CHPT RESULTS FOR HVP AND A NEW EVALUATION OF THE PION LOOP CONTRIBUTION

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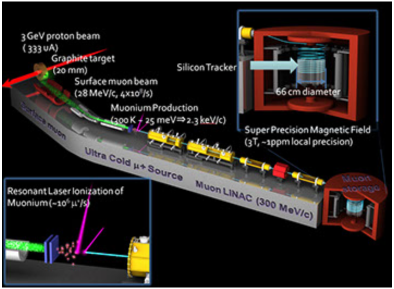
Towards high precision muon g-2/EDM measurement at J-PARC 28-29 November 2016

Why do we do this?

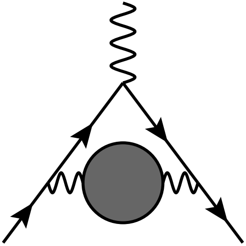
The muon $a_\mu = \dfrac{g-2}{2}$ will be measured more precisely



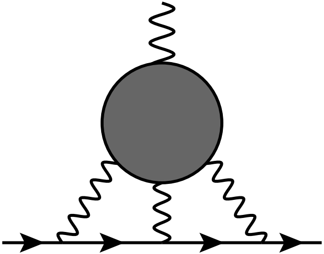
J-PARC

Fermilab

Hadronic contributions



HVP



HLbL

- The blobs are hadronic contributions
- I will present some results on both HVP and HLbL
- There are higher order contributions of both types

This talk

- HLbL (models)
  - J. Bijnens and J. Relefors, "Pion light-by-light contributions to the muon $g-2$," JHEP **1609** (2016) 113 [arXiv:1608.01454 [hep-ph]].
  - Disconnected contributions are (expected to be) large
  - A new evalution of the pion loop
- HVP (Chiral perturbation theory (ChPT))
  - J. Bijnens and J. Relefors, "Connected, Disconnected and Strange Quark Contributions to HVP," JHEP **1611** (2016) 086 [arXiv:1609.01573 [hep-lat]].

    An estimate of the disconnected and strange quark contributions
  - Finite volume corrections

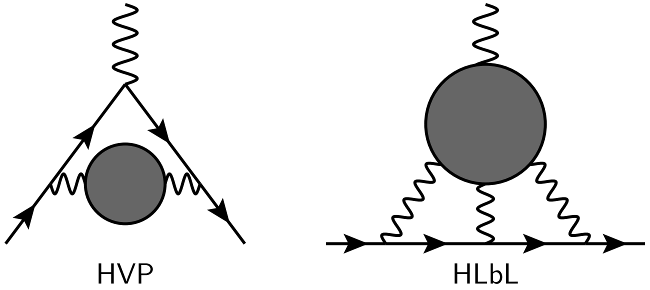
    J. Bijnens and J. Relefors, to be published
- All can be found in the PhD thesis of Johan Relefors (with some smaller changes)

To ChPT or not to ChPT

- ChPT = Effective field theory describing the lowest order pseudo-scalar representation
- or the (pseudo) Goldstone bosons from spontaneous breaking of chiral symmetry.
- Describes pions, kaons and etas at low-energies
- It's an effective field theory: new parameters or LECs at each new order
- Recent review of LECs:

  JB, Ecker,Ann.Rev.Nucl.Part.Sci. 64 (2014) 149 [arXiv:1405.6488]
- a_μ is a very low-energy quantity, why not just calculate it in ChPT?

To ChPT or not to ChPT

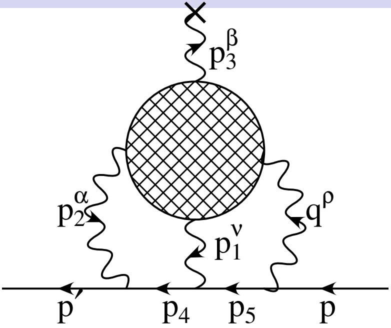


- Fill the blobs with pions and kaons
- Lowest order for both HVP and HLbL:
  pure pion loop (or scalar QED): well defined answer
- NLO: the blob is nicely finite
  but not after the muon/photon integrations
- Needs a counterterm (NLO LEC) that is the muon $g-2$

To ChPT or not to ChPT

- So need more than ChPT
- Experiment
- Dispersion relations
- lattice QCD
- Models
- ChPT can be used to put constraints, help understanding results and estimate not evaluated parts,. . .

HLbL: the main object to calculate



- Muon line and photons: well known
- The blob: fill in with hadrons/QCD
- Trouble: low and high energy very mixed
- Double counting needs to be avoided: hadron exchanges versus quarks

A separation proposal: a start

E. de Rafael, "Hadronic contributions to the muon g-2 and low-energy QCD," Phys. Lett. **B322** (1994) 239-246. [hep-ph/9311316].

- Use ChPT p counting and large N_c
- p^4, order 1: pion-loop
- p^8, order N_c: quark-loop and heavier meson exchanges
- p^6, order N_c: pion exchange

Does not fully solve the problem
only short-distance part of quark-loop is really p^8
but it's a start

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Implemented by two groups in the 1990s:

- Hayakawa, Kinoshita, Sanda: meson models, pion loop using hidden local symmetry, quark-loop with VMD, calculation in Minkowski space (HKS)
- JB, Pallante, Prades: Try using as much as possible a consistent model-approach, ENJL, calculation in Euclidean space (BPP)

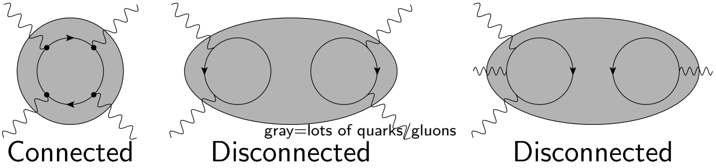
Papers: BPP and HKS

- JB, E. Pallante and J. Prades
  - “Comment on the pion pole part of the light-by-light contribution to the muon g-2,” Nucl. Phys. B **626** (2002) 410 [arXiv:hep-ph/0112255].
  - “Analysis of the Hadronic Light-by-Light Contributions to the Muon $g-2$,” Nucl. Phys. B **474** (1996) 379 [arXiv:hep-ph/9511388].
  - “Hadronic light by light contributions to the muon g-2 in the large N_c limit,” Phys. Rev. Lett. **75** (1995) 1447 [Erratum-ibid. **75** (1995) 3781] [arXiv:hep-ph/9505251].
- Hayakawa, Kinoshita, (Sanda)
  - “Pseudoscalar pole terms in the hadronic light by light scattering contribution to muon g - 2,” Phys. Rev. **D57** (1998) 465-477. [hep-ph/9708227], Erratum-ibid.D66 (2002) 019902[hep-ph/0112102].
  - “Hadronic light by light scattering contribution to muon g-2,” Phys. Rev. **D54** (1996) 3137-3153. [hep-ph/9601310].
  - “Hadronic light by light scattering effect on muon g-2,” Phys. Rev. Lett. **75** (1995) 790-793. [hep-ph/9503463].

