LUND UNIVERSITY

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ε'/ε in the Chiral Limit

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Long collaboration on nonleptonic matrix-elements with J. Prades, Granada, and some with E. Pallante, Barcelona

JHEP **0006** (2000) 035 [hep-ph/0005189].
JHEP **0001** (2000) 002 [hep-ph/9911392].
JHEP **9901** (1999) 023 [hep-ph/9811472].
Nucl. Phys. **B521** (1998) 305 [hep-ph/9801326]. (with EP)
Nucl. Phys. **B490** (1997) 239 [hep-ph/9610360].
Nucl. Phys. **B474** (1996) 379 [hep-ph/9511388]. (with EP)
Nucl. Phys. **B444** (1995) 523 [hep-ph/9502363].
Phys. Rev. Lett. **75** (1995) 1447 [hep-ph/9505251]. (with EP)
Phys. Lett. **B342** (1995) 331 [hep-ph/9409255].

- Introduction
- Short-distance enhancement
- X-boson or fictitious gauge boson scheme
- Long-distance
- Results

$K \to \pi\pi$ decays in isospin amplitudes:

$$A[K^0 \to \pi^0\pi^0] \equiv \sqrt{\frac{1}{3}}A_0 - \sqrt{\frac{2}{3}}A_2 \qquad (1)$$

$$A[K^0 \to \pi^+\pi^-] \equiv \sqrt{\frac{1}{3}}A_0 + \frac{1}{\sqrt{6}}A_2 \qquad (2)$$

$$A[K^+ \to \pi^+\pi^0] \equiv \frac{\sqrt{3}}{2}A_2 \qquad (3)$$

$$\left|\frac{A_0}{A_2}\right| = 22.1 \qquad \text{and the naive gives} \qquad \sqrt{2}$$

$$\left|\frac{A_0}{A_2}\right| = 16.4 \ (p^2 \text{ part})$$

From a fit to $K \to \pi\pi$ and $K \to \pi\pi\pi$ up to order p^4, removing m_s, m_u, m_d, final state rescattering,... J. Kambor, J. Missimer, and D. Wyler, Phys. Lett. B 261 (1991) 496.

For later use: $A_I = -i a_I e^{i\delta_I}$

For observables we now define:

$$\varepsilon = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})}$$

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$$

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}$$

and

$$\varepsilon' = \frac{1}{\sqrt{2}} \left(\frac{A(K_L \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})} - \varepsilon \frac{A(K_S \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})} \right)$$

$$\left. \begin{array}{ll} |\mathrm{Re} a_0| >> |\mathrm{Re} a_2| >> |\mathrm{Im} a_0|, |\mathrm{Im} a_2| & \\ |\varepsilon|, |\tilde{\varepsilon}| << 1 & |\varepsilon'| << |\varepsilon| \end{array} \right\} \Longrightarrow$$

$$\varepsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \frac{\mathrm{Re} a_2}{\mathrm{Re} a_0} \left(\frac{\mathrm{Im} a_2}{\mathrm{Re} a_2} - \frac{\mathrm{Im} a_0}{\mathrm{Re} a_0} \right)$$

Now using $\Delta m = m_L - m_S \approx \frac{\Delta\Gamma}{2}$ and $\Gamma_L << \Gamma_S$ and Γ_{12} dominated by $\pi\pi$

$$\varepsilon = \frac{1}{\sqrt{2}} e^{i\pi/4} \left(\frac{\mathrm{Im} M_{12}}{\Delta m} + \frac{\mathrm{Im} a_0}{\mathrm{Re} a_0} \right)$$

