# Chiral Meson Physics at Two Loops

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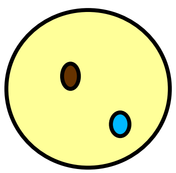
# Overview

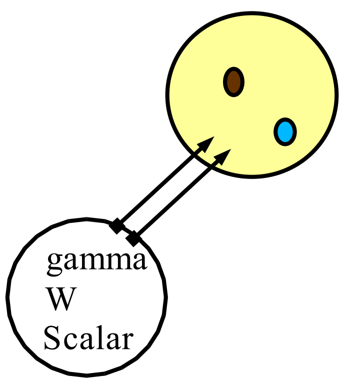
- Introduction
- Why (Effective) Field Theory
- Chiral Perturbation Theory
- ChPT and lattice QCD
- Two Loop: General
- Two Loop: Two Flavours
- Two Loop: PQChPT
- Two Loop: Three Flavours
  - General fitting strategy and some comments
  - $\pi\pi$, $\pi K$ and scalar form factors
  - $K_{\ell 3}$ and $V_{us}$
- Conclusions

# Introduction

Simplest hadrons : ground state mesons : $\pi^\pm$, $\pi^0$, $K$ and $\eta$.



- Minimum number of constituents
- Lightest state: spatially simplest
- Chiral Symmetry: $m_q \to 0 \Longrightarrow m_M \to 0$
- Masses and Decay Constants Simple




- Form Factors Next Simplest property
- Spatial information via Fourier Transform
- Different probes $\Longrightarrow$ different properties

# Why (Effective) Field Theory?

Effective:

- Use right degrees of freedom : essence of (most) physics
- Gap in the spectrum $\Longrightarrow$ separation of scales
- With lower d.o.f.: build most general Lagrangian

$$\left.\begin{array}{l} \text{➠ } \infty\sharp \text{ parameters} \\ \text{➠ Where did predictivity go ?} \end{array}\right\} \Longrightarrow \text{power counting}$$

# Why (Effective) Field Theory?

## Field Theory

- Only known way to combine QM and special relativity
- Taylor series does not work (convergence radius zero)
- Continuum of excitation states to be taken into account

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- model-independent and systematic: ALL effects at given order included
- Theory $\Longrightarrow$ errors can be estimated

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- model-independent and systematic: ALL effects at given order included
- Theory $\Longrightarrow$ errors can be estimated
- Many parameters (but possible modelspace is large)
- Expansion might not converge (often still useful for model classification)

# Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting

Expected breakdown scale: Resonances, so $M_\rho$ or higher depending on the channel

## Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $$\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[ i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left( \bar{q}_R q_L + \bar{q}_L q_R \right) \right]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

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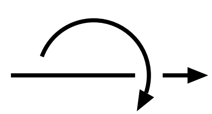
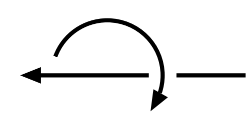
So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Can also see that via



$$v < c,\ m_q \neq 0 \Longrightarrow$$
$$v = c,\ m_q = 0 \not\Longrightarrow$$

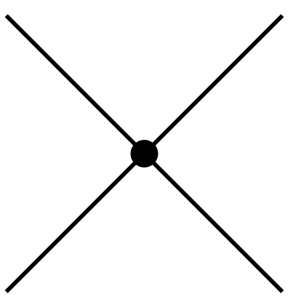


# Chiral Perturbation Theory

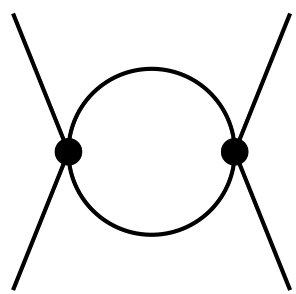
$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

Power counting in momenta:



$p^2$

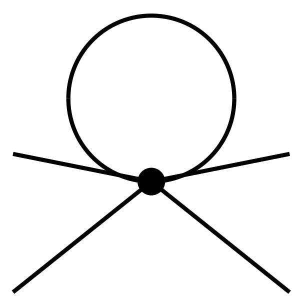


$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$



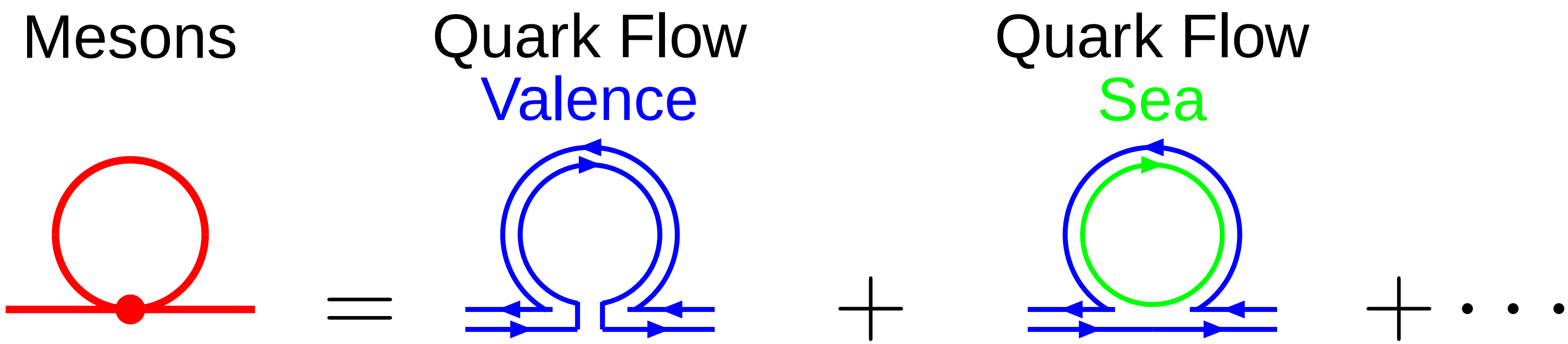
$1/p^2$

$\int d^4p$ $\quad p^4$



$(p^2)\,(1/p^2)\,p^4 = p^4$

# ChPT and Lattice QCD




Valence is easy to deal with in lattice QCD
Sea is very difficult

They can be treated separately: i.e. different quark masses

Partially Quenched ChPT (PQChPT)

# Two Loop: General

## Lagrangian Structure:

| | 2 flavour | | 3 flavour | | 3+3 PQChPT | |
|---|---|---|---|---|---|---|
| $p^2$ | $F, B$ | 2 | $F_0, B_0$ | 2 | $F_0, B_0$ | 2 |
| $p^4$ | $l_i^r, h_i^r$ | 7+3 | $L_i^r, H_i^r$ | 10+2 | $\hat{L}_i^r, \hat{H}_i^r$ | 11+2 |
| $p^6$ | $c_i^r$ | 53+4 | $C_i^r$ | 90+4 | $K_i^r$ | 112+3 |

