HADRNIC LIGHT-BY-LIGHT FOR THE
MUON ANOMALOUS MAGNETIC MOMENT

Study of Strongly Interacting Matter
HadronPhysics

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- **Final experimental paper:**

- **Review 1:**

- **Review 2:**

- **Review 3:**
Literature

Lectures:

“Final” HLbL number:
Muon $g - 2$: measurement

**LIFE OF A MUON: THE g-2 EXPERIMENT**

- Protons from AGS.
- Hit Target.
- Pions, weighing 1/6 proton, are created.
- Muons are tiny magnets spinning on axis like tops.
- Pions decay to muons.
- Muons are fed into a uniform, doughnut-shaped magnetic field and travel in a circle.
- After each circle, muon's spin axis changes by 12°, yet it keeps on traveling in the same direction.
- One of 24 detectors see an electron, giving the muon spin direction; $g-2$ is this angle, divided by the magnetic field the muon is traveling through in the ring.
- After circling the ring many times, muons spontaneously decay to electron, (plus neutrinos,) in the direction of the muon spin.
Muon $g - 2$: measurement
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Muon $g - 2$: measurement


2001: $\mu^-$, others $\mu^+$
Muon $g - 2$: overview

- in terms of the anomaly $a_\mu = (g - 2)/2$
- Data dominated by BNL E821 (statistics)(systematic)
  \[ a_{\mu^+}^{\text{exp}} = 11659204(6)(5) \times 10^{-10} \]
  \[ a_{\mu^-}^{\text{exp}} = 11659215(8)(3) \times 10^{-10} \]
  \[ a_\mu^{\text{exp}} = 11659208.9(5.4)(3.3) \times 10^{-10} \]
- Theory is off somewhat (electroweak)(LO had)(HO had)
  \[ a_\mu^{\text{SM}} = 11659180.2(0.2)(4.2)(2.6) \times 10^{-10} \]
- $\Delta a_\mu = a^{\text{exp}}_\mu - a^{\text{SM}}_\mu = 28.7(6.3)(4.9) \times 10^{-10}$ (PDG)
- E821 goes to Fermilab, expect factor of four in precision
- Note: $g$ agrees to $10^{-8}$ with theory
- Many BSM models **CAN** predict a value in this range
  (often a lot more or a lot less)
- Numbers taken from PDG2012, see references there
Muon $g - 2$: QED

\[ a_\mu^{\text{QED}} = \frac{\alpha}{2\pi} + 0.765857410(27) \left( \frac{\alpha}{\pi} \right)^2 + 24.05050964(43) \left( \frac{\alpha}{\pi} \right)^3 
+ 130.8055(80) \left( \frac{\alpha}{\pi} \right)^4 + 663(20) \left( \frac{\alpha}{\pi} \right)^5 + \ldots \]

- First three loops known analytically
- Four-loops fully done numerically
- Five loops estimate
- Kinoshita, Laporta, Remiddi, Schwinger, …
- $\alpha$ fixed from the electron $g - 2$: $\alpha = 1/137.035999084(51)$
- $a_\mu^{\text{QED}} = 11658471.809(0.015) \times 10^{-10}$
- Light-by-light surprisingly large: $2670 \times 10^{-10}$

\[ e = 20.95, \mu = 0.37, \tau = 0.002 \]
Muon $g - 2$: Electroweak

- $a_{\mu}^{\text{EW}}[1\text{-loop}] = 19.48 \times 10^{-10}$
- $a_{\mu}^{\text{EW}}[2\text{-loop}] = -4.07(0.10)(0.18) \times 10^{-10}$
- $a_{\mu}^{\text{EW}} = 15.4(0.1)(0.2) \times 10^{-10}$ (triangle)(Higgs mass)
- 2-loop large because of the large logarithms in the triangle anomaly present here
- SM anomaly cancellation very needed
- “anomaly” as part of the “anomaly”
Muon $g - 2$: LO hadronic

\[ a_{\mu}^{\text{LO had}} = \frac{1}{3} \left( \frac{\alpha}{\pi} \right)^2 \int_{m^2_{\pi}}^{\infty} ds \frac{K(s)}{s} R^{(0)}(s) \]

- $R^{(0)}(s)$ bare cross-section ratio
- $\sigma(e^+ e^- \rightarrow \text{hadrons}) / \sigma(e^+ e^- \rightarrow \mu^+ \mu^-)$
- Bare, many different evaluations, ...
- $e^+ e^-$ versus $\tau$-decays
- $a_{\mu}^{\text{LO Had}} = 692.3(4.2)(0.3) \times 10^{-10}$ (exp)(pert. QCD)
LO hadronic

LO hadronic

LO hadronic

Contribution

Error


• Direct measurements
• Radiative return: radiate hard photon to get lower $E_{cms}$
• Radiative corrections
• $\tau$-decay: isospin breaking
• can be improved experimentally
A comment on models

- \( a_{\mu}^{\text{LOHad}} = 692.3(4.2) \times 10^{-10} \)

