ππ and πK at Two Loops

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Overview


Introduction

Why (Effective) Field Theory

Chiral Perturbation Theory

Two Loop: General

Two Loop: Three Flavours
  - General fitting strategy and some comments
    - $\pi\pi, \pi K$

Conclusions
**Introduction**

- $\pi\pi$ and $\pi K$ scattering are basic strong processes. Study them as precise as possible.

- Earlier: $\pi\pi$ proved that two-flavour case of $\langle \bar{q}q \rangle$.

- Three flavour case: works or problems with strange quark loops (in scalar sector)?

- $\pi K$ excellent place to study this.

- Need precise calculations also here.
Effective: Use right degrees of freedom: essence of (most) physics
- Gap in the spectrum $\implies$ separation of scales
- With lower d.o.f.: build most general Lagrangian

$\Rightarrow \infty$ parameters
$\Rightarrow$ Where did predictivity go? $\implies$ power counting
Why (Effective) Field Theory?

Field Theory

- Only known way to combine QM and special relativity
- Taylor series does not work (convergence radius zero)
- Continuum of excitation states to be taken into account
- Off-shell effects fully under control: these effects are there as new free parameters
- Model-independent and systematic: ALL effects at given order included
- Theory \( \implies \) errors can be estimated
- Many parameters (but possible modelspace is large)
- Expansion might not converge (often still useful for model classification)
Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so $M_\rho$ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L D q_L + i\bar{q}_R D q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Can also see that via $v < c, m_q \neq 0 \implies v = c, m_q = 0 \nRightarrow$
Chiral Perturbation Theory

\[ \langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0 \]

\( SU(3)_L \times SU(3)_R \) broken spontaneously to \( SU(3)_V \)

8 generators broken \( \implies \) 8 massless degrees of freedom and interaction vanishes at zero momentum

Power counting in momenta:

\[
\begin{align*}
\int d^4 p & \\
\end{align*}
\]

\[
\begin{align*}
1/p^2 & \\
p^4 & \\
\end{align*}
\]

\[
\begin{align*}
(p^2)^2 (1/p^2)^2 p^4 & = p^4 \\
(p^2) (1/p^2) p^4 & = p^4
\end{align*}
\]
Two Loop: General

Lagrangian Structure:

<table>
<thead>
<tr>
<th></th>
<th>2 flavour</th>
<th>3 flavour</th>
<th>3+3 PQChPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p^2)</td>
<td>(F, B)</td>
<td>(F_0, B_0)</td>
<td>(F_0, B_0)</td>
</tr>
<tr>
<td>(p^4)</td>
<td>(l_i^r, h_i^r)</td>
<td>(L_i^r, H_i^r)</td>
<td>(\hat{L}_i^r, \hat{H}_i^r)</td>
</tr>
<tr>
<td>(p^6)</td>
<td>(c_i^r)</td>
<td>(C_i^r)</td>
<td>(K_i^r)</td>
</tr>
</tbody>
</table>

- \(p^2\): Weinberg 1966
- \(p^4\): Gasser, Leutwyler 84,85
- \(p^6\): JB, Colangelo, Ecker 99,00

Note:
- All infinities known
- Two Flavour Most Things Done
Three Flavours at Two Loop

\[ \Pi_{VV\pi}, \Pi_{VV\eta}, \Pi_{VVK} \]
\[ \Pi_{VV\rho\omega} \]
\[ \Pi_{AA\pi}, \Pi_{AA\eta}, F_\pi, F_\eta, m_\pi, m_\eta \]
\[ \Pi_{SS} \]
\[ \Pi_{VVK}, \Pi_{AAK}, F_K, m_K \]
\[ K_4, \langle \bar{q}q \rangle \]
\[ F_M, m_M, \langle \bar{q}q \rangle (m_u \neq m_d) \]
\[ F_{V\pi}, F_{VK^+}, F_{VK^0} \]
\[ K_3 \]
\[ F_{S\pi}, F_{SK} \text{ (includes } \sigma\text{-terms)} \]
\[ K, \pi \rightarrow \ell \nu \gamma \]
\[ \pi\pi \]
\[ \pi K \]

Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera

Maltman

Kambor, Golowich; Amorós, JB, Talavera

Moussallam \[ L_4^r, L_6^r \]

Amorós, JB, Talavera

Amorós, JB, Talavera \[ L_1^r, L_2^r, L_3^r \]

Amorós, JB, Talavera \[ L_5^r, L_7^r, L_8^r, m_u/m_d \]

Post, Schilcher; JB, Talavera \[ L_9^r \]

Post, Schilcher; JB, Talavera \[ V_{us} \]

