



LUND
UNIVERSITY



Chiral Meson Physics at Two Loops

Johan Bijnens

Lund University

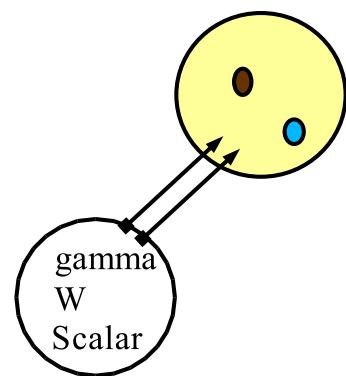
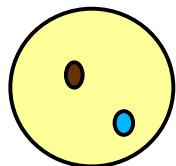
`bijnens@thep.lu.se`

Overview

- Introduction
- Why (Effective) Field Theory
- Chiral Perturbation Theory
- ChPT and lattice QCD
- Two Loop: General
- Two Loop: Two Flavours
- Two Loop: PQChPT
- Two Loop: Three Flavours
 - General fitting strategy and some comments
 - $\pi\pi$, πK and scalar form factors
 - $K_{\ell 3}$ and V_{us}
- Conclusions

Introduction

Simplest hadrons : ground state mesons : π^\pm , π^0 , K and η .



- Minimum number of constituents
- Lightest state: spatially simplest
- Chiral Symmetry: $m_q \rightarrow 0 \implies m_M \rightarrow 0$
- Masses and Decay Constants Simple

- Form Factors Next Simplest property
- Spatial information via Fourier Transform
- Different probes \implies different properties

Why (Effective) Field Theory?

- Effective:**
- Use right degrees of freedom : essence of (most) physics
 - Gap in the spectrum \implies separation of scales
 - With lower d.o.f.: build most general Lagrangian

$\implies \infty \#$ parameters
 \implies Where did predictivity go ? } \implies power counting

Why (Effective) Field Theory?

Field Theory

- ➡ Only known way to combine QM and special relativity
- ➡ Taylor series does not work (convergence radius zero)
- ➡ Continuum of excitation states to be taken into account

Why (Effective) Field Theory?

Field Theory

- ➡ Only known way to combine QM and special relativity
- ➡ Taylor series does not work (convergence radius zero)
- ➡ Continuum of excitation states to be taken into account
- ➡ Off-shell effects fully under control: these effects are there as new free parameters
- ➡ model-independent and systematic: ALL effects at given order included
- ➡ Theory \implies errors can be estimated

Why (Effective) Field Theory?

Field Theory

- ➡ Only known way to combine QM and special relativity
- ➡ Taylor series does not work (convergence radius zero)
- ➡ Continuum of excitation states to be taken into account
- ➡ Off-shell effects fully under control: these effects are there as new free parameters
- ➡ model-independent and systematic: ALL effects at given order included
- ➡ Theory \implies errors can be estimated
- ➡ Many parameters (but possible modelspace is large)
- ➡ Expansion might not converge (often still useful for model classification)

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But
$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But
$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$$

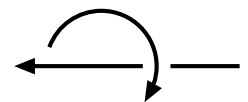
So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Can also see that via



$$v < c, m_q \neq 0 \implies$$

$$v = c, m_q = 0 \not\Rightarrow$$



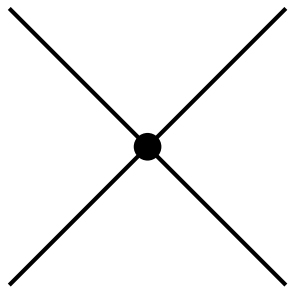
Chiral Perturbation Theory

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$

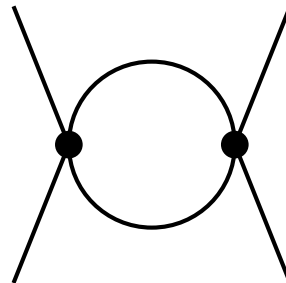
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken \implies 8 massless degrees of freedom
and interaction vanishes at zero momentum

Power counting in momenta:



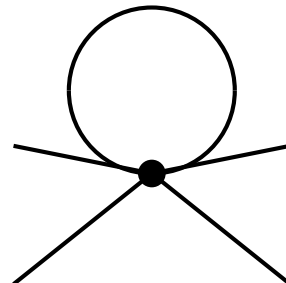
$$p^2$$



$$(p^2)^2 (1/p^2)^2 p^4 = p^4$$



$$1/p^2$$

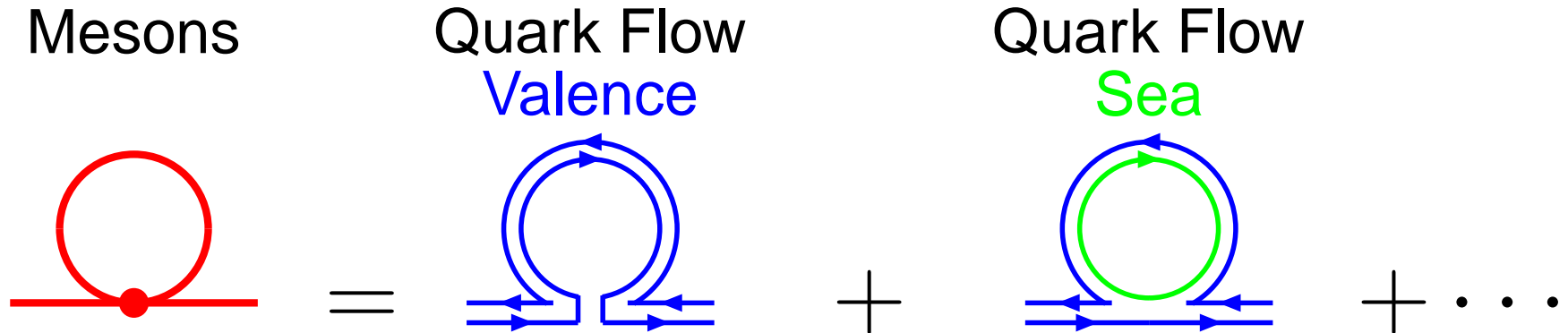


$$(p^2) (1/p^2) p^4 = p^4$$

$$\int d^4 p$$

$$p^4$$

ChPT and Lattice QCD



Valence is *easy* to deal with in lattice QCD

Sea is *very difficult*

They can be treated separately: i.e. different quark masses

Partially Quenched ChPT (PQChPT)

Two Loop: General

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	\hat{L}_i^r, \hat{H}_i^r	11+2
p^6	c_i^r	53+4	C_i^r	90+4	K_i^r	112+3

p^2 : Weinberg 1966

p^4 : Gasser, Leutwyler 84,85

p^6 : JB, Colangelo, Ecker 99,00

Note {

- ▣ replica method \implies PQ obtained from N_F flavour
- ▣ All infinities known
- ▣ 3 flavour is a special case of 3+3 PQ:
 $\hat{L}_i^r, K_i^r \rightarrow L_i^r, C_i^r$

