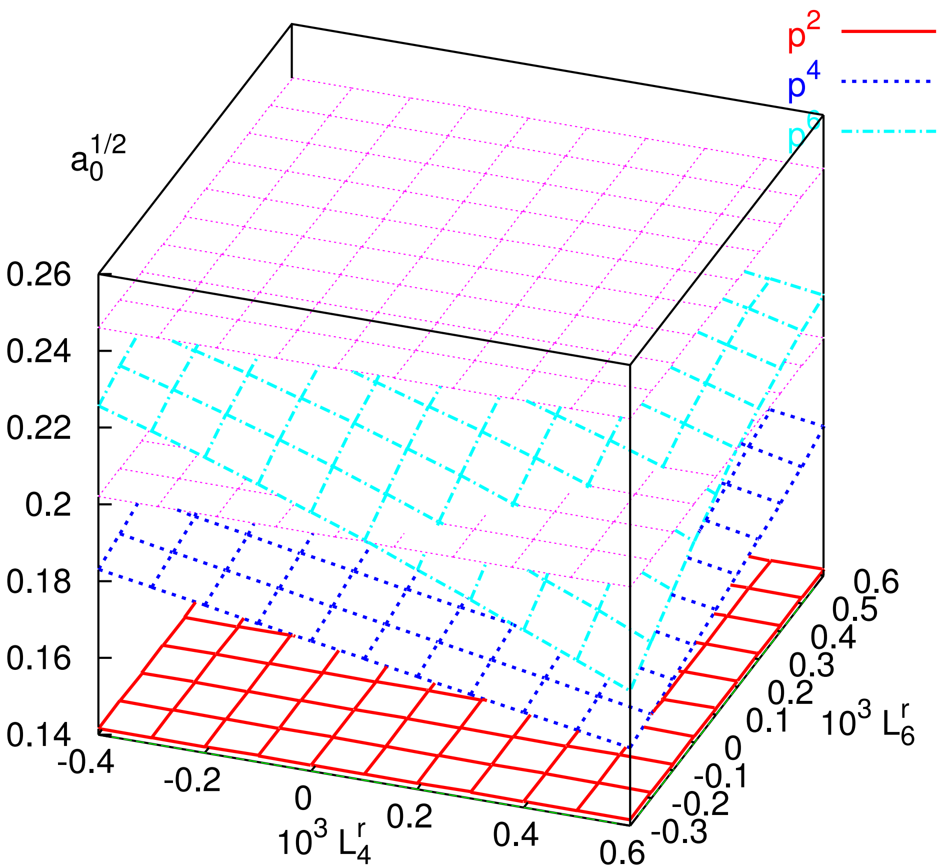


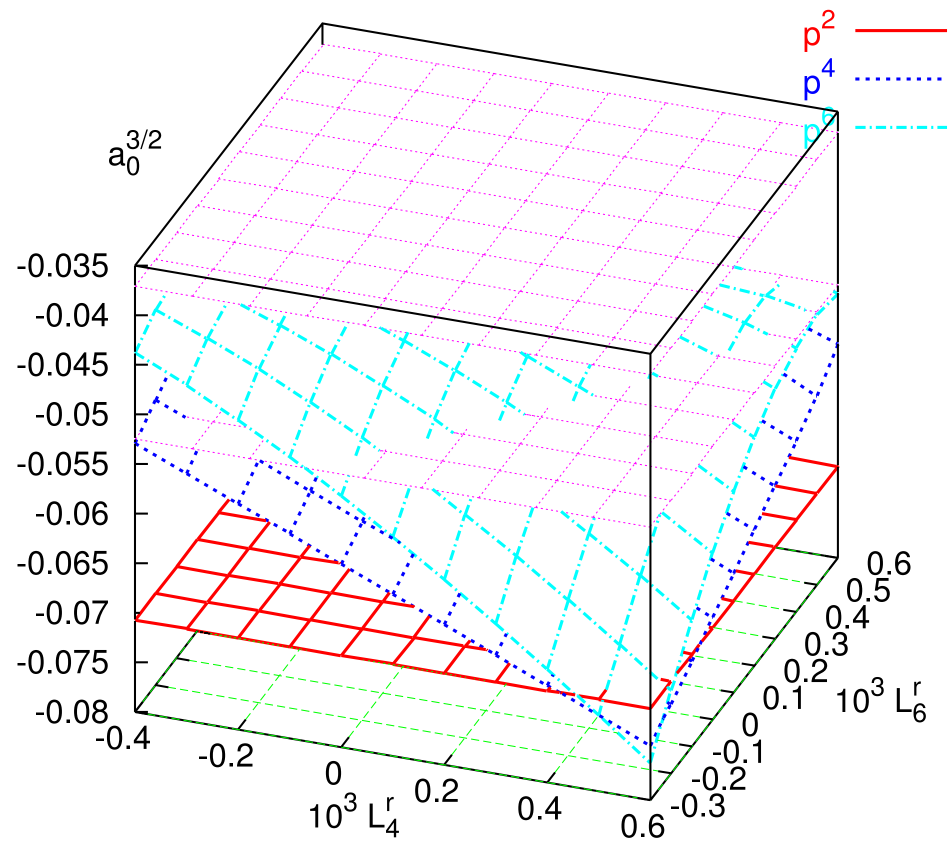
$C_1 = 1.104 \pm 0.009$, $C_2 = 1.120 \pm 0.027$