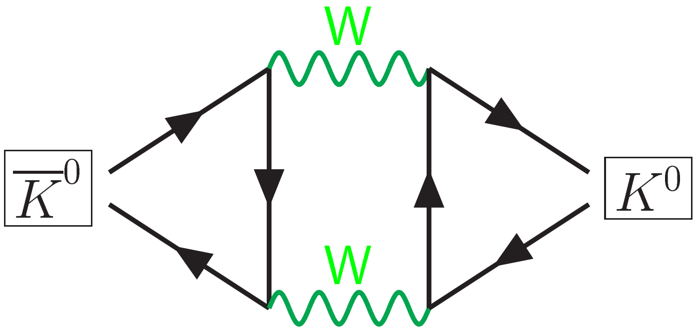
$$\eta_{+-} = \varepsilon + \varepsilon'$$

$$\eta_{00} = \varepsilon - 2\varepsilon'$$

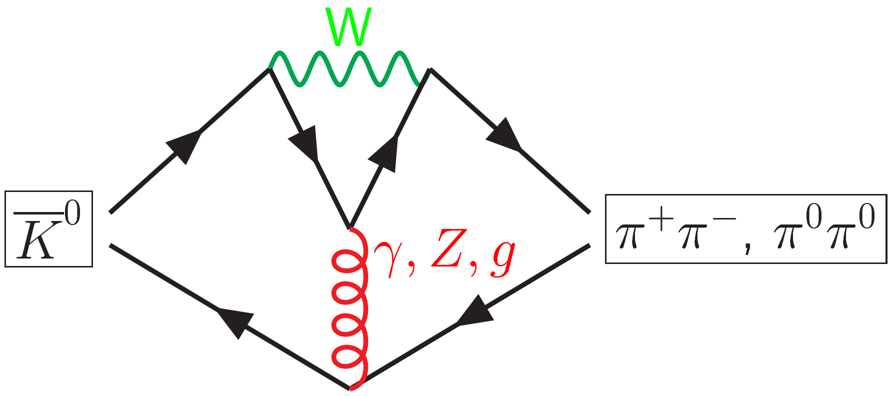
ε describes the indirect part ε' the direct part of CP-violation.

$|\varepsilon| = 2.28 \cdot 10^{-3}$

from



AND from



we get ε'

Experiment: $\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{1}{6}\left\{1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right\}$

NA31	$(23.0 \pm 6.5) \times 10^{-4}$
E731	$(7.4 \pm 5.9) \times 10^{-4}$
KTeV	$(28.0 \pm 4.1) \times 10^{-4}$
NA48 97	$(18.5 \pm 7.3) \times 10^{-4}$
NA48 98	$(12.2 \pm 4.9) \times 10^{-4}$
ALL	$(19.3 \pm 2.4) \times 10^{-4}$

$(\chi^2/dof = 11.1/5)$

From quarks to mesons in the weak interactions

ENERGY SCALE	FIELDS	Effective Theory
M_W	W, Z, γ, g; τ, μ, e, ν_ℓ; t, b, c, s, u, d	Standard Model
	$\Downarrow$ using OPE	
$\lesssim m_c$	$\gamma, g;\ \mu, e, \nu_\ell$; s, d, u	QCD,QED,$\mathcal{H}_{\text{eff}}^{\lvert\Delta S\rvert=1,2}$
	$\Downarrow$???	
M_K	$\gamma;\ \mu, e, \nu_\ell$; $\pi,\ K,\ \eta$	CHPT

OPE and renormalization group needed:
$\log\frac{M_W^2}{M_K^2}$ and $\log\frac{M_t^2}{M_K^2}$ are large
$\Rightarrow$ need to resum their effects.

??? we will split up in several steps later

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i \left(z_i - y_i \frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} \right) Q_i$$

