HADRONIC LIGHT-BY-LIGHT FOR THE MUON ANOMALOUS MAGNETIC MOMENT

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LUND UNIVERSITY

Overview

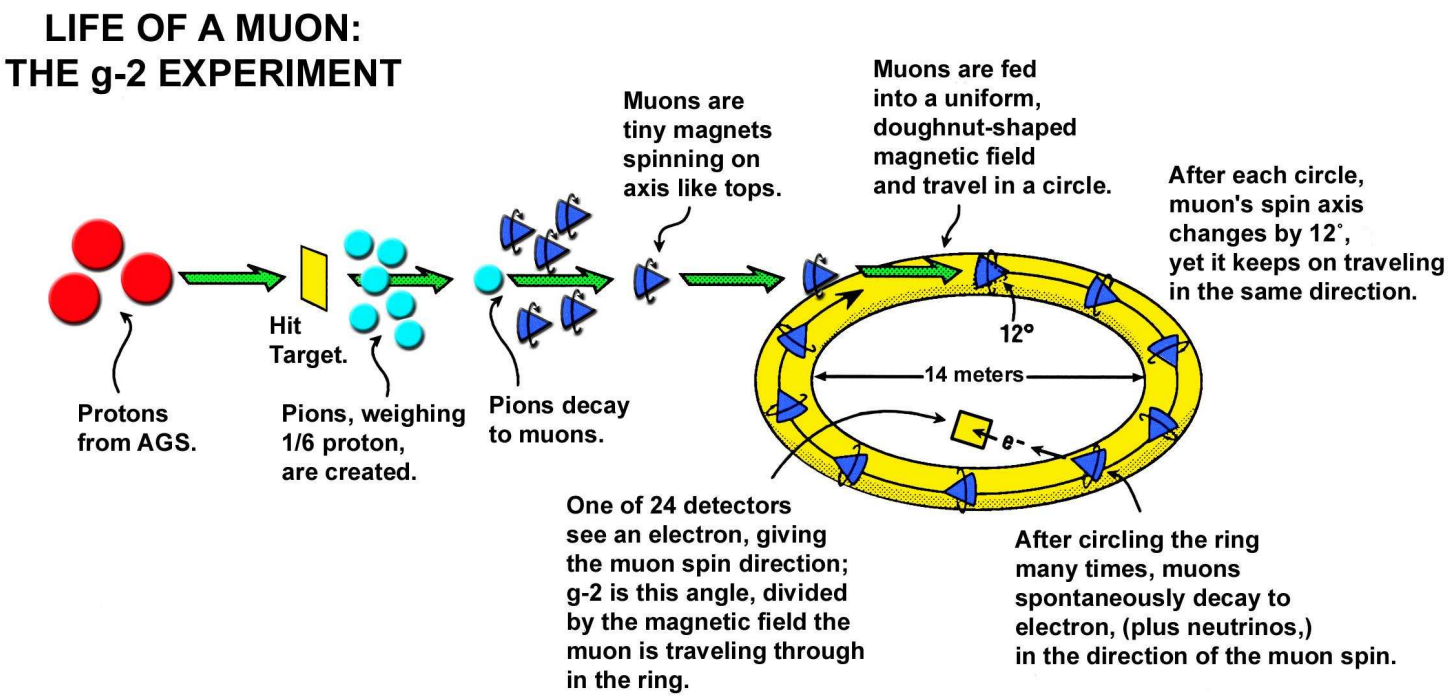
Literature

- Final experimental paper:
  G. W. Bennett *et al.* [Muon G-2 Collaboration], "Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL," Phys. Rev. D **73** (2006) 072003 [hep-ex/0602035].
- Review 1:
  F. J. M. Farley and Y. K. Semertzidis, "The 47 years of muon g-2," Prog. Part. Nucl. Phys. **52** (2004) .
- Review 2:
  J. P. Miller, E. de Rafael and B. L. Roberts, "Muon (g-2): Experiment and theory," Rept. Prog. Phys. **70** (2007) 795 [hep-ph/0703049].
- Review 3:
  F. Jegerlehner and A. Nyffeler, "The Muon g-2," Phys. Rept. **477** (2009) 1 [arXiv:0902.3360 [hep-ph]].

Literature

- Lectures:
  M. Knecht, Lect. Notes Phys. **629** (2004) 37 [hep-ph/0307239].
- "Final" HLBL number:
  - J. Bijnens and J. Prades, Mod. Phys. Lett. A **22** (2007) 767 [hep-ph/0702170].
  - J. Prades, E. de Rafael and A. Vainshtein, "Hadronic Light-by-Light Scattering Contribution to the Muon Anomalous Magnetic Moment," (Advanced series on directions in high energy physics. 20) [arXiv:0901.0306 [hep-ph]].

Muon $g-2$: measurement



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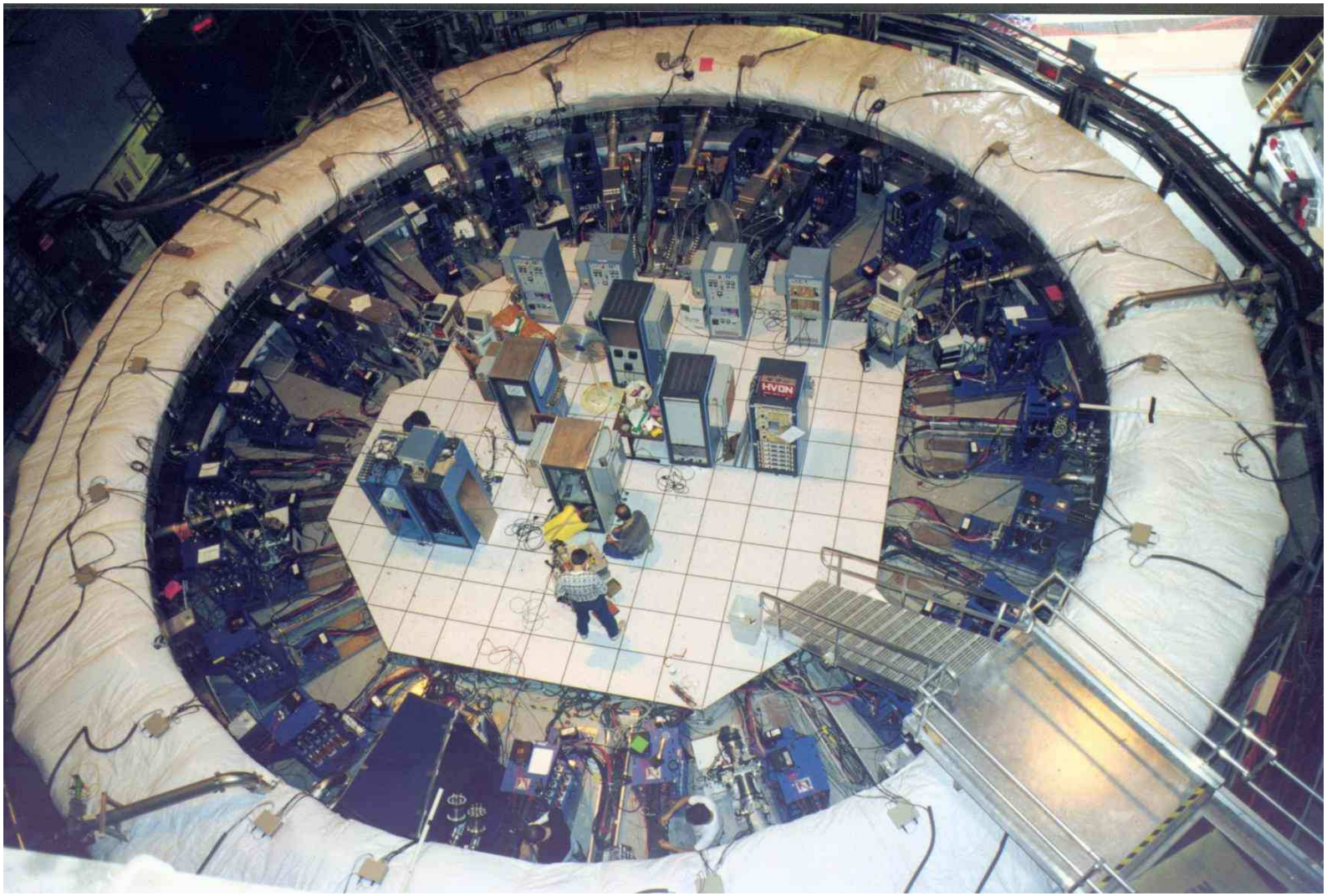

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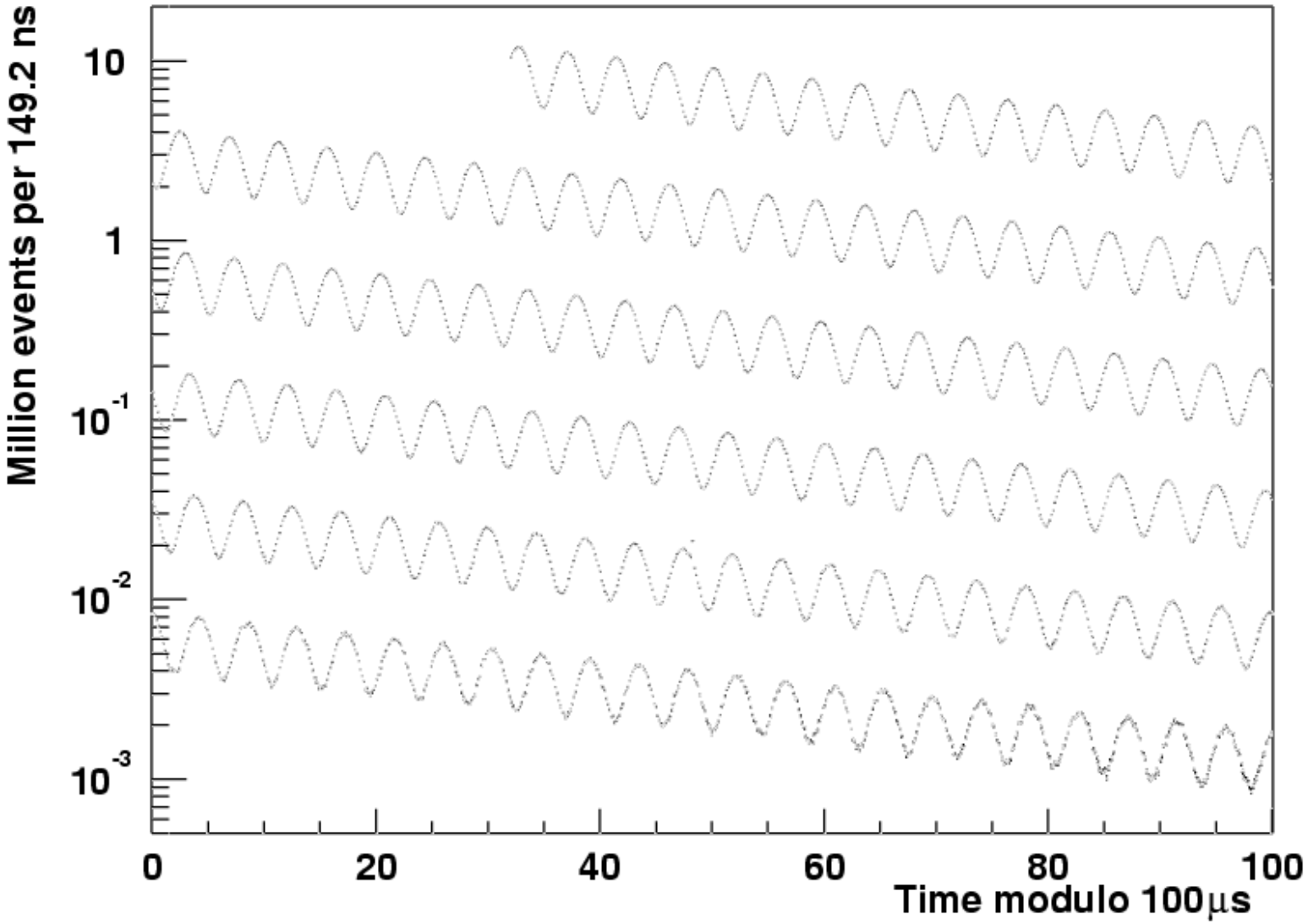