- Rewrite: \( a_{\mu}^{\text{LOHad}} = 8\alpha^2 \int \frac{ds}{s} K(s) \frac{1}{\pi} \text{Im} \Pi(s) \)

- Rho: \( \frac{1}{\pi} \text{Im} \Pi(s) = \frac{2}{3} f_V^2 M_V^2 \delta(s - M_V^2) + \frac{2}{3} \frac{N_c}{12\pi^2} \theta(s - s_0) \)

- \( K(s) \approx \frac{m_\mu^2}{3t} \ (s \gg m_\mu^2) \)

- \( a_{\mu}^{\text{LOHad}} = \frac{16}{9} \alpha^2 \left( f_V^2 \frac{m_\mu^2}{M_V^2} + \frac{N_c}{12\pi^2} \frac{m_\mu^2}{s_0} \right) \)

- Put in \( f_V \approx 0.17, \ M_V \approx 0.77 \text{ GeV} \) and \( s_0 \approx 1.35 \text{ GeV}^2 \)

- \( a_{\mu}^{\text{LOHad}} \approx (450 + 95 \approx 545) \times 10^{-10} \)

- Many models have this within 30%
Muon $g - 2$: HO hadronic

- Two main types of contributions

- HO HVP is like LO Had but a more complicated function
  \[ K(s) \, a_{\mu}^{\text{HO HVP}} = -9.84(0.06) \times 10^{-10} \]

- HLbL is the real problem: best estimate now:
  \[ a_{\mu}^{\text{HLbL}} = 10.5(2.6) \times 10^{-10} \]

- Note that the sum is very small: but not an indication of the error
Summary of Muon $g - 2$ contributions

<table>
<thead>
<tr>
<th></th>
<th>$10^{10} a_\mu$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>exp</td>
<td>11 659 208.9</td>
<td>6.3</td>
</tr>
<tr>
<td>theory</td>
<td>11 659 180.2</td>
<td>5.0</td>
</tr>
<tr>
<td>QED</td>
<td>11 658 471.8</td>
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<tr>
<td>EW</td>
<td>15.4</td>
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<tr>
<td>LO Had</td>
<td>692.3</td>
<td>4.2</td>
</tr>
<tr>
<td>HO HVP</td>
<td>$-9.8$</td>
<td>0.1</td>
</tr>
<tr>
<td>HLbL</td>
<td>10.5</td>
<td>2.6</td>
</tr>
<tr>
<td>difference</td>
<td>28.7</td>
<td>8.1</td>
</tr>
</tbody>
</table>

- Error on LO had all $e^+ e^-$ based OK
- $\tau$ based 2 $\sigma$
- Error on HLbL
- Errors added quadratically
- 3.5 $\sigma$
- Difference: 4% of LO Had
- 270% of HLbL
- 1% of leptonic LbL

Generic SUSY: $12.3 \times 10^{-10} \left( \frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \tan \beta$

$M_{\text{SUSY}} \approx 66 \text{ GeV} \sqrt{\tan \beta}$
Our object

- Muon line and photons: well known
- The blob: fill in with hadrons/QCD
- Trouble: low and high energy very mixed
- Double counting needs to be avoided: hadron exchanges versus quarks
A separation proposal: a start


- Use ChPT $p$ counting and large $N_c$
  - $p^4$, order 1: pion-loop
  - $p^8$, order $N_c$: quark-loop and heavier meson exchanges
  - $p^6$, order $N_c$: pion exchange

Does not fully solve the problem
only short-distance part of quark-loop is really $p^8$
but it’s a start
A separation proposal: a start


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  - $p^6$, order $N_c$: pion exchange

Implemented by two groups in the 1990s:

- Hayakawa, Kinoshita, Sanda: meson models, pion loop using hidden local symmetry, quark-loop with VMD, calculation in Minkowski space
- JB, Pallante, Prades: Try using as much as possible a consistent model-approach, calculation in Euclidean space
Papers: BPP and HKS

- **JB, E. Pallante and J. Prades**

- **Hayakawa, Kinoshita, (Sanda)**
Differences

- **HK(S)**
  - Purely hadronic exchanges
  - quark-loop with hadronic VMD
  - Studied dependence of everything on $m_V$

- **BPP**
  - Use the ENJL as an overall model to have a similar uncertainty on all low-energy parts
  - repair some of the worst short-comings
  - Add the short-distance quark-loop
  - Study of cut-off dependence