Two Flavours at Two Loop

Most Calculations Done

$$\gamma\gamma \rightarrow \pi^0\pi^0$$

Bellucci, Gasser, Sainio

$$\gamma\gamma \rightarrow \pi^+\pi^-, F_\pi, m_\pi$$

Bürigi

$$\pi\pi\text{-scattering}, F_\pi, m_\pi$$

JB, Colangelo, Ecker, Gasser, Sainio

$$F_{V\pi}(t), F_{S\pi}$$

JB, Colangelo, Talavera

$$\pi \rightarrow \ell\nu\gamma$$

JB, Talavera

- Reasonable convergence
- c_i^r not important for many threshold quantities

⇒ combined with dispersive methods, Roy equations, etc.
for very precise calculations

PQChPT at Two Loop

Subject just beginning

valence equal mass, 3 sea equal mass:

$m_{\pi^+}^2$: JB, Danielsson, Lähde, hep-lat/0406017

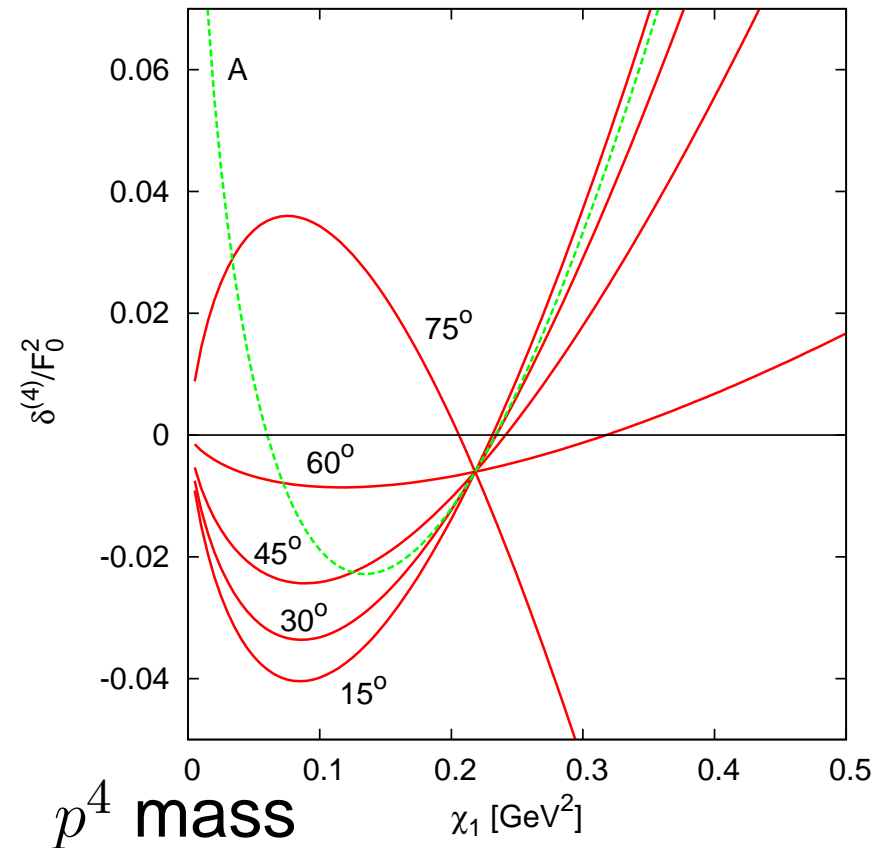
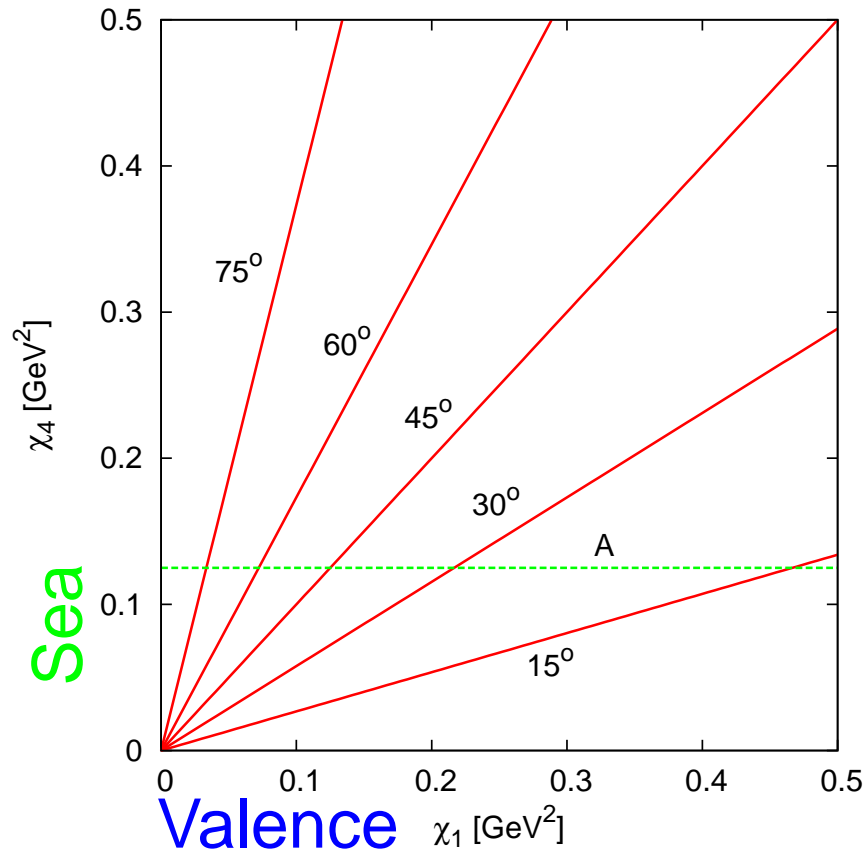
$F_{\pi^+}^2$: JB, Lähde, soon

Planned: all the other mass combinations

Actual Calculations: {

- ▣ heavy use of FORM *Vermaseren*
- ▣ use PQ without super Φ_0 in supersymmetric formalism
- ▣ Main problem: sheer size of the expressions