Some main observations

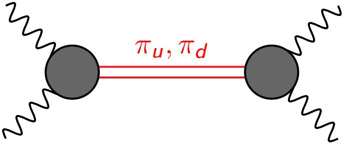
- The largest constribution is π^0 (and η, η') exchange
  - For the pseudo-scalar exchange most evaluations are in reasonable agreement
  - That will be used for an estimate of disconnected/connected
- The pion loop can be sizable but a large difference between the two evaluations
  - For the pure pion loop part, even larger numbers have been proposed by Engel, Ramsey-Musolf
  - Discussed in my second part
  - Another approach is the dispersive by Colangelo et al.
- There are other contributions but the sum is smaller than the leading pseudo-scalar exchange
- I interpret present HLbL lattice results as saying that no major contributions have been missed in the model estimates (but error and actual value might still change)

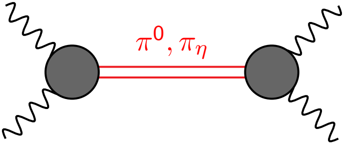
Disconnected/Connected



- Estimate the full result with pseudo-scalar exchange
- Connected diagrams only:
  - the gluon exchanges responsible for $U(1)_A$ breaking are not included at all
  - η' becomes light, mainly $(\bar{u}u + \bar{d}d)/\sqrt{2}$ (π_η) and has the same mass as the pion
  - Or the two-light states are π_u $(\bar{u}u)$ and π_d $(\bar{d}d)$
  - η becomes mainly $\bar{s}s$ and much heavier than the pion (and thus small contribution)
- Assume that couplings are not affected (not too bad experimentally)

Disconnected/Connected





- Two flavour case only: up and down quarks (three flavour not more difficult, just more numbers)
- Meson couplings to two-photons is via quark-loop
- Look at charge factors for Connected
  - As "quark-loop": $q_u^4 + q_d^4 = \frac{17}{81}$
  - As π_u, π_d: $q_u^2 q_u^2 + q_d^2 q_d^2 = \frac{17}{81}$
  - As π^0, π_η: $\left(\frac{q_u^2 - q_d^2}{\sqrt{2}}\right)^2 + \left(\frac{q_u^2 + q_d^2}{\sqrt{2}}\right)^2 = \frac{9}{162} + \frac{25}{162} = \frac{17}{81}$
- Include $U(1)_A$ breaking: π_η heavy
  - π^0: $\left(\frac{q_u^2 - q_d^2}{\sqrt{2}}\right)^2 = \frac{9}{162}$

Disconnected/Connected

- So in this limit:
  - Two-flavour case
  - $U(1)_A$ breaking makes π_η infinitely heavy
  - Full result dominated by pseudo-scalar exchange
  - $U(1)_A$ breaking does not affect couplings
  - Connected: $\frac{34}{162}$
    Disconnected: $-\frac{25}{162}$
    Sum: $\frac{9}{162}$
- All assumptions get corrections but final conclusion stays

The disconnected contribution is expected to be large and of opposite sign with significant cancellations

- Argument used to go from large-N_c to π^0, η, η' in
  JB, Pallante, Prades, Nucl. Phys. B **474** (1996) 379 [arXiv:hep-ph/9511388]
- This form: JB, Relefors, JHEP **1609** (2016) 113 [arXiv:1608.01454]

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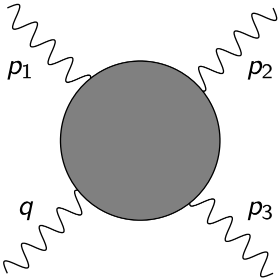
General properties

$$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3) \quad = $$



Actually we really need $\left.\dfrac{\delta \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$

General properties

$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)$:

- In general 138 Lorentz structures (but only 28 contribute to $g-2$)
- Using $q_\rho \Pi^{\rho\nu\alpha\beta} = p_{1\nu}\Pi^{\rho\nu\alpha\beta} = p_{2\alpha}\Pi^{\rho\nu\alpha\beta} = p_{3\beta}\Pi^{\rho\nu\alpha\beta} = 0$ 43 gauge invariant structures
- Bose symmetry relates some of them
- All depend on p_1^2, p_2^2 and q^2, but before derivative and $p_3 \to 0$ also $p_3^2, p_1 \cdot p_2, p_1 \cdot p_3$
- Actually 2 less but singular basis Fischer et al.
- Compare HVP: one function, one variable
- General calculation from experiment: how difficult: Colangelo
- In four photon measurement: lepton contribution

General properties

$\int \frac{d^4 p_1}{(2\pi)^4} \int \frac{d^4 p_2}{(2\pi)^4}$ plus loops inside the hadronic part

- 8 dimensional integral, three trivial,
- 5 remain: $p_1^2, p_2^2, p_1 \cdot p_2, p_1 \cdot p_\mu, p_2 \cdot p_\mu$
- Rotate to Euclidean space:
  - Easier separation of long and short-distance
  - Artefacts (confinement) in models smeared out.
- More recent: can do two more using Gegenbauer techniques Knecht-Nyffeler, Jegerlehner-Nyffeler,JB–Zahiri-Abyaneh–Relefors
- P_1^2, P_2^2 and Q^2 remain
- study $a_\mu^{\rm X} = \int dl_{P_1} dl_{P_2} a_\mu^{\rm XLL} = \int dl_{P_1} dl_{P_2} dl_Q a_\mu^{\rm XLLQ}$ $l_P = \ln(P/{\rm GeV})$, to see where the contributions are
- Study the dependence on the cut-off for the photons

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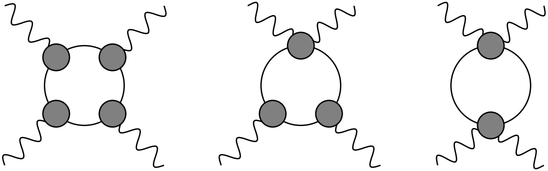
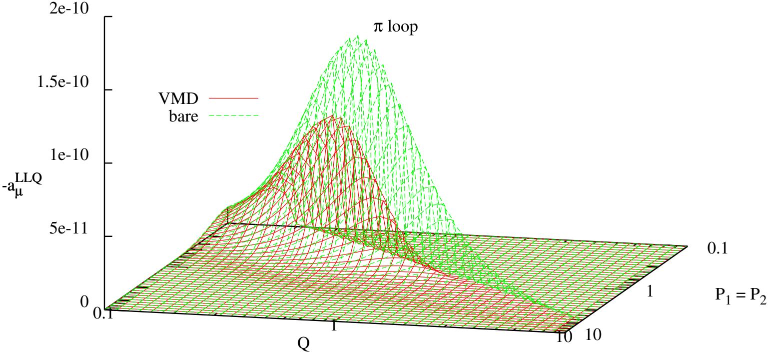
π-loop