$p^2$: Weinberg 1966

$p^4$: Gasser, Leutwyler 84,85

$p^6$: JB, Colangelo, Ecker 99,00

Note
- replica method $\Longrightarrow$ PQ obtained from $N_F$ flavour
- All infinities known
- 3 flavour is a special case of 3+3 PQ:

  $\hat{L}_i^r, K_i^r \to L_i^r, C_i^r$

# Two Flavours at Two Loop

Most Calculations Done

| | |
|---|---|
| $\gamma\gamma \to \pi^0\pi^0$ | Bellucci, Gasser, Sainio |
| $\gamma\gamma \to \pi^+\pi^-$, $F_\pi$, $m_\pi$ | Bürgi |
| $\pi\pi$-scattering, $F_\pi$, $m_\pi$ | JB,Colangelo,Ecker,Gasser,Sainio |
| $F_{V\pi}(t)$, $F_{S\pi}$ | JB, Colangelo, Talavera |
| $\pi \to \ell\nu\gamma$ | JB, Talavera |

- Reasonable convergence
- $c_i^r$ not important for many threshold quantities

$\Longrightarrow$ combined with dispersive methods, Roy equations, etc.

for very precise calculations

# PQChPT at Two Loop

Subject just beginning

valence equal mass, 3 sea equal mass:

$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0406017

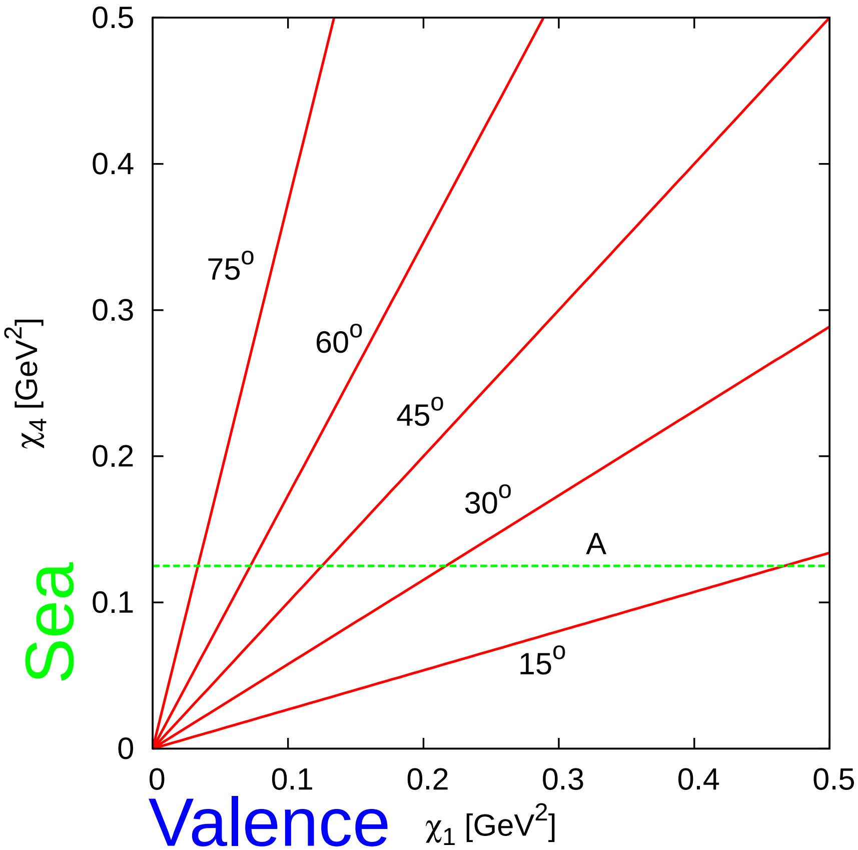
$F^2_{\pi^+}$: JB,Lähde, soon

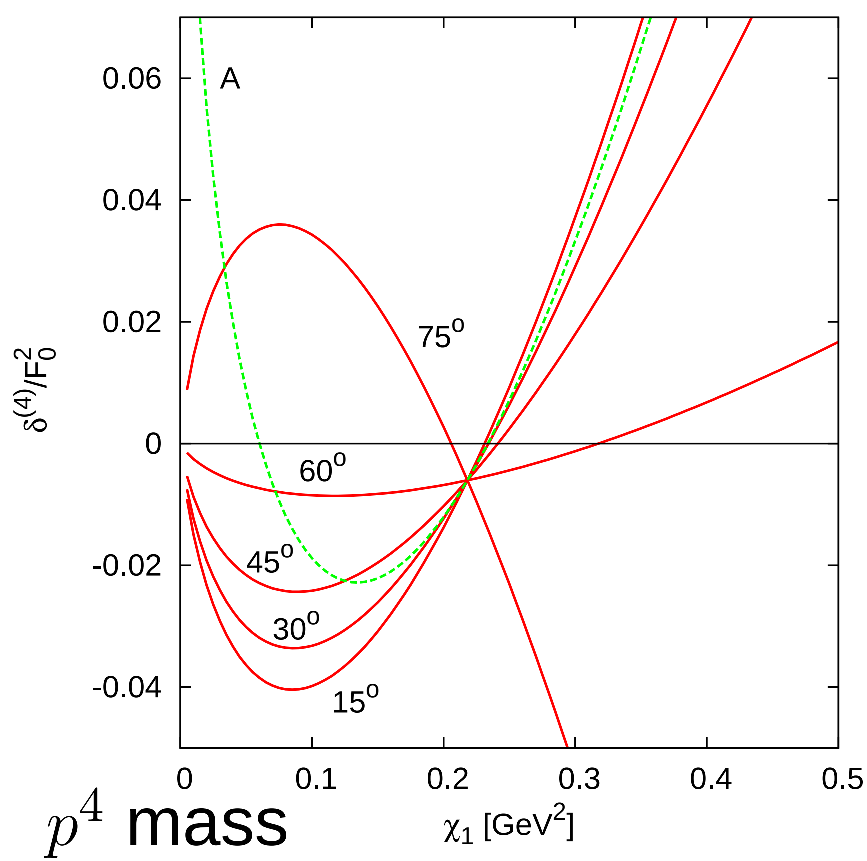
Planned: all the other mass combinations

Actual Calculations:

- heavy use of FORM Vermaseren
- use PQ without super $\Phi_0$ in supersymmetric formalism
- Main problem: sheer size of the expressions

# PQChPT at Two Loop

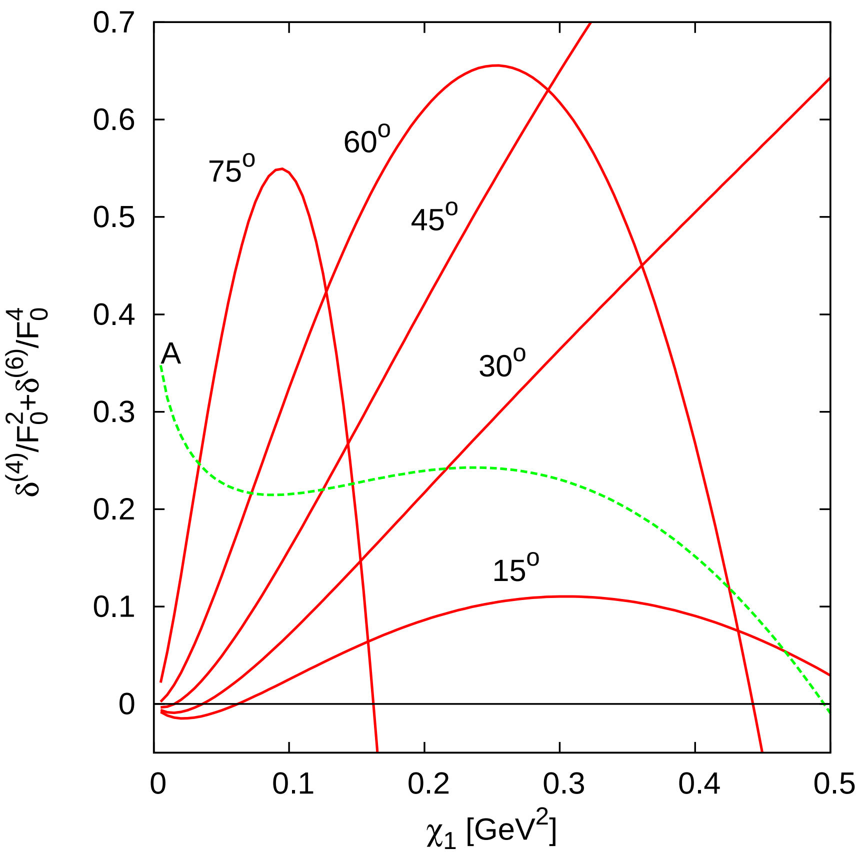


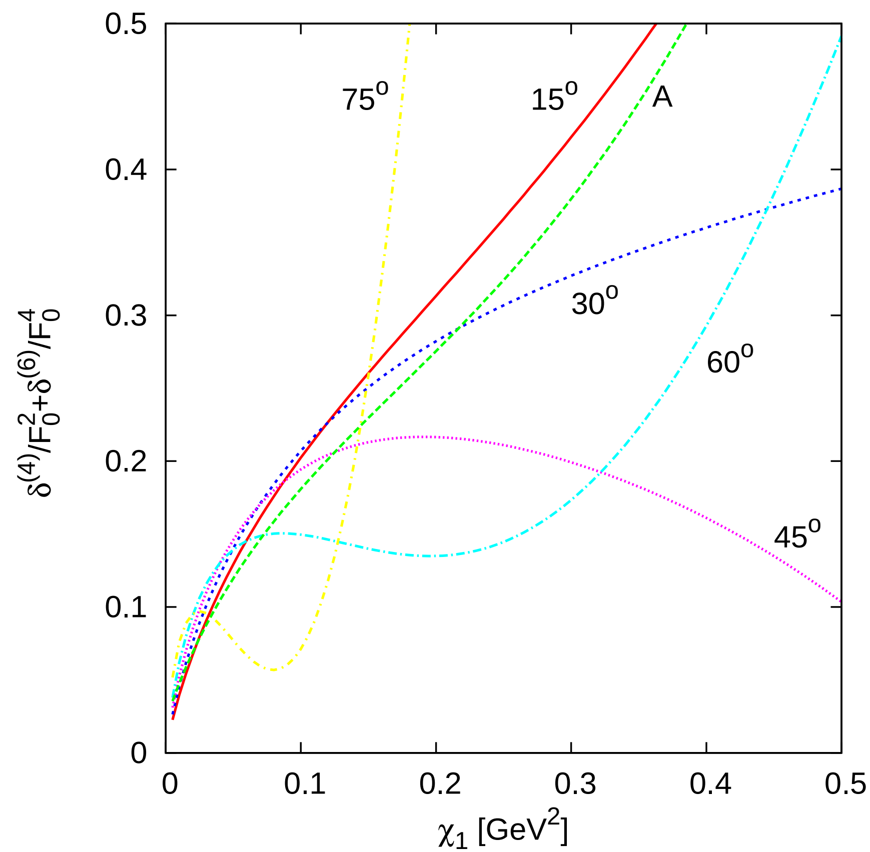




$$\chi_i = 2B_0 m_i = m_M^{2(0)}$$

$0.3\ GeV^2 \approx (550\ MeV)^2 \sim$ border ChPT

# PQChPT at Two Loops




$p^4 + p^6$ relative correction mass decay constant

(preliminary)

# Three Flavours at Two Loop

| | | |
|---|---|---|
| $\Pi_{VV\pi}$, $\Pi_{VV\eta}$, $\Pi_{VVk}$ | Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera | |
| $\Pi_{AA\pi}$, $\Pi_{AA\eta}$, $F_\pi$, $F_\eta$, $m_\pi$, $m_\eta$ | Kambor, Golowich; Amorós, JB, Talavera | |
| $\Pi_{SS}$ | Moussallam | $L_4^r, L_6^r$ |
| $\Pi_{VVK}$, $\Pi_{AAK}$, $F_K$, $m_K$ | Amorós, JB, Talavera | |
| $K_{\ell 4}$ | Amorós, JB, Talavera | $L_1^r, L_2^r, L_3^r$ |
| $F_M$, $m_M$ $(m_u \neq m_d)$ | Amorós, JB, Talavera | $L_{5,7,8}^r, m_u/m_d$ |
| $F_{V\pi}$, $F_{VK^+}$, $F_{VK^0}$ | Post, Schilcher; JB, Talavera | $L_9^r$ |
| $K_{\ell 3}$ | Post, Schilcher; JB, Talavera | $V_{us}$ |
| $F_{S\pi}$, $F_{SK}$ | JB, Dhonte | $L_4^r, L_6^r$ |
| $K, \pi \to \ell\nu\gamma$ | Geng, Ho, Wu | $L_{10}^r$ |
| $\pi\pi$ | JB,Dhonte,Talavera | |
| $\pi K$ | JB,Dhonte,Talavera | |