$$\begin{aligned}
Q_1^u &= (\bar{s}_\alpha \gamma_\mu u_\beta)_L (\bar{u}_\beta \gamma^\mu d_\alpha)_L \quad Q_1^c = (\bar{s}_\alpha \gamma_\mu c_\beta)_L (\bar{c}_\beta \gamma^\mu d_\alpha)_L \\
Q_2^u &= (\bar{s}_\alpha \gamma_\mu u_\alpha)_L (\bar{u}_\beta \gamma^\mu d_\beta)_L \quad Q_2^c = (\bar{s}_\alpha \gamma_\mu c_\alpha)_L (\bar{c}_\beta \gamma^\mu d_\beta)_L \\
Q_3 &= (\bar{s}_\alpha \gamma_\mu d_\alpha)_L \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu q_\beta)_L \\
Q_4 &= (\bar{s}_\alpha \gamma_\mu d_\beta)_L \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu q_\alpha)_L \\
Q_5 &= (\bar{s}_\alpha \gamma_\mu d_\alpha)_L \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu q_\beta)_R \\
Q_6 &= (\bar{s}_\alpha \gamma_\mu d_\beta)_L \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu q_\alpha)_R \\
Q_7 &= (\bar{s}_\alpha \gamma_\mu d_\alpha)_L \sum_{q=u,d,s,c,b} \frac{3}{2} e_q (\bar{q}_\beta \gamma^\mu q_\beta)_R \\
Q_8 &= (\bar{s}_\alpha \gamma_\mu d_\beta)_L \sum_{q=u,d,s,c,b} \frac{3}{2} e_q (\bar{q}_\beta \gamma^\mu q_\alpha)_R \\
Q_9 &= (\bar{s}_\alpha \gamma_\mu d_\alpha)_L \sum_{q=u,d,s,c,b} \frac{3}{2} e_q (\bar{q}_\beta \gamma^\mu q_\beta)_L \\
Q_{10} &= (\bar{s}_\alpha \gamma_\mu d_\beta)_L \sum_{q=u,d,s,c,b} \frac{3}{2} e_q (\bar{q}_\beta \gamma^\mu q_\alpha)_L
\end{aligned}$$

with $(\bar{q}\gamma_\mu q')_{(L,R)} = \bar{q}\gamma_\mu(1 \mp \gamma_5)q'$; α and β are colour indices.

Results in NDR

$m_t(m_t) = 166\ GeV$, $\alpha = 1/137.0$, $\alpha_S(m_Z) = 0.1186$

	$\mu_{OPE} = 0.9$ GeV	$\mu_X = 0.9$ GeV
z_1	-0.490	-0.788
z_2	1.266	1.457
z_3	0.0092	0.0086
z_4	-0.0265	-0.0101
z_5	0.0065	0.0029
z_6	-0.0270	-0.0149
z_7	$2.6\ 10^{-5}$	0.0002
z_8	$5.3\ 10^{-5}$	$6.8\ 10^{-5}$
z_9	$5.3\ 10^{-5}$	0.0003
z_{10}	$-3.6\ 10^{-5}$	$-8.7\ 10^{-5}$
y_1	0.	0.
y_2	0.	0.
y_3	0.0287	0.0399
y_4	-0.0532	-0.0572
y_5	0.0018	0.0112
y_6	-0.0995	-0.1223
y_7	$-0.9\ 10^{-5}$	-0.00016
y_8	0.0013	0.0018
y_9	-0.0105	-0.0121
y_{10}	0.0041	0.0065

z_1 and z_2 have changed much from 0 and 1 $\Rightarrow$: short-distance contribution to $\Delta I = 1/2$ Rule.