Phys.Rev. D73 (2006) 072003 [hep-ex/0602035]

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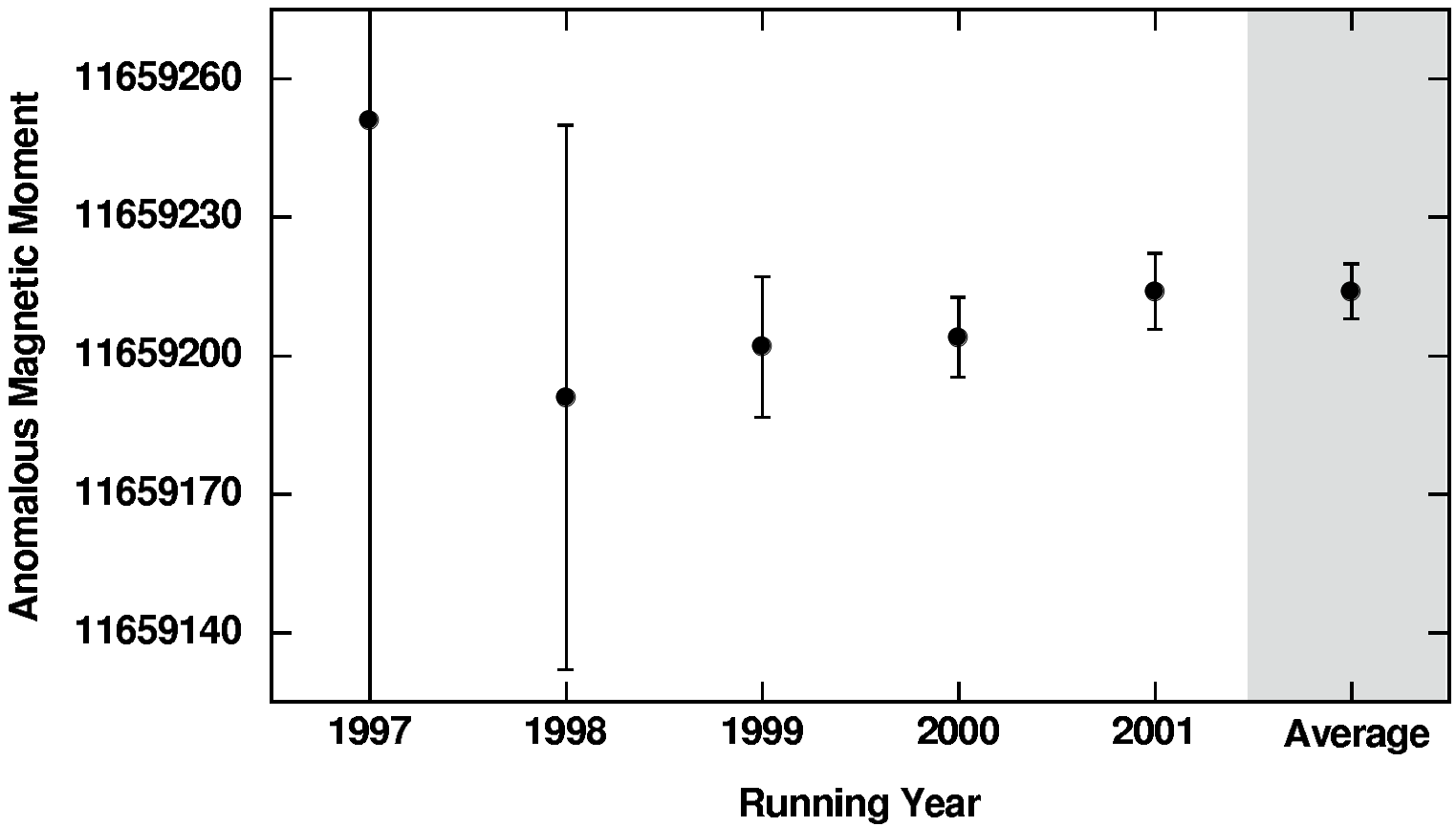


Phys.Rev. D73 (2006) 072003 [hep-ex/0602035]

2001: μ^-, others μ^+

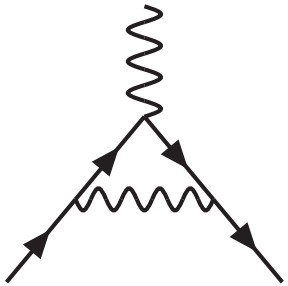
Muon $g-2$: overview

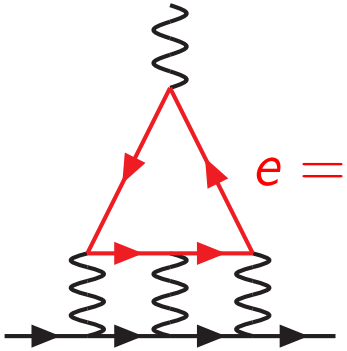
- in terms of the anomaly $a_\mu = (g-2)/2$
- Data dominated by BNL E821 (statistics)(systematic)
  $a_{\mu^+}^{\rm exp} = 11659204(6)(5) \times 10^{-10}$
  $a_{\mu^-}^{\rm exp} = 11659215(8)(3) \times 10^{-10}$
  $a_{\mu}^{\rm exp} = 11659208.9(5.4)(3.3) \times 10^{-10}$
- Theory is off somewhat (electroweak)(LO had)(HO had)
  $a_\mu^{\rm SM} = 11659180.2(0.2)(4.2)(2.6) \times 10^{-10}$
- $\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 28.7(6.3)(4.9) \times 10^{-10}$ (PDG)
- E821 goes to Fermilab, expect factor of four in precision
- Note: g agrees to 10^{-8} with theory
- Many BSM models CAN predict a value in this range (often a lot more or a lot less)
- Numbers taken from PDG2012, see references there

Muon $g-2$: QED

$$a_\mu^{\rm QED} = \frac{\alpha}{2\pi} + 0.765857410(27)\left(\frac{\alpha}{\pi}\right)^2 + 24.05050964(43)\left(\frac{\alpha}{\pi}\right)^3 + 130.8055(80)\left(\frac{\alpha}{\pi}\right)^4 + 663(20)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

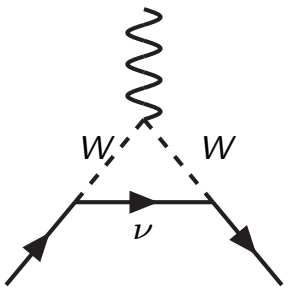
- First three loops known analytically
- four-loops fully done numerically
- Five loops estimate
- Kinoshita, Laporta, Remiddi, Schwinger,...
- α fixed from the electron $g-2$: $\alpha = 1/137.035999084(51)$
- $a_\mu^{\rm QED} = 11658471.809(0.015) \times 10^{-10}$
- Light-by-light surprisingly large: 2670×10^{-10}

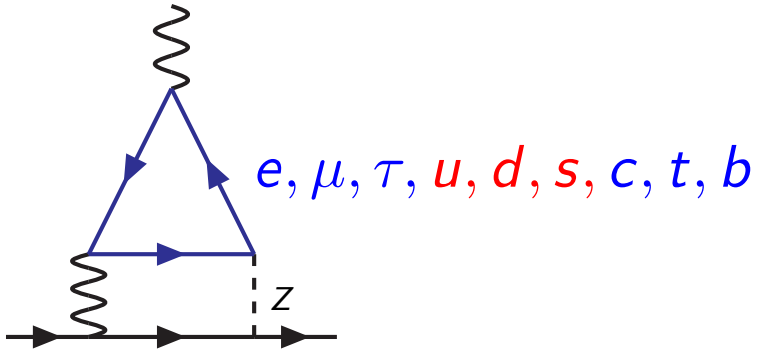




$e = 20.95, \mu = 0.37, \tau = 0.002$

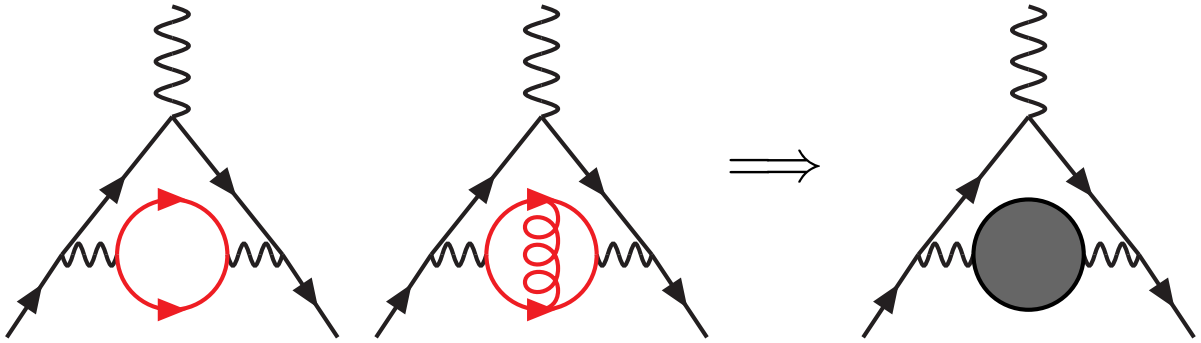
Muon $g-2$: Electroweak





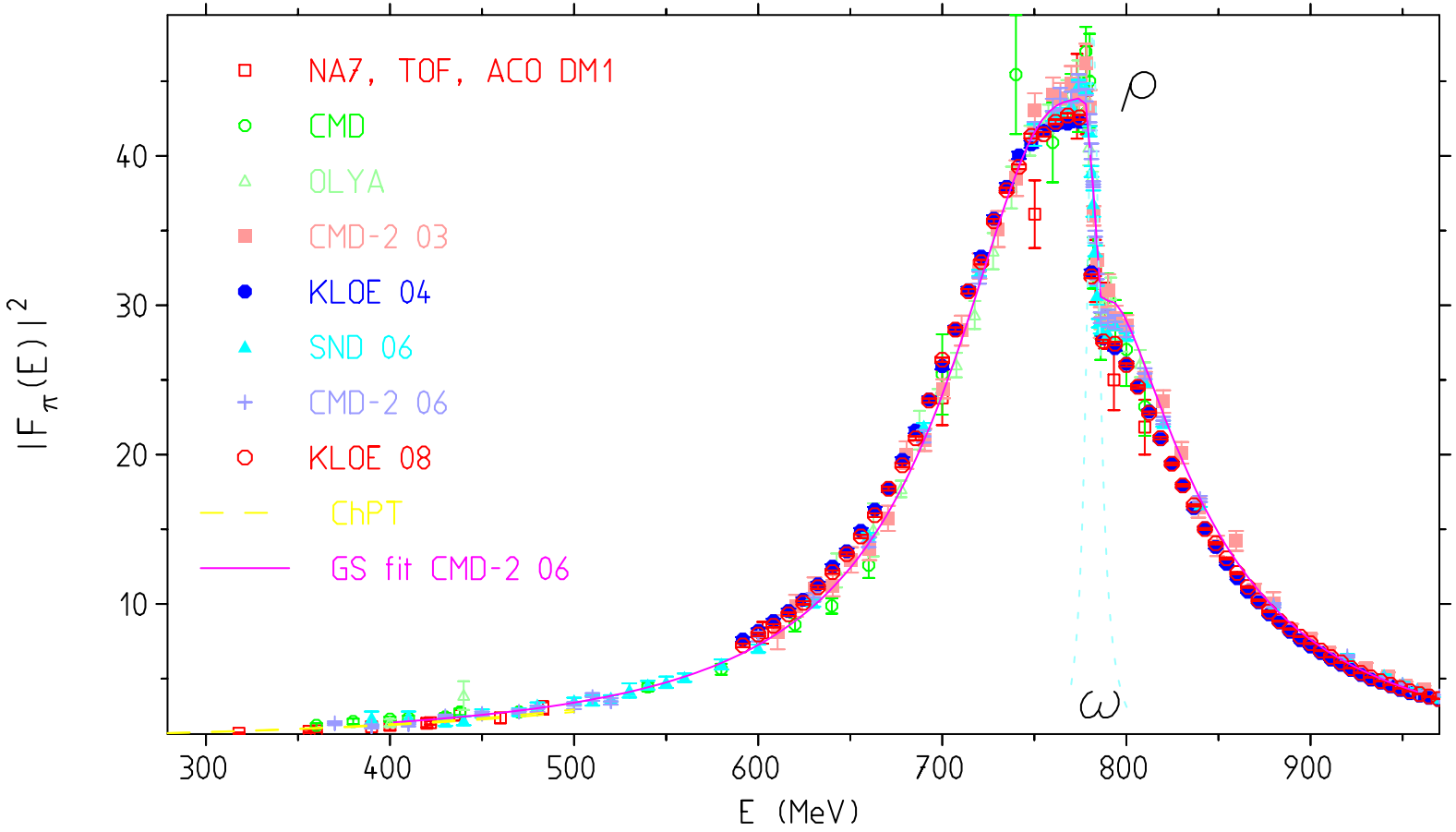
- $a_\mu^{\rm EW}[1{-}{\rm loop}] = 19.48 \times 10^{-10}$
- $a_\mu^{\rm EW}[2{-}{\rm loop}] = -4.07(0.10)(0.18) \times 10^{-10}$
- $a_\mu^{\rm EW} = 15.4(0.1)(0.2) \times 10^{-10}$ (triangle)(Higgs mass)
- 2-loop large because of the large logarithms in the triangle anomaly present here
- SM anomaly cancellation very needed
- "anomaly" as part of the "anomaly"