- **Sign mistake**
  - HKS: Euclidean versus Minkowski $\varepsilon^{\mu\nu\alpha\beta}$
  - BPP: notes all correct sign, program had wrong sign, probably minus sign from fermion loop not removed
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The overall

\[ a_{\mu}^{\text{HLbL}} = \frac{-1}{48 m_\mu} \text{tr}[ (\not{p} + m_\mu) M^{\lambda\beta}(0) (\not{p} + m_\mu) [\gamma_\lambda, \gamma_\beta] ]. \]

\[ M^{\lambda\beta}(p_3) = |e|^6 \int \frac{d^4 p_1}{(2\pi)^4} \int \frac{d^4 p_2}{(2\pi)^4} \frac{1}{q^2 p_1^2 p_2^2 (p_4^2 - m_\mu^2)(p_5^2 - m_\mu^2)} \times \left[ \frac{\delta \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}} \right] \gamma_\alpha(p_4 + m_\mu) \gamma_\nu(p_5 + m_\mu) \gamma_\rho. \]

- We used: \( \Pi^{\rho\nu\alpha\lambda}(p_1, p_2, p_3) = -p_{3\beta} \frac{\delta \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}. \)
- Can calculate at \( p_3 = 0 \) but must take derivative
- derivative in quark-loop: each permutation finite
- Four point function of \( V_i^\mu(x) \equiv \sum_i Q_i \left[ \bar{q}_i(x) \gamma^\mu q_i(x) \right] \)

\[ \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3) \equiv i^3 \int d^4 x \int d^4 y \int d^4 z e^{i(p_1 \cdot x + p_2 \cdot y + p_3 \cdot z)} \times \left\langle 0 \right| T \left( V_\alpha^\rho(0) V_\nu^\nu(x) V_\lambda^\alpha(y) V_\beta^\beta(z) \right) \left| 0 \right\rangle \]
General properties

\[ \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3): \]

- In general 138 Lorentz structures (but only 32 contribute to \( g - 2 \))
- Using \( q_{\rho} \Pi^{\rho\nu\alpha\beta} = p_{1\nu} \Pi^{\rho\nu\alpha\beta} = p_{2\alpha} \Pi^{\rho\nu\alpha\beta} = p_{3\beta} \Pi^{\rho\nu\alpha\beta} = 0 \)
- 43 gauge invariant structures
- Bose symmetry relates some of them
- All depend on \( p_1^2, p_2^2 \) and \( q^2 \), but before derivative and \( p_3 \to 0 \) also \( p_3^2, p_1 \cdot p_2, p_1 \cdot p_3, p_2 \cdot p_3 \)
- Compare HVP: one function, one variable
- General calculation from experiment difficult to see how
- In four photon measurement: lepton contribution
General properties

\[ \int \frac{d^4 p_1}{(2\pi)^4} \int \frac{d^4 p_2}{(2\pi)^4} \] plus loops inside the hadronic part

- 8 dimensional integral, three trivial,
- 5 remain: \( p_1^2, p_2^2, p_1 \cdot p_2, p_1 \cdot p_\mu, p_2 \cdot p_\mu \)
- Rotate to Euclidean space:
  - Easier separation of long and short-distance
  - Artefacts (confinement) in models smeared out.

More recent: can do two more using Gegenbauer techniques Knecht-Nyffeler, Jegerlehner-Nyffeler, JB–Zahiri-Abyaneh–Relefors

\( P_1^2, P_2^2 \) and \( Q^2 \) remain

study \( a_\mu^X = \int dl_{P_1} dl_{P_2} a_\mu^{XLL} = \int dl_{P_1} dl_{P_2} dl_{Q} a_\mu^{XLLQ} \)

\( l_P = \ln \left( P/\text{GeV} \right) \), to see where the contributions are
General properties

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ENJL: our main model

\[ \mathcal{L}_{\text{ENJL}} = \bar{q}^\alpha \left\{ i \gamma^\mu \left( \partial_\mu - i v_\mu - i a_\mu \gamma_5 \right) - (\mathcal{M} + s - i p \gamma_5) \right\} q^\alpha \]
\[ + 2 g_S \left( \bar{q}_R^\alpha q_L^\beta \right) \left( \bar{q}_L^\beta q_R^\alpha \right) \]
\[ - g_V \left[ \left( \bar{q}_L^\alpha \gamma^\mu q_L^\beta \right) \left( \bar{q}_L^\beta \gamma_\mu q_L^\alpha \right) + \left( \bar{q}_R^\alpha \gamma^\mu q_R^\beta \right) \left( \bar{q}_R^\beta \gamma_\mu q_R^\alpha \right) \right] \]

\bullet \bar{q} \equiv (\bar{u}, \bar{d}, \bar{s})

\bullet v_\mu, a_\mu, s, p: \text{external vector, axial-vector, scalar and pseudoscalar matrix sources}

\bullet \mathcal{M} \text{ is the quark-mass matrix.}

\bullet g_V \equiv \frac{8 \pi^2 G_V(\Lambda)}{N_c \Lambda^2}, \quad g_S \equiv \frac{4 \pi^2 G_S(\Lambda)}{N_c \Lambda^2}.