Large enhancement of y_6 and y_8

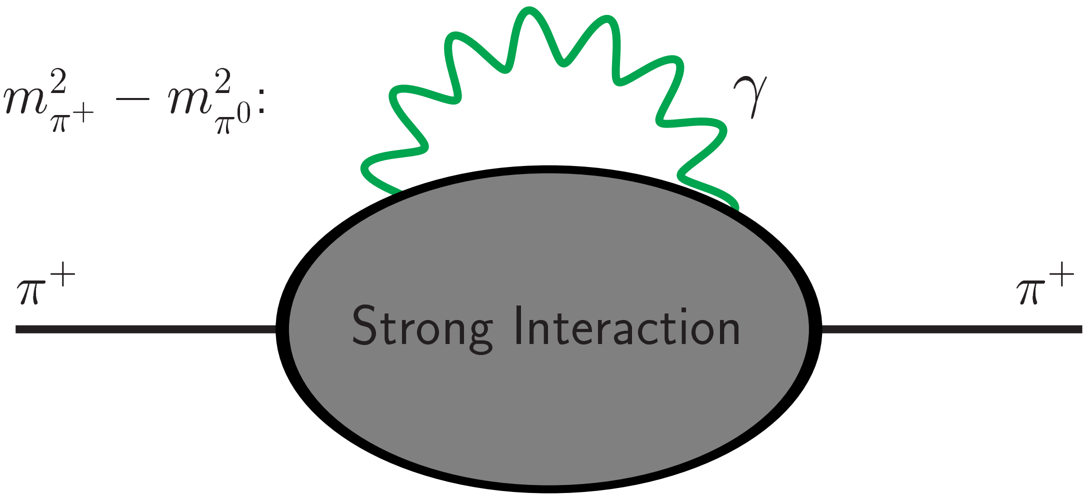
The Matrix-Elements

Remember: C_i depend on μ_{OPE} and on definition of Q_i
Solve in various ways:

- Stay in QCD $\Rightarrow$ Lattice calculations: Sachrajda (not fully usable at present)
- Give up $\Rightarrow$ Naive factorization.
- Improved factorization
- X-boson method (or fictitious boson method)
- Large N_c
  - CHPT: Bardeen-Buras-Gérard; Dortmund
  - ENJL: JB, J. Prades
  - LMD: Peris, de Rafael

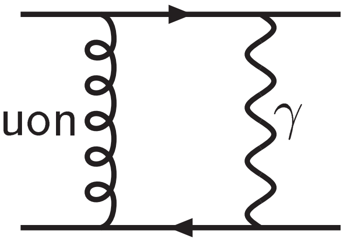
The X-boson method: preamble

Simpler: $m^2_{\pi^+} - m^2_{\pi^0}$:



Matrix Element involves: $M = \int_0^\infty dq^2_\gamma$.

Split integral at μ^2:

$\int_{\mu^2}^\infty dq^2_\gamma \longrightarrow$ gluon  $\longrightarrow (\bar{q}q)(\bar{q}q)$ Estimate in large N_c

$\Longrightarrow m^2_{\pi^+} - m^2_{\pi^0} = \frac{3\alpha_S\alpha_e}{\mu^2 F^4}\langle\bar{q}q\rangle^2$ Bardeen, Bijnens, Gérard(BBG)

$\int_0^{\mu^2} dq^2_\gamma \longrightarrow$ CHPT p^2 or p^4 (BBG) or ENJL (BP)

or LMD approximation (Knecht et al.)

Main points: photon couplings known EVERYWHERE

Good identification of scale: μ is physical

good matching and numbers obtained

The X-boson method

Improved factorization is scheme- and scale-independent BUT r_1 depends on the quark/gluon state chosen.

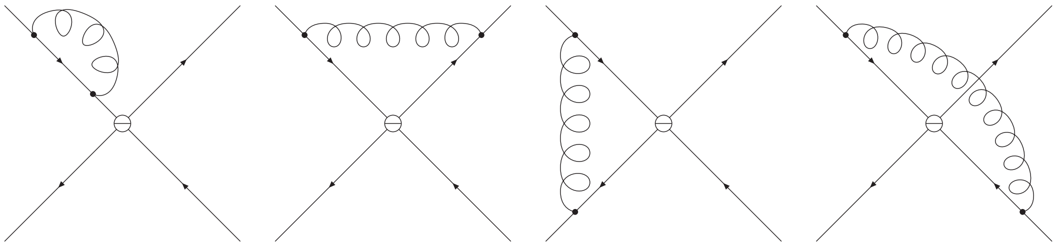
Ecker $\Longrightarrow$ currents identifiable across boundaries.