Muon $g-2$: LO hadronic



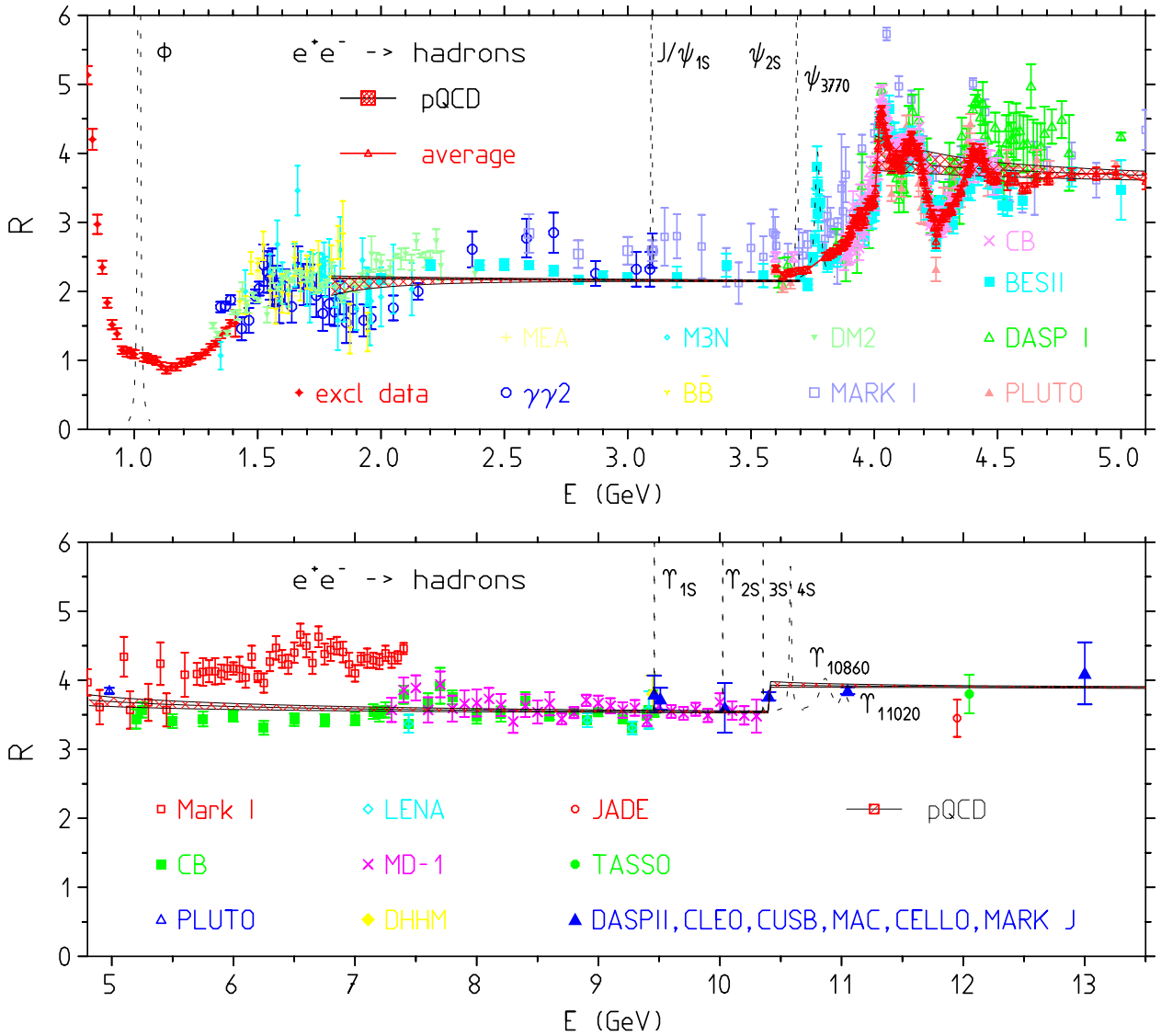
- $a_\mu^{\rm LOhad} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R^{(0)}(s)$
- $R^{(0)}(s)$ bare cross-section ratio $\dfrac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$
- Bare, many different evaluations,...
- e^+e^- versus τ-decays
- $a_\mu^{\rm LOHad} = 692.3(4.2)(0.3) \times 10^{-10}$ (exp)(pert. QCD)

LO hadronic



Jegerlehner, Nyffeler Phys.Rept. 477 (2009) 1-110 [0902.3360]

LO hadronic



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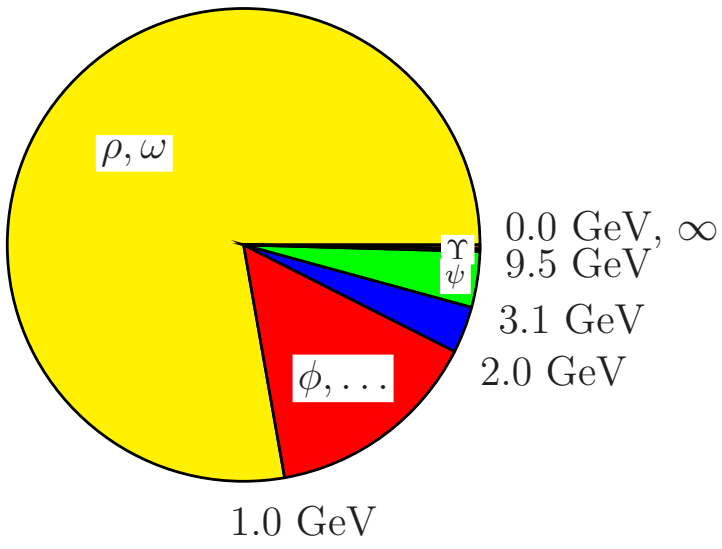
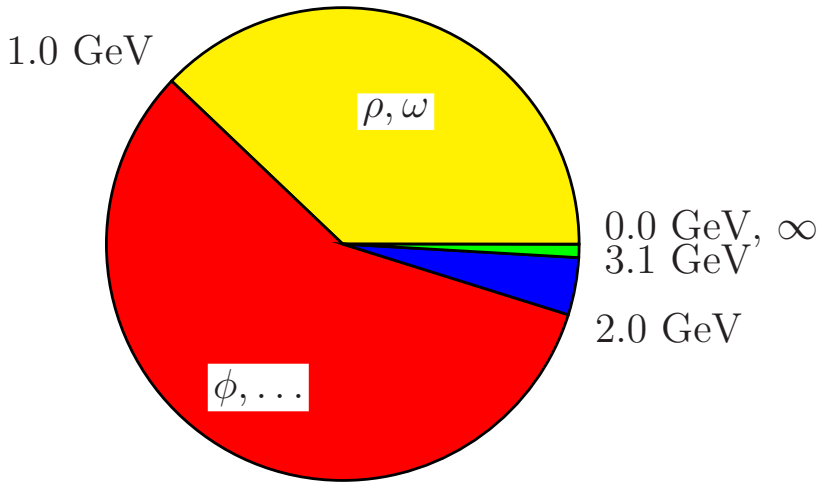
LO hadronic



Contribution



Error

Jegerlehner, Nyffeler Phys.Rept. 477 (2009) 1-110 [0902.3360]

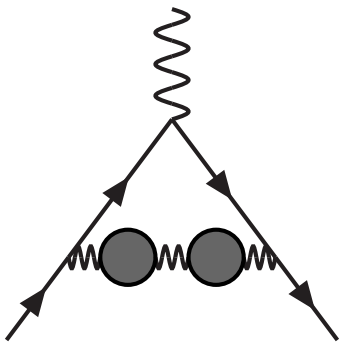
- Direct measurements
- Radiative return: radiate hard photon to get lower E_{cms}
- Radiative corrections
- τ-decay: isospin breaking
- can be improved experimentally

A comment on models

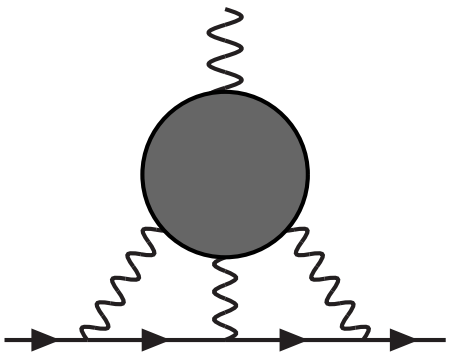
- $a_\mu^{\rm LOHad} = 692.3(4.2) \times 10^{-10}$
- Rewrite: $a_\mu^{\rm LOHad} = 8\alpha^2 \int \frac{ds}{s} K(s) \frac{1}{\pi} {\rm Im}\Pi(s)$
- Rho: $\frac{1}{\pi}{\rm Im}\Pi(s) = \frac{2}{3} f_V^2 M_V^2 \delta(s - M_V^2) + \frac{2}{3}\frac{N_c}{12\pi^2}\theta(s - s_0)$
- $K(s) \approx \frac{m_\mu^2}{3t}$ $(s \gg m_\mu^2)$
- $a_\mu^{\rm LOHad} = \frac{16}{9}\alpha^2 \left(f_V^2 \frac{m_\mu^2}{M_V^2} + \frac{N_c}{12\pi^2}\frac{m_\mu^2}{s_0} \right)$
- Put in $f_V \approx 0.17$, $M_V \approx 0.77$ GeV and $s_0 \approx 1.35\ {\rm GeV}^2$
- $a_\mu^{\rm LOHad} \approx (450 + 95 \approx 545) \times 10^{-10}$
- Many models have this within 30%

Muon $g-2$: HO hadronic

- Two main types of contributions



HO HVP



HLbL

- HO HVP is like LO Had but a more complicated function $K(s)$ $a_\mu^{\mathrm{HO\ HVP}} = -9.84(0.06) \times 10^{-10}$
- HLbL is the real problem: best estimate now: $a_\mu^{\mathrm{HLbL}} = 10.5(2.6) \times 10^{-10}$
- Note that the sum is very small: but not an indication of the error