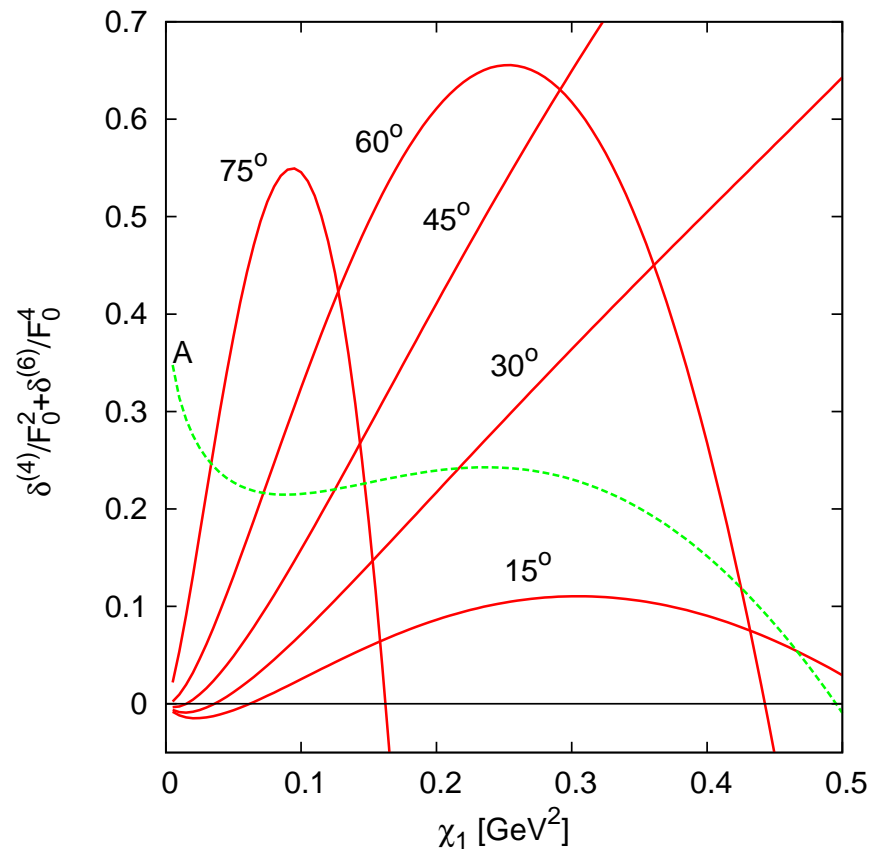
PQChPT at Two Loop



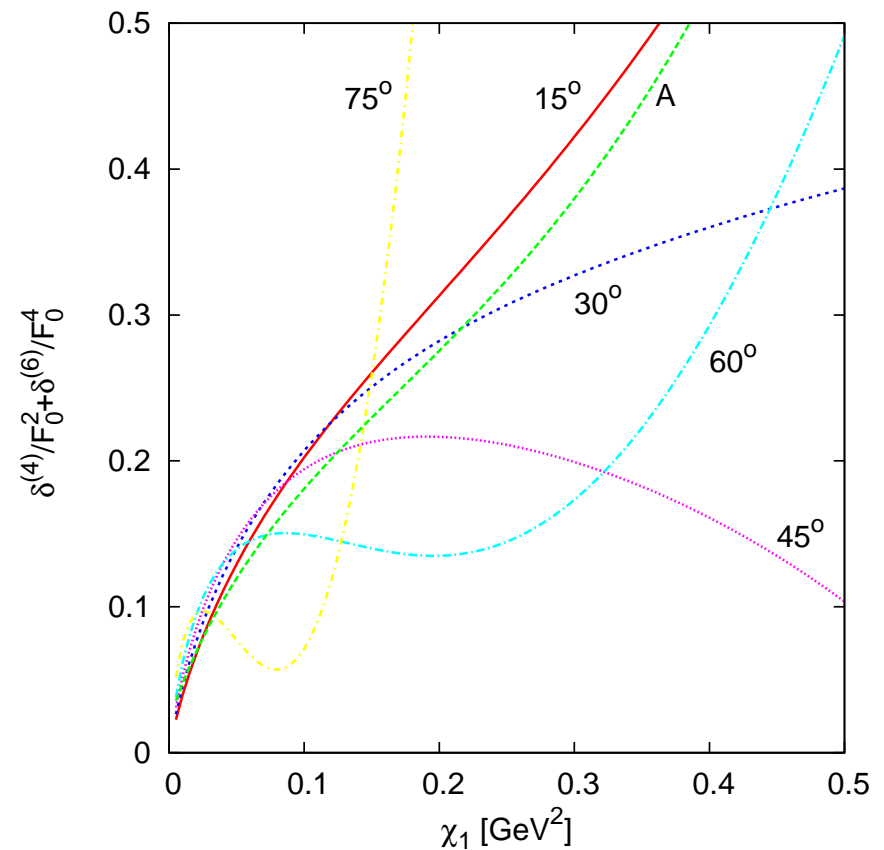
$$\chi_i = 2B_0 m_i = m_M^{2(0)}$$

$$0.3 \text{ GeV}^2 \approx (550 \text{ MeV})^2 \sim \text{border ChPT}$$

PQChPT at Two Loops



$p^4 + p^6$ relative correction mass



decay constant
(preliminary)

Three Flavours at Two Loop

$\Pi_{VV\pi}, \Pi_{VV\eta}, \Pi_{VVk}$ Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera

$\Pi_{AA\pi}, \Pi_{AA\eta}, F_\pi, F_\eta, m_\pi, m_\eta$ Kambor, Golowich; Amorós, JB, Talavera

Π_{SS} Moussallam L_4^r, L_6^r

$\Pi_{VVK}, \Pi_{AAK}, F_K, m_K$ Amorós, JB, Talavera

$K_{\ell 4}$ Amorós, JB, Talavera L_1^r, L_2^r, L_3^r

$F_M, m_M (m_u \neq m_d)$ Amorós, JB, Talavera $L_{5,7,8}^r, m_u/m_d$

$F_{V\pi}, F_{VK^+}, F_{VK^0}$ Post, Schilcher; JB, Talavera L_9^r

$K_{\ell 3}$ Post, Schilcher; JB, Talavera V_{us}

$F_{S\pi}, F_{SK}$ JB, Dhonte L_4^r, L_6^r

$K, \pi \rightarrow \ell\nu\gamma$ Geng, Ho, Wu L_{10}^r

$\pi\pi$ JB, Dhonte, Talavera

πK JB, Dhonte, Talavera

General Strategy and some comments

- Find enough inputs from experiment
- C_i^r :
 - kinematical dependence: agree well with single resonance saturation
 - quark mass+kinematical: if vector dominated, seems to be OK
 - quark mass+kinematical: if scalar dominated: which scalars? (not σ)
 - quark masses: which scalars? unrealistically large estimates
- in p^6 physical or lowest order masses: thresholds in right place requires physical

General Strategy and some comments

Inputs:

$K_{\ell 4}$: $F(0)$, $G(0)$, λ

$m_{\pi^0}^2$, m_{η}^2 , $m_{K^+}^2$, $m_{K^0}^2$

F_{π^+}

F_{K^+} / F_{π^+}

m_s / \hat{m}

L_4^r , L_6^r

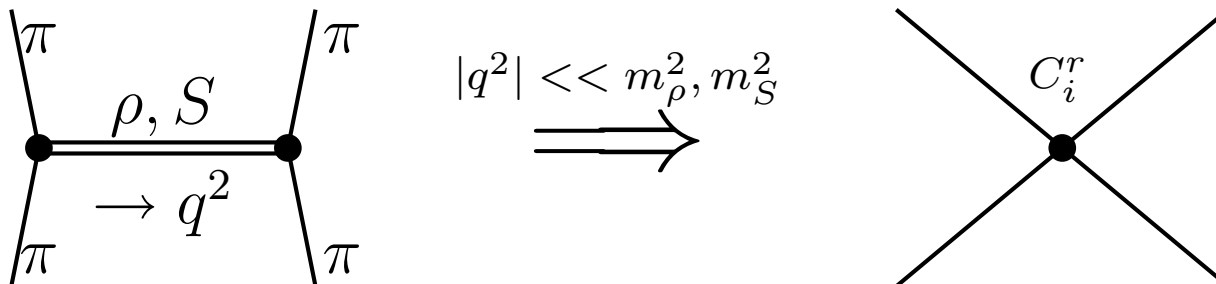
C_i^r from single resonance approximation

E865 BNL

em with Dashen violation

24 (26)

$\hat{m} = (m_u + m_d) / 2$



General Strategy and some comments

	fi t 10	same p^4	fi t B	fi t D
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69
$10^3 L_3^r$	-2.53 ± 0.37	-2.91	-2.31	-2.33
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54

- ▣ errors are very correlated
- ▣ $\mu = 770$ MeV; 550 or 1000 within errors
- ▣ varying C_i^r factor 2 about errors
- ▣ $L_4^r, L_6^r \approx -0.3, \dots, 0.6 \cdot 10^{-3}$ OK
- ▣ fi t B: small corrections to pion “sigma” term, fi t scalar radius
- ▣ fi t D: fi t $\pi\pi$ and πK thresholds

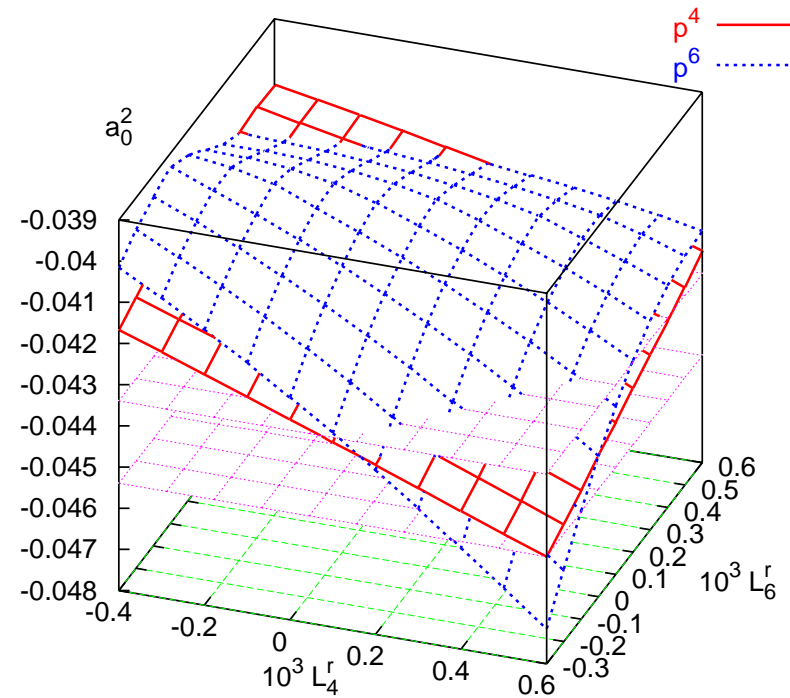
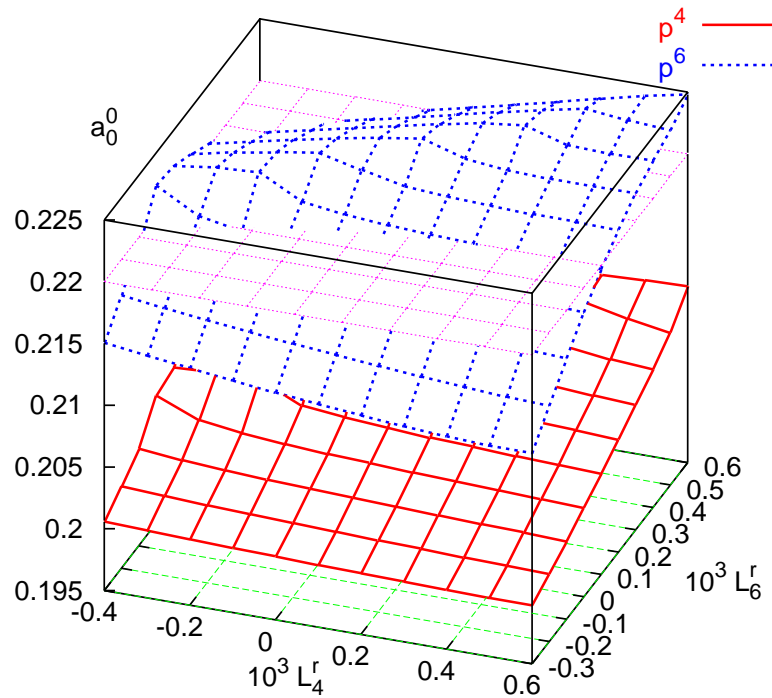
General Strategy and some comments

	fit 10	same p^4	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
$m_\pi^2: p^4, p^6$	0.006,0.258	0.009, $\equiv 0$	-0.138,0.009	-0.091,0.133
$m_K^2: p^4, p^6$	0.007,0.306	0.075, $\equiv 0$	-0.149,0.094	-0.096,0.201
$m_\eta^2: p^4, p^6$	-0.052,0.318	0.013, $\equiv 0$	-0.197,0.073	-0.151,0.197
m_u/m_d	0.45 ± 0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}: p^4, p^6$	0.169,0.051	0.22, $\equiv 0$	0.153,0.067	0.159,0.061