Simpler case of π^+-π^0 mass difference first

$$\mathcal{H}_{\text{eff}}^{\Delta S=2} \Longrightarrow g_X X^{\mu}_{\Delta S=2} \left(\bar{s}_\alpha \gamma_\mu d_\alpha\right)_L$$

with M_X such that $\alpha_S \log \frac{M_X}{\mu}$ is small.

OPE matrix element :

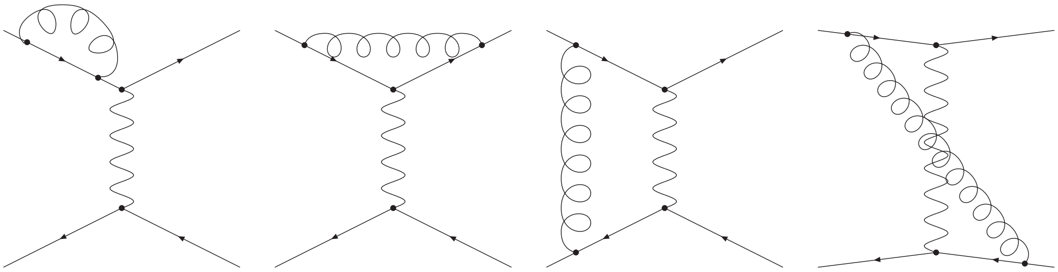


$$iC_D \left[\left(1 + \alpha_S(\nu) F(q_i)\right) S_1 + \left(1 + \alpha_S(\nu) F'(q_i)\right) S_2 \right]$$

$$C_D = -C(\nu) \left(1 + \frac{\alpha_S(\nu)}{\pi} \left[\frac{\gamma_1}{2} \ln\left(\frac{2 q_1 \cdot q_2}{\nu^2}\right) + r_1 \right] \right)$$

S_1 and S_2 the tree level matrix elements $(\bar{s}\gamma^\mu d)_L (\bar{s}\gamma_\mu d)_L$

X matrix element :



$$iC_C\left[\left(1+\alpha_S(\mu_C)F(q_i)\right)S_1+\left(1+\alpha_S(\mu_C)F'(q_i)\right)S_2\right]+\mathcal{O}(M_X^{-4})$$

$$C_C=\frac{-g_X^2}{M_X^2}\left(1+\frac{\alpha_S(\mu_C)}{\pi}\left[\frac{\gamma_1}{2}\ln\left(\frac{2q_1\cdot q_2}{M_X^2}\right)+\tilde{r}_1\right]\right)$$

$F(q_i)$ and $F'(q_i)$ cancel, r_1 cancels scheme dependence and $\tilde{r}_1$ changes to X-boson current scheme.