\bullet G_V, G_S \text{ are dimensionless and valid up to } \Lambda

\bullet \text{No confinement but has good pion, vector meson and OK axial vector-meson phenomenology}
**ENJL: our main model**


- Gap equation: chiral symmetry spontaneously broken

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\[ \text{Generates poles, i.e. mesons via bubble resummation} \]

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ENJL: our main model

- Can be thought of as a very simple rainbow and ladder approximation in the DSE equation with constant kernels for the one-gluon exchange
- Parameters fit via $F_{\pi}$, $L_i^r$, vector meson properties, ...
- $G_S = 1.216$, $G_V = 1.263$, $\Lambda = 1.16$ GeV
- has $M_Q = 263$ MeV
- Has a number of decent matchings to short-distance, e.g. $\Pi_V - \Pi_A$ but fails in others.
- Generates always VMD in external legs (but with a twist)
- Hook together general processes by one-loop vertices and bubble-chain propagators
Separation of contributions

- Quark loop with external bubble-chains
- \( \approx \) Quark-loop with VMD

- Also internal bubble chain
- \( \approx \) meson exchange
- Note that vertices have structure
- Off-shell effect in model included
\[ \pi^0 \text{ exchange} \]

- \[ "\pi^0" = \frac{1}{(p^2 - m_{\pi}^2)} \]
- The blobs need to be modelled, and in e.g. ENJL contain corrections also to the \( \frac{1}{(p^2 - m_{\pi}^2)} \)
- Pointlike has a logarithmic divergence
- Numbers \( \pi^0 \), but also \( \eta, \eta' \)
\( \pi^0 \) exchange

<table>
<thead>
<tr>
<th>Cutoff (GeV)</th>
<th>( a_\mu \times 10^{10} )</th>
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</thead>
<tbody>
<tr>
<td>0.5</td>
<td>4.92(2)</td>
</tr>
<tr>
<td>0.7</td>
<td>7.68(4)</td>
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<tr>
<td>1.0</td>
<td>11.15(7)</td>
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<td>2.0</td>
<td>21.3(2)</td>
</tr>
<tr>
<td>4.0</td>
<td>32.7(5)</td>
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Point-like ENJL–VMD

<table>
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<tr>
<th>Cutoff (GeV)</th>
<th>( a_\mu \times 10^{10} )</th>
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<tbody>
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<td>3.29(2)</td>
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<tr>
<td>0.7</td>
<td>4.24(4)</td>
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<td>1.0</td>
<td>4.90(5)</td>
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<td>2.0</td>
<td>5.63(8)</td>
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<tr>
<td>4.0</td>
<td>6.22(17)</td>
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Pointlike VMD

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<tr>
<td>0.7</td>
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<tr>
<td>1.0</td>
<td>5.18(3)</td>
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<tr>
<td>2.0</td>
<td>5.62(5)</td>
</tr>
<tr>
<td>4.0</td>
<td>5.58(5)</td>
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Transverse VMD

<table>
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<th>Cutoff (GeV)</th>
<th>( a_\mu \times 10^{10} )</th>
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<tbody>
<tr>
<td>0.5</td>
<td>3.60(3)</td>
</tr>
<tr>
<td>0.7</td>
<td>4.73(4)</td>
</tr>
<tr>
<td>1.0</td>
<td>5.61(6)</td>
</tr>
<tr>
<td>2.0</td>
<td>6.39(9)</td>
</tr>
<tr>
<td>4.0</td>
<td>6.59(16)</td>
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</tbody>
</table>

CELLO-VMD

<table>
<thead>
<tr>
<th>Cutoff (GeV)</th>
<th>( a_\mu \times 10^{10} )</th>
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<tbody>
<tr>
<td>0.5</td>
<td>3.53(2)</td>
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<tr>
<td>0.7</td>
<td>4.57(4)</td>
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<tr>
<td>1.0</td>
<td>5.29(5)</td>
</tr>
<tr>
<td>2.0</td>
<td>5.89(8)</td>
</tr>
<tr>
<td>4.0</td>
<td>6.02(10)</td>
</tr>
</tbody>
</table>

BPP: All in reasonable agreement \( a_\mu^{\pi^0} = 5.9 \times 10^{-10} \)
$\pi^0$ exchange

- **BPP:** $a_{\mu}^{\pi^0} = 5.9(0.9) \times 10^{-10}$
- **Nonlocal quark model:** $a_{\mu}^{\pi^0} = 6.27 \times 10^{-10}$
  

- **DSE model:** $a_{\mu}^{\pi^0} = 5.75 \times 10^{-10}$
  

- **LMD+V:** $a_{\mu}^{\pi^0} = (5.8 - 6.3) \times 10^{-10}$
  

- **Formfactor inspired by AdS/QCD:** $a_{\mu}^{\pi^0} = 6.54 \cdot 10^{-10}$
  

- **Constraint via magnetic susceptibility:** $a_{\mu}^{\pi^0} = 7.2 \times 10^{-10}$
  