Colangelo, Gasser, Leutwyler

$C_1 = C_2 = 1$ at order p^2

πK



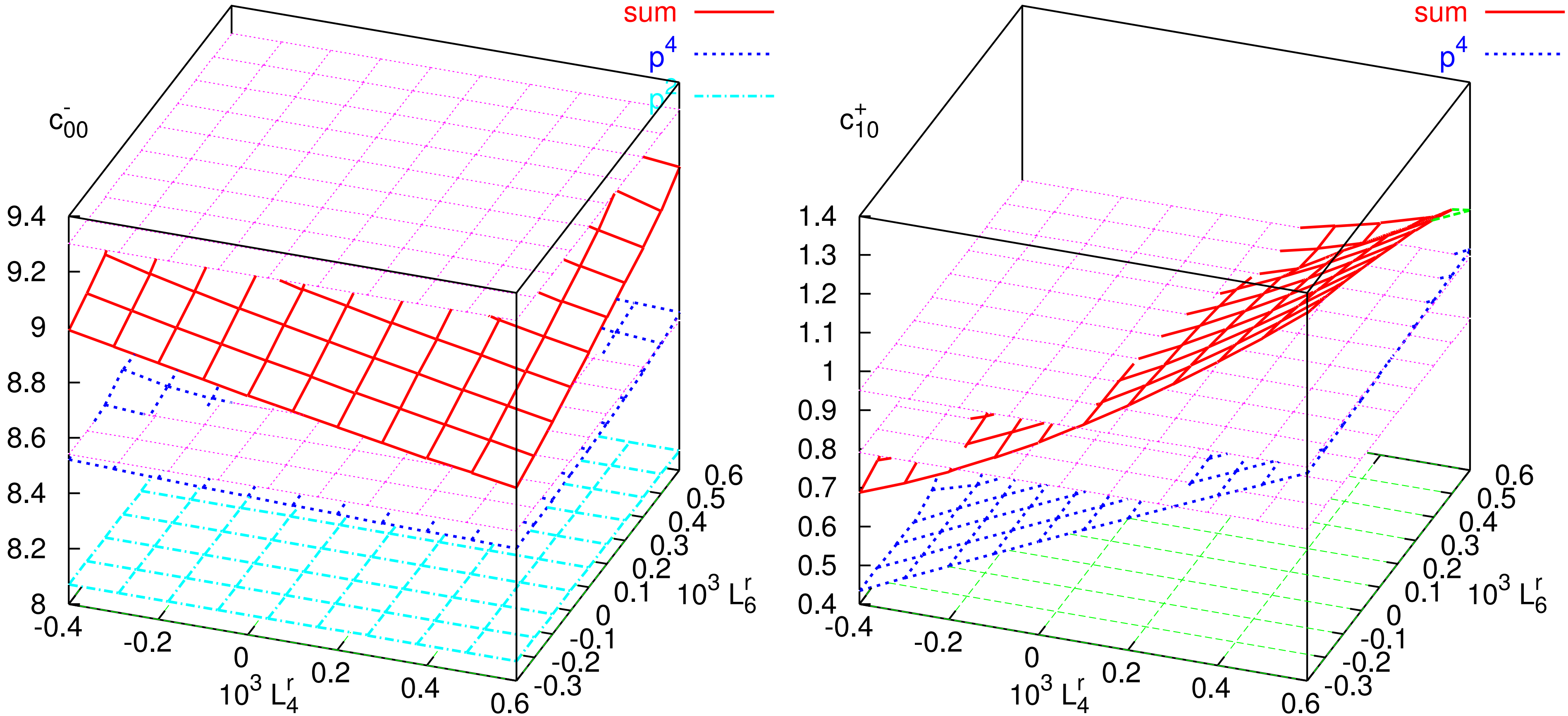


$a_0^{1/2} = 0.224 \pm 0.022$, $a^2_{3/2} = -0.0448 \pm 0.0077$

Büttiker, Descotes-Genon, Moussallam

$a_0^{1/2} = 0.142$ $a_0^2 = -0.0708$ at order p^2

πK subthreshold parameters



$c^+_{10} = 0.87 \pm 0.08$, $c^-_{00} = 8.92 \pm 0.38$

Büttiker, Descotes-Genon, Moussallam

πK subthreshold parameters

	Vector	Scalar	Sum Reso	chiral order	p^2	p^4	p^6
c^+_{00}	−0.02	0.13	0.11	2	0	0.122	0.007
c^+_{10}	0.018	−0.063	−0.045	2	0.5704	−0.113	0.460