Summary of Muon $g-2$ contributions

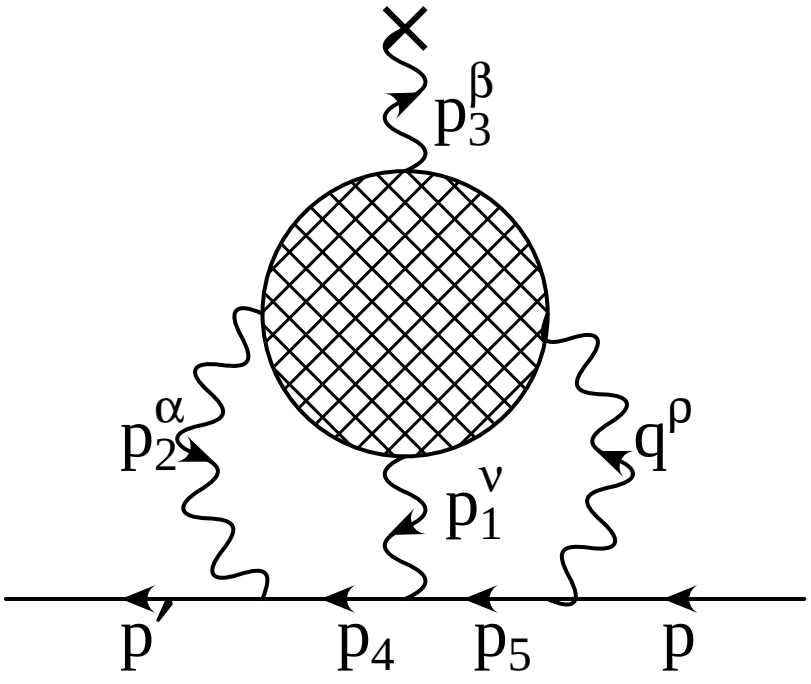
	$10^{10} a_\mu$	
exp	11 659 208.9	6.3
theory	11 659 180.2	5.0
QED	11 658 471.8	0.0
EW	15.4	0.2
LO Had	692.3	4.2
HO HVP	−9.8	0.1
HLbL	10.5	2.6
difference	28.7	8.1

- Error on LO had
  all e^+e^- based OK
  τ based 2 σ
- Error on HLbL
- Errors added quadratically
- 3.5 σ
- Difference:
  4% of LO Had
  270% of HLbL
  1% of leptonic LbL

Generic SUSY: $12.3 \times 10^{-10} \left(\frac{100\ \mathrm{GeV}}{M_{SUSY}}\right)^2 \tan\beta$

$M_{SUSY} \approx 66\ \mathrm{GeV}\sqrt{\tan\beta}$

Our object



- Muon line and photons: well known
- The blob: fill in with hadrons/QCD
- Trouble: low and high energy very mixed
- Double counting needs to be avoided: hadron exchanges versus quarks

A separation proposal: a start

E. de Rafael, "Hadronic contributions to the muon g-2 and low-energy QCD," Phys. Lett. **B322** (1994) 239-246. [hep-ph/9311316].

- Use ChPT p counting and large N_c
- p^4, order 1: pion-loop
- p^8, order N_c: quark-loop and heavier meson exchanges
- p^6, order N_c: pion exchange

Does not fully solve the problem
only short-distance part of quark-loop is really p^8
but it's a start

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Implemented by two groups in the 1990s:

- Hayakawa, Kinoshita, Sanda: meson models, pion loop using hidden local symmetry, quark-loop with VMD, calculation in Minkowski space
- JB, Pallante, Prades: Try using as much as possible a consistent model-approach, calculation in Euclidean space

Papers: BPP and HKS

- JB, E. Pallante and J. Prades
  - "Comment on the pion pole part of the light-by-light contribution to the muon g-2," Nucl. Phys. B **626** (2002) 410 [arXiv:hep-ph/0112255].
  - "Analysis of the Hadronic Light-by-Light Contributions to the Muon $g-2$," Nucl. Phys. B **474** (1996) 379 [arXiv:hep-ph/9511388].
  - "Hadronic light by light contributions to the muon g-2 in the large N_c limit," Phys. Rev. Lett. **75** (1995) 1447 [Erratum-ibid. **75** (1995) 3781] [arXiv:hep-ph/9505251].
- Hayakawa, Kinoshita, (Sanda)
  - "Pseudoscalar pole terms in the hadronic light by light scattering contribution to muon g - 2," Phys. Rev. **D57** (1998) 465-477. [hep-ph/9708227], Erratum-ibid.D66 (2002) 019902[hep-ph/0112102].
  - "Hadronic light by light scattering contribution to muon g-2," Phys. Rev. **D54** (1996) 3137-3153. [hep-ph/9601310].
  - "Hadronic light by light scattering effect on muon g-2," Phys. Rev. Lett. **75** (1995) 790-793. [hep-ph/9503463].

Differences

- HK(S)
  - Purely hadronic exchanges
  - quark-loop with hadronic VMD
  - Studied dependence of everything on m_V
- BPP
  - Use the ENJL as an overall model to have a similar uncertainty on all low-energy parts
  - repair some of the worst short-comings
  - Add the short-distance quark-loop
  - Study of cut-off dependence
- Sign mistake
  - HKS: Euclidean versus Minkowski $\varepsilon^{\mu\nu\alpha\beta}$
  - BPP: notes all correct sign, program had wrong sign, probably minus sign from fermion loop not removed

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The overall

$$a_\mu^{\rm HLbL} = \frac{-1}{48 m_\mu} {\rm tr}[(\not p + m_\mu) M^{\lambda\beta}(0)\,(\not p + m_\mu)[\gamma_\lambda, \gamma_\beta]]\,.$$

$$\begin{aligned} M^{\lambda\beta}(p_3) &= |e|^6 \int \frac{{\rm d}^4 p_1}{(2\pi)^4} \int \frac{{\rm d}^4 p_2}{(2\pi)^4} \frac{1}{q^2\, p_1^2\, p_2^2 (p_4^2 - m_\mu^2)\,(p_5^2 - m_\mu^2)} \\ &\times \left[\frac{\delta \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\right] \gamma_\alpha (\not p_4 + m_\mu) \gamma_\nu (\not p_5 + m_\mu) \gamma_\rho\,. \end{aligned}$$

- We used: $\Pi^{\rho\nu\alpha\lambda}(p_1, p_2, p_3) = -p_{3\beta} \dfrac{\delta \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\,.$
- Can calculate at $p_3 = 0$ but must take derivative
- derivative in quark-loop: each permutation finite
- Four point function of $V_i^\mu(x) \equiv \sum_i Q_i\, [\bar q_i(x) \gamma^\mu q_i(x)]$

$$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3) \equiv i^3 \int {\rm d}^4 x \int {\rm d}^4 y \int {\rm d}^4 z\, e^{i(p_1\cdot x + p_2 \cdot y + p_3 \cdot z)} \times \langle 0 | T \left(V_a^\rho(0) V_b^\nu(x) V_c^\alpha(y) V_d^\beta(z) \right) | 0 \rangle$$

General properties

$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)$:

- In general 138 Lorentz structures (but only 32 contribute to $g-2$)
- Using $q_\rho \Pi^{\rho\nu\alpha\beta} = p_{1\nu}\Pi^{\rho\nu\alpha\beta} = p_{2\alpha}\Pi^{\rho\nu\alpha\beta} = p_{3\beta}\Pi^{\rho\nu\alpha\beta} = 0$ 43 gauge invariant structures
- Bose symmetry relates some of them
- All depend on p_1^2, p_2^2 and q^2, but before derivative and $p_3 \to 0$ also $p_3^2, p_1 \cdot p_2, p_1 \cdot p_3, p_2 \cdot p_3$
- Compare HVP: one function, one variable
- General calculation from experiment difficult to see how
- In four photon measurement: lepton contribution

General properties

$\int \frac{d^4 p_1}{(2\pi)^4} \int \frac{d^4 p_2}{(2\pi)^4}$ plus loops inside the hadronic part

- 8 dimensional integral, three trivial,
- 5 remain: $p_1^2, p_2^2, p_1 \cdot p_2, p_1 \cdot p_\mu, p_2 \cdot p_\mu$
- Rotate to Euclidean space:
  - Easier separation of long and short-distance
  - Artefacts (confinement) in models smeared out.
- More recent: can do two more using Gegenbauer techniques Knecht-Nyffeler, Jegerlehner-Nyffeler,JB–Zahiri-Abyaneh–Relefors
- P_1^2, P_2^2 and Q^2 remain
- study $a_\mu^{\mathrm{X}} = \int dl_{P_1} dl_{P_2} a_\mu^{\mathrm{XLL}} = \int dl_{P_1} dl_{P_2} dl_Q a_\mu^{\mathrm{XLLQ}}$ $l_P = \ln(P/\mathrm{GeV})$, to see where the contributions are

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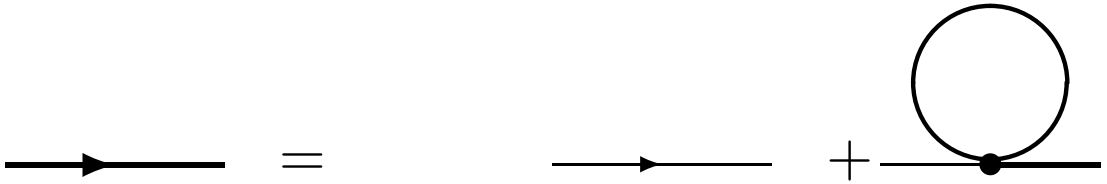
ENJL: our main model

$$\begin{aligned}\mathcal{L}_{\text{ENJL}} &= \overline{q}^{\alpha}\left\{i\gamma^{\mu}\left(\partial_{\mu}-iv_{\mu}-ia_{\mu}\gamma_5\right)-\left(\mathcal{M}+s-ip\gamma_5\right)\right\}q^{\alpha} \\ &\quad +2\,g_S\left(\overline{q}_R^{\alpha}q_L^{\beta}\right)\left(\overline{q}_L^{\beta}q_R^{\alpha}\right) \\ &\quad -g_V\left[\left(\overline{q}_L^{\alpha}\gamma^{\mu}q_L^{\beta}\right)\left(\overline{q}_L^{\beta}\gamma_{\mu}q_L^{\alpha}\right)+\left(\overline{q}_R^{\alpha}\gamma^{\mu}q_R^{\beta}\right)\left(\overline{q}_R^{\beta}\gamma_{\mu}q_R^{\alpha}\right)\right]\end{aligned}$$

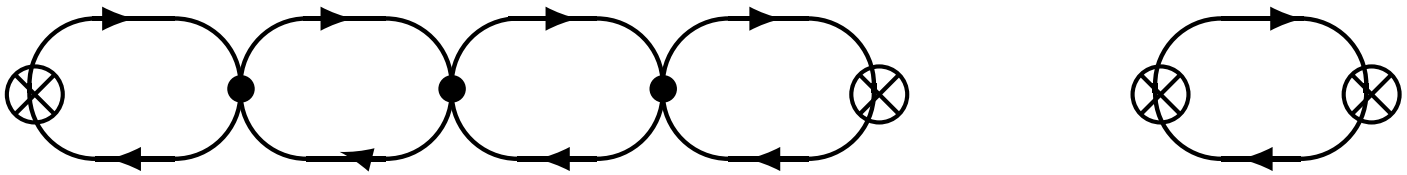
- $\overline{q}\equiv\left(\overline{u},\overline{d},\overline{s}\right)$
- v_μ, a_μ, s, p: external vector, axial-vector, scalar and pseudoscalar matrix sources
- $\mathcal{M}$ is the quark-mass matrix.
- $g_V\equiv\frac{8\pi^2G_V(\Lambda)}{N_c\Lambda^2}, \qquad g_S\equiv\frac{4\pi^2G_S(\Lambda)}{N_c\Lambda^2}\,.$
- G_V, G_S are dimensionless and valid up to Λ
- No confinement but has good pion, vector meson and OK axial vector-meson phenomenology

ENJL: our main model

- (this) ENJL JB, Bruno, de Rafael, Nucl. Phys. B390 (1993) 501 [hep-ph/9206236]; JB, Phys. Rep. 265 (1996) 369 [hep-ph/9502335] (review)
- Gap equation: chiral symmetry spontaneously broken