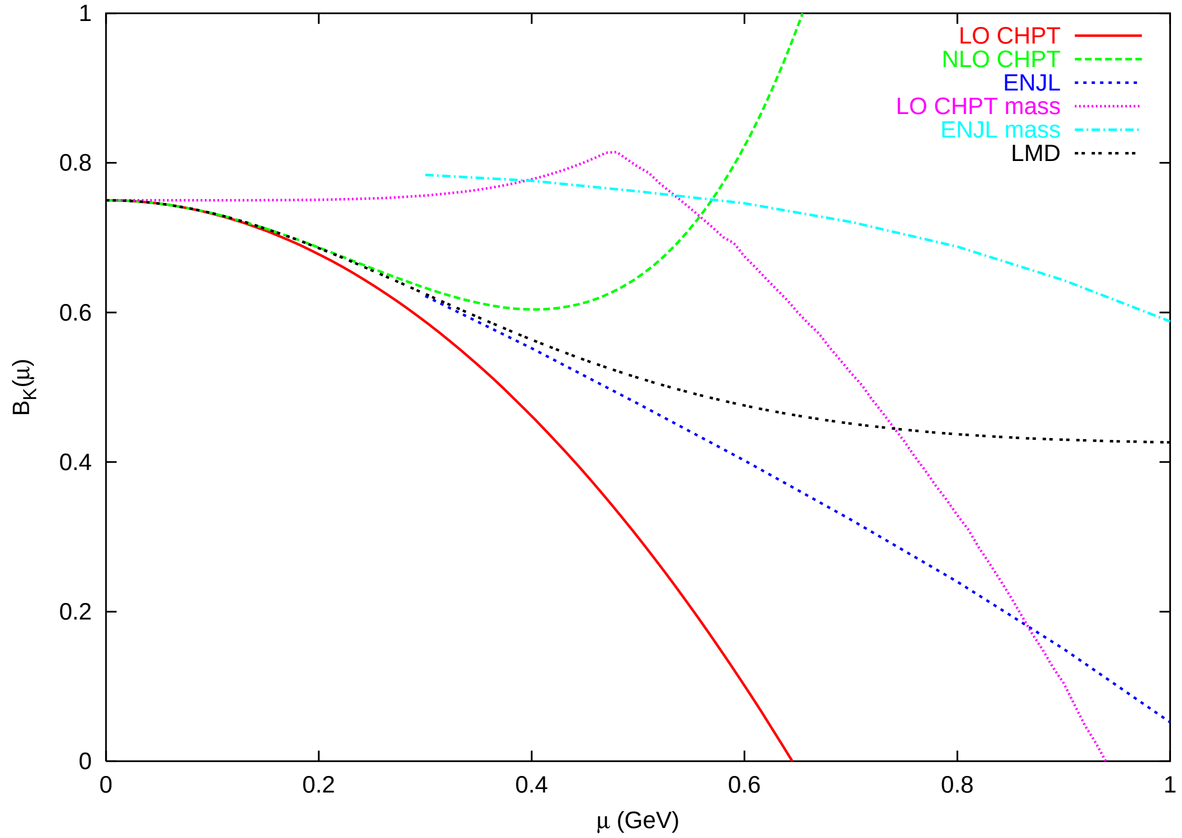
g_X is now scale, scheme and external quark-gluon state independent.

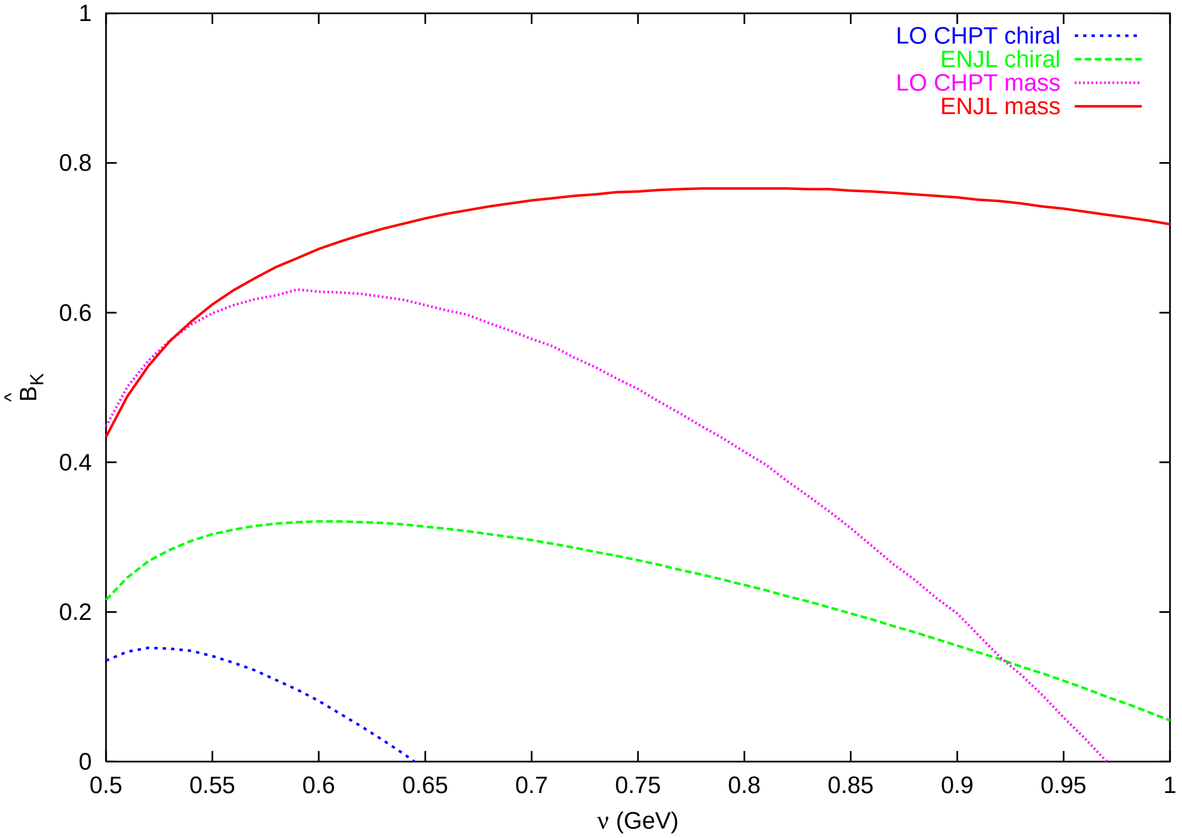
g_X depends on the precise scheme used for the vector, axial-vector current.