# General Strategy and some comments

- Find enough inputs from experiment
- $C_i^r$:
  - kinematical dependence: agree well with single resonance saturation
  - quark mass+kinematical: if vector dominated, seems to be OK
  - quark mass+kinematical: if scalar dominated: which scalars? (not $\sigma$)
  - quark masses: which scalars? unrealistically large estimates
- in $p^6$ physical or lowest order masses: thresholds in right place requires physical

# General Strategy and some comments

Inputs:

$K_{\ell 4}$: $F(0)$, $G(0)$, $\lambda$ E865 BNL

$m^2_{\pi^0}$, $m^2_\eta$, $m^2_{K^+}$, $m^2_{K^0}$ em with Dashen violation

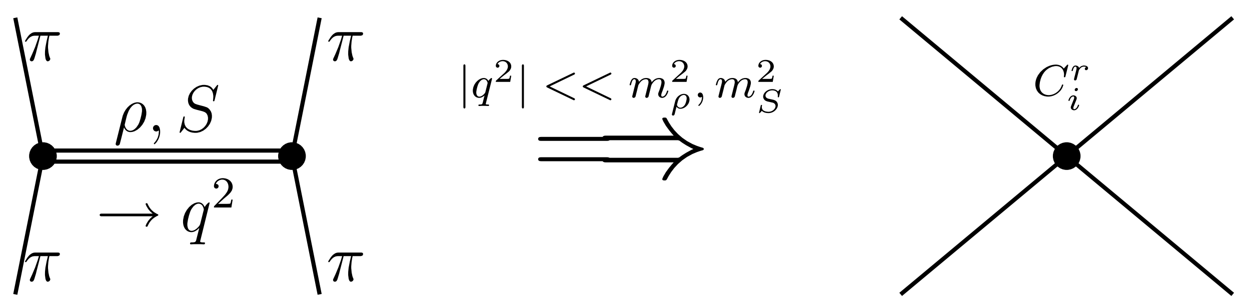
$F_{\pi^+}$

$F_{K^+}/F_{\pi^+}$

$m_s/\hat{m}$ 24 (26) $\hat{m} = (m_u + m_d)/2$

$L^r_4$, $L^r_6$

$C^r_i$ from single resonance approximation



# General Strategy and some comments

| | fit 10 | same $p^4$ | fit B | fit D |
|---|---|---|---|---|
| $10^3 L_1^r$ | $0.43 \pm 0.12$ | $0.38$ | $0.44$ | $0.44$ |
| $10^3 L_2^r$ | $0.73 \pm 0.12$ | $1.59$ | $0.60$ | $0.69$ |
| $10^3 L_3^r$ | $-2.53 \pm 0.37$ | $-2.91$ | $-2.31$ | $-2.33$ |
| $10^3 L_4^r$ | $\equiv 0$ | $\equiv 0$ | $\equiv 0.5$ | $\equiv 0.2$ |
| $10^3 L_5^r$ | $0.97 \pm 0.11$ | $1.46$ | $0.82$ | $0.88$ |
| $10^3 L_6^r$ | $\equiv 0$ | $\equiv 0$ | $\equiv 0.1$ | $\equiv 0$ |
| $10^3 L_7^r$ | $-0.31 \pm 0.14$ | $-0.49$ | $-0.26$ | $-0.28$ |
| $10^3 L_8^r$ | $0.60 \pm 0.18$ | $1.00$ | $0.50$ | $0.54$ |

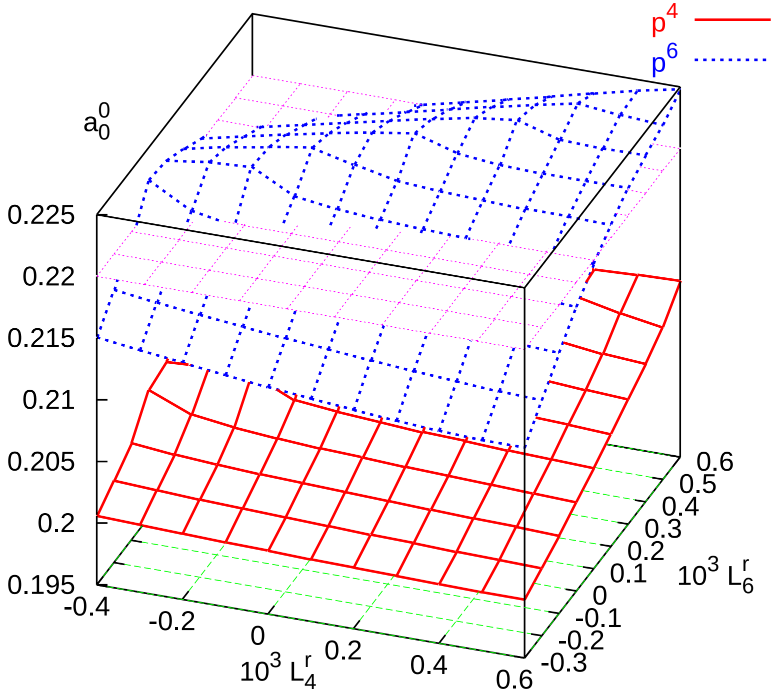
- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying $C_i^r$ factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \dots, 0.6\ 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius
- fit D: fit $\pi\pi$ and $\pi K$ thresholds