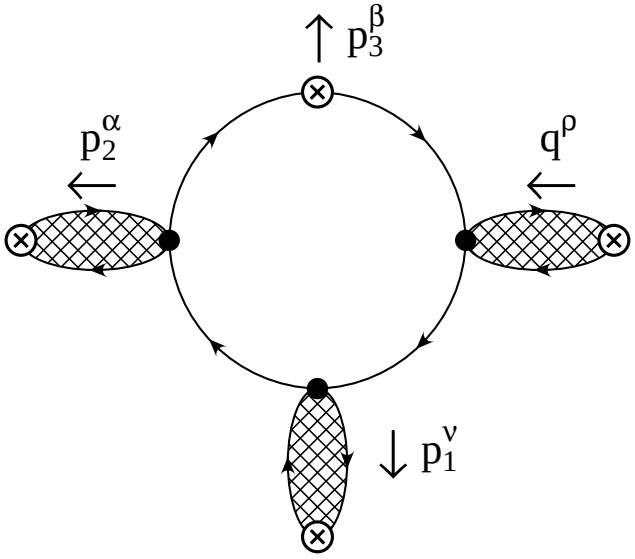
- Generates poles, i.e. mesons via bubble resummation



ENJL: our main model

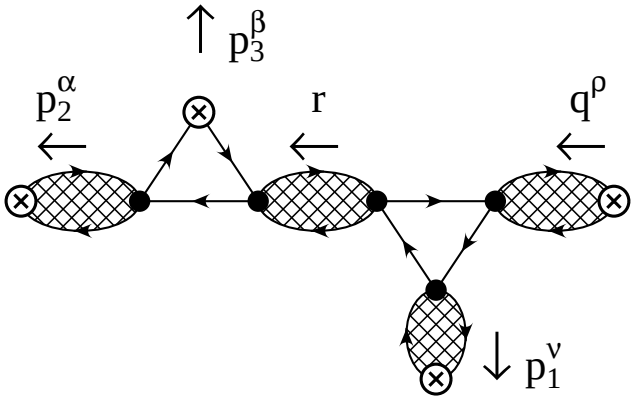
- Can be thought of as a very simple rainbow and ladder approximation in the DSE equation with constant kernels for the one-gluon exchange
- Parameters fit via F_π, L_i^r, vector meson properties,...
- $G_S = 1.216$, $G_V = 1.263$, $\Lambda = 1.16$ GeV
- has $M_Q = 263$ MeV
- Has a number of decent matchings to short-distance, e.g. $\Pi_V - \Pi_A$ but fails in others.
- Generates always VMD in external legs (but with a twist)
- Hook together general processes by one-loop vertices and bubble-chain propagators

Separation of contributions



(a)

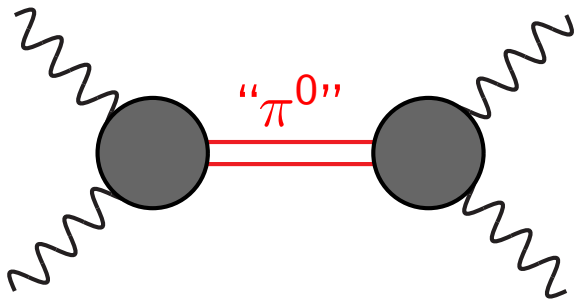
- Quark loop with external bubble-chains
- $\approx$ Quark-loop with VMD



(b)

- Also internal bubble chain
- $\approx$ meson exchange
- Note that vertices have structure
- Off-shell effect in model included

π^0 exchange



- "π^0" $= 1/(p^2 - m_\pi^2)$
- The blobs need to be modelled, and in e.g. ENJL contain corrections also to the $1/(p^2 - m_\pi^2)$
- Pointlike has a logarithmic divergence
- Numbers π^0, but also η, η'

π^0 exchange

	$a_\mu \times 10^{10}$				
Cutoff (GeV)	Point-like	ENJL–VMD	Pointlike VMD	Transverse VMD	CELLO-VMD
0.5	4.92(2)	3.29(2)	3.46(2)	3.60(3)	3.53(2)
0.7	7.68(4)	4.24(4)	4.49(3)	4.73(4)	4.57(4)
1.0	11.15(7)	4.90(5)	5.18(3)	5.61(6)	5.29(5)
2.0	21.3(2)	5.63(8)	5.62(5)	6.39(9)	5.89(8)
4.0	32.7(5)	6.22(17)	5.58(5)	6.59(16)	6.02(10)

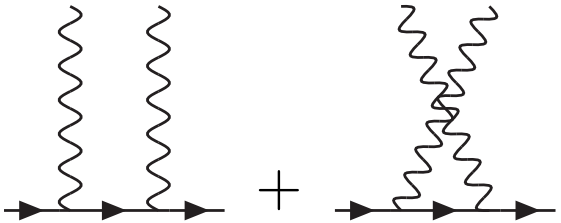
BPP: All in reasonable agreement $a_\mu^{\pi^0} = 5.9 \times 10^{-10}$

π^0 exchange

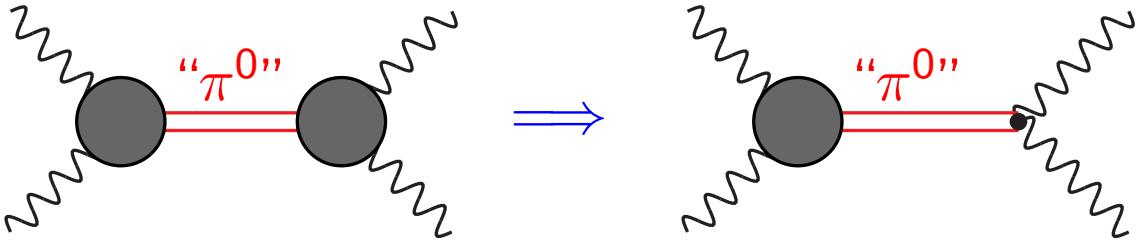
- BPP: $a_\mu^{\pi^0} = 5.9(0.9) \times 10^{-10}$
- Nonlocal quark model: $a_\mu^{\pi^0} = 6.27 \times 10^{-10}$
  A. E. Dorokhov, W. Broniowski, Phys. Rev. **D78** (2008) 073011. [0805.0760]]
- DSE model: $a_\mu^{\pi^0} = 5.75 \times 10^{-10}$
  T. Goecke, C. S. Fischer and R. Williams, Phys. Rev. D **83** (2011) 094006 [arXiv:1012.3886 [hep-ph]]
- LMD+V: $a_\mu^{\pi^0} = (5.8 - 6.3) \times 10^{-10}$
  M. Knecht, A. Nyffeler, Phys. Rev. **D65**(2002)073034, [hep-ph/0111058]
- Formfactor inspired by AdS/QCD: $a_\mu^{\pi^0} = 6.54 \cdot 10^{-10}$
  L. Cappiello, O. Cata and G. D'Ambrosio, Phys. Rev. D **83** (2011) 093006 [arXiv:1009.1161 [hep-ph]]
- Constraint via magnetic susceptibility: $a_\mu^{\pi^0} = 7.2 \times 10^{-10}$
  A. Nyffeler, Phys. Rev. D **79** (2009) 073012 [arXiv:0901.1172 [hep-ph]].
- All except possibly last in good agreement

MV short-distance: π^0 exchange

- K. Melnikov, A. Vainshtein, Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment revisited, Phys. Rev. **D70** (2004) 113006. [hep-ph/0312226]
- take $P_1^2 \approx P_2^2 \gg Q^2$: Leading term in OPE of two vector currents is proportional to axial current
- $\Pi^{\rho\nu\alpha\beta} \propto \frac{P_\rho}{P_1^2} \langle 0 | T \left(J_{A\nu} J_{V\alpha} J_{V\beta} \right) | 0 \rangle$
- J_A comes from

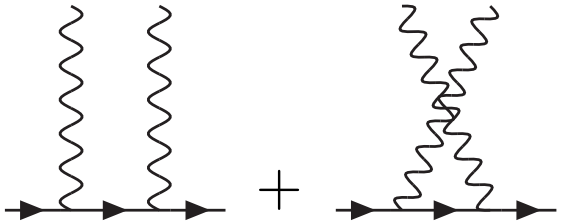


- AVV triangle anomaly: extra info
- Implemented via setting one blob = 1



- $a_\mu^{\pi^0} = 7.7 \times 10^{-10}$

π^0 exchange

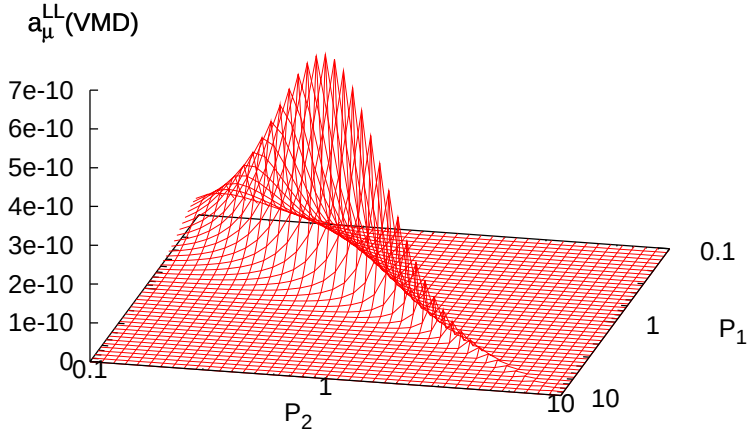
- The pointlike vertex implements shortdistance part, not only π^0-exchange
- 

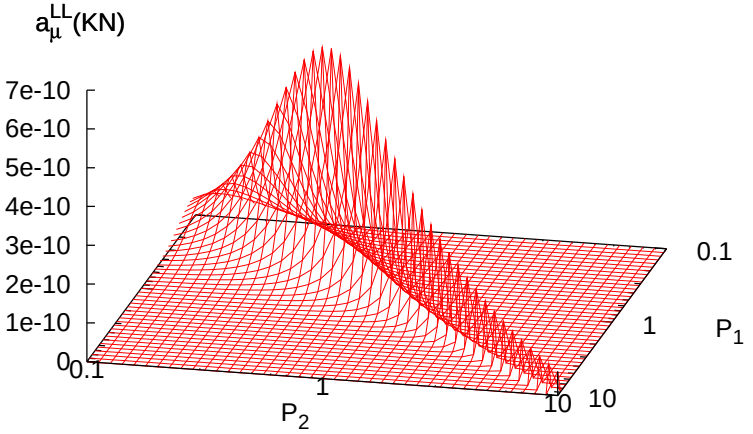
  Are these part of the quark-loop? See also in

  Dorokhov,Broniowski, Phys.Rev. D78(2008)07301
- BPP quarkloop + π^0-exchange $\approx$ MV π^0-exchange

π^0 exchange

- Which momentum regimes important studied: JB and J. Prades, Mod. Phys. Lett. A **22** (2007) 767 [hep-ph/0702170]
- $a_\mu = \int dl_1 dl_2 a_\mu^{LL}$ with $l_i = \log(P_i/GeV)$





Checking which momentum regions do what (but would need three dimensional)

Pseudoscalar exchange

- Point-like VMD: π^0 η and η' give 5.58, 1.38, 1.04.
- Models that include $U(1)_A$ breaking give similar ratios
- Pure large N_c models use this ratio
- The MV argument should give some enhancement over the full VMD like models
- Total pseudo-scalar exchange is about $a_\mu^{PS} = 8 - 10 \times 10^{-10}$
- AdS/QCD estimate (includes excited pseudo-scalars) $a_\mu^{PS} = 10.7 \times 10^{-10}$

  D. K. Hong and D. Kim, Phys. Lett. B **680** (2009) 480 [arXiv:0904.4042 [hep-ph]]

Axial-vector exchange exchange

Cut-off Λ (GeV)	$a_\mu \times 10^{10}$ from Axial-Vector Exchange $\mathcal{O}(N_c)$
0.5	0.05(0.01)
0.7	0.07(0.01)
1.0	0.13(0.01)
2.0	0.24(0.02)
4.0	0.59(0.07)

There is some pseudo-scalar exchange piece here as well, off-shell not quite clear what is what.