▣▣▣▣ $m_u = 0$ always very far from the fits

▣▣▣▣ F_0 : pion decay constant in the chiral limit

$\pi\pi$

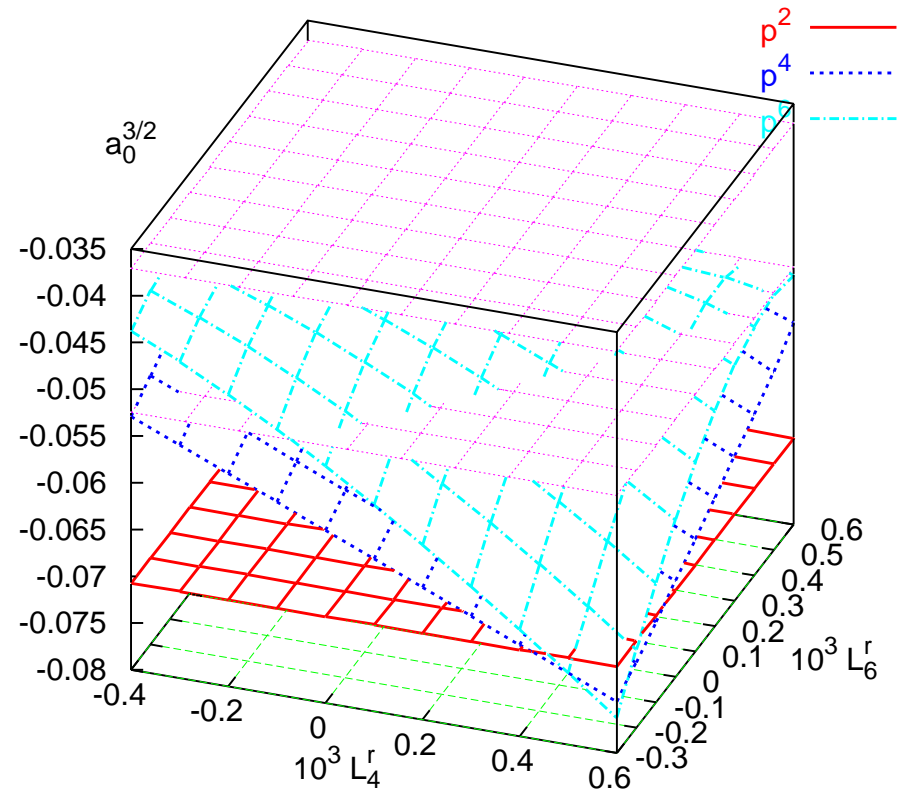
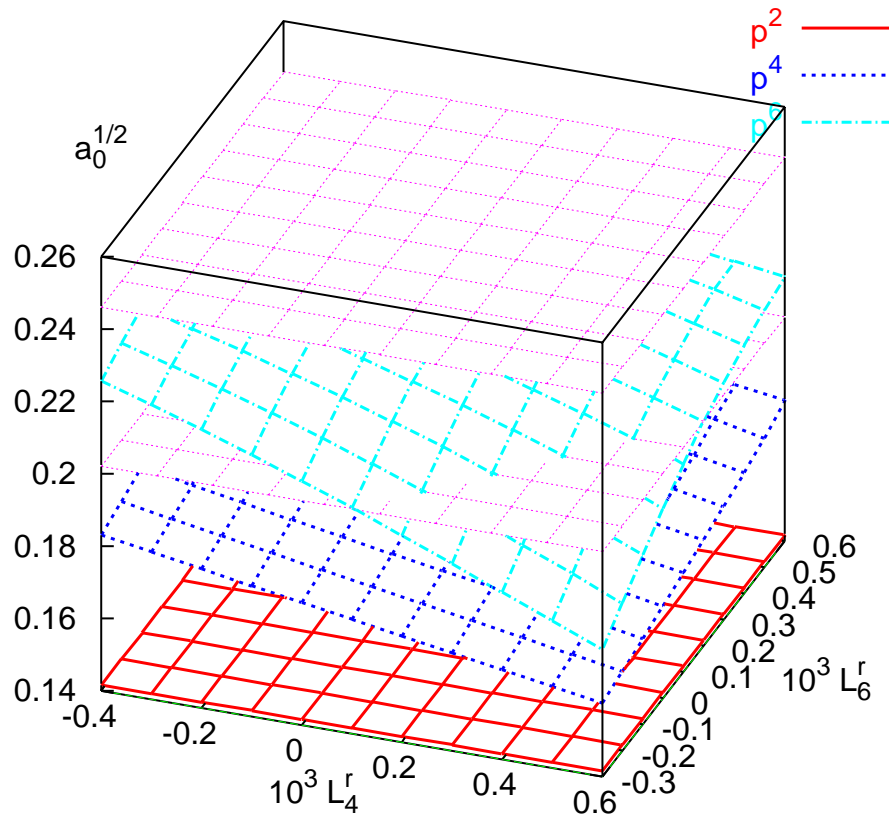