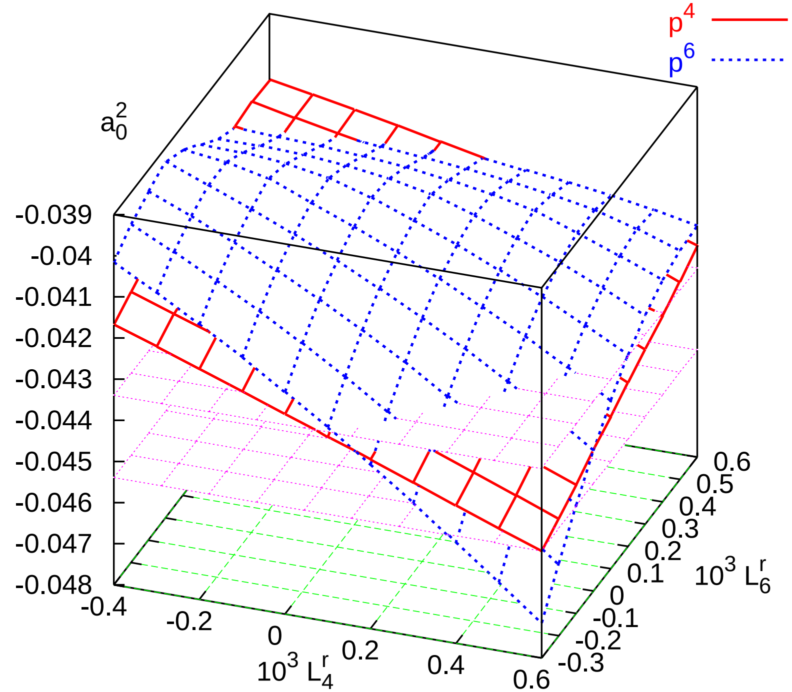
# General Strategy and some comments

| | fit 10 | same $p^4$ | fit B | fit D |
|---|---|---|---|---|
| $2B_0\hat{m}/m_\pi^2$ | 0.736 | 0.991 | 1.129 | 0.958 |
| $m_\pi^2$: $p^4, p^6$ | 0.006,0.258 | 0.009,$\equiv 0$ | $-$0.138,0.009 | $-$0.091,0.133 |
| $m_K^2$: $p^4, p^6$ | 0.007,0.306 | 0.075,$\equiv 0$ | $-$0.149,0.094 | $-$0.096,0.201 |
| $m_\eta^2$: $p^4, p^6$ | $-$0.052,0.318 | 0.013,$\equiv 0$ | $-$0.197,0.073 | $-$0.151,0.197 |
| $m_u/m_d$ | 0.45$\pm$0.05 | 0.52 | 0.52 | 0.50 |
| $F_0$ [MeV] | 87.7 | 81.1 | 70.4 | 80.4 |
| $\frac{F_K}{F_\pi}$: $p^4, p^6$ | 0.169,0.051 | 0.22,$\equiv 0$ | 0.153,0.067 | 0.159,0.061 |

- $m_u = 0$ always very far from the fits
- $F_0$: pion decay constant in the chiral limit