c^-_{00}	0.21	0.17	0.38	2	8.070	0.311	0.017
c^+_{20}	−0.0053	0.0023	−0.0030	4	—	0.0256	−0.0254
c^-_{10}	−0.11	−0.04	−0.15	4	—	−0.0254	0.121
c^+_{01}	−0.27	0.28	0.01	4	—	1.667	1.492
c^+_{30}	0.00026	0.00010	0.00036	6	—	0.00121	0.00071
c^-_{20}	0.0037	0.00060	0.0043	6	—	0.00478	0.00320
c^+_{11}	0.017	−0.008	0.009	6	—	−0.126	−0.006
c^-_{01}	0.25	0.04	0.29	6	—	0.229	0.196

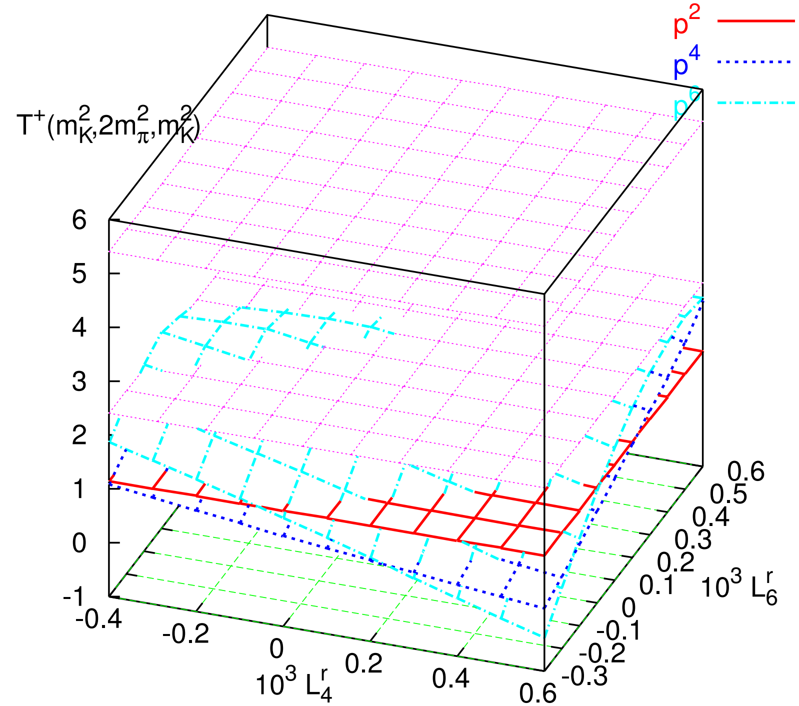
Resonance contributions, units: $m_{\pi^+}^{2i+2j}$ (c^+_{ij}) and $m_{\pi^+}^{2i+2j+1}$ (c^-_{ij})