$$a_0^0 = 0.220 \pm 0.005, \quad a_0^2 = -0.0444 \pm 0.0010$$

Colangelo, Gasser, Leutwyler

$$a_0^0 = 0.159 \quad a_0^2 = -0.0454 \quad \text{at order } p^2$$

πK

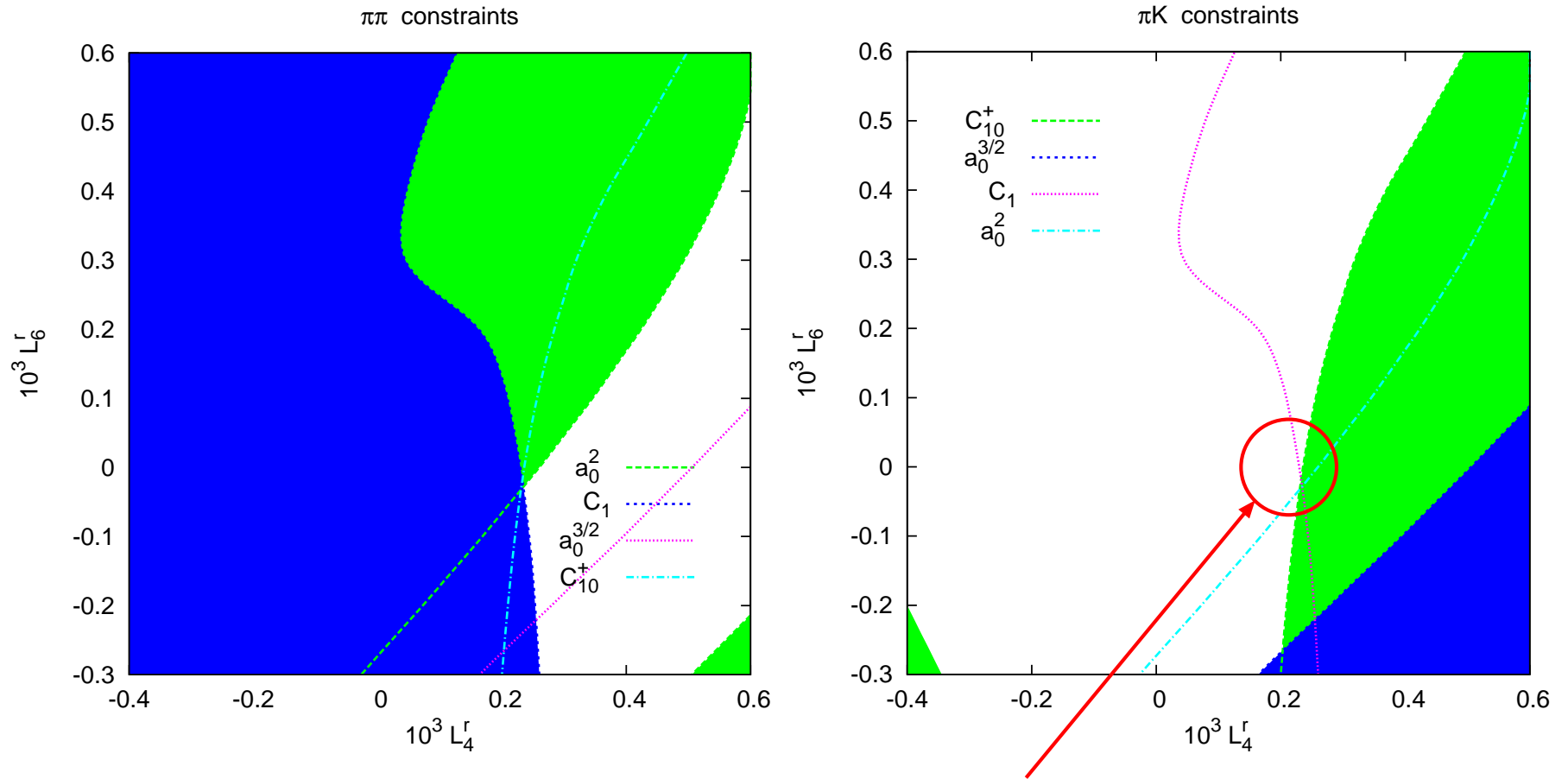


$$a_0^{1/2} = 0.224 \pm 0.022, \quad a_{3/2}^2 = -0.0448 \pm 0.0077$$

Büttiker, Descotes-Genon, Moussallam

$$a_0^{1/2} = 0.142 \quad a_0^2 = -0.0708 \quad \text{at order } p^2$$

$\pi\pi$ and πK



preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor:

$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization:

$$f_{+,0}(t) = f_+(0) \left(1 + \lambda_{+,0} \frac{t}{m_\pi^2} \right)$$

$|V_{us}|$:

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor:

$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization:

$$f_{+,0}(t) = f_+(0) \left(1 + \lambda_{+,0} \frac{t}{m_\pi^2} \right)$$

$|V_{us}|$: ● Know theoretically $f_+(0) = 1 + \dots$

- Short distance correction to G_F from G_μ **Marciano-Sirlin**
- Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$
- Isospin Breaking **Leuwylor-Roos** $\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.022$ **In**

Progress

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor:

$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization:

$$f_{+,0}(t) = f_+(0) \left(1 + \lambda_{+,0} \frac{t}{m_\pi^2} \right)$$

$|V_{us}|$: ● Know theoretically $f_+(0) = 1 + \dots$

● Short distance correction to G_F from G_μ **Marciano-Sirlin**

● Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$

● Isospin Breaking **Leuwylor-Roos** $\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.022$ **In**

Progress

● Know experimentally $f_+(0)$

● Radiative Corrections: **Cirigliano et al., hep-ph/0110153**

● Parametrize form-factor: is linear enough for $f_+(t)$?

$K_{\ell 3}$ Definitions and V_{us}

Scalar formfactor:

$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization:

$$f_{+,0}(t) = f_+(0) \left(1 + \lambda_{+,0} \frac{t}{m_\pi^2} \right)$$

$|V_{us}|$: ● Know theoretically $f_+(0) = 1 + \dots$

● Short distance correction to G_F from G_μ **Marciano-Sirlin**

● Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$

● Isospin Breaking **Leuwylor-Roos** $\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.022$ **In**

Progress

● Know experimentally $f_+(0)$

● Radiative Corrections: **Cirigliano et al., hep-ph/0110153**

● Parametrize form-factor: is linear enough for $f_+(t)$?

PDG2002: $|V_{ud}| = 0.9734 \pm 0.0008$, $|V_{us}| = 0.2196 \pm 0.0026$

$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) = 0.9957 \pm 0.0019$

$f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

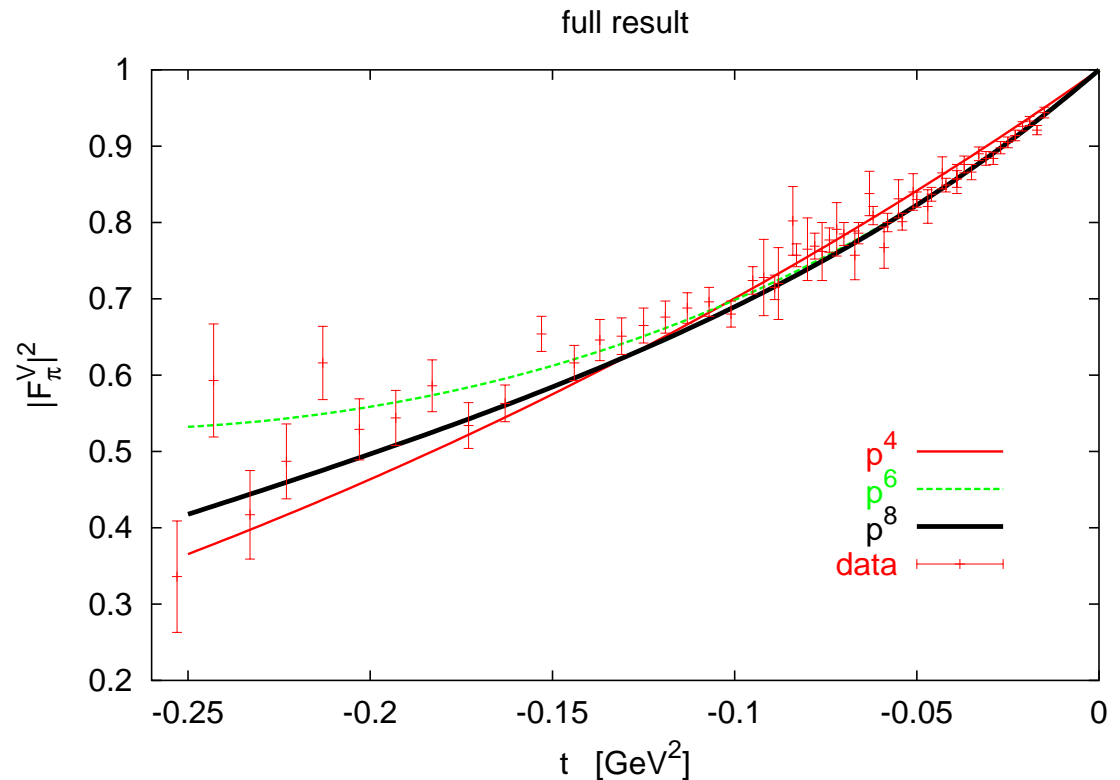
$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4} (-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