JB, Dhonte \[ L_4^r, L_6^r \]

Geng, Ho, Wu \[ L_{10}^r \]

JB, Dhonte, Talavera

JB, Dhonte, Talavera
General Strategy and some comments

- Find enough inputs from experiment
- $C_i^r$:
  - kinematical dependence: agree well with single resonance saturation
  - quark mass+kinematical: if vector dominated, seems to be OK
  - quark mass+kinematical: if scalar dominated: which scalars? (not $\sigma$)
  - quark masses: which scalars? unrealistically large estimates
- in $p^6$ physical or lowest order masses: thresholds in right place requires physical
General Strategy and some comments

Inputs:

\( K_{\ell 4} : F(0), G(0), \lambda \)

\( m_{\pi^0}^2, m_\eta^2, m_{K^+}^2, m_{K^0}^2 \)

\( F_{\pi^+} \)

\( F_{K^+/F_{\pi^+}} \)

E865 BNL em with Dashen violation
Inputs:

\[ K_{4\ell} : F(0), G(0), \lambda \]

\[ m_{\pi^0}^2, m_\eta^2, m_{K^+}^2, m_{K^0}^2 \]

\[ F_{\pi^+} \]

\[ F_{K^+}/F_{\pi^+} \]

\[ m_s/\hat{m} \]

\[ L_4^r, L_6^r \]

\[ \hat{m} = (m_u + m_d)/2 \]

E865 BNL em with Dashen violation

\[ \hat{m} = (m_u + m_d)/2 \]

\[ \Rightarrow \text{other } L_i^r \text{ vary correlated} \]
General Strategy and some comments

Inputs:

\[ K_{\ell 4}: F(0), G(0), \lambda \]
\[ m_{\pi^0}^2, m_{\eta}^2, m_{K^+}^2, m_{K^0}^2 \]
\[ F_{\pi^+} \]
\[ F_{K^+} / F_{\pi^+} \]
\[ m_s / \hat{m} \]
\[ L_{4r}, L_{6r} \]

\[ \hat{m} = (m_u + m_d) / 2 \]

Vary \( \Rightarrow \) other \( L_{i}^{r} \) vary correlated

\( C_{i}^{r} \) from single resonance approximation

\[ \pi \rightarrow q^2 \]
\[ \rho, S \]
\[ |q^2| << m_{\rho}^2, m_{S}^2 \]

\( \pi \pi \) and \( \pi K \) at Two Loops
## General Strategy: fit results

<table>
<thead>
<tr>
<th></th>
<th>fit 10</th>
<th>same $p^4$</th>
<th>fit B</th>
<th>fit D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3L_1^r$</td>
<td>0.43 ± 0.12</td>
<td>0.38</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>$10^3L_2^r$</td>
<td>0.73 ± 0.12</td>
<td>1.59</td>
<td>0.60</td>
<td>0.69</td>
</tr>
<tr>
<td>$10^3L_3^r$</td>
<td>−2.53 ± 0.37</td>
<td>−2.91</td>
<td>−2.31</td>
<td>−2.33</td>
</tr>
<tr>
<td>$10^3L_4^r$</td>
<td>≡ 0</td>
<td>≡ 0</td>
<td>≡ 0.5</td>
<td>≡ 0.2</td>
</tr>
<tr>
<td>$10^3L_5^r$</td>
<td>0.97 ± 0.11</td>
<td>1.46</td>
<td>0.82</td>
<td>0.88</td>
</tr>
<tr>
<td>$10^3L_6^r$</td>
<td>≡ 0</td>
<td>≡ 0</td>
<td>≡ 0.1</td>
<td>≡ 0</td>
</tr>
<tr>
<td>$10^3L_7^r$</td>
<td>−0.31 ± 0.14</td>
<td>−0.49</td>
<td>−0.26</td>
<td>−0.28</td>
</tr>
<tr>
<td>$10^3L_8^r$</td>
<td>0.60 ± 0.18</td>
<td>1.00</td>
<td>0.50</td>
<td>0.54</td>
</tr>
</tbody>
</table>

- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying $C_i^r$ factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6 \times 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and $\pi K$ thresholds
### General Strategy: some outputs