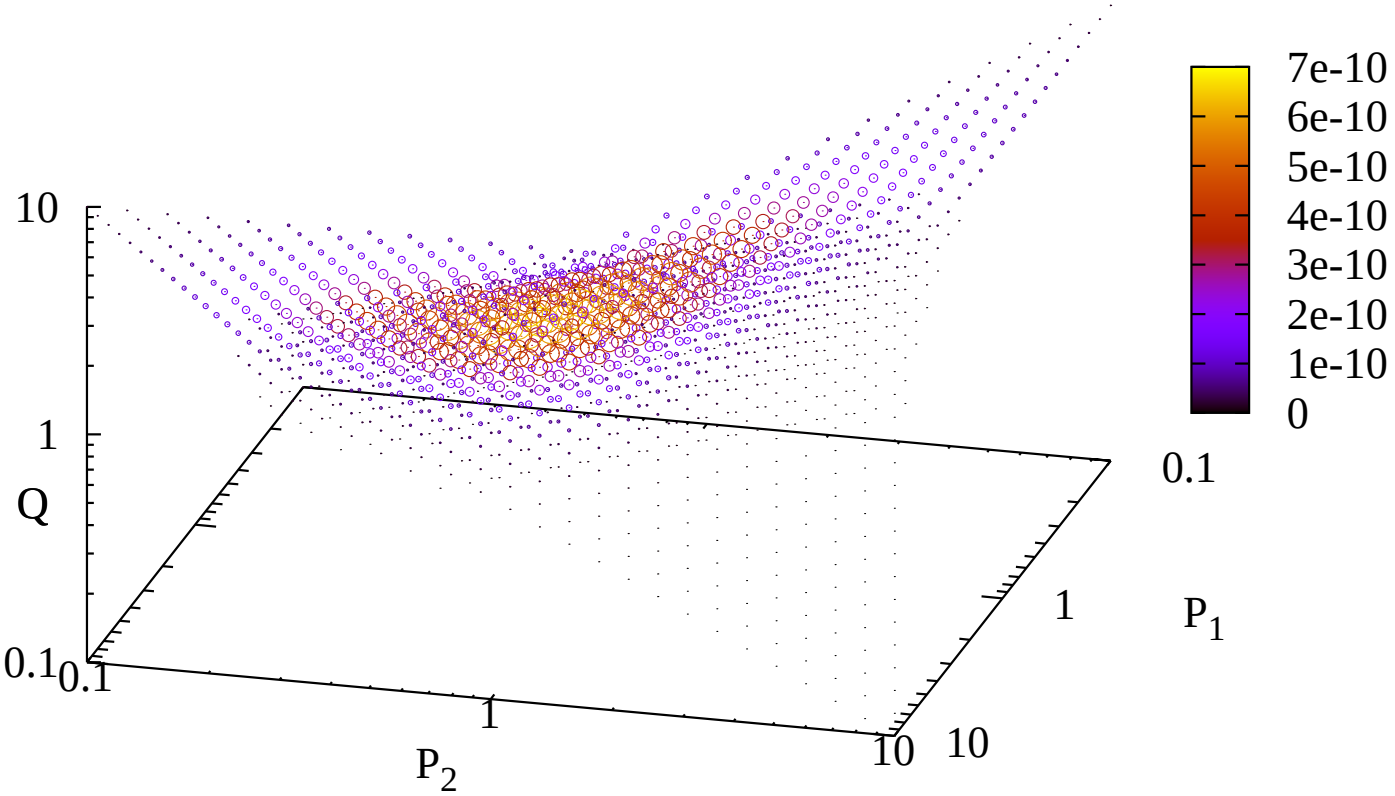
- $a_\mu^{\text{axial}} = 0.6 \times 10^{-10}$
- MV: short distance enhancement + mixing (both enhance about the same)
  $a_\mu^{\text{axial}} = 2.2 \times 10^{-10}$

Pure quark loop

Cut-off Λ (GeV)	$a_\mu \times 10^7$ Electron Loop	$a_\mu \times 10^9$ Muon Loop	$a_\mu \times 10^9$ Constituent Quark Loop
0.5	2.41(8)	2.41(3)	0.395(4)
0.7	2.60(10)	3.09(7)	0.705(9)
1.0	2.59(7)	3.76(9)	1.10(2)
2.0	2.60(6)	4.54(9)	1.81(5)
4.0	2.75(9)	4.60(11)	2.27(7)
8.0	2.57(6)	4.84(13)	2.58(7)
Known Results	2.6252(4)	4.65	2.37(16)

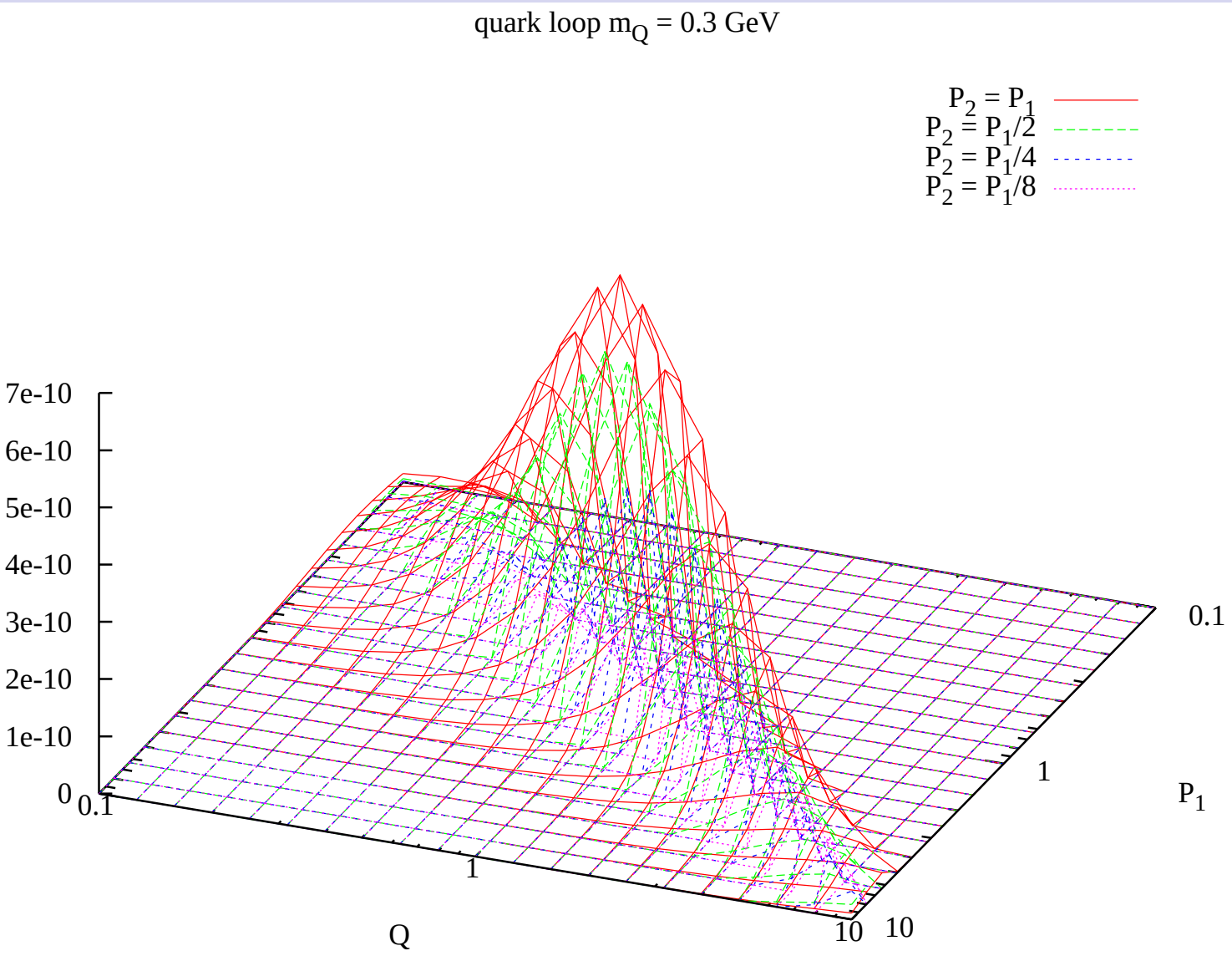
- M_Q : 300 MeV
- now known fully analytically
- Us: 5+(3-1) integrals extra are Feynman parameters
- Slow convergence:
  - electron: all at 500 MeV
  - Muon: only half at 500 MeV, at 1 GeV still 20% missing
  - 300 MeV quark: at 2 GeV still 25% missing

Pure quark loop: momentum area



- This plots $a_\mu^{\rm ql} = \int dl_{P_1} dl_{P_2} dl_Q \, a_\mu^{\rm LLQ}$
- Succeeded in 3D plot but was useless
- JB-Zahiri-Abyaneh, work in progress

Pure quark loop: momentum area



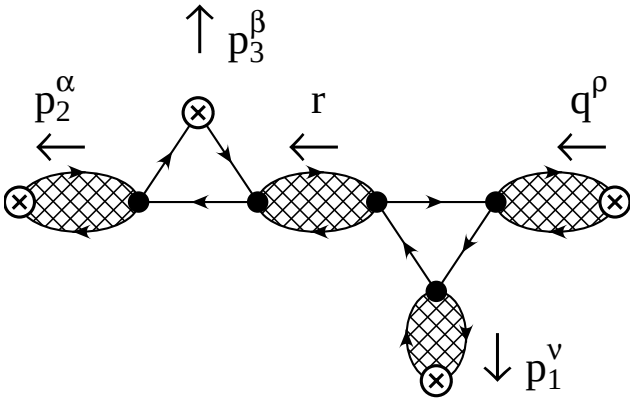
Most from $P_1 \approx P_2 \approx Q$, sizable large momentum part

ENJL quark-loop

Cut-off Λ GeV	$a_\mu \times 10^{10}$ VMD	$a_\mu \times 10^{10}$ ENJL	$a_\mu \times 10^{10}$ masscut	$a_\mu \times 10^{10}$ sum ENJL+masscut
0.5	0.48	0.78	2.46	3.2
0.7	0.72	1.14	1.13	2.3
1.0	0.87	1.44	0.59	2.0
2.0	0.98	1.78	0.13	1.9
4.0	0.98	1.98	0.03	2.0
8.0	0.98	2.00	.005	2.0

- Very stable
- ENJL cuts off slower than pure VMD
- masscut: $M_Q = \Lambda$ to have short-distance and no problem with momentum regions
- Quite stable in region 1-4 GeV

ENJL: scalar



- $$\Pi^{\rho\nu\alpha\beta} = \overline{\Pi}^{VVS}_{ab}(p_1, r) g_S \left(1 + g_S \Pi^S(r)\right) \overline{\Pi}^{SVV}_{cd}(p_2, p_3) \mathcal{V}^{abcd\rho\nu\alpha\beta} + \text{permutations}$$
- $g_S\left(1 + g_S \Pi_S\right) = \frac{g_A(r^2)(2M_Q)^2}{2f^2(r^2)} \frac{1}{M_S^2(r^2) - r^2}$
- $\mathcal{V}^{abcd\rho\nu\alpha\beta}$: ENJL VMD legs
- In ENJL only scalar+quark-loop properly chiral invariant

ENJL: scalar/QL

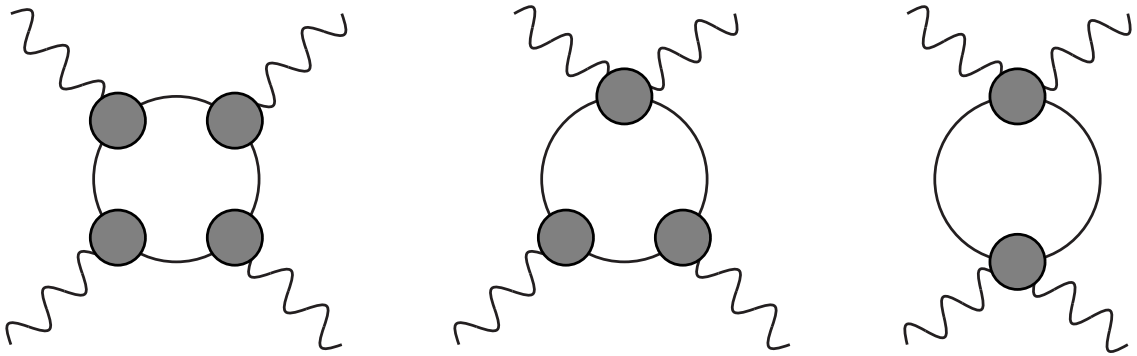
Cut-off Λ GeV	$a_\mu \times 10^{10}$ Quark-loop VMD	$a_\mu \times 10^{10}$ Quark-loop ENJL	$a_\mu \times 10^{10}$ Scalar Exchange
0.5	0.48	0.78	−0.22
0.7	0.72	1.14	−0.46
1.0	0.87	1.44	−0.60
2.0	0.98	1.78	−0.68
4.0	0.98	1.98	−0.68
8.0	0.98	2.00	−0.68