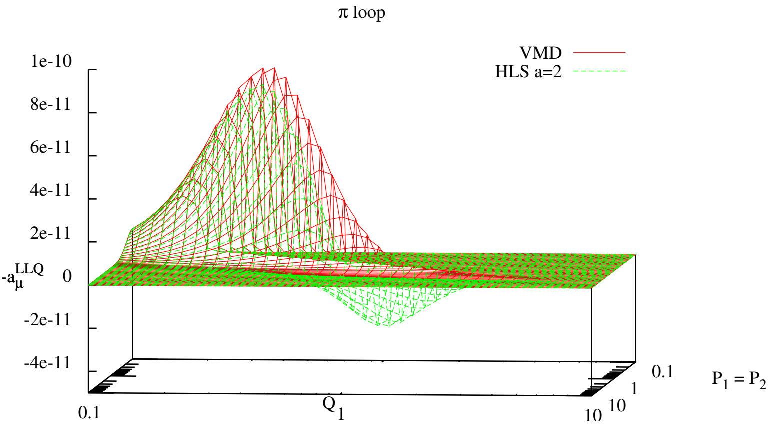
- A bare π-loop (sQED) give about $-4 \cdot 10^{-10}$
- The $\pi\pi\gamma^*$ vertex is always done using VMD
- $\pi\pi\gamma^*\gamma^*$ vertex two choices:
  - Hidden local symmetry model: only one γ has VMD
  - Full VMD
  - Both are chirally symmetric
  - The HLS model used has problems with π^+-π^0 mass difference (due to not having an a_1)
- Final numbers quite different: -0.45 and -1.9 ($\times 10^{-10}$)
- For BPP stopped at 1 GeV but within 10% of higher Λ

π loop: Bare vs VMD



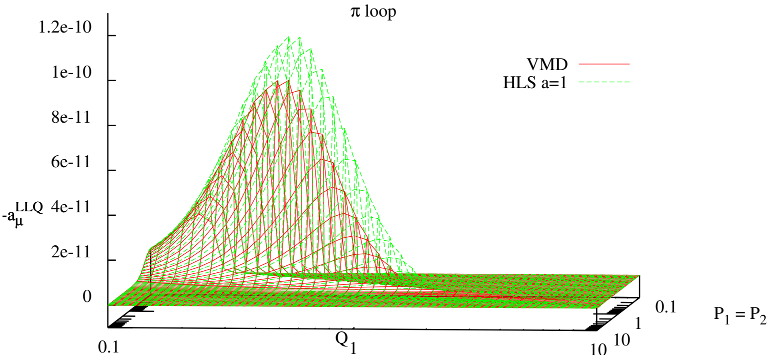
- plotted a_μ^{LLQ} for $P_1 = P_2$
- $a_\mu = \int dl_{P_1} dl_{P_2} dl_Q \, a_\mu^{LLQ}$
- $l_Q = \log(Q/1 \text{ GeV})$

π loop: VMD vs HLS



Usual HLS, $a = 2$

π loop: VMD vs HLS



HLS with $a = 1$, satisfies more short-distance constraints

π loop

- $\pi\pi\gamma^*\gamma^*$ for $q_1^2 = q_2^2$ has a short-distance constraint from the OPE as well.
- HLS does not satisfy it
- full VMD does: so probably better estimate
- Ramsey-Musolf suggested to do pure ChPT for the π loop
  K. T. Engel and M. J. Ramsey-Musolf, Phys. Lett. B **738** (2014) 123 [arXiv:1309.2225 [hep-ph]].
- Polarizability ($L_9 + L_{10}$) up to 10%, charge radius 30% at low energies, more at higher
- Both HLS and VMD have charge radius effect but not polarizability

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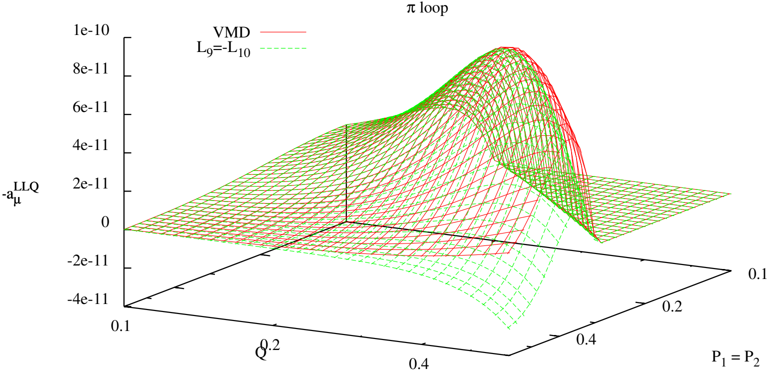
π loop: L_9, L_{10}

- ChPT for muon $g-2$ at order p^6 is not powercounting finite so no prediction for a_μ exists.
- But can be used to study the low momentum end of the integral over P_1, P_2, Q
- The four-photon amplitude is finite still at two-loop order (counterterms start at order p^8)
- Add L_9 and L_{10} vertices to the bare pion loop:

  JB, Relefors, Zahiri-Abyaneh, 1208.3548,1208.2554,1308.2575,1510.05796

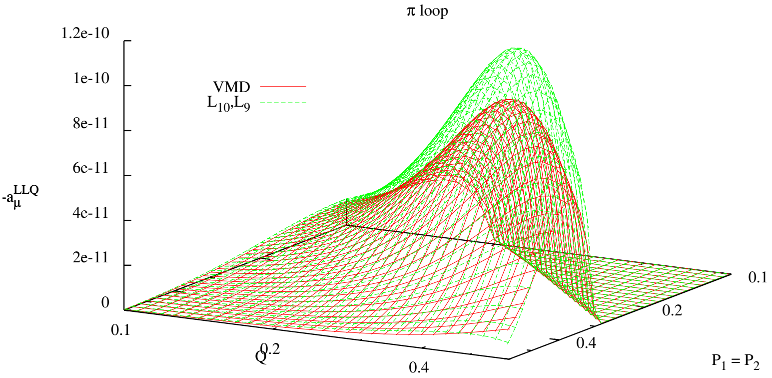
  JB, Relefors, JHEP **1609** (2016) 113 [arXiv:1608.01454 [hep-ph]].

π loop: VMD vs charge radius



low scale, charge radius effect well reproduced

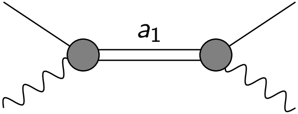
π loop: VMD vs L_9 and L_{10}



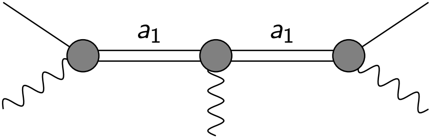
- $L_9 + L_{10} \neq 0$ gives an enhancement of 10-15%
- To do it fully need to get a model: include a_1

Include a_1

- $L_9 + L_{10}$ effect is from



- But to get gauge invariance correctly need



Include a_1

- Consistency problem: full a_1-loop?
- Treat a_1 and ρ classical and π quantum: there must be a π that closes the loop
  Argument: integrate out ρ and a_1 classically, then do pion loops with the resulting Lagrangian
- To avoid problems: representation without a_1-π mixing
- Check for curiosity what happens if we add a_1-loop

Include a_1

- Use antisymmetric vector representation for a_1 and ρ
- Fields $A_{\mu\nu}$, $V_{\mu\nu}$ (nonets)
- Kinetic terms: $-\frac{1}{2}\left\langle \nabla^\lambda V_{\lambda\mu}\nabla_\nu V^{\nu\mu} - \frac{M_V^2}{2}V_{\mu\nu}V^{\mu\nu}\right\rangle$

$$-\frac{1}{2}\left\langle \nabla^\lambda A_{\lambda\mu}\nabla_\nu A^{\nu\mu} - \frac{M_A^2}{2}A_{\mu\nu}A^{\mu\nu}\right\rangle$$