<table>
<thead>
<tr>
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<th>fit 10</th>
<th>same $p^4$</th>
<th>fit B</th>
<th>fit D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2B_0 \hat{m}/m_\pi^2$</td>
<td>0.736</td>
<td>0.991</td>
<td>1.129</td>
<td>0.958</td>
</tr>
<tr>
<td>$m_\pi^2: p^4, p^6$</td>
<td>0.006,0.258</td>
<td>0.009,≡ 0</td>
<td>−0.138,0.009</td>
<td>−0.091,0.133</td>
</tr>
<tr>
<td>$m_K^2: p^4, p^6$</td>
<td>0.007,0.306</td>
<td>0.075,≡ 0</td>
<td>−0.149,0.094</td>
<td>−0.096,0.201</td>
</tr>
<tr>
<td>$m_\eta^2: p^4, p^6$</td>
<td>−0.052,0.318</td>
<td>0.013,≡ 0</td>
<td>−0.197,0.073</td>
<td>−0.151,0.197</td>
</tr>
<tr>
<td>$m_u/m_d$</td>
<td>0.45±0.05</td>
<td>0.52</td>
<td>0.52</td>
<td>0.50</td>
</tr>
</tbody>
</table>

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$F_0$ [MeV]</td>
<td>87.7</td>
<td>81.1</td>
<td>70.4</td>
<td>80.4</td>
</tr>
<tr>
<td>$F_K/F_\pi: p^4, p^6$</td>
<td>0.169,0.051</td>
<td>0.22,≡ 0</td>
<td>0.153,0.067</td>
<td>0.159,0.061</td>
</tr>
</tbody>
</table>

- Pattern of mass corrections can vary a lot
- $F_K/F_\pi$ always OK expansion
- $m_u = 0$ always very far from the fits
- $F_0$: pion decay constant in the chiral limit
\begin{align*}
  a_0^0 &= 0.220 \pm 0.005, \quad a_0^2 = -0.0444 \pm 0.0010 \\
  \text{Colangelo, Gasser, Leutwyler} \\
  a_0^0 &= 0.159 \quad a_0^2 = -0.0454 \text{ at order } p^2
\end{align*}
\[ C_1 = 1.104 \pm 0.009, \quad C_2 = 1.120 \pm 0.027 \]

Colangelo, Gasser, Leutwyler

\[ C_1 = C_2 = 1 \] at order \( p^2 \)
\[ a_0^{1/2} = 0.224 \pm 0.022, \quad a_0^{3/2} = -0.0448 \pm 0.0077 \]

Büttiker, Descotes-Genon, Moussallam

\[ a_0^{1/2} = 0.142 \quad a_0^2 = -0.0708 \text{ at order } p^2 \]
\[ c_{10}^+ = 0.87 \pm 0.08, \quad c_{00}^- = 8.92 \pm 0.38 \]

Büttiker, Descotes-Genon, Moussallam
\[ \pi K \] subthreshold parameters

<table>
<thead>
<tr>
<th></th>
<th>Vector</th>
<th>Scalar</th>
<th>Sum Reso</th>
<th>chiral order</th>
<th>( p^2 )</th>
<th>( p^4 )</th>
<th>( p^6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{00}^+ )</td>
<td>-0.02</td>
<td>0.13</td>
<td>0.11</td>
<td>2</td>
<td>0</td>
<td>0.122</td>
<td>0.007</td>
</tr>
<tr>
<td>( c_{10}^+ )</td>
<td>0.018</td>
<td>-0.063</td>
<td>-0.045</td>
<td>2</td>
<td>0.5704</td>
<td>-0.113</td>
<td>0.460</td>
</tr>
<tr>
<td>( c_{00}^- )</td>
<td>0.21</td>
<td>0.17</td>
<td>0.38</td>
<td>2</td>
<td>8.070</td>
<td>0.311</td>
<td>0.017</td>
</tr>
<tr>
<td>( c_{20}^+ )</td>
<td>-0.0053</td>
<td>0.0023</td>
<td>-0.0030</td>
<td>4</td>
<td>—</td>
<td>0.0256</td>
<td>-0.0254</td>
</tr>
<tr>
<td>( c_{10}^- )</td>
<td>-0.11</td>
<td>-0.04</td>
<td>-0.15</td>
<td>4</td>
<td>—</td>
<td>-0.0254</td>
<td>0.121</td>
</tr>
<tr>
<td>( c_{01}^+ )</td>
<td>-0.27</td>
<td>0.28</td>
<td>0.01</td>
<td>4</td>
<td>—</td>
<td>1.667</td>
<td>1.492</td>
</tr>
<tr>
<td>( c_{30}^+ )</td>
<td>0.00026</td>
<td>0.00010</td>
<td>0.00036</td>
<td>6</td>
<td>—</td>
<td>0.00121</td>
<td>0.00071</td>
</tr>
<tr>
<td>( c_{20}^- )</td>
<td>0.0037</td>
<td>0.00060</td>
<td>0.0043</td>
<td>6</td>
<td>—</td>
<td>0.00478</td>
<td>0.00320</td>
</tr>
<tr>
<td>( c_{11}^+ )</td>
<td>0.017</td>
<td>-0.008</td>
<td>0.009</td>
<td>6</td>
<td>—</td>
<td>-0.126</td>
<td>-0.006</td>
</tr>
<tr>
<td>( c_{01}^- )</td>
<td>0.25</td>
<td>0.04</td>
<td>0.29</td>
<td>6</td>
<td>—</td>
<td>0.229</td>
<td>0.196</td>
</tr>
</tbody>
</table>