- Note: ENJL+scalar (BPP) $\approx$ Quark-loop VMD (HKS)
- $M_S \approx 620$ MeV certainly an overestimate for real scalars
- If scalar is σ: related to pion loop part?
- quark-loop: $a_\mu^{ql} \approx 1 \times 10^{-10}$ bare $a_\mu^{ql} = 2.37 \times 10^{-10}$

Quark loop DSE

- DSE model: $a_\mu^{ql} = 13.6(5.9) \times 10^{-10}$ T. Goecke, C. S. Fischer and R. Williams, Phys. Rev. D **83** (2011) 094006 [arXiv:1012.3886 [hep-ph]]
- Not a full calculation (yet) but includes an estimate of some of the missing parts
- Note: a lot larger than bare quark loop with constituent mass
- I am puzzled: this DSE model (Maris-Roberts) does reproduce a lot of low-energy phenomenology. I would have guessed that it would be very similar to ENJL in its results.
- Can one find something in between full DSE and ENJL that is easier to handle?
- Error found in calculation, still not finalized: preliminary $a_\mu^{ql} = 10.7(0.2) \times 10^{-10}$ T. Goecke, C. S. Fischer and R. Williams, arXiv:1210.1759 [hep-ph]

π and K-loop



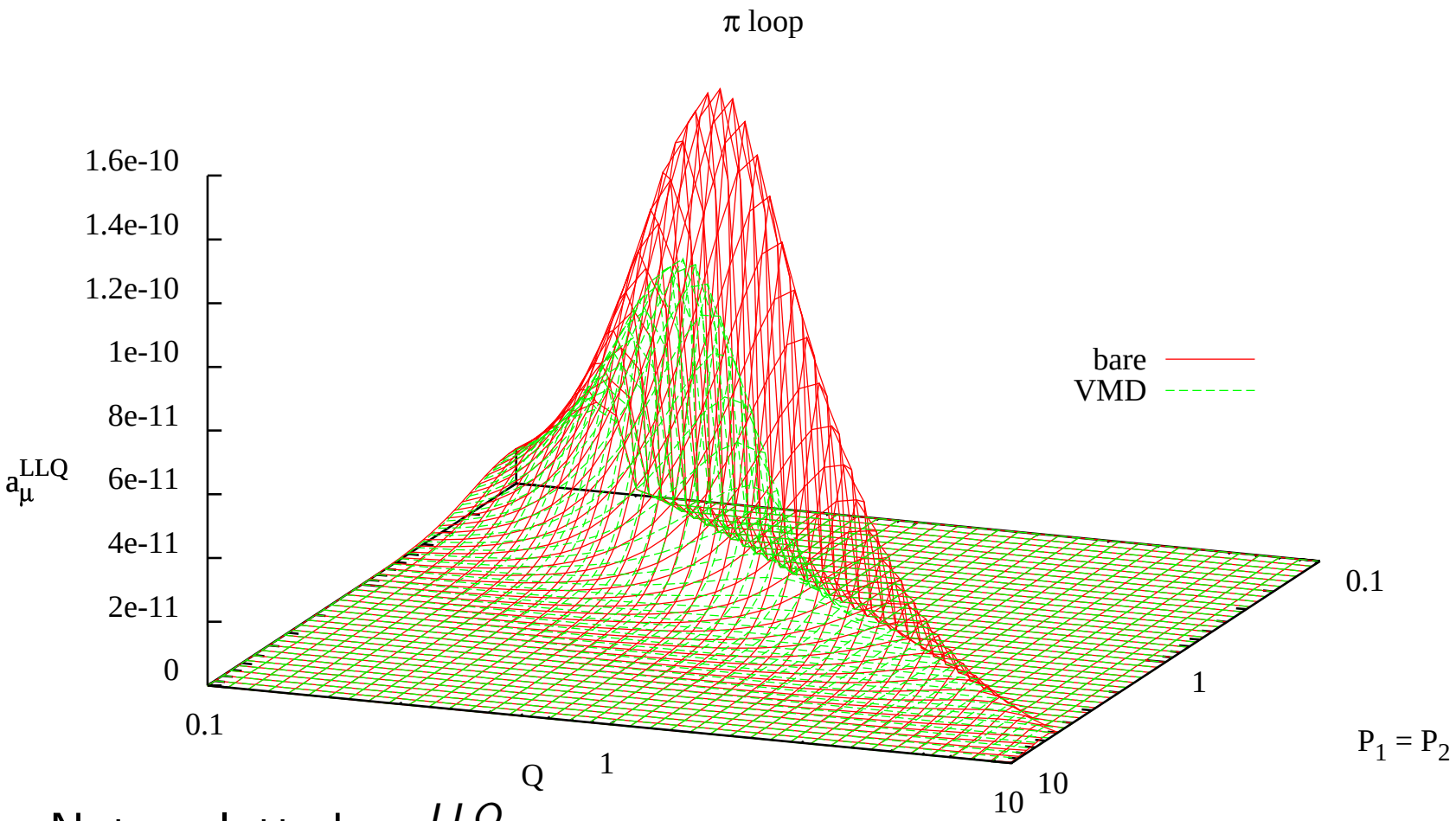
- The $\pi\pi\gamma^*$ vertex is always done using VMD
- $\pi\pi\gamma^*\gamma^*$ vertex two choices:
  - Hidden local symmetry model: only one γ has VMD
  - Full VMD
  - Both are chirally symmetric
  - Check if they live up to MV short distance (Full VMD does, HLS does not)
  - The HLS model used has problems with π^+-π^0 mass difference (due to not having an a_1)
- Final numbers quite different: -0.045 and -0.19
- For BPP stopped at 1 GeV but within 10% of higher Λ

π and K-loop

Cut-off	$10^{10} a_\mu$				
GeV	π bare	π VMD	π ENJL	π HLS	K ENJL
0.5	$-1.71(7)$	$-1.16(3)$	$-1.20(0.03)$	$-1.05(0.01)$	$-0.020(0.001)$
0.6	$-2.03(8)$	$-1.41(4)$	$-1.42(0.03)$	$-1.15(0.01)$	$-0.026(0.001)$
0.7	$-2.41(9)$	$-1.46(4)$	$-1.56(0.03)$	$-1.17(0.01)$	$-0.034(0.001)$
0.8	$-2.64(9)$	$-1.57(6)$	$-1.67(0.04)$	$-1.16(0.01)$	$-0.042(0.001)$
1.0	$-2.97(12)$	$-1.59(15)$	$-1.81(0.05)$	$-1.07(0.01)$	$-0.048(0.002)$
2.0	$-3.82(18)$	$-1.70(7)$	$-2.16(0.06)$	$-0.68(0.01)$	$-0.087(0.005)$
4.0	$-4.12(18)$	$-1.66(6)$	$-2.18(0.07)$	$-0.50(0.01)$	$-0.099(0.005)$

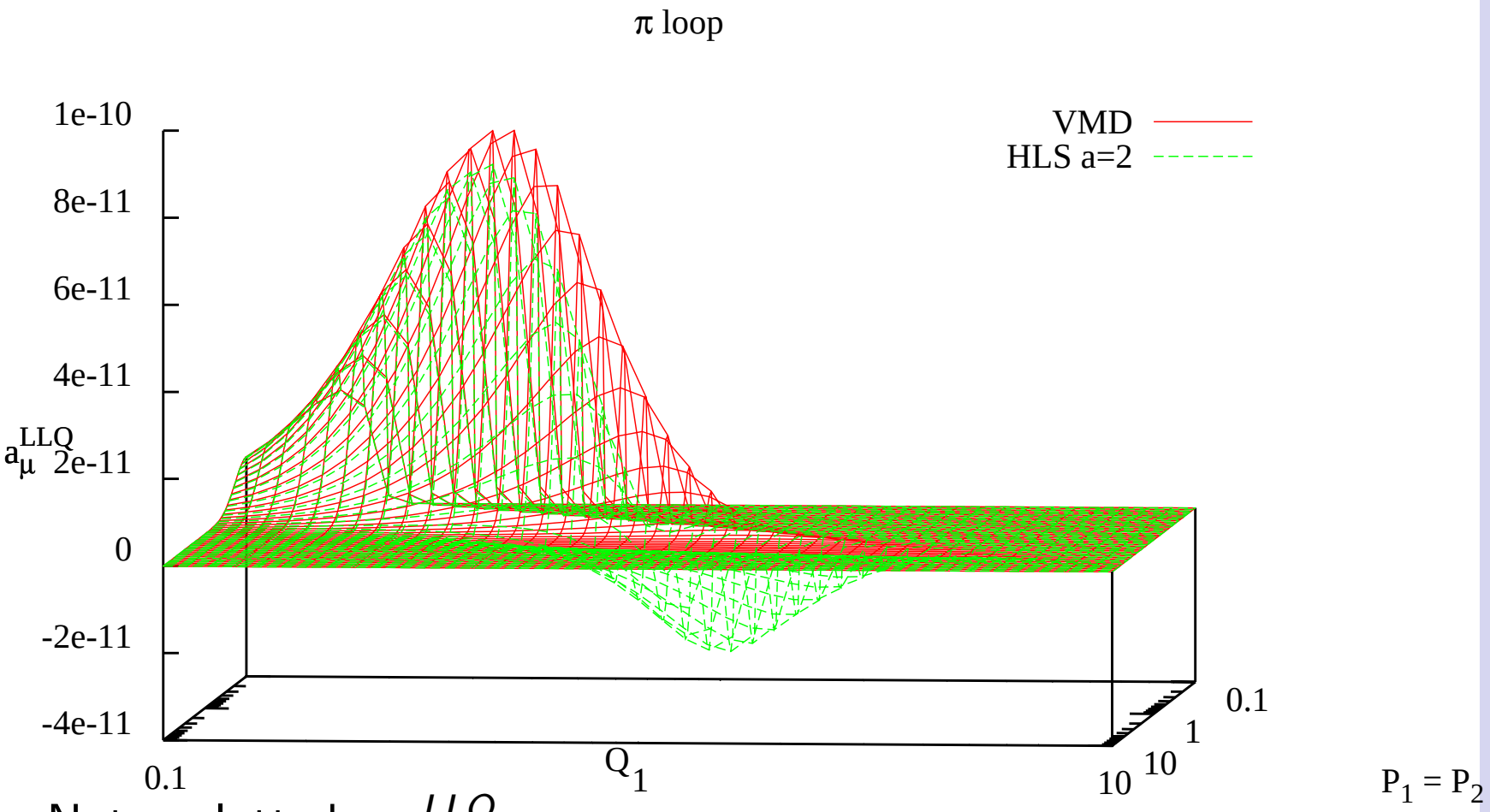
- HLS JB-Zahiri-Abyaneh
- note the suppression by the propagators

π loop: Bare vs VMD



Note: plotted $-a_\mu^{LLQ}$

π loop: VMD vs HLS



Note: plotted $-a_\mu^{LLQ}$

π loop

- $\pi\pi\gamma^*\gamma^*$ for $q_1^2 = q_2^2$ has a short-distance constraint from the OPE as well.
- HLS does not satisfy it
- full VMD does: so probably better estimate
- Ramsey-Musolf suggested to do pure ChPT for the π loop
  K. T. Engel, H. H. Patel and M. J. Ramsey-Musolf, "Hadronic light-by-light scattering and the pion polarizability," Phys. Rev. D **86** (2012) 037502 [arXiv:1201.0809 [hep-ph]].
- So far ChPT at p^4 done for four-point function in limit $p_1, p_2, q \ll m_\pi$ (Euler-Heisenberg plus next order)
- Polarizability $(L_9 + L_{10})$ up to 10%, charge radius 30%
- Both HLS and VMD have charge radius effect but not polarizability