All except possibly last in good agreement
MV short-distance: $\pi^0$ exchange

- take $P_1^2 \approx P_2^2 \gg Q^2$: Leading term in OPE of two vector currents is proportional to axial current
  \[
  \Pi^{\rho\nu\alpha\beta} \propto \frac{P_\rho}{P_1^2} \langle 0 | T (J_{A\nu} J_{V\alpha} J_{V\beta}) | 0 \rangle
  \]
- $J_A$ comes from
- AVV triangle anomaly: extra info
- Implemented via setting one blob = 1
  \[
  a_{\mu}^{\pi^0} = 7.7 \times 10^{-10}
  \]
$\pi^0$ exchange

- The pointlike vertex implements shortdistance part, not only $\pi^0$-exchange

\[ \begin{array}{c}
\pi^- + K^0 \\
\end{array} \]

Are these part of the quark-loop? See also in Dorokhov, Broniowski, Phys. Rev. D78(2008)07301

- BPP quarkloop + $\pi^0$-exchange $\approx$ MV $\pi^0$-exchange
π⁰ exchange


\[ a_\mu = \int dl_1 dl_2 a_{\mu LL}^{LL} \text{ with } l_i = \log(P_i/\text{GeV}) \]

Checking which momentum regions do what (but would need three dimensional)
Pseudoscalar exchange

- Point-like VMD: $\pi^0$, $\eta$ and $\eta'$ give 5.58, 1.38, 1.04.
- Models that include $U(1)_A$ breaking give similar ratios.
- Pure large $N_c$ models use this ratio.
- The MV argument should give some enhancement over the full VMD like models.
- Total pseudo-scalar exchange is about
  \[ a_{\mu}^{PS} = 8 - 10 \times 10^{-10} \]
- AdS/QCD estimate (includes excited pseudo-scalars)
  \[ a_{\mu}^{PS} = 10.7 \times 10^{-10} \]

Axial-vector exchange exchange

<table>
<thead>
<tr>
<th>Cut-off $\Lambda$ (GeV)</th>
<th>$a_\mu \times 10^{10}$ from Axial-Vector Exchange $\mathcal{O}(N_c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.05(0.01)</td>
</tr>
<tr>
<td>0.7</td>
<td>0.07(0.01)</td>
</tr>
<tr>
<td>1.0</td>
<td>0.13(0.01)</td>
</tr>
<tr>
<td>2.0</td>
<td>0.24(0.02)</td>
</tr>
<tr>
<td>4.0</td>
<td>0.59(0.07)</td>
</tr>
</tbody>
</table>

There is some pseudo-scalar exchange piece here as well, off-shell not quite clear what is what.

- $a_{\mu}^{\text{axial}} = 0.6 \times 10^{-10}$
- MV: short distance enhancement + mixing (both enhance about the same)
  
  $a_{\mu}^{\text{axial}} = 2.2 \times 10^{-10}$
Pure quark loop

<table>
<thead>
<tr>
<th>Cut-off $\Lambda$ (GeV)</th>
<th>$a_\mu \times 10^7$</th>
<th>$a_\mu \times 10^9$</th>
<th>$a_\mu \times 10^9$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Electron Loop</td>
<td>Muon Loop</td>
<td>Constituent Quark Loop</td>
</tr>
<tr>
<td>0.5</td>
<td>2.41(8)</td>
<td>2.41(3)</td>
<td>0.395(4)</td>
</tr>
<tr>
<td>0.7</td>
<td>2.60(10)</td>
<td>3.09(7)</td>
<td>0.705(9)</td>
</tr>
<tr>
<td>1.0</td>
<td>2.59(7)</td>
<td>3.76(9)</td>
<td>1.10(2)</td>
</tr>
<tr>
<td>2.0</td>
<td>2.60(6)</td>
<td>4.54(9)</td>
<td>1.81(5)</td>
</tr>
<tr>
<td>4.0</td>
<td>2.75(9)</td>
<td>4.60(11)</td>
<td>2.27(7)</td>
</tr>
<tr>
<td>8.0</td>
<td>2.57(6)</td>
<td>4.84(13)</td>
<td>2.58(7)</td>
</tr>
<tr>
<td>Known Results</td>
<td>2.6252(4)</td>
<td>4.65</td>
<td>2.37(16)</td>
</tr>
</tbody>
</table>

- $M_Q$: 300 MeV
- now known fully analytically
- Us: 5+(3-1) integrals extra are Feynman parameters
- Slow convergence:
  - electron: all at 500 MeV
  - Muon: only half at 500 MeV, at 1 GeV still 20% missing
  - 300 MeV quark: at 2 GeV still 25% missing
Pure quark loop: momentum area

This plots $a_{\mu}^{ql} = \int dl_{P_1} dl_{P_2} dl_Q a_{\mu}^{LLQ}$

Succeeded in 3D plot but was useless

JB-Zahiri-Abyaneh, work in progress
Pure quark loop: momentum area

Hadronic
Light-by-light
for muon g-2

Johan Bijnens

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Most from \( P_1 \approx P_2 \approx Q \), sizable large momentum part
**ENJL quark-loop**