Pion electromagnetic Form factor:
JB, Talavera

$$L_9^r = 0.00593 \pm 0.00043$$

$$-4C_{88}^r + 4C_{90}^r = 0.00022 \pm 0.00002$$

$$\text{VMD: } R_{+1}^{K\pi} \approx -4 \cdot 10^{-5} \text{ GeV}^2$$



ChPT fit to $f_+(t)$

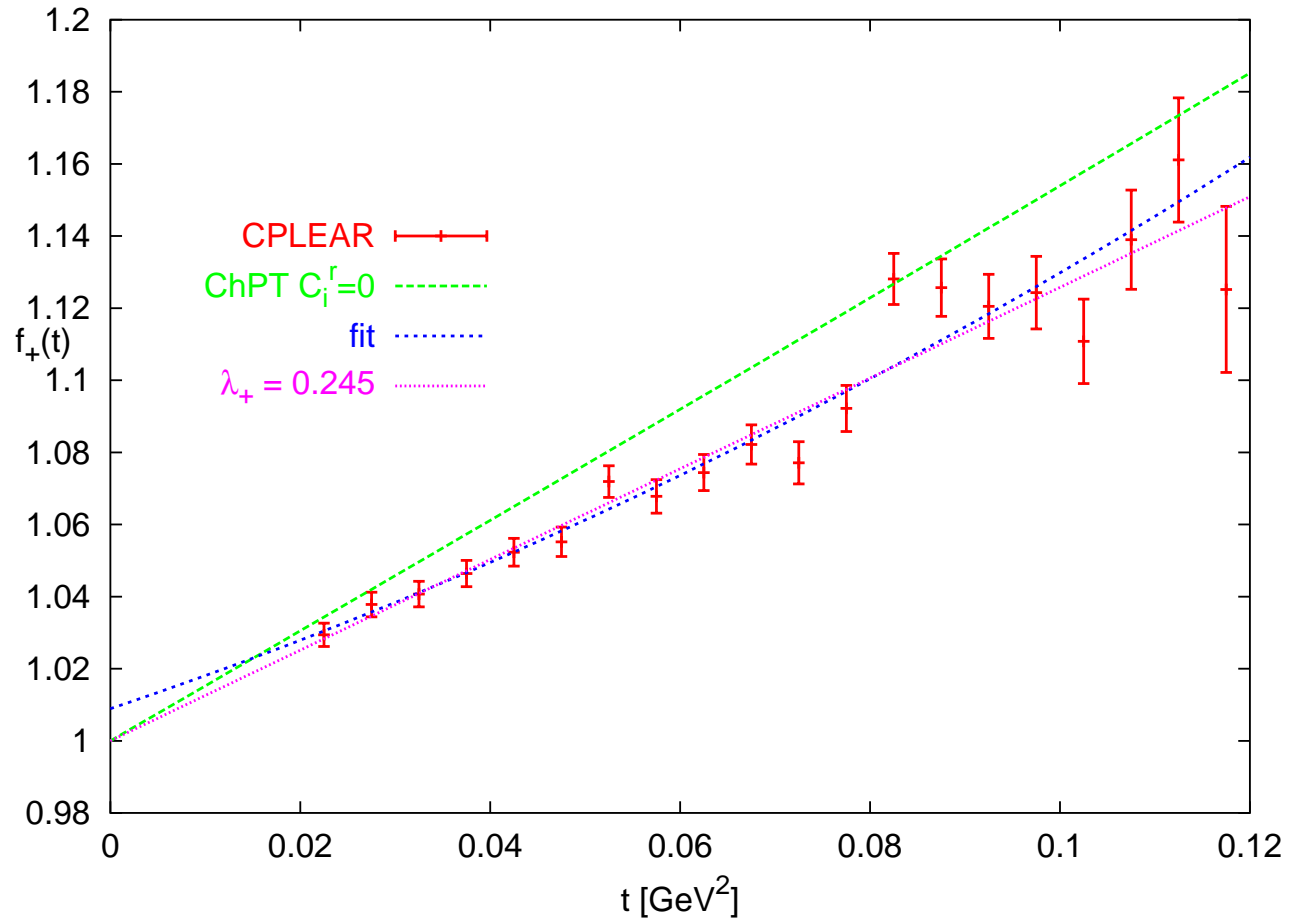
$$\Rightarrow R_{+1}^{K\pi} = -(4.7 \pm 0.5) 10^{-5} \text{ GeV}^2$$