Chiral order at which they first have tree level contributions

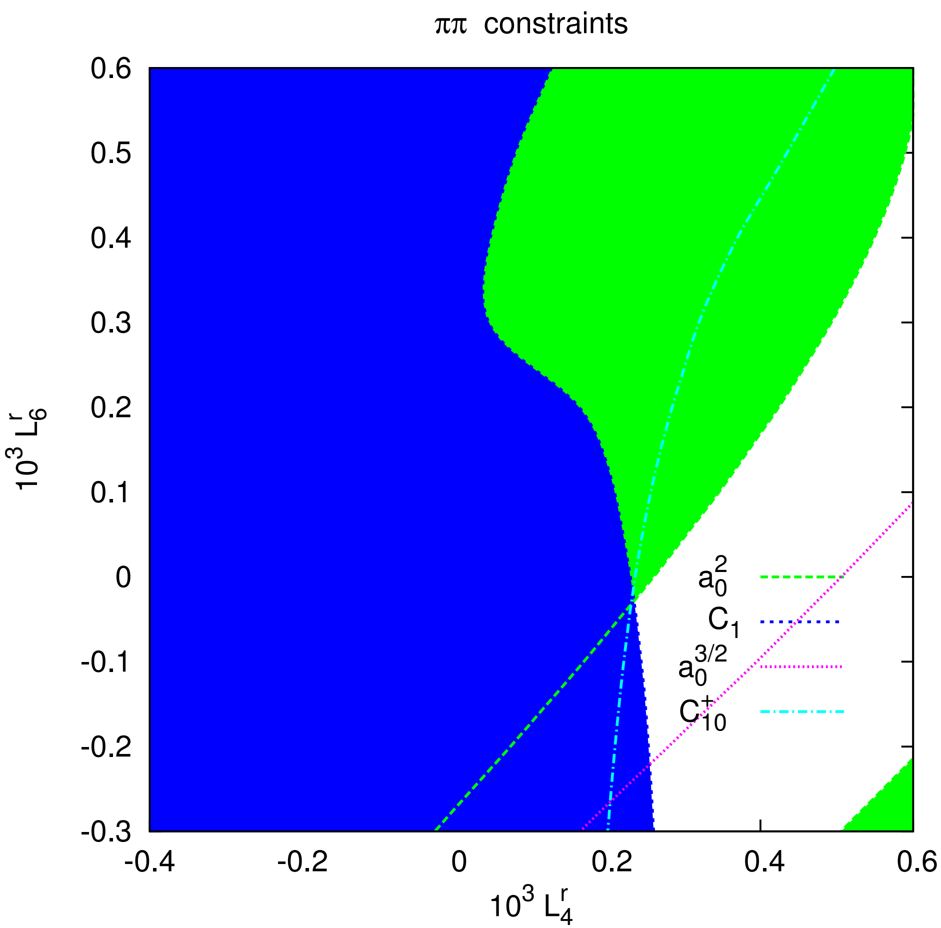
Contributions with the $L_i^r = C_i^r = 0$ at $\mu = 0.77$ GeV.