<table>
<thead>
<tr>
<th>Cut-off $\Lambda$ (GeV)</th>
<th>$a_\mu \times 10^{10}$ VMD</th>
<th>$a_\mu \times 10^{10}$ ENJL</th>
<th>$a_\mu \times 10^{10}$ masscut</th>
<th>$a_\mu \times 10^{10}$ ENJL + masscut sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.48</td>
<td>0.78</td>
<td>2.46</td>
<td>3.2</td>
</tr>
<tr>
<td>0.7</td>
<td>0.72</td>
<td>1.14</td>
<td>1.13</td>
<td>2.3</td>
</tr>
<tr>
<td>1.0</td>
<td>0.87</td>
<td>1.44</td>
<td>0.59</td>
<td>2.0</td>
</tr>
<tr>
<td>2.0</td>
<td>0.98</td>
<td>1.78</td>
<td>0.13</td>
<td>1.9</td>
</tr>
<tr>
<td>4.0</td>
<td>0.98</td>
<td>1.98</td>
<td>0.03</td>
<td>2.0</td>
</tr>
<tr>
<td>8.0</td>
<td>0.98</td>
<td>2.00</td>
<td>0.005</td>
<td>2.0</td>
</tr>
</tbody>
</table>

- Very stable
- ENJL cuts off slower than pure VMD
- masscut: $M_Q = \Lambda$ to have short-distance and no problem with momentum regions
- Quite stable in region 1-4 GeV
ENJL: scalar

\[ \Pi_{\rho\nu\alpha\beta} = \prod_{ab}^{\text{VVS}} (p_1, r) g_S \left( 1 + g_S \Pi^S (r) \right) \prod_{cd}^{\text{SVV}} (p_2, p_3) \gamma_{abcd\rho\nu\alpha\beta} + \text{permutations} \]

\[ g_S \left( 1 + g_S \Pi^S \right) = \frac{g_A (r^2)(2M_Q)^2}{2f^2(r^2)} \frac{1}{M_S^2(r^2) - r^2} \]

\( \gamma_{abcd\rho\nu\alpha\beta} \): ENJL VMD legs

In ENJL only scalar + quark-loop properly chiral invariant
ENJL: scalar/QL

<table>
<thead>
<tr>
<th>Cut-off $\Lambda$ (GeV)</th>
<th>$a_\mu \times 10^{10}$ Quark-loop VMD</th>
<th>$a_\mu \times 10^{10}$ Quark-loop ENJL</th>
<th>$a_\mu \times 10^{10}$ Scalar Exchange</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.48</td>
<td>0.78</td>
<td>−0.22</td>
</tr>
<tr>
<td>0.7</td>
<td>0.72</td>
<td>1.14</td>
<td>−0.46</td>
</tr>
<tr>
<td>1.0</td>
<td>0.87</td>
<td>1.44</td>
<td>−0.60</td>
</tr>
<tr>
<td>2.0</td>
<td>0.98</td>
<td>1.78</td>
<td>−0.68</td>
</tr>
<tr>
<td>4.0</td>
<td>0.98</td>
<td>1.98</td>
<td>−0.68</td>
</tr>
<tr>
<td>8.0</td>
<td>0.98</td>
<td>2.00</td>
<td>−0.68</td>
</tr>
</tbody>
</table>

- Note: ENJL+scalar (BPP) ≈ Quark-loop VMD (HKS)
- $M_S \approx 620$ MeV certainly an overestimate for real scalars
- If scalar is $\sigma$: related to pion loop part?
- quark-loop: $a_{\mu q}^{ql} \approx 1 \times 10^{-10}$ bare $a_{\mu q}^{ql} = 2.37 \times 10^{-10}$
Quark loop DSE


  - Not a full calculation (yet) but includes an estimate of some of the missing parts

  - Note: a lot larger than bare quark loop with constituent mass

  - I am puzzled: this DSE model (Maris-Roberts) does reproduce a lot of low-energy phenomenology. I would have guessed that it would be very similar to ENJL in its results.

  - Can one find something in between full DSE and ENJL that is easier to handle?

  - Error found in calculation, still not finalized: preliminary $a^{ql}_{\mu} = 10.7(0.2) \times 10^{-10}$ T. Goecke, C. S. Fischer and R. Williams, arXiv:1210.1759 [hep-ph]
\( \pi \) and \( K \)-loop

- The \( \pi \pi \gamma^* \) vertex is always done using VMD
- \( \pi \pi \gamma^* \gamma^* \) vertex two choices:
  - Hidden local symmetry model: only one \( \gamma \) has VMD
  - Full VMD
  - Both are chirally symmetric
  - Check if they live up to MV short distance (Full VMD does, HLS does not)
  - The HLS model used has problems with \( \pi^+ - \pi^0 \) mass difference (due to not having an \( a_1 \))

- Final numbers quite different: \(-0.045\) and \(-0.19\)
- For BPP stopped at 1 GeV but within 10% of higher \( \Lambda \)
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a_1-exchange