- Terms that give contributions to the L_i^r:

$$\frac{F_V}{2\sqrt{2}}\left\langle f_{+\mu\nu}V^{\mu\nu}\right\rangle + \frac{iG_V}{\sqrt{2}}\left\langle V^{\mu\nu}u_\mu u_\nu\right\rangle + \frac{F_A}{2\sqrt{2}}\left\langle f_{-\mu\nu}A^{\mu\nu}\right\rangle$$

- $L_9 = \frac{F_V G_V}{2M_V^2}$, $L_{10} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$
- Weinberg sum rules: (Chiral limit)

$$F_V^2 = F_A^2 + F_\pi^2 \qquad\qquad F_V^2 M_V^2 = F_A^2 M_A^2$$

- VMD for $\pi\pi\gamma$: $\qquad F_V G_V = F_\pi^2$

$V_{\mu\nu}$ only

- $\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)$ is not finite (but was also not finite for HLS)
- But $\left.\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ also not finite (but was finite for HLS)
- Derivative one finite for $G_V = F_V/2$
- Surprise: $g-2$ identical to HLS with $a = \frac{F_V^2}{F_\pi^2}$
- Yes I know, different representations are identical BUT they do differ in higher order terms and even in what is higher order
- Same comments as for HLS numerics

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$V_{\mu\nu}$ and $A_{\mu\nu}$

- Add a_1
- Calculate a lot
- $\left.\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ finite for:
  - $G_V = F_V = 0$ and $F_A^2 = -2F_\pi^2$
  - If adding full a_1-loop $G_V = F_V = 0$ and $F_A^2 = -F_\pi^2$

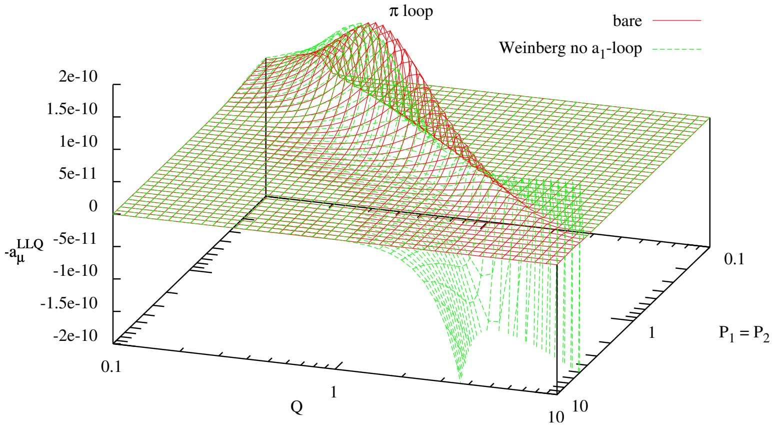
$V_{\mu\nu}$ and $A_{\mu\nu}$

- Start by adding $\rho a_1 \pi$ vertices
- $\lambda_1 \langle [V^{\mu\nu}, A_{\mu\nu}] \chi_- \rangle + \lambda_2 \langle [V^{\mu\nu}, A_{\nu\alpha}] h_\mu{}^\nu \rangle$
  $+\lambda_3 \langle i [\nabla^\mu V_{\mu\nu}, A_{\nu\alpha}] u_\alpha \rangle + \lambda_4 \langle i [\nabla_\alpha V_{\mu\nu}, A_{\alpha\nu}] u^\mu \rangle$
  $+\lambda_5 \langle i [\nabla^\alpha V_{\mu\nu}, A_{\mu\nu}] u_\alpha \rangle + \lambda_6 \langle i [V^{\mu\nu}, A_{\mu\nu}] f_-{}^\alpha{}_\nu \rangle$
  $+\lambda_7 \langle i V_{\mu\nu} A^{\mu\rho} A^\nu{}_\rho \rangle$
- All lowest dimensional vertices of their respective type
- Not all independent, there are three relations
- Follow from the constraints on $V_{\mu\nu}$ and $A_{\mu\nu}$ (thanks to Stefan Leupold)

$V_{\mu\nu}$ and $A_{\mu\nu}$: big disappointment

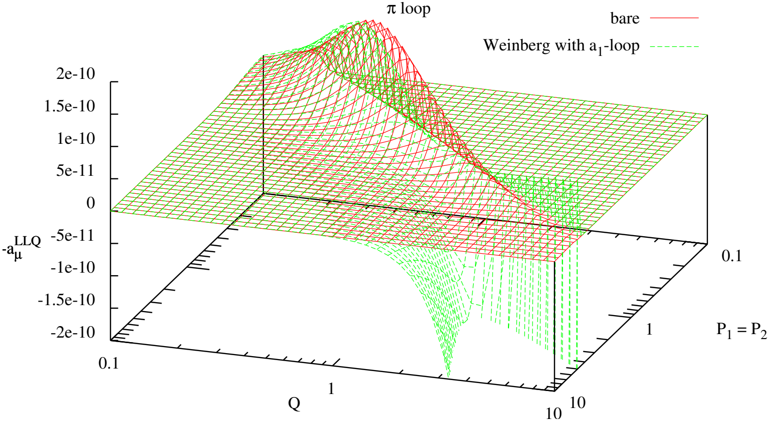
- Work a whole lot
- $\left.\frac{\delta \Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ not obviously finite
- Work a lot more
- Prove that $\left.\frac{\delta \Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ finite, only same solutions as before
- Try the combination that show up in $g-2$ only
- Work a lot
- Again, only same solutions as before
- Small loophole left: after the integration for $g-2$ could be finite but many funny functions of m_π, m_μ, M_V and M_A show up.

a_1-loop: cases with good L_9 and L_{10}



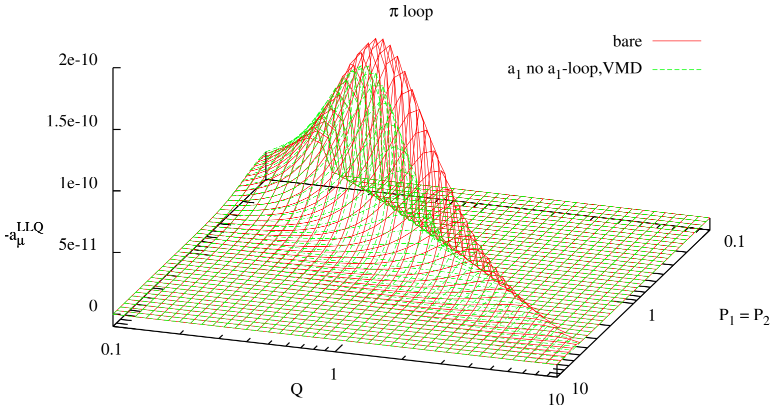
- Add F_V, G_V and F_A
- Fix values by Weinberg sum rules and VMD in $\gamma^*\pi\pi$
- no a_1-loop