# $\pi\pi$



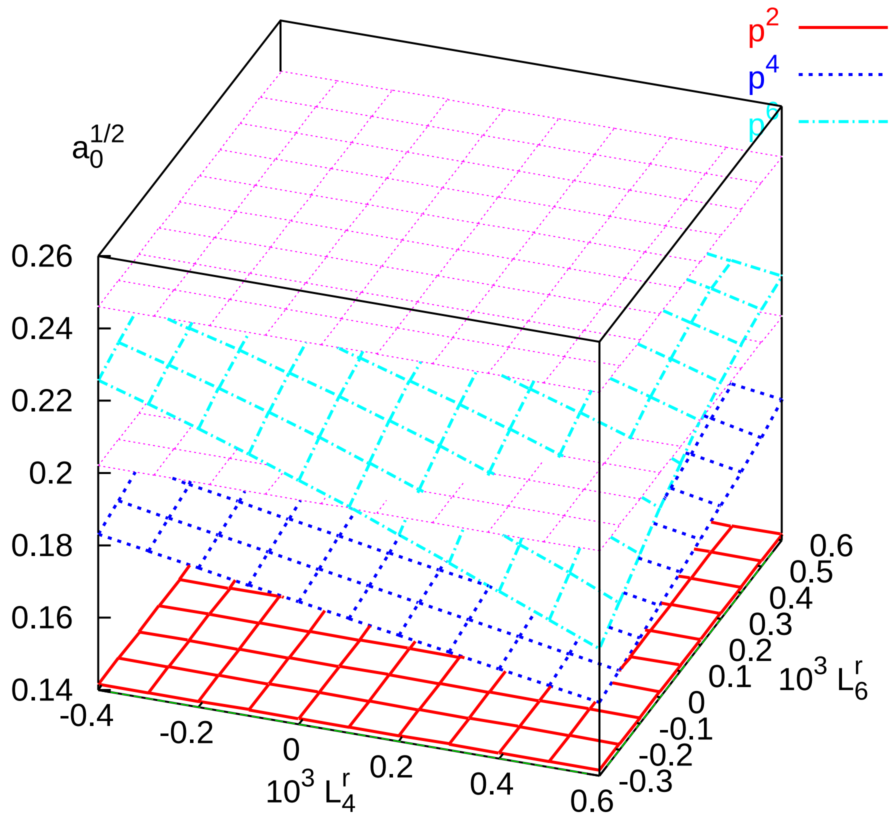



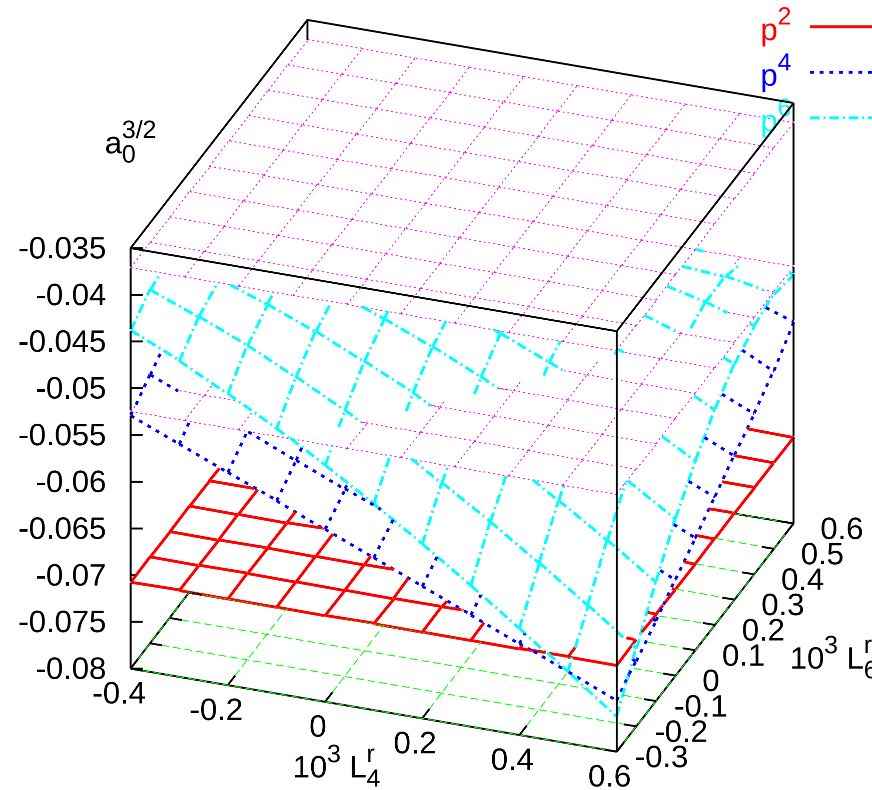

$a_0^0 = 0.220 \pm 0.005$, $a_0^2 = -0.0444 \pm 0.0010$

Colangelo, Gasser, Leutwyler

$a_0^0 = 0.159$ $a_0^2 = -0.0454$ at order $p^2$

# $\pi K$



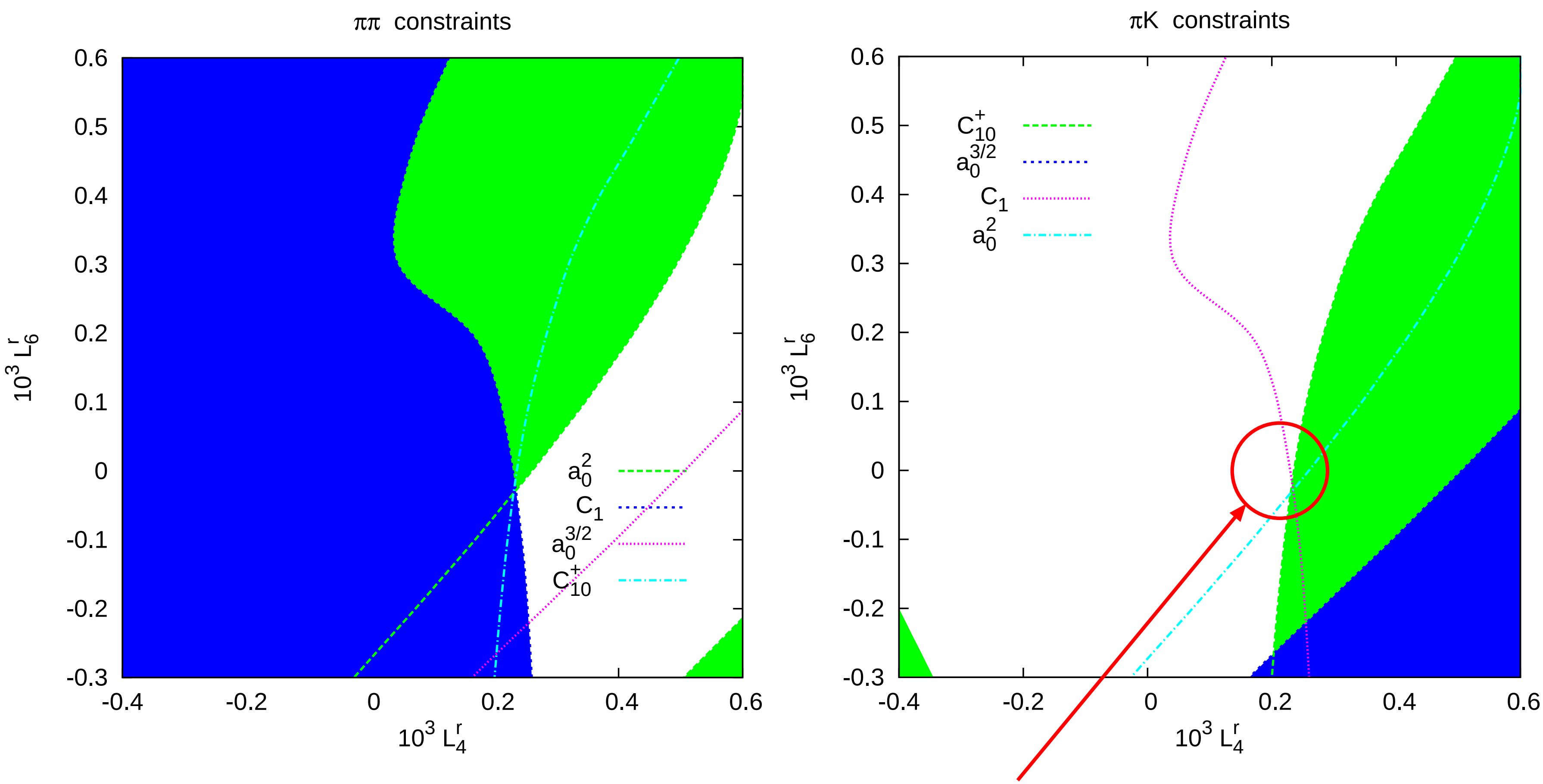




$a_0^{1/2} = 0.224 \pm 0.022$, $a^2_{3/2} = -0.0448 \pm 0.0077$

Büttiker, Descotes-Genon, Moussallam

$a_0^{1/2} = 0.142 \; a_0^2 = -0.0708$ at order $p^2$

# $\pi\pi$ and $\pi K$




preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

# $K_{\ell 3}$ Definitions and $V_{us}$

Scalar formfactor:
$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization:
$$f_{+,0}(t) = f_+(0)\left(1 + \lambda_{+,0}\frac{t}{m_\pi^2}\right)$$

$|V_{us}|$:

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$|V_{us}|$:

- Know theoretically $f_+(0) = 1 + \cdots$
  - Short distance correction to $G_F$ from $G_\mu$ Marciano-Sirlin
  - Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$
  - Isospin Breaking Leuwyler-Roos $\dfrac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.022$ In Progress

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PDG2002: $|V_{ud}| = 0.9734 \pm 0.0008$, $|V_{us}| = 0.2196 \pm 0.0026$
$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) = 0.9957 \pm 0.0019$