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π and K-loop

<table>
<thead>
<tr>
<th>Cut-off (GeV)</th>
<th>π bare</th>
<th>π VMD</th>
<th>π ENJL</th>
<th>π HLS</th>
<th>K ENJL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>−1.71(7)</td>
<td>−1.16(3)</td>
<td>−1.20(0.03)</td>
<td>−1.05(0.01)</td>
<td>−0.020(0.001)</td>
</tr>
<tr>
<td>0.6</td>
<td>−2.03(8)</td>
<td>−1.41(4)</td>
<td>−1.42(0.03)</td>
<td>−1.15(0.01)</td>
<td>−0.026(0.001)</td>
</tr>
<tr>
<td>0.7</td>
<td>−2.41(9)</td>
<td>−1.46(4)</td>
<td>−1.56(0.03)</td>
<td>−1.17(0.01)</td>
<td>−0.034(0.001)</td>
</tr>
<tr>
<td>0.8</td>
<td>−2.64(9)</td>
<td>−1.57(6)</td>
<td>−1.67(0.04)</td>
<td>−1.16(0.01)</td>
<td>−0.042(0.001)</td>
</tr>
<tr>
<td>1.0</td>
<td>−2.97(12)</td>
<td>−1.59(15)</td>
<td>−1.81(0.05)</td>
<td>−1.07(0.01)</td>
<td>−0.048(0.002)</td>
</tr>
<tr>
<td>2.0</td>
<td>−3.82(18)</td>
<td>−1.70(7)</td>
<td>−2.16(0.06)</td>
<td>−0.68(0.01)</td>
<td>−0.087(0.005)</td>
</tr>
<tr>
<td>4.0</td>
<td>−4.12(18)</td>
<td>−1.66(6)</td>
<td>−2.18(0.07)</td>
<td>−0.50(0.01)</td>
<td>−0.099(0.005)</td>
</tr>
</tbody>
</table>

- **HLS JB-Zahiri-Abyaneh**
- note the suppression by the propagators
π loop: Bare vs VMD

Note: plotted $-a_{\mu}^{LLQ}$
π loop: VMD vs HLS

Note: plotted $-a_{\mu}^{LLQ}$
π loop

- \(\pi\pi\gamma^*\gamma^*\) for \(q_1^2 = q_2^2\) has a short-distance constraint from the OPE as well.
- HLS does not satisfy it
- full VMD does: so probably better estimate
- Ramsey-Musolf suggested to do pure ChPT for the \(\pi\) loop
- So far ChPT at \(p^4\) done for four-point function in limit \(p_1, p_2, q \ll m_\pi\) (Euler-Heisenberg plus next order)
- Polarizability \((L_9 + L_{10})\) up to 10%, charge radius 30%
- Both HLS and VMD have charge radius effect but not polarizability
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- Polarizability \( (L_9 + L_{10}) \) up to 10\%, charge radius 30\%
- Both HLS and VMD have charge radius effect but not polarizability
\( \pi \) loop: \( L_9, L_{10} \)

- ChPT for muon \( g - 2 \) at order \( p^6 \) is not powercounting finite so no prediction for \( a_\mu \) exists.
- But can be used to study the low momentum end of the integral over \( P_1, P_2, Q \)
- The four-photon amplitude is finite still at two-loop order (counterterms start at order \( p^8 \))
- Add \( L_9 \) and \( L_{10} \) vertices to the bare pion loop
  - JB-Zahiri-Abyaneh
- Program the Euler-Heisenberg plus NLO result of Ramsey-Musolf et al. into our programs for \( a_\mu \)
- Bare pion-loop and \( L_9, L_{10} \) part in limit \( p_1, p_2, q \ll m_\pi \) agree with Euler-Heisenberg plus next order analytically
- Numerics very preliminary
ChPT for muon $g - 2$ at order $p^6$ is not powercounting finite so no prediction for $a_\mu$ exists.

But can be used to study the low momentum end of the integral over $P_1, P_2, Q$.

The four-photon amplitude is finite still at two-loop order (counterterms start at order $p^8$).

Add $L_9$ and $L_{10}$ vertices to the bare pion loop

JB-Zahiri-Abyaneh

Program the Euler-Heisenberg plus NLO result of Ramsey-Musolf et al. into our programs for $a_\mu$

Bare pion-loop and $L_9, L_{10}$ part in limit $p_1, p_2, q \ll m_\pi$ agree with Euler-Heisenberg plus next order analytically.