a_1-loop: cases with good L_9 and L_{10}



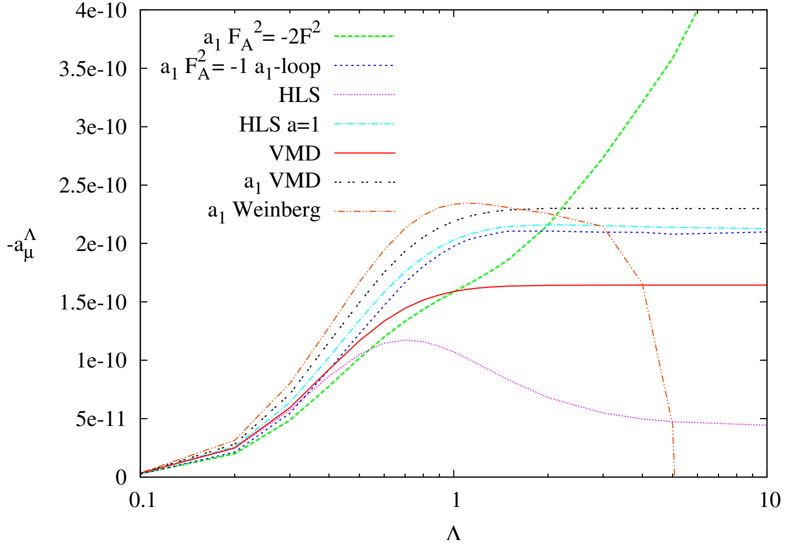
- Add F_V, G_V and F_A
- Fix values by Weinberg sum rules and VMD in $\gamma^*\pi\pi$
- With a_1-loop (is different plot!!)

a_1-loop: cases with good L_9 and L_{10}



- Add a_1 with $F_A^2 = +F_\pi^2$
- Add the full VMD as done earlier for the bare pion loop

Integration results



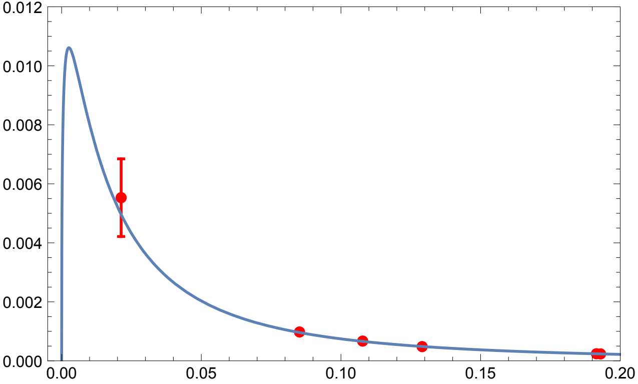
$P_1, P_2, Q \leq \Lambda$

Integration results with a_1

- Problem: get high energy behaviour good enough
- But all models with reasonable L_9 and L_{10} fall way inside the error quoted earlier $(-1.9 \pm 1.3)\ 10^{-10}$
- Tentative conclusion: Use hadrons only below about 1 GeV: $a_\mu^{\pi-\text{loop}} = (-2.0 \pm 0.5)\ 10^{-10}$
- Note that Engel and Ramsey-Musolf, arXiv:1309.2225 is a bit more pessimistic quoting numbers from $(-1.1 \text{ to } -7.1)\ 10^{-10}$

Two-point: Why

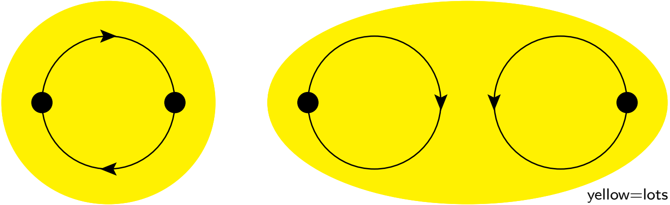
Muon: $a_\mu = (g-2)/2$ and $a_\mu^{\mathrm{LO,HVP}} = \int_0^\infty dQ^2 f\left(Q^2\right)\hat{\Pi}\left(Q^2\right)$



plot: $f\left(Q^2\right)\hat{\Pi}\left(Q^2\right)$ with $Q^2 = -q^2$ in GeV^2

Figure and data: Aubin, Blum, Chau, Golterman, Peris, Tu, Phys. Rev. D93 (2016) 054508 [arXiv:1512.07555]

Two-point: Connected versus disconnected



- $\Pi^{\mu\nu}_{ab}(q) \equiv i \int d^4x e^{iq\cdot x} \langle T(j^\mu_a(x) j^{\nu\dagger}_a(0)) \rangle$
- $j^\mu_{\pi^+} = \bar{d}\gamma^\mu u$
- $j^\mu_u = \bar{u}\gamma^\mu u, \quad j^\mu_d = \bar{d}\gamma^\mu d, \quad j^\mu_s = \bar{s}\gamma^\mu s$
- $j^\mu_e = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \frac{1}{3}\bar{s}\gamma^\mu s$
- Study in ChPT at one-loop:

  Della Morte, Jüttner, JHEP 1011 (2010) 154 [arXiv:1009.3783]
- Extend to two-loops and some more:

  JB, J. Relefors, JHEP **1611** (2016) 086 [arXiv:1609.01573]

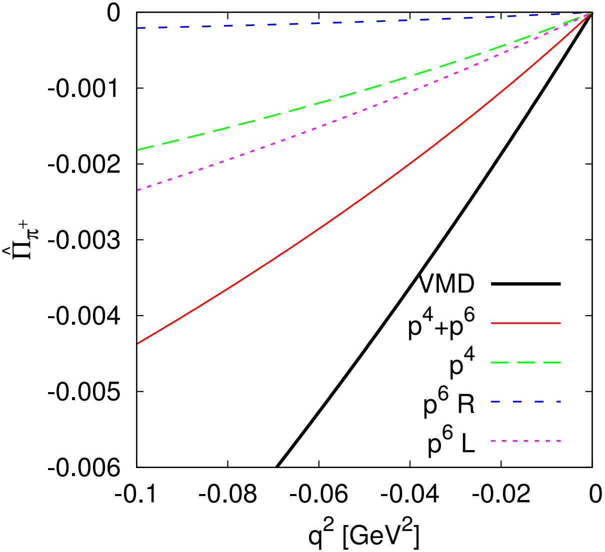
Two-point: Connected versus disconnected

- Include also singlet part of the vector current
- There are new terms in the Lagrangian
- p^4 only one more: $\langle L_{\mu\nu}\rangle\langle L^{\mu\nu}\rangle + \langle R_{\mu\nu}\rangle\langle R^{\mu\nu}\rangle$
- $\Longrightarrow$ The pure singlet vector current does not couple to mesons until p^6
- $\Longrightarrow$ Loop diagrams involving the pure singlet vector current only appear at p^8 (implies relations)
- $\Longrightarrow$ Loop diagrams with singlet vector and WZW only at p^{10}
- p^6 (no full classification, just some examples) $\langle D_\rho L_{\mu\nu}\rangle\langle D^\rho L^{\mu\nu}\rangle + \langle D_\rho R_{\mu\nu}\rangle\langle D^\rho R^{\mu\nu}\rangle$, $\langle L_{\mu\nu}\rangle\langle L^{\mu\nu}\chi^\dagger U\rangle + \langle R_{\mu\nu}\rangle\langle R^{\mu\nu}\chi U^\dagger\rangle$,...
- Results at two-loop order, unquenched isospin limit