# $f_+(t)$ Theory

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

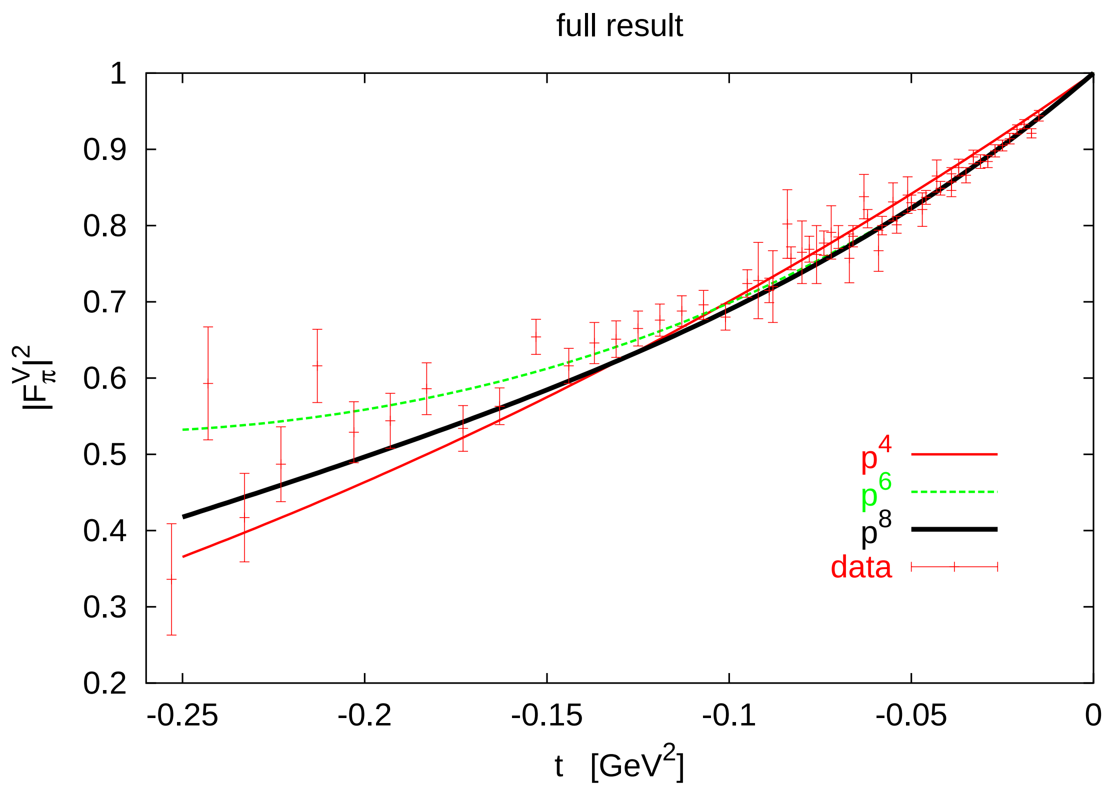
$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4}(-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