Resonance contributions, units: \( m_{\pi^+}^{2i+2j} (c_{ij}^+) \) and \( m_{\pi^+}^{2i+2j+1} (c_{ij}^-) \)

Chiral order at which they first have tree level contributions

Contributions with the \( L_i^r = C_i^r = 0 \) at \( \mu = 0.77 \) GeV.
\( \pi K \) subthreshold parameters

<table>
<thead>
<tr>
<th></th>
<th>Fit 10</th>
<th>BDM</th>
<th>Lang</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{00}^+ )</td>
<td>0.278</td>
<td>2.01 ± 1.10</td>
<td>−0.52 ± 2.03</td>
</tr>
<tr>
<td>( c_{10}^+ )</td>
<td>0.898</td>
<td>0.87 ± 0.08</td>
<td>0.55 ± 0.07</td>
</tr>
<tr>
<td>( c_{00}^- )</td>
<td>8.99</td>
<td>8.92 ± 0.38</td>
<td>7.31 ± 0.90</td>
</tr>
<tr>
<td>( c_{20}^+ )</td>
<td>0.003</td>
<td>0.024 ± 0.006</td>
<td></td>
</tr>
<tr>
<td>( c_{10}^- )</td>
<td>0.088</td>
<td>0.31 ± 0.01</td>
<td>0.21 ± 0.04</td>
</tr>
<tr>
<td>( c_{01}^+ )</td>
<td>3.8</td>
<td>2.07 ± 0.10</td>
<td>2.06 ± 0.22</td>
</tr>
<tr>
<td>( c_{30}^+ )</td>
<td>0.0025</td>
<td>0.0034 ± 0.0008</td>
<td></td>
</tr>
<tr>
<td>( c_{20}^- )</td>
<td>0.013</td>
<td>0.0085 ± 0.0001</td>
<td></td>
</tr>
<tr>
<td>( c_{11}^+ )</td>
<td>−0.10</td>
<td>−0.066 ± 0.010</td>
<td></td>
</tr>
<tr>
<td>( c_{01}^- )</td>
<td>0.71</td>
<td>0.62 ± 0.06</td>
<td>0.51 ± 0.10</td>
</tr>
<tr>
<td>( c_{02}^+ )</td>
<td>0.23</td>
<td>0.34 ± 0.03</td>
<td></td>
</tr>
</tbody>
</table>
\( \pi \pi \) and \( \pi K \)

\( \pi \pi \) constraints

Preferred region: \( t \leq 10^{-3} \)

\( L^f = 10^3 L_4 \)

- \( a_0^2 \)
- \( C_1 \)
- \( a_0^{3/2} \)
- \( C_{10} \)
$\pi\pi$ and $\pi K$

$\pi\pi$ constraints

$10^3 L_6^r$

$-0.3$ $-0.2$ $-0.1$ $0$ $0.1$ $0.2$ $0.3$ $0.4$ $0.5$ $0.6$

$10^3 L_4^r$

$-0.3$ $-0.2$ $-0.1$ $0$ $0.1$ $0.2$ $0.3$ $0.4$ $0.5$ $0.6$

$\pi K$ constraints

$10^3 L_6^r$

$-0.3$ $-0.2$ $-0.1$ $0$ $0.1$ $0.2$ $0.3$ $0.4$ $0.5$ $0.6$

$10^3 L_4^r$

$-0.3$ $-0.2$ $-0.1$ $0$ $0.1$ $0.2$ $0.3$ $0.4$ $0.5$ $0.6$

Preferred region: $t_D$: $10^3 L_6^r$: $0$, $10^3 L_4^r$: $0$.
preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$
Conclusions

Three flavour ChPT at 2 loops doing fine: much progress

- many calculations done
- things seem to work but convergence is fairly slow
- “kinematical” and “vector” $C^r_i$ seem to be OK
- $L_4^r, L_6^r$ nonzero but reasonable for large $N_c$
- $\eta \rightarrow 3\pi$, isobreaking in $K_{\ell 3}$: parts done
Conclusions

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$\pi K$ open problems

- Cleaning up $C^r_i$ contributions and uncertainties
- Properly predicting threshold parameters