Numerics very preliminary.
Note: plotted $-a_{\mu}^{LLQ}$, low scale, charge radius effect well reproduced
Note: plotted $-a_{\mu}^{LLQ}$, gives an enhancement of 10-15%
Work in progress

- add the $a_1$ to the pion loop.
- **JB-Relefors** in progress
- Use antisymmetric (axial-)vector formalism: easier to add different terms
  
  \[ \mathcal{L} = \frac{i G_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle + \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f^{\mu\nu} \rangle \]

  $\rho\pi\pi$ and $\rho-\gamma$ couplings

  Finite result for $g - 2$ requires $G_V = F_V/2$

  Then same results as HLS with $a = F_V^2/F^2$
### Summary: ENJL vs PdRV

<table>
<thead>
<tr>
<th>Contribution</th>
<th>BPP</th>
<th>PdRV arXiv:0901.0306</th>
</tr>
</thead>
<tbody>
<tr>
<td>quark-loop</td>
<td>$(2.1 \pm 0.3) \cdot 10^{-10}$</td>
<td>(11.4 ± 1.3) \cdot 10^{-10}</td>
</tr>
<tr>
<td>pseudo-scalar</td>
<td>$(8.5 \pm 1.3) \cdot 10^{-10}$</td>
<td></td>
</tr>
<tr>
<td>axial-vector</td>
<td>$(0.25 \pm 0.1) \cdot 10^{-10}$</td>
<td>$(1.5 \pm 1.0) \cdot 10^{-10}$</td>
</tr>
<tr>
<td>scalar</td>
<td>$(-0.68 \pm 0.2) \cdot 10^{-10}$</td>
<td>$(0.7 \pm 0.7) \cdot 10^{-10}$</td>
</tr>
<tr>
<td>$\pi K$-loop</td>
<td>$(-1.9 \pm 1.3) \cdot 10^{-10}$</td>
<td>$(-1.9 \pm 1.9) \cdot 10^{-10}$</td>
</tr>
<tr>
<td>errors/sum</td>
<td>linearly</td>
<td>quadratically</td>
</tr>
<tr>
<td></td>
<td>$(8.3 \pm 3.2) \cdot 10^{-10}$</td>
<td>$(10.5 \pm 2.6) \cdot 10^{-10}$</td>
</tr>
</tbody>
</table>
What can we do more?

- The ENJL model can certainly be improved:
  - Chiral nonlocal quark-model (like nonlocal ENJL): so far only $\pi^0$-exchange done
  - DSE: $\pi^0$-exchange similar to everyone else, quark-loop very different, looking forward to final results
- More resonances models should be tried, AdS/QCD is one approach, $R_{\chi T}$ (Valencia et al.) possible,…
- Note short-distance matching must be done in many channels, there are theorems JB,Gamiz,Lipartia,Prades that with only a few resonances this requires compromises
- $\pi$-loop: HLS smaller than double VMD (understood) models with $\rho$ and $a_1$ (in progress)
What can we do more?

- Constraints from experiment:
  
  
  Studying three formfactors $P\gamma^*\gamma^*$ in $P \rightarrow \ell^+\ell^-\ell'^+\ell'^-$, $e^+e^- \rightarrow e^+e^- P$ exact tree level and for $g - 2$ (but beware sign):
  
  - Conclusion: possible but VERY difficult
  - Two $\gamma^*$ off-shell not so important for our choice of form-factor

- All information on hadrons and 1-2-3-4 off-shell photons is welcome: constrain the models

- More short-distance constraints: MV, Nyffeler integrate with all contributions, not just $\pi^0$-exchange

- Need a new overall evaluation with consistent approach.

- Lattice has done first steps
What can we do more?

- Constraints from experiment:
  
  
  Studying three formfactors \( P\gamma^*\gamma^* \) in \( P \to \ell^+\ell^-\ell'^+\ell'^- \), \( e^+e^- \to e^+e^- \) \( P \) exact tree level and for \( g - 2 \) (but beware sign):
  
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- More short-distance constraints: MV, Nyffeler integrate with all contributions, not just \( \pi^0 \)-exchange

- Need a new overall evaluation with consistent approach.

- Lattice has done first steps
Summary of Muon $g - 2$ contributions

<table>
<thead>
<tr>
<th></th>
<th>$10^{10} a_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>exp</td>
<td>11 659 208.9</td>
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<tr>
<td>theory</td>
<td>11 659 180.2</td>
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<tr>
<td>QED</td>
<td>11 658 471.8</td>
</tr>
<tr>
<td>EW</td>
<td>15.4</td>
</tr>
<tr>
<td>LO Had</td>
<td>692.3</td>
</tr>
<tr>
<td>HO HVP</td>
<td>-9.8</td>
</tr>
<tr>
<td>HLbL</td>
<td>10.5</td>
</tr>
<tr>
<td>difference</td>
<td>28.7</td>
</tr>
</tbody>
</table>

- Error on LO had all $e^+ e^-$ based OK
- $\tau$ based 2 $\sigma$
- Error on HLbL
- Errors added quadratically
- $3.5 \sigma$
- Difference:
  - 4% of LO Had
  - 270% of HLbL
  - 1% of leptonic LbL

Generic SUSY: $12.3 \times 10^{-10} \left( \frac{100 \text{ GeV}}{M_{SUSY}} \right)^2 \tan \beta$

$M_{SUSY} \approx 66 \text{ GeV} \sqrt{\tan \beta}$