Pion electromagnetic Form factor:
JB, Talavera

$L_9^r = 0.00593 \pm 0.00043$

$-4C_{88}^r + 4C_{90}^r = 0.00022 \pm 0.00002$

VMD: $R_{+1}^{K\pi} \approx -4\,10^{-5}$ GeV$^2$



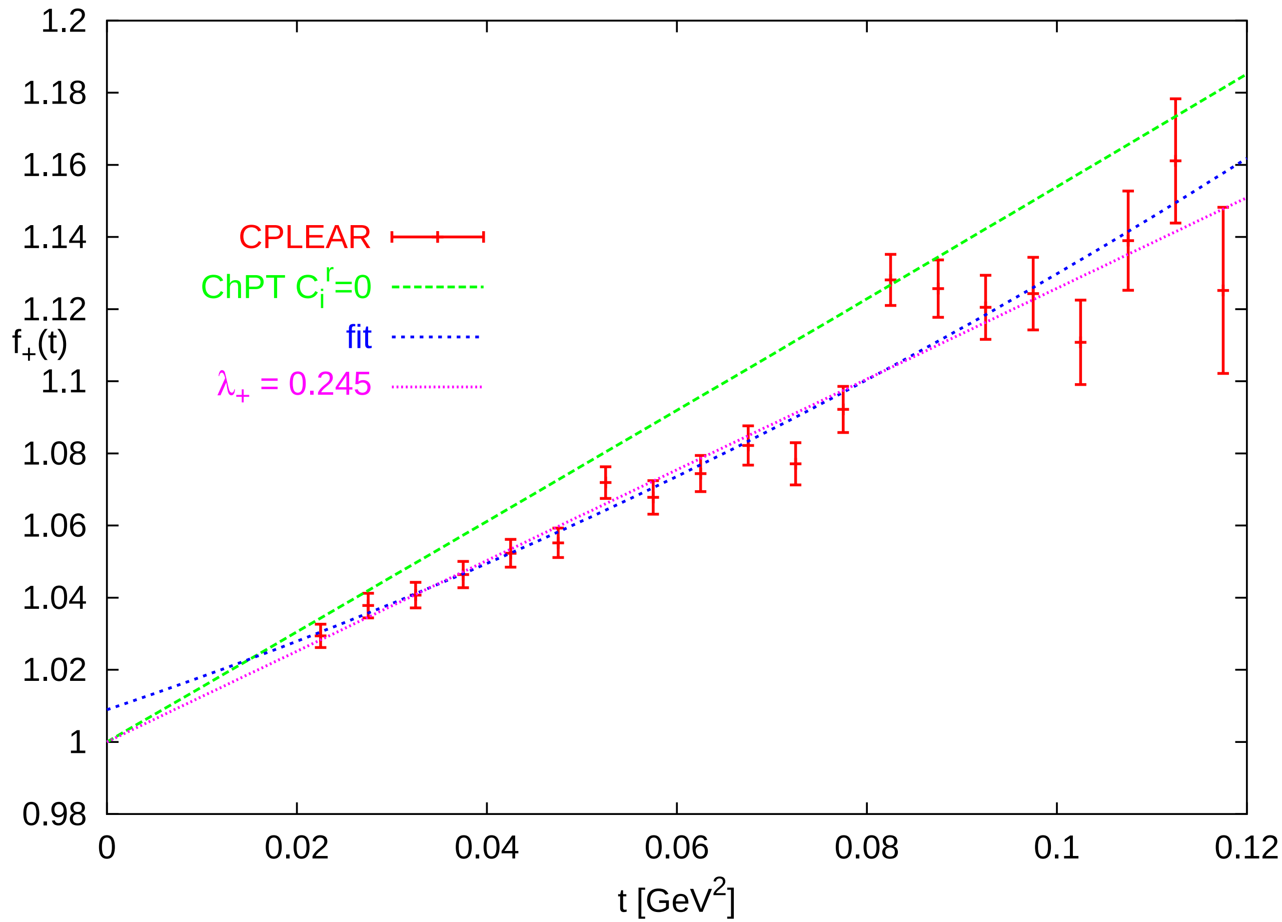
# ChPT fit to $f_+(t)$

$\Rightarrow R^{K\pi}_{+1} =$
$-(4.7\pm 0.5)\, 10^{-5}$ GeV$^2$

$\left(c_+ = 3.2 \text{ GeV}^{-4}\right)$
fixed by ChPT

$\Rightarrow a_+ = 1.009 \pm 0.004$

$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$



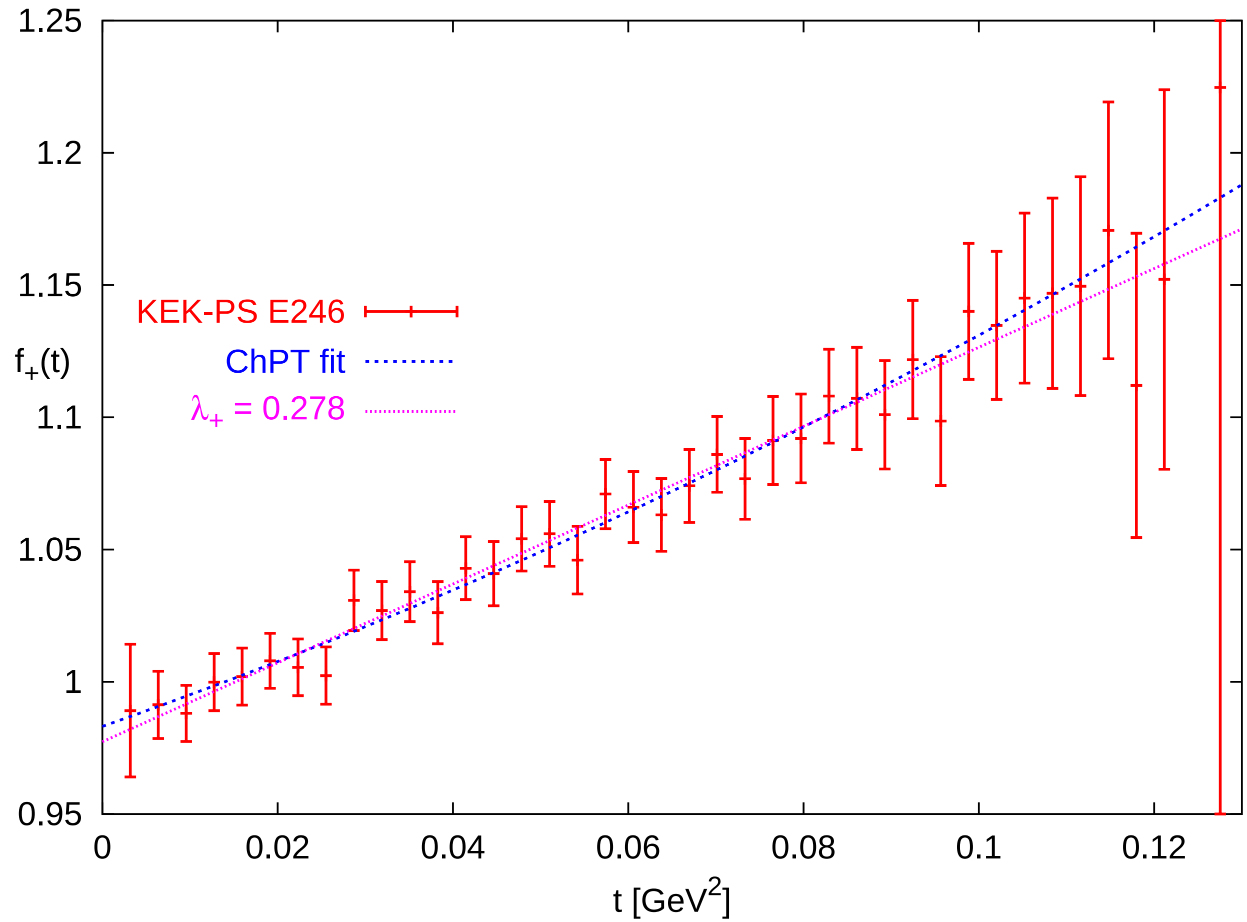
# ChPT fit to $f_+(t)$

$\Rightarrow R^{K\pi}_{+1} =$
$-2.5\ 10^{-5}$ GeV$^2$

$\left(c_+ = 3.2 \text{ GeV}^{-4}\right)$
fi xed by ChPT

$\Rightarrow a_+ = 1.006$

$\Rightarrow \lambda_+ = 0.0214 \pm 0.0018$



# $f_0(t)$

Main Result:

$$
\begin{aligned}
f_0(t) \quad = \quad & 1 - \frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 \\
& +8\frac{t}{F_\pi^4}(2C_{12}^r + C_{34}^r)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}\left(F_K/F_\pi - 1\right) \\
& -\frac{8}{F_\pi^4}t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
\end{aligned}
$$

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& + 8\frac{t}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}\left(F_K/F_\pi - 1\right) \\
& - \frac{8}{F_\pi^4} t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.
\end{aligned}
$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO $C_i^r$ and only depend on the $L_i^r$ at order $p^6$

$\Longrightarrow$

All needed parameters can be determined experimentally

# $f_0(t)$

Main Result:

$$f_0(t) = 1 - \frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 + 8\frac{t}{F_\pi^4}(2C_{12}^r + C_{34}^r)\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}(F_K/F_\pi - 1) - \frac{8}{F_\pi^4}t^2 C_{12}^r + \overline{\Delta}(t) + \Delta(0)\,.$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO $C_i^r$ and only depend on the $L_i^r$ at order $p^6$

$\Longrightarrow$

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r]\,.$$

$$\Delta(0) = -0.0227\,(p^4) + 0.0113\,(p^6 \text{ pure loop}) + 0.0033\,(p^6 L_i^r)$$

# $V_{us}$ present status

- More Theory: Dispersion theory relates slopes and curvature in $f_0(t)$ Jamin, Oller, Pich

  $C_{12}^r + C_{34}^r = 3.2 \pm 1.5\ 10^{-6} \Longrightarrow f_+(0)_{p^6} = 0.002 \pm 0.009$.

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  - 2003: E865 $K_{\ell 3}^+$ branching ratio: strong increase
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    - Curvature seen, reasonable agreement with ChPT
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- Expect both more theory and experiment (KLOE,NA48)

the unitarity problem might be on the way out

# Conclusions

- 2 flavour ChPT at 2 loops (almost) finished subject
- 3 flavour ChPT at 2 loops
  - many calculations done
  - things seem to work but convergence is fairly slow
  - "kinematical" and "vector" $C_i^r$ seem to be OK
  - $L_4^r, L_6^r$ nonzero but reasonable for large $N_c$
  - $\eta \to 3\pi$, isobreaking in $K_{\ell 3}$: parts done
- PQChPT at 2 loops: subject just beginning
- $K_{\ell 3}$ an example of results even with all the $C_i^r$