Two-point: Connected versus disconnected

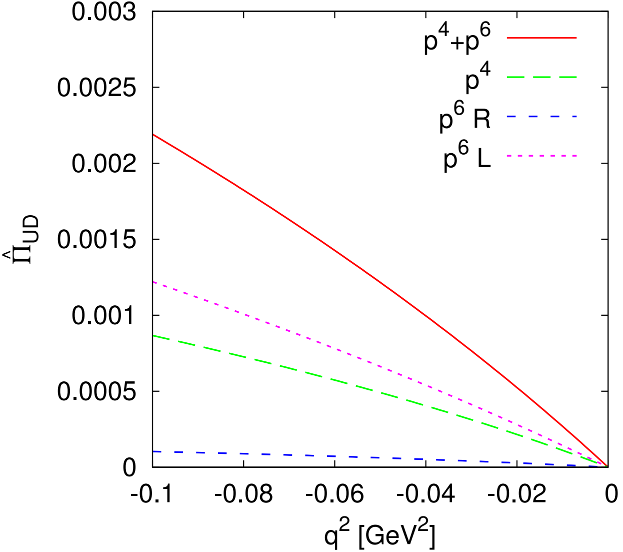
- $\Pi^{\mu\nu}_{\pi^+\pi^+}$: only connected
- $\Pi^{\mu\nu}_{ud}$: only disconnected
- $\Pi^{\mu\nu}_{uu} = \Pi^{\mu\nu}_{\pi^+\pi^+} + \Pi^{\mu\nu}_{ud}$
- $\Pi^{\mu\nu}_{ee} = \frac{5}{9}\Pi^{\mu\nu}_{\pi^+\pi^+} + \frac{1}{9}\Pi^{\mu\nu}_{ud}$
- Infinite volume (and the *ab* considered here): $\Pi^{\mu\nu}_{ab} = \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi^{(1)}_{ab}$
- Large N_c + VMD estimate: $\Pi^{(1)}_{\pi^+\pi^+} = \frac{4F_\pi^2}{M_V^2 - q^2}$
- Plots on next pages are for $\Pi^{(1)}_{ab0}(q^2) = \Pi^{(1)}_{ab}(q^2) - \Pi^{(1)}_{ab}(0)$
- At p^4 the extra LEC cancels, at p^6 there are new LEC contributions, but no new ones in the loop parts

Two-point: Connected versus disconnected



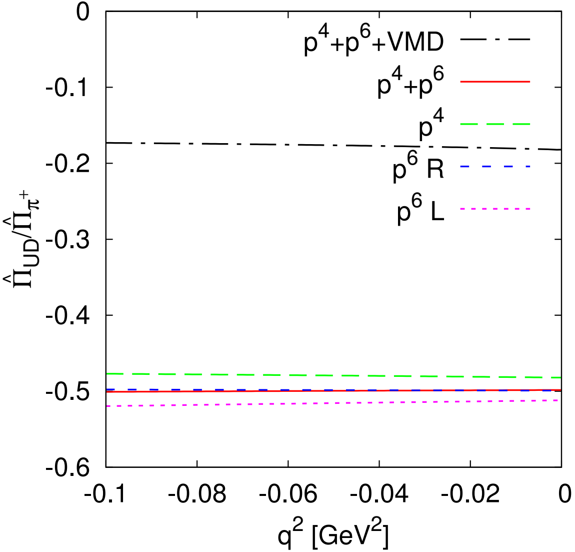
- Connected
- p^6 is large
- Due to the L_i^r loops

Two-point: Connected versus disconnected



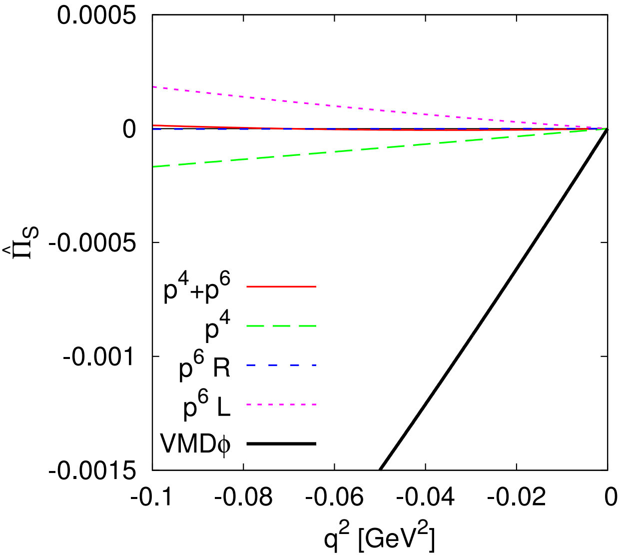
- Disconnected
- p^6 is large
- Due to the L_i^r loops
- about $-\frac{1}{2}$ connected
- $-\frac{1}{10}$ is from $\Pi^{(1)}_{ee} = \frac{5}{9}\Pi^{(1)}_{\pi^+\pi^+} + \frac{1}{9}\Pi^{(1)}_{ud}$

Two-point: Connected versus disconnected



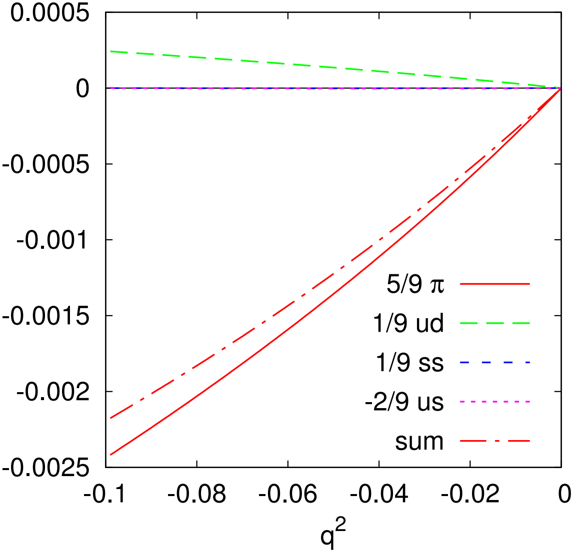
- p^4 and p^6 pion part exactly $-\frac{1}{2}$
- not true for unsubtracted at p^4(LEC)
- not true for pure LEC at p^6

Two-point: Including strange



- π connected u,d
- *ud* disconnected u,d
- *ss* strange current
- *us* mixed strange-u,d
- strange part is very small: $q^2 = 0$ subtraction (only kaon loops) p^4 and p^6 cancel largely

Two-point: with strange, electromagnetic current



- π connected u,d
- ud disconnected u,d
- ss strange current
- us mixed s–u,d
- new p^6 LEC cancels

Comparison with other results

- Time moments, Taylor expansion
- $\hat{\Pi}(q)^2 = \Pi_1 q^2 - \Pi_2 q^4 + \cdots$
- Lattice data from Miura presentation Lattice2106 and HPQCD
- Phenomenological HLS fit (Benayoun et al)