πK subthreshold parameters

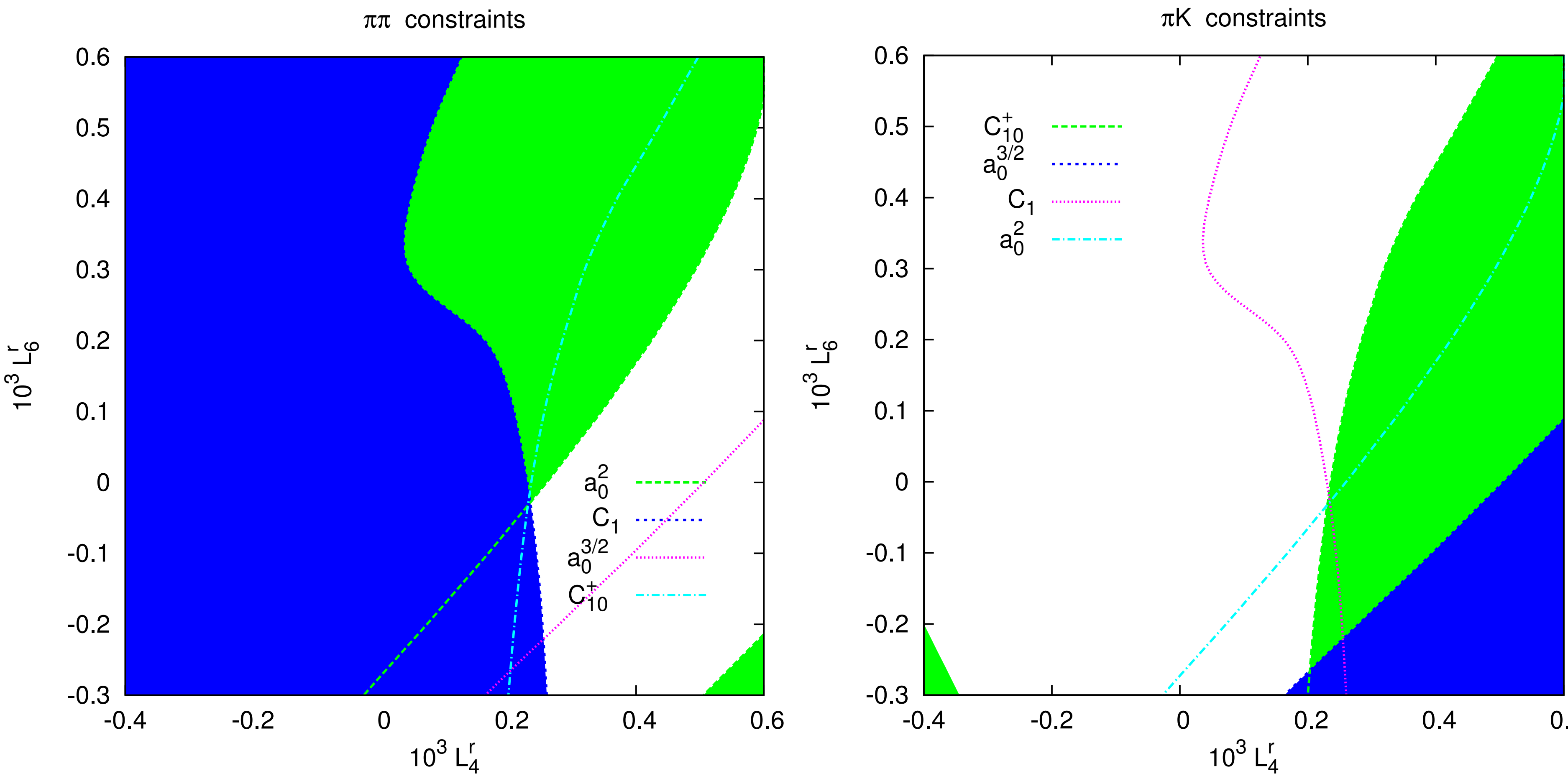
	Fit 10	BDM	Lang
c_{00}^{+}	0.278	2.01 ± 1.10	-0.52 ± 2.03
c_{10}^{+}	0.898	0.87 ± 0.08	0.55 ± 0.07
c_{00}^{-}	8.99	8.92 ± 0.38	7.31 ± 0.90
c_{20}^{+}	0.003	0.024 ± 0.006	
c_{10}^{-}	0.088	0.31 ± 0.01	0.21 ± 0.04
c_{01}^{+}	3.8	2.07 ± 0.10	2.06 ± 0.22
c_{30}^{+}	0.0025	0.0034 ± 0.0008	
c_{20}^{-}	0.013	0.0085 ± 0.0001	
c_{11}^{+}	-0.10	-0.066 ± 0.010	
c_{01}^{-}	0.71	0.62 ± 0.06	0.51 ± 0.10
c_{02}^{+}	0.23	0.34 ± 0.03	



$\pi\pi$ and πK


ππ constraints
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.2
0
0.2
0.4
0.6
$10^3 L_6^r$
$10^3 L_4^r$
a_0^2
C_1
$a_0^{3/2}$
C_{10}^+