$$\left(c_+ = 3.2 \text{ GeV}^{-4} \right)$$

fixed by ChPT

$$\Rightarrow a_+ = 1.009 \pm 0.004$$

$$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$$



ChPT fit to $f_+(t)$

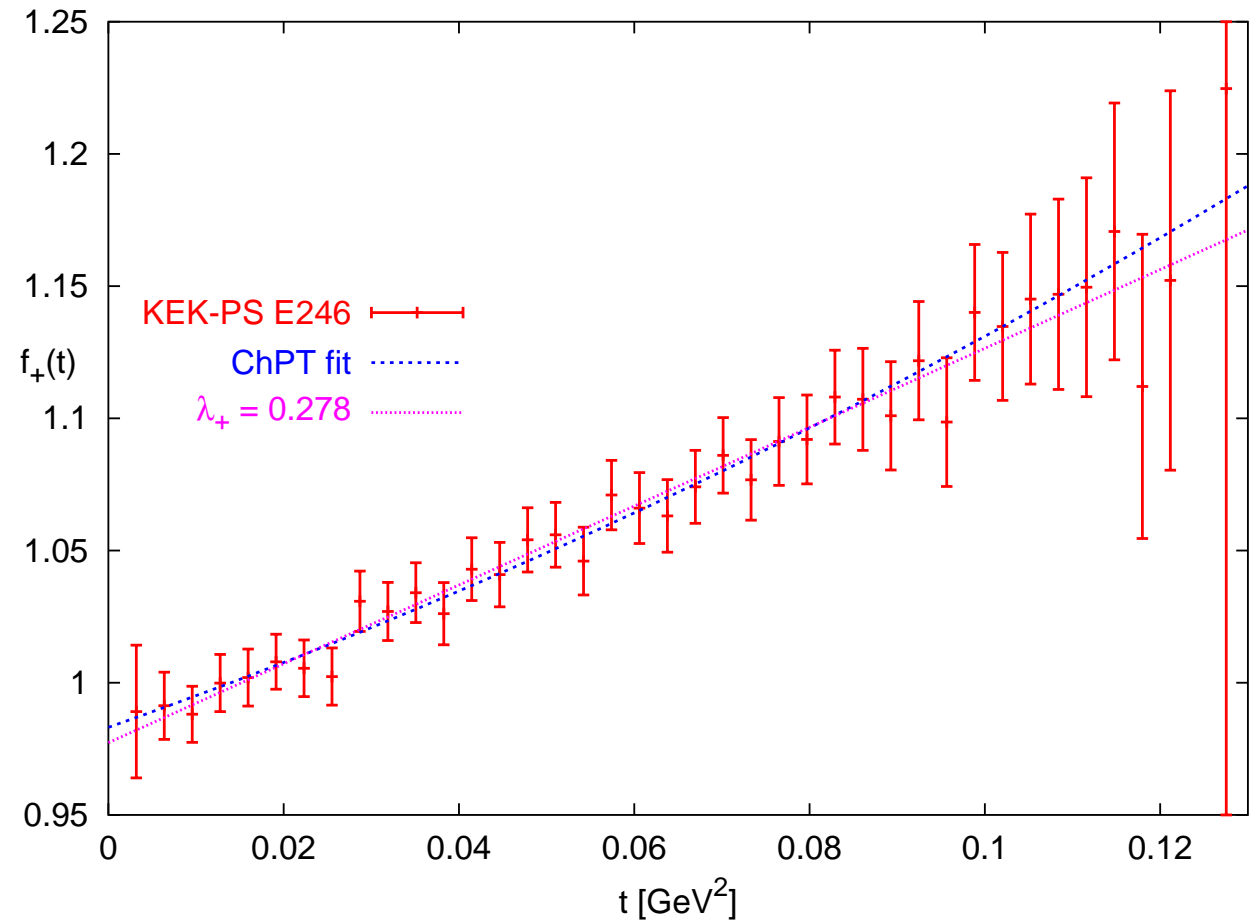
$$\Rightarrow R_{+1}^{K\pi} = -2.5 \cdot 10^{-5} \text{ GeV}^2$$

$$(c_+ = 3.2 \text{ GeV}^{-4})$$

fixed by ChPT

$$\Rightarrow a_+ = 1.006$$

$$\Rightarrow \lambda_+ = 0.0214 \pm 0.0018$$



$f_0(t)$

Main Result:

$$\begin{aligned} f_0(t) = & 1 - \frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 \\ & + 8 \frac{t}{F_\pi^4} (2C_{12}^r + C_{34}^r) (m_K^2 + m_\pi^2) + \frac{t}{m_K^2 - m_\pi^2} (F_K/F_\pi - 1) \\ & - \frac{8}{F_\pi^4} t^2 C_{12}^r + \bar{\Delta}(t) + \Delta(0). \end{aligned}$$

$f_0(t)$

Main Result:

$$\begin{aligned} f_0(t) = & 1 - \frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 \\ & + 8 \frac{t}{F_\pi^4} (2C_{12}^r + C_{34}^r) (m_K^2 + m_\pi^2) + \frac{t}{m_K^2 - m_\pi^2} (F_K/F_\pi - 1) \\ & - \frac{8}{F_\pi^4} t^2 C_{12}^r + \bar{\Delta}(t) + \Delta(0). \end{aligned}$$

$\bar{\Delta}(t)$ and $\Delta(0)$ contain **NO** C_i^r and only depend on the L_i^r at order p^6

\implies

All needed parameters can be determined experimentally

$f_0(t)$

Main Result:

$$\begin{aligned} f_0(t) = & 1 - \frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) (m_K^2 - m_\pi^2)^2 \\ & + 8 \frac{t}{F_\pi^4} (2C_{12}^r + C_{34}^r) (m_K^2 + m_\pi^2) + \frac{t}{m_K^2 - m_\pi^2} (F_K/F_\pi - 1) \\ & - \frac{8}{F_\pi^4} t^2 C_{12}^r + \bar{\Delta}(t) + \Delta(0). \end{aligned}$$

$\bar{\Delta}(t)$ and $\Delta(0)$ contain **NO** C_i^r and only depend on the L_i^r at order p^6

\implies

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r].$$

$$\Delta(0) = -0.0227 (p^4) + 0.0113 (p^6 \text{ pure loop}) + 0.0033 (p^6 L_i^r)$$

V_{us} present status

- **More Theory:** Dispersion theory relates slopes and curvature in $f_0(t)$ Jamin, Oller, Pich

$$C_{12}^r + C_{34}^r = 3.2 \pm 1.5 \cdot 10^{-6} \implies f_+(0)_{p^6} = 0.002 \pm 0.009.$$

V_{us} present status

- **More Theory:** Dispersion theory relates slopes and curvature in $f_0(t)$ Jamin, Oller, Pich
$$C_{12}^r + C_{34}^r = 3.2 \pm 1.5 \cdot 10^{-6} \implies f_+(0)_{p^6} = 0.002 \pm 0.009.$$
- **More Experiment:**
 - 2003: E865 $K_{\ell 3}^+$ branching ratio: strong increase
 - 2004: KTeV $K_{\ell 3}^0$ branching ratio: strong increase
 - 2004: formfactor data KTeV (K^0) and ISTRA+ (K^+):
 - **Curvature seen**, reasonable agreement with ChPT
 - λ_0 : old discrepancies gone?

V_{us} present status

- **More Theory:** Dispersion theory relates slopes and curvature in $f_0(t)$ Jamin, Oller, Pich
 $C_{12}^r + C_{34}^r = 3.2 \pm 1.5 \cdot 10^{-6} \implies f_+(0)_{p^6} = 0.002 \pm 0.009.$
- **More Experiment:**
 - 2003: E865 $K_{\ell 3}^+$ branching ratio: strong increase
 - 2004: KTeV $K_{\ell 3}^0$ branching ratio: strong increase
 - 2004: formfactor data KTeV (K^0) and ISTRA+ (K^+):
 - **Curvature seen**, reasonable agreement with ChPT
 - λ_0 : old discrepancies gone?
- Expect both more theory and experiment (KLOE, NA48)

the unitarity problem might be on the way out

Conclusions

- 2 flavour ChPT at 2 loops (almost) finished subject
- 3 flavour ChPT at 2 loops
 - many calculations done
 - things seem to work but convergence is fairly slow
 - “kinematical” and “vector” C_i^r seem to be OK
 - L_4^r, L_6^r nonzero but reasonable for large N_c
 - $\eta \rightarrow 3\pi$, isobreaking in $K_{\ell 3}$: parts done
- PQChPT at 2 loops: subject just beginning
- $K_{\ell 3}$ an example of results even with all the C_i^r