(See J. Bijnens and J. Prades, JHEP **0001** (2000) 002 [hep-ph/9911392])

$\Delta S=1$ case is more complicated (10 by 10 matrices.)

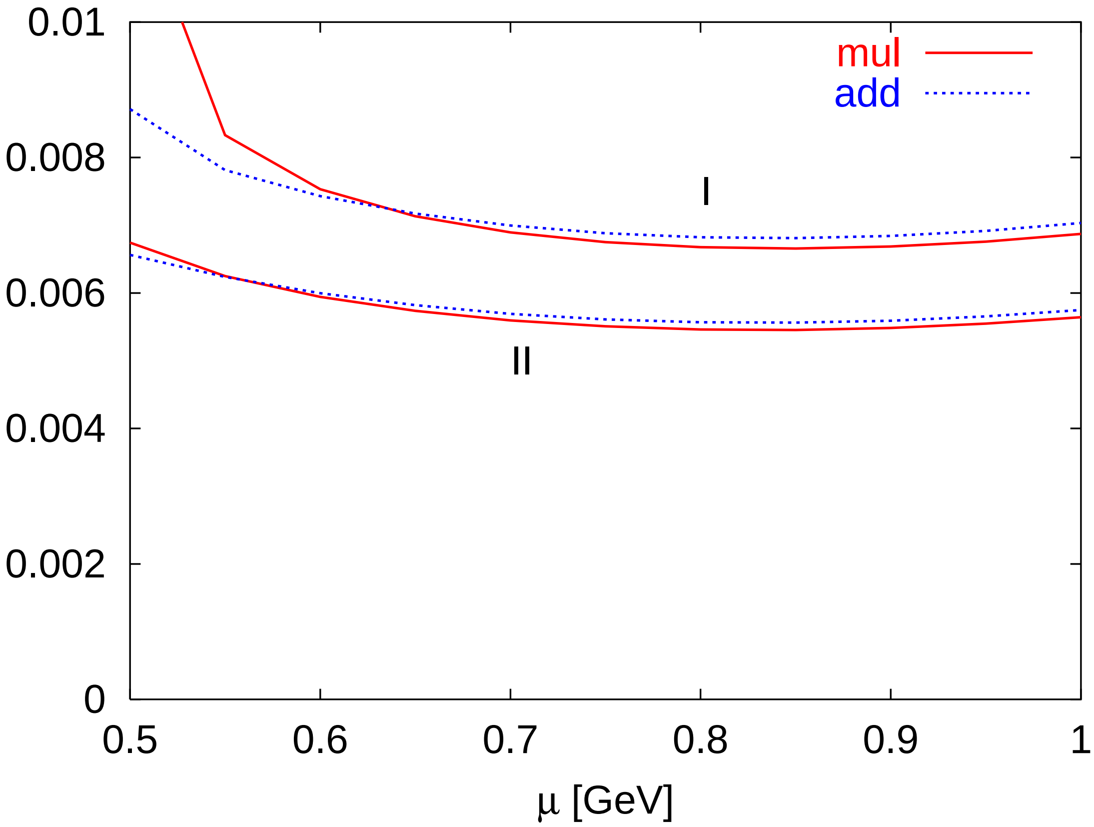
1. Introduce a set of fictitious gauge bosons: X
2. $\alpha_S \log M_X \mu$ does not need resumming.
3. X-bosons must be uncolored.
4. Only perturbative QCD and OPE used so far.
5. $r_1-\tilde{r}_1=-\frac{11}{12}$