Comparing with other results

Reference	Π_A	Π_1 (GeV^{-2})	Π_2 (GeV^{-4})
Π^{VMD}	$\hat{\Pi}_{\pi^+}$	0.0967	−0.163
p^4	$\hat{\Pi}_{\pi^+}$	0.0240	−0.091
p^6 R	$\hat{\Pi}_{\pi^+}$	0.0031	−0.014
p^6 L	$\hat{\Pi}_{\pi^+}$	0.0286	−0.067
sum	$\hat{\Pi}_{\pi^+}$	0.152	−0.336
Miura Lattice2016	$\hat{\Pi}_{\pi^+}$	0.1657(16)(18)	−0.297(10)(05)
HPQCD	$\hat{\Pi}_{\pi^+}$	0.1460(22)	−0.2228(65)
p^4	$\hat{\Pi}_{UD}$	−0.0116	0.045
p^6 R	$\hat{\Pi}_{UD}$	−0.0015	0.007
p^6 L	$\hat{\Pi}_{UD}$	−0.0146	0.032
sum	$\hat{\Pi}_{UD}$	−0.0278	0.085
Miura Lattice2016	$\hat{\Pi}_{UD}$	−0.015(2)(1)	0.046(10)(04)

Connected and Disconnected:

size difference (-few%) understood from ChPT and VMD

Comparing with other results

Reference	Π_A	Π_1 (GeV^{-2})	Π_2 (GeV^{-4})
$\Pi^{VMD\phi}$	$\hat{\Pi}_S$	0.0314	−0.030
p^4	$\hat{\Pi}_S$	0.0017	−0.001
p^6 R	$\hat{\Pi}_S$	0.0000	0.000
p^6 L	$\hat{\Pi}_S$	−0.0013	−0.005
sum	$\hat{\Pi}_S$	0.0318	−0.035
Miura Lattice2016	$\hat{\Pi}_S$	0.0657(1)(2)	−0.0532(1)(3)
HPQCD	$\hat{\Pi}_S$	0.06625(74)	−0.0526(11)
our result	$\hat{\Pi}_{EM}$	0.0852	−0.182
Benayoun et al	$\hat{\Pi}_{EM}$	0.0990(7)	−0.206(2)
Miura Lattice2016	$\hat{\Pi}_{EM}$	0.0972(2)(1)	−0.166(6)(3)

Strange and Total electromagnetic

Twisted boundary conditions

- On a lattice at finite volume $p^i = 2\pi n^i/L$: very few momenta directly accessible
- Put a constraint on certain quark fields in some directions: $q(x^i + L) = e^{i\theta^i_q} q(x^i)$
- Then momenta are $p^i = \theta^i/L + 2\pi n^i/L$. Allows to map out momentum space on the lattice much better

  Bedaque,...
- Small note:
  - Beware what people call momentum: is θ^i/L included or not?
  - Reason: a colour singlet gauge transformation $G^S_\mu \to G^S_\mu - \partial_\mu \epsilon(x)$, $q(x) \to e^{i\epsilon(x)} q(x)$, $\epsilon(x) = -\theta^i_q x^i/L$
  - Boundary condition Twisted $\Leftrightarrow$ constant background field+periodic

Twisted boundary conditions: Drawbacks

Drawbacks:

- Box: Rotation invariance $\rightarrow$ cubic invariance
- Twisting: reduces symmetry further
- Can only be done for the connected part

Consequences:

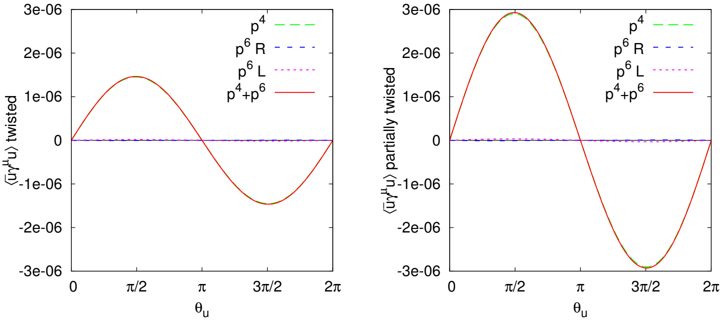
- $m^2(\vec{p}^2) = E^2 - \vec{p}^2$ is not constant
- There are typically more form-factors
- In general: quantities depend on more (all) components of the momenta
- Charge conjugation involves a change in momentum

Two-point function: twisted boundary conditions

JB, Relefors, JHEP 05 (201)4 015 [arXiv:1402.1385]

- $\int_V \frac{d^d k}{(2\pi)^d} \frac{k_\mu}{k^2 - m^2} \neq 0$
- $\langle \bar{u}\gamma^\mu u \rangle \neq 0$
- $j^\mu_{\pi^+} = \bar{d}\gamma^\mu u$
  satisfies $\partial_\mu \left\langle T(j^\mu_{\pi^+}(x) j^{\nu\dagger}_{\pi^+}(0))\right\rangle = \delta^{(4)}(x) \left\langle \bar{d}\gamma^\nu d - \bar{u}\gamma^\nu u \right\rangle$
- $\Pi^{\mu\nu}_a(q) \equiv i \int d^4 x e^{iq\cdot x} \left\langle T(j^\mu_a(x) j^{\nu\dagger}_a(0))\right\rangle$
  Satisfies WT identity. $q_\mu \Pi^{\mu\nu}_{\pi^+} = \left\langle \bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d \right\rangle$
- ChPT at one-loop satisfies this
  see also Aubin et al, Phys.Rev. D88 (2013) 7, 074505 [arXiv:1307.4701]
- two-loop in partially quenched: JB, Relefors, in preparation
  satisfies the WT identity (as it should)

$\langle \bar{u}\gamma^\mu u \rangle$



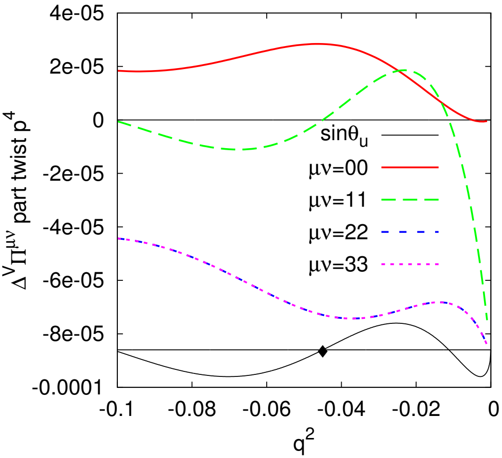
Fully twisted | Partially twisted

$\theta_u = (0, \theta_u, 0, 0)$, all others untwisted

$m_\pi L = 4$ (ratio at p^4=2 up to kaon loops)

For comparison: $\langle \bar{u}u \rangle^V \approx -2.4\ 10^{-5}\ \text{GeV}^3$
$\langle \bar{u}u \rangle \approx -1.2\ 10^{-2}\ \text{GeV}^3$