$\pi\pi$ and πK



ππ constraints
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.2
0
0.2
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a_0^2
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C_{10}^+
πK constraints
0.6
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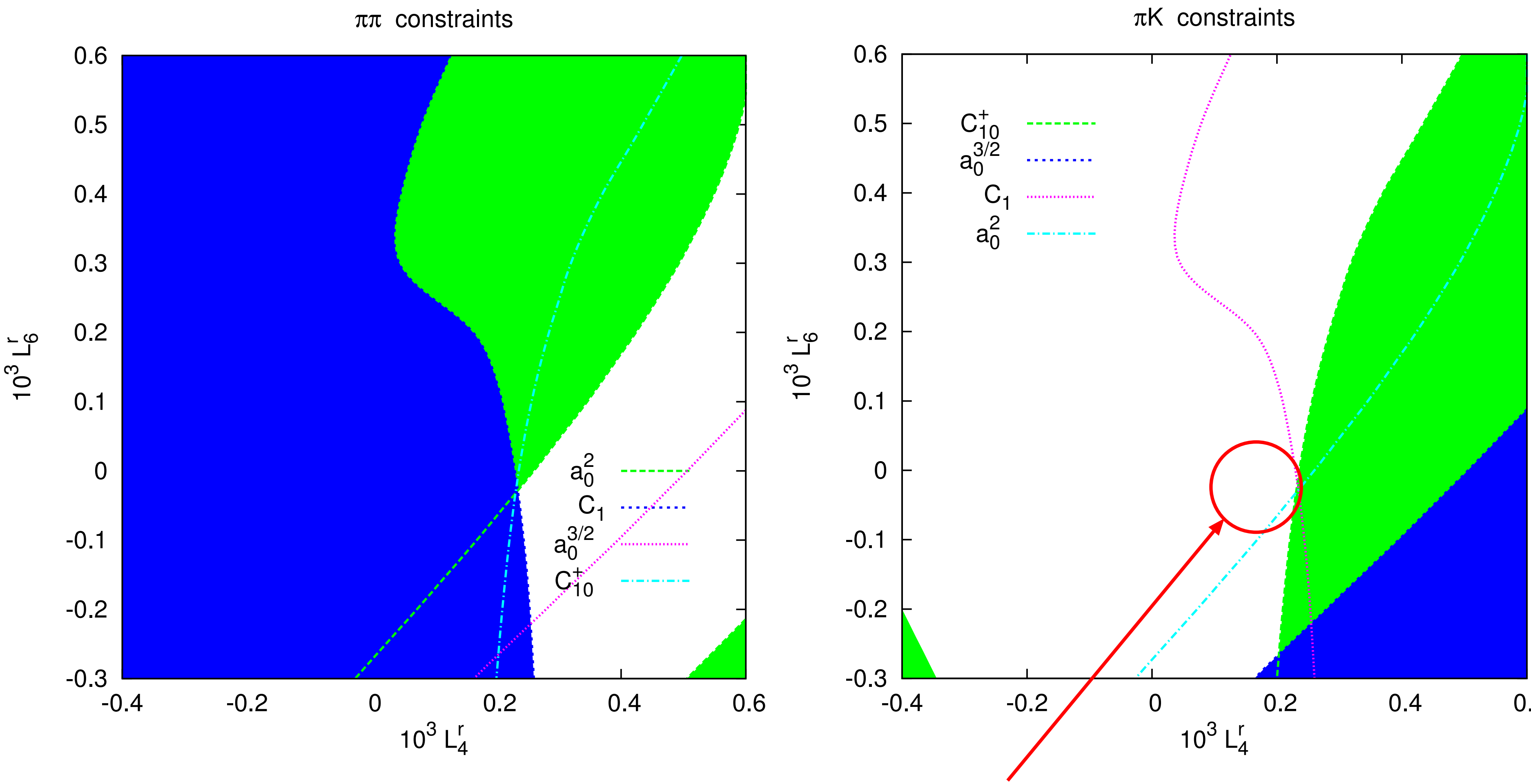
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.2
0
0.2
0.4
0.6
$10^3 L_6^r$
$10^3 L_4^r$
C_{10}^+
$a_0^{3/2}$
C_1
a_0^2

$\pi\pi$ and πK



preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

Conclusions

Three flavour ChPT at 2 loops doing fine: much progress

- many calculations done
- things seem to work but convergence is fairly slow
- “kinematical” and “vector” C_i^r seem to be OK
- L_4^r, L_6^r nonzero but reasonable for large N_c
- $\eta \to 3\pi$, isobreaking in $K_{\ell 3}$: parts done

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πK open problems

- Cleaning up C_i^r contributions and uncertainties
- Properly predicting threshold parameters