$$\hat{B}_K^\chi = 0.32 \pm 0.06\ (\alpha_S) \pm 0.12\ (\text{model})$$
$$\hat{B}_K = 0.77 \pm 0.05\ (\alpha_S) \pm 0.05\ (\text{model})$$

ε'/ε in the chiral limit



Used $G_{27} = 0.48$, Re$G_8 = 6.2$
ε we get within 10% of the experimental value

$$\left(\frac{\varepsilon'}{\varepsilon}\right)^{\chi} = (7.4 - 1.9) \cdot 10^{-3} = 5.5 \cdot 10^{-3}$$

- $B_6 \approx 2.5$ not $\lesssim 1.5$
- $B_8 \approx 1.3$ OK but not $B_8 \approx B_6$
- FSI
- isospin breaking: Ω

$$|\varepsilon'_K| \simeq \frac{1}{\sqrt{2}}\frac{\text{Re}\, a_2}{\text{Re}\, a_0}\left(-\frac{\text{Im}\, a_0}{\text{Re}\, a_0}+\frac{\text{Im}\, a_2}{\text{Re}\, a_2}\right)$$

B-parameters:

Method	$B_6^{(1/2)NDR}(2\text{GeV})$	$B_7^{(3/2)NDR}(2\text{GeV})$	$B_8^{(3/2)NDR}(2\text{GeV})$
This work [χ limit]	2.5 ± 0.4	0.8 ± 0.1	1.35 ± 0.20
LMD	–	0.94	–
Dispersive	–	0.78 ± 0.17	1.6 ± 0.4
sum rules Narison	2.99 ± 0.34	0.7 ± 0.2	1.70 ± 0.39
CHPT Dortmund	$1.5 \sim 1.7$	$-0.1 \sim 0.09$	$0.4 \sim 0.7$
Lattice	–	$0.5 \sim 0.8$	$0.7 \sim 1.1$
Chiral QM Bertolini	$1.2 \sim 1.7$	~ 0.9	~ 0.9

FSI:

$$\left.\begin{array}{l}\text{Re}a_0,\ \text{Re}a_2 \text{ from full experiment}\\ \text{Im}a_0,\ \text{Im}a_2 \text{ to } p^2\end{array}\right\} \Longrightarrow \begin{array}{l}\text{first term extra}\\ \text{suppressed}\end{array}$$

For us: FSI mainly act on prefactor

Isospin breaking: π^0,η and η' mix $+\ldots$

$$\frac{\Delta\text{Im}a_2}{\text{Re}a_2} \approx \Omega\frac{\text{Im}a_0}{\text{Re}a_0} \quad \text{with} \quad \Omega \approx 0.16 \pm 0.03$$

Including last two main corrections

$$\left|\frac{\varepsilon'}{\varepsilon}\right| = (5.4 - 2.3)\cdot 10^{-3} = (3.1 \pm ??)\cdot 10^{-3}$$