Two-point: partially twisted, one-loop



$q = \left(0, \sqrt{-q^2}, 0, 0\right)$

$\Pi^{22} = \Pi^{33}$

$\vec{\theta}_u = L\, q$

$m_{\pi 0} L = 4$

$m_{\pi 0} = 0.135$ GeV

$-q^2 \Pi^{(1)}_{\rm VMD} = \frac{-4q^2 F_\pi^2}{M_V^2 - q^2}$

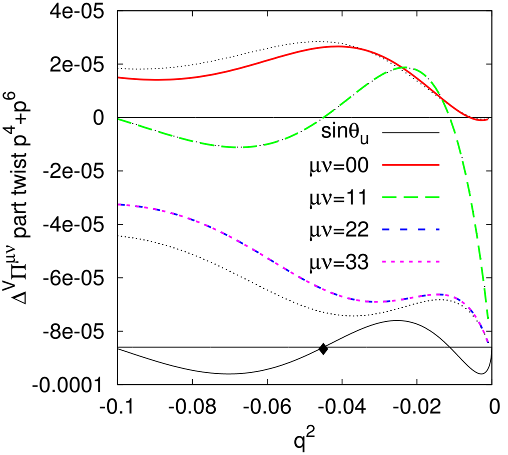
$\approx 5\text{e-}3 \cdot \frac{q^2}{0.1}$

diamond: periodic

Note: $\Pi^{\mu\nu}(0) \neq 0$

Correction is at the % level

Two-point: partially twisted, with two-loop



$q = \left(0, \sqrt{-q^2}, 0, 0\right)$

$\Pi^{22} = \Pi^{33}$

$\vec{\theta}_u = L\,q$

$m_{\pi 0} L = 4$

$m_{\pi 0} = 0.135$ GeV

$-q^2 \Pi^{(1)}_{\mathrm{VMD}} = \frac{-4q^2 F_\pi^2}{M_V^2 - q^2}$

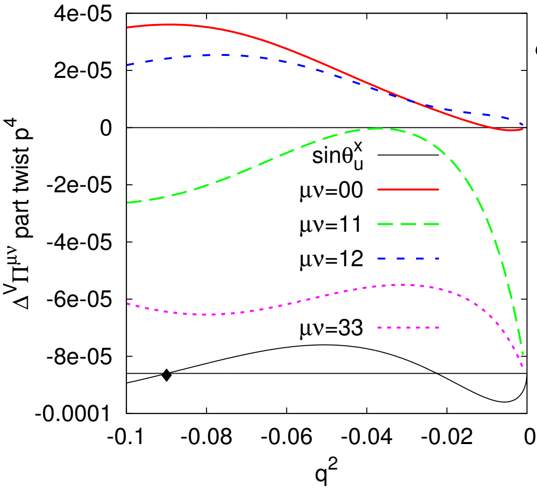
$\approx 5\text{e-}3 \cdot \frac{q^2}{0.1}$

diamond: periodic

Note: $\Pi^{\mu\nu}(0) \neq 0$

Correction from two loop is reasonable (thin lines are p^4)

Two-point: partially twisted, one-loop



$$q = \left(0, \frac{\sqrt{-q^2}}{\sqrt{2}}, \frac{\sqrt{-q^2}}{\sqrt{2}}, 0\right)$$

$\Pi^{11} = \Pi^{22}$

$\vec{\theta}_u = L\, q$

$m_{\pi 0} L = 4$

$m_{\pi 0} = 0.135$ GeV

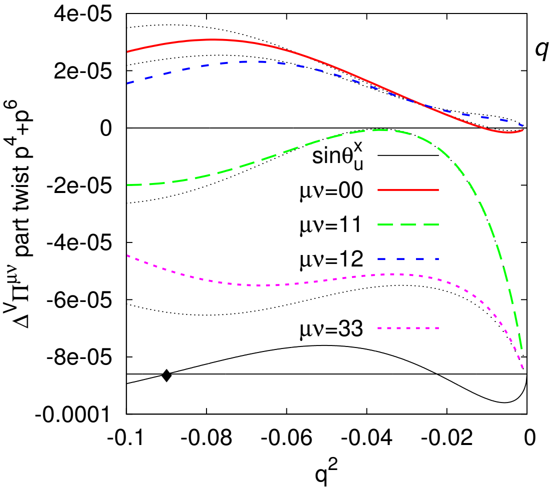
$$-q^2\Pi^{(1)}_{\mathrm{VMD}} = \frac{-4q^2F_\pi^2}{M_V^2-q^2}$$
$$\approx 5\text{e-}3\cdot\frac{q^2}{0.1}$$

diamond: periodic

Note: $\Pi^{\mu\nu}(0) \neq 0$

Correction is at the % level

Two-point: partially twisted, two-loop



$$q = \left(0, \frac{\sqrt{-q^2}}{\sqrt{2}}, \frac{\sqrt{-q^2}}{\sqrt{2}}, 0\right)$$

$\Pi^{11} = \Pi^{22}$

$\vec{\theta}_u = L\,q$

$m_{\pi 0} L = 4$

$m_{\pi 0} = 0.135$ GeV

$$-q^2\Pi^{(1)}_{\mathrm{VMD}} = \frac{-4q^2F_\pi^2}{M_V^2 - q^2} \approx 5\text{e-}3\cdot\frac{q^2}{0.1}$$

diamond: periodic

Note: $\Pi^{\mu\nu}(0) \neq 0$

Two loop correction again reasonable (thin lines are p^4)

Two-point: untwisted, connected part two-loop

$q/(2\pi/L)$		Π_U^{00} $[10^{-5}\ \text{GeV}^2]$	Π_U^{11} $[10^{-5}\ \text{GeV}^2]$	Π_U^{33} $[10^{-5}\ \text{GeV}^2]$	
(0,0,0,0)	p^4	0.000	-8.785	-8.785	$\Pi_U^{22} = \Pi_U^{33}$
	p^6 R	0.000	0.045	0.045	
	p^6 L	0.000	-0.102	-0.102	
	sum	0.000	-8.842	-8.842	
(0,1,0,0)	p^4	2.840	0.000	-7.294	$\Pi_U^{22} = \Pi_U^{33}$
	p^6 R	-0.091	0.000	0.223	
	p^6 L	-0.117	0.000	0.633	
	sum	2.632	0.000	-6.438	
(0,1,1,0)	p^4	3.604	-2.415	-6.442	$\Pi_U^{22} = \Pi_U^{11}$
	p^6 R	-0.195	0.128	0.338	$\Pi_U^{12} = -\Pi_U^{11}$
	p^6 L	-0.458	0.376	1.144	$\Pi_U^{21} = -\Pi_U^{11}$
	sum	2.951	-1.911	-4.960	

- $q^2 = 0, 0.045, 0.09\ \text{GeV}^2$
- Disconnected $= -1/2$ shown (with very small differences)

Conclusions

Showed you results for:

- Hadronic Light by Light
  - Expect large and opposite sign disconnected contributions
  - A new evaluation of the pion loop including effect of polarizability
- Hadronic vacuum polarization: vector two-point function
  - Connected versus disconnected at two-loops
  - Connected: twisting and finite volume at two-loops
    - The WI are satisfied exactly
    - The corrections are sizable for present lattices (few%) but two-loop corrections were normal size
    - Be careful: ChPT must exactly correspond to your lattice calculation
  - Our results can help with chiral extrapolation
  - Programs will be available via CHIRON http://www.thep.lu.se/~bijnens/chiron/ sometime later this year (manual writing takes time)