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π loop: L_9, L_{10}

- ChPT for muon $g-2$ at order p^6 is not powercounting finite so no prediction for a_μ exists.
- But can be used to study the low momentum end of the integral over P_1, P_2, Q
- The four-photon amplitude is finite still at two-loop order (counterterms start at order p^8)
- Add L_9 and L_{10} vertices to the bare pion loop JB-Zahiri-Abyaneh
- Program the Euler-Heisenberg plus NLO result of Ramsey-Musolf et al. into our programs for a_μ
- Bare pion-loop and L_9, L_{10} part in limit $p_1, p_2, q \ll m_\pi$ agree with Euler-Heisenberg plus next order analytically
- Numerics very preliminary

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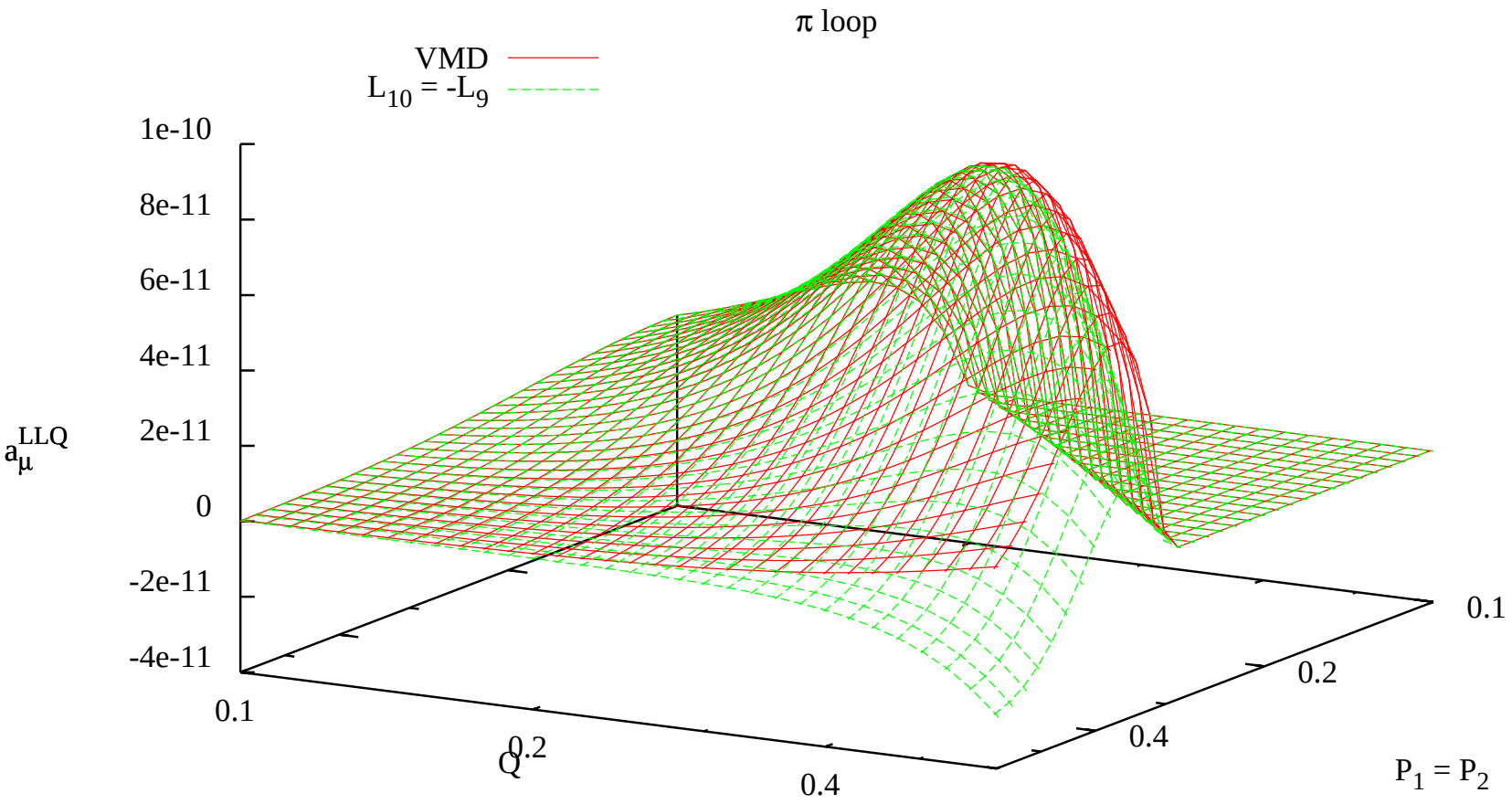
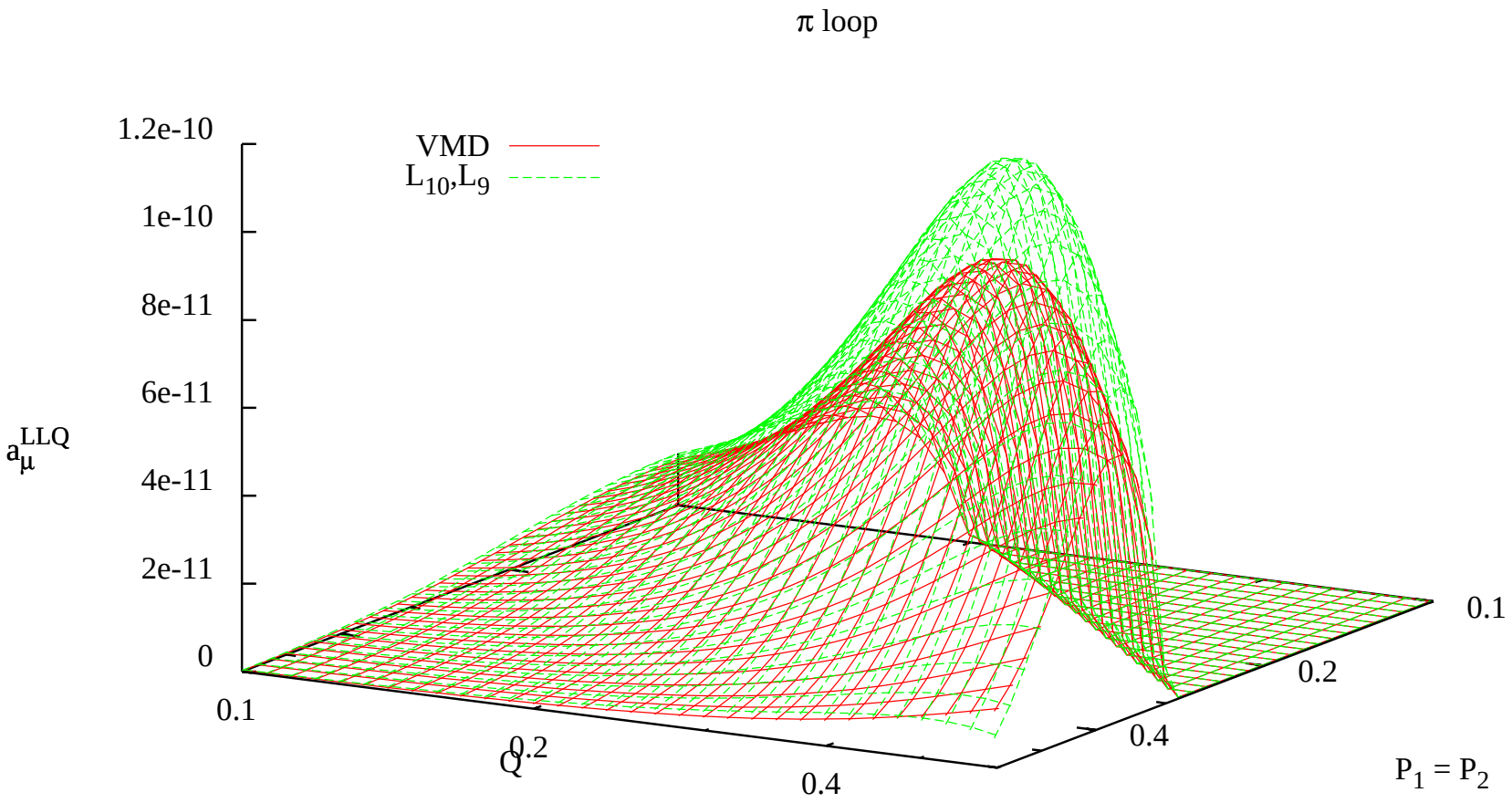
π loop: VMD vs charge radius



Note: plotted $-a_\mu^{LLQ}$, low scale, charge radius effect well reproduced

π loop: VMD vs L_9 and L_{10}



Note: plotted $-a_\mu^{LLQ}$, gives an enhancement of 10-15%

Work in progress

- add the a_1 to the pion loop.
- JB-Relefors in progress
- Use antisymmetric (axial-)vector formalism: easier to add different terms
- $\mathcal{L} = \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle + \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f^{\mu\nu} \rangle$
  $\rho\pi\pi$ and ρ-γ couplings
- Finite result for $g-2$ requires $G_V = F_V/2$
- Then same results as HLS with $a = F_V^2/F^2$

Summary: ENJL vc PdRV

	BPP	PdRV arXiv:0901.0306
quark-loop	$(2.1 \pm 0.3) \cdot 10^{-10}$	—
pseudo-scalar	$(8.5 \pm 1.3) \cdot 10^{-10}$	$(11.4 \pm 1.3) \cdot 10^{-10}$
axial-vector	$(0.25 \pm 0.1) \cdot 10^{-10}$	$(1.5 \pm 1.0) \cdot 10^{-10}$
scalar	$(-0.68 \pm 0.2) \cdot 10^{-10}$	$(-0.7 \pm 0.7) \cdot 10^{-10}$
πK-loop	$(-1.9 \pm 1.3) \cdot 10^{-10}$	$(-1.9 \pm 1.9) \cdot 10^{-10}$
errors	linearly	quadratically
sum	$(8.3 \pm 3.2) \cdot 10^{-10}$	$(10.5 \pm 2.6) \cdot 10^{-10}$

What can we do more?

- The ENJL model can certainly be improved:
  - Chiral nonlocal quark-model (like nonlocal ENJL): so far only π^0-exchange done
  - DSE: π^0-exchange similar to everyone else, quark-loop very different, looking forward to final results
- More resonances models should be tried, AdS/QCD is one approach, RχT (Valencia *et al.*) possible,. . .
- Note short-distance matching must be done in many channels, there are theorems JB,Gamiz,Lipartia,Prades that with only a few resonances this requires compromises
- π-loop: HLS smaller than double VMD (understood) models with ρ and a_1 (in progress)

What can we do more?

- Constraints from experiment:

  J. Bijnens and F. Persson, hep-ph/hep-ph/0106130
  Studying three formfactors $P\gamma^*\gamma^*$ in $P \to \ell^+\ell^-\ell'^+\ell'^-$, $e^+e^- \to e^+e^-P$ exact tree level and for $g-2$ (but beware sign):
  - Conclusion: possible but VERY difficult
  - Two γ^* off-shell not so important for our choice of form-factor
- All information on hadrons and 1-2-3-4 off-shell photons is welcome: constrain the models
- More short-distance constraints: MV, Nyffeler integrate with all contributions, not just π^0-exchange
- Need a new overall evaluation with consistent approach.
- Lattice has done first steps

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Summary of Muon $g-2$ contributions

	$10^{10} a_\mu$	
exp	11 659 208.9	6.3
theory	11 659 180.2	5.0
QED	11 658 471.8	0.0
EW	15.4	0.2
LO Had	692.3	4.2
HO HVP	−9.8	0.1
HLbL	10.5	2.6
difference	28.7	8.1

- Error on LO had
  all e^+e^- based OK
  τ based 2 σ
- Error on HLbL
- Errors added quadratically
- 3.5 σ
- Difference:
  4% of LO Had
  270% of HLbL
  1% of leptonic LbL

Generic SUSY: $12.3 \times 10^{-10} \left(\frac{100\ \mathrm{GeV}}{M_{SUSY}}\right)^2 \tan\beta$

$M_{SUSY} \approx 66\ \mathrm{GeV}\sqrt{\